Strategy-Proof Spectrum Allocation among Multiple Operators in Wireless Networks†

Indu Yadav, Ankur A. Kulkarni, Abhay Karandikar

***Abstract*—The exponentially increasing demand for data necessitates efficient spectrum allocation among operators in wireless networks. In this paper, we address the spectrum allocation problem among non-cooperative operators via auctions. The classical Vickrey-Clarke-Groves (VCG) approach provides the framework for a strategy-proof and social welfare maximizing auction. However, the VCG mechanism has high computational complexity, which makes it infeasible for practical implementation. In this work, we propose sealed bid auction mechanisms for spectrum allocation, which are computationally tractable. These can be used for spectrum allocation by performing auctions at shorter intervals to cater to the dynamic load variation in the network. We establish that the proposed algorithm is strategy-proof for the uniform demand scenario. Furthermore, for non-uniform demand, we propose an algorithm that satisfies weak strategy-proofness. Here, we also consider non-linear increase in the marginal valuations with demand. Simulation results are also presented to exhibit the performance comparison of the proposed algorithms with VCG and other existing mechanisms.**

***Index Terms*—Dynamic Spectrum Allocation, Strategy-proof Auctions, Mechanism Design**

I. Introduction

With recent advancements in wireless communication technologies, the telecom market has witnessed exponential growth in data traffic in the past few decades. As per the current trends, mobile data traffic is expected to increase more than 5 times by 2024 [2]. Globally, Fifth Generation (5G) technology will further escalate the amount of data traffic. With the rapid development of smart devices, the end user data rate requirements have also become stringent. Satisfying the increasing number of end users with the desired Quality of Services (QoSs) has further contributed to the crisis of limited, scarce, and expensive "spectrum" and an efficient utilization of spectrum is a requirement which cannot be ignored.

Traditionally, the spectrum is allocated statically on lease for long durations such as one year or more to the service providers[1]. Usually, service providers estimate the peak traffic conditions of the network and calculate the quantum of spectrum accordingly. However, the peak traffic requirements arise sporadically in the network. This leads to the under-utilization of spectrum in the long run. Therefore, the static allocation technique of spectrum is inefficient in terms of spectrum utilization and not suitable to meet the requirements of the next-generation networks. Moreover, it has been shown that the traffic conditions in a wireless network vary as a function of time and location [3]. For instance, while on any regular day, peak traffic in residential areas is more likely to occur in the evening, in office areas one may observe peak traffic during business hours. Thus, the wireless networks observe a significant peak in the average traffic ratio [4].

To address the issue of inefficient spectrum usage a computationally efficient spectrum allocation mechanism is required so that the spectrum can be allocated in dynamic fashion considering the spatial and temporal traffic variations in the network. Auctions are commonly preferred for spectrum allocation among multiple operators. In our work, we focus on computationally efficient spectrum allocation mechanisms for spectrum distribution among multiple operators, to ensure that the spectrum is allocated quickly as per service providers' demands.

In general, the spectrum is allocated among the operators using sealed bid auction format. In sealed bid auctions, interested buyers send their valuations for the object in a closed envelope along with the demand, to the auctioneer. Thus, the privacy of the valuation and the demand for the object is ensured for each service provider. In spectrum auctions, spectrum valuation for a service provider depends on the desired bandwidth and other factors such as the number of subscribers and the services desired by the subscribers. Hence, the spectrum valuation is a private information of an operator which is not known to the auctioneer. Generally, the participants in any auction are selfish and are likely to misreport the actual valuation to the auctioneer if there is an incentive to do so. Hence, ensuring the strategy-proofness of auctions is of significant importance [5]. An auction is said to be strategy-proof if any operator does not gain on deviating from the true or actual value of their demands of the spectrum. This implies that even if an operator misreports its valuation, it can never achieve utility greater than that of the true valuation.

Strategy-proof auctions not only compel the participants to reveal their true valuations but also make the process of spectrum allocation easier for the auctioneer and the operators. The operators are neither required to perform complex computations nor they have to invest time to determine the optimal bidding strategy to maximize their utility gains. Hence, it makes the process of resource allocation faster by removing the time and the computational overhead. Moreover, strategy-proofness also increases the number of participants in an auction.

In spectrum auctions, three properties are of utmost impor-

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[1]The terminologies "service provider" or "operator" have been used interchangeably throughout the paper

tance: strategy-proofness, low computational complexity, and optimality of allocation to maximize the social welfare [6]. Unfortunately, achieving all three properties simultaneously in an auction is provably NP-Hard [7]. Vickrey Clarke Groves (VCG) [8]–[10] is a well-known mechanism which proposes a framework for guaranteeing strategy-proof behavior in auctions with optimal allocation strategy, but it is computationally infeasible in large networks [6].

Various Dynamic Spectrum Allocation (DSA) mechanisms proposed in the literature are designed for single parameter environment [6]. Thus, the existing literature is focused on the dynamic spectrum allocation at the base station level, where it is assumed that the individual base stations participate in the auction. However, in today's wireless networks, a base station is not an independent entity and may not have the long term view of traffic to estimate the resource requirements and the corresponding valuations.

In practice, spectrum allocation is performed at the operator level, where multiple base stations are associated with an operator. Operators estimate the resource requirements and the corresponding bid at each base station as per the traffic in the wireless network. Since multiple base stations are associated with an operator, the operator has multiple bids and requirements corresponding to each of its base stations. Unfortunately, devising a computationally efficient strategy-proof spectrum allocation mechanism becomes much more difficult as operators report a vector of bids corresponding to the associated base stations. Thus, an operator may misreport the valuation and demand at a few BSs to increase the overall utility gain. To the best of our knowledge, none of the existing works e.g., [7], [11]–[13], address the dynamic spectrum allocation at the operator level.

We focus on designing efficient strategy-proof mechanisms which are suitable for implementation in short durations to handle the spatio-temporal load variations of the network. First, we propose a strategy-proof mechanism where the demand at each BS is of a single channel. Next, we extend the mechanism for multiple channel availability with non-uniform channel requirement (demand) across the BSs and linearly increasing valuations with demand. Here, we discuss the scenario when strategy-proofness of the mechanism may not be ensured. Finally, we propose Non-uniform Demand Weakly Strategy-proof Auction Mechanism (NUD-WSPAM) where BSs of an operator may have different demands and per channel valuation is non-increasing at each BS. Here, we introduce the concept of weak strategy-proofness. We also prove the individual rationality, monotonicity, and weak strategy-proofness of NUD-WSPAM.

Monte Carlo simulations are performed in MATLAB [14] to evaluate the performances of the proposed spectrum allocation mechanisms. Using simulation results, social welfare and spectrum utilization of the proposed algorithms in comparison to other algorithms in the literature e.g., [11] are also evaluated. Simulations are also performed for large network sizes (i.e., a large number of BSs) to validate the applicability in practical scenarios.

A. Related Work

In this section, we review some related work on Dynamic Spectrum Access (DSA). Auction-based spectrum allocation approaches have been extensively studied in the literature [15]–[21]. As stated above, achieving strategy-proof optimal allocation and computational feasibility in a mechanism is NP-Hard. In [20], the authors present a DSA mechanism in cellular networks that achieves near-optimal allocation for revenue maximization using a greedy graph coloring approach. The authors in [15] study real-time spectrum allocation mechanism. Though the mechanisms proposed in [20], [15] are computationally feasible in terms of implementation, they are not guaranteed to be strategy-proof. In [21], the authors propose a mechanism that ensures a certain fair chance of spectrum allocation along with the maximization of social welfare. In [22], the authors propose a revenue maximization mechanism for spectrum allocation. For revenue maximization, the combination of well known Vickrey-Clarke-Groves (VCG) [8]–[10] mechanism and Myerson's Lemma [23] are studied. In [7], the authors proposed VERITAS, a sealed bid strategy-proof auction mechanism that follows a certain monotonicity behavior. The authors in [11] propose another strategy-proof mechanism SMALL which groups non-conflicting base stations and sacrifices the base station(s) corresponding to the lowest bid in the winner group. SMALL has better allocation efficiency than that of the algorithm proposed in [7]. In [15], the authors propose an auction-based approach for fine-grained (i.e., a channel is sliced into smaller frequencies) channel allocation. However, it does not satisfy the strategy-proofness property. As interference is one of the major concerns in wireless, the authors in [17], propose an auction-based power allocation mechanism. However, it fails to be strategy-proof.

Both VERITAS [7] and SMALL [11] assume that the channel valuation increases linearly with the demand. In [12], [24], [25], strategy-proof double auction mechanisms are studied. The authors in [26], [27] studies auction-based approaches for DSA in cognitive networks. In [16], the game-theoretic aspect of the DSA in cognitive networks is explored.

The authors in [28] consider adaptive-width spectrum allocation problem where the channel valuation is a non-increasing function of the demand. To take the decrease in valuation with the demand into account, strategy-proof mechanism SPECIAL is proposed. Here, it is assumed that all the base stations bid for all the channels available for auction. To improve the social welfare and revenue of VERITAS, the concept of the reserve price in valuation is incorporated in [13].

Most of the existing works are centered on designing a computationally feasible strategy-proof spectrum auction mechanism for non-cooperative base station participation in auctions. Moreover, [7], [11], [12], [20], [21], [24], [25], [27], [28] consider base stations with uniform channel demand. However, only a few works [7], [11], [28] consider multiple channel demand across the BSs. Except [28], all the works assume that the channel valuation scales linearly with the demand, which may not be true in general as throughput may not increase linearly as a function of bandwidth.

To the best of our knowledge, none of the previous works has considered the operators as the players in the spectrum auction. In comparison, in our work, we consider that non-cooperative and rational operators participate in spectrum auctions and each operator has multiple BSs. Our work also considers non-uniform channel requirement at the BSs.

B. Contributions

In this paper, we investigate the problem of spectrum allocation using sealed bid auction across multiple BSs of coexisting operators in a geographical region. We summarize our contributions as follows.

- We consider the problem of spectrum allocation among coexisting multiple operators in a region. The base stations associated with each operator are used to provide services to the end users. We formulate the problem in multi-parameter environment to maximize the total social welfare of the auction. This has not been addressed in the literature so far.
- We propose a strategy-proof spectrum allocation mechanism at the operator level, where strategy-proofness holds for a set of valuations submitted to auctioneer corresponding to each operator.
- We propose computationally efficient auction mechanism which is applicable to perform auction repeatedly in short durations as per traffic variation.
- We propose a generalized spectrum allocation mechanism that is weakly strategy-proof even if the spectrum demands are not the same across the BSs of an operator. Further, we also consider the case where the channel valuation may not be linearly increasing with the demand of the channels at a base station.
- We analytically prove that the proposed mechanism follows monotonicity, individual rationality and (weak) strategy-proofness.
- We compare the performance of the proposed mechanism with various mechanisms in small as well as in large networks using Monte Carlo simulations.

The rest of the paper is organized as follows. Section II describes the system model and preliminaries of strategy-proof auctions. In Section III, we propose a mechanism for single channel allocation. Section IV proposes an extension of the mechanism proposed in Section III and describes how it fails to be strategy-proof through an example. In Section V, the generalized strategy-proof spectrum allocation mechanism is presented. We summarize the proposed mechanisms in Section II-B. In Section VI, we evaluate the performance of proposed mechanisms through simulations. In Section VII, we conclude the paper.

II. System Model

We consider a geographical region where multiple operators provide services to the end users. Multiple BSs are associated with each operator in the given region. The system model (Fig. 1) comprises a controller for each operator, set of BSs associated with the operators, auctioneer, and spectrum database. There are two decision making devices, controllers, and auctioneer in the system. Each operator has a controller

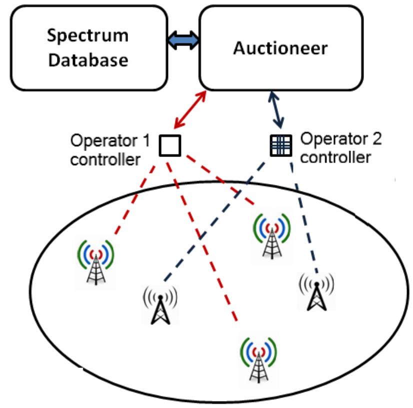


Fig. 1: Illustration of system model.

which determines the number of channels (demand) required and the valuation of channels at the BSs associated with the operator. The demand and the valuation may vary over time depending on the traffic conditions of the wireless network. The operators communicate their spectrum demand and valuation at each base station through the controller. The information of the number of channels available for allocation is contained in the spectrum database. We assume that the channels are of equal bandwidth and are orthogonal. Since orthogonal channels do not have overlapping frequency bands, simultaneous operations on orthogonal channels do not cause interference. Auctioneer is another decision making entity, which decides who should get the spectrum (channel) and what should be the appropriate price for providing exclusive 'right to use' channel to an operator.

In our work, unlike the other existing works, operators are bidders (players) instead of individual BSs in the wireless network. Each operator communicates a vector of bids and demands to the auctioneer via the controller for the BSs associated with it.

Other assumptions made in our system model are as follows.

• We assume that an auctioneer has knowledge of the topology in the geographical region. Therefore, the overall conflict graph consisting of all the BSs participating in the auction is available to the auctioneer.

• We assume all channels are homogeneous in characteristics and act as substitutes. Thus, the bid or valuation is channel independent.

• We consider that operators employ Fractional Frequency Reuse (FFR) techniques to cancel interference across its own BSs. Therefore, the same frequency band (channel) can be allocated to the BSs of an operator. Hence, any base station of an operator would experience interference only from the BSs associated with other operators in the given region. We also assume that the channel requirement for each BS is arrived at after including the impact of the interference coordination technique.

We capture the interference among the BSs of the operators with the help of a graph $\mathcal{G} = (V, \mathcal{E})$, that is obtained from the knowledge of the topology in the geographical region, where V represents the set of vertices (nodes), and $\mathcal{E}$ represents the set of edges in the graph. The set of vertices in the graph correspond to the BSs of various operators in the region. Any two base stations are said to interfere with each other if the geographical distance between them is less than a predetermined value d. In this case, there is an edge between them in the graph. Two interfering BSs (nodes) cannot be assigned the same channel concurrently.

A. Background on Auctions

1) Strategy-Proof Spectrum Auctions: In conventional auctions, once an object is allocated to a buyer, it cannot be allocated to other buyers. However, in spectrum auctions, the same spectrum (frequency band) can be reused or reallocated after a certain fixed distance depending on the coverage area of BSs. This implies that any two BSs can be assigned the same frequency band if they do not interfere with each other. This feature provides an advantage in terms of spectrum utilization, but it is more challenging to achieve strategy-proof spectrum auction. Second price auction mechanism [5] ensures strategy-proof behavior in conventional auctions. However, the same is not guaranteed in the spectrum auctions [7]. In second-price auctions, the object goes to the highest bidder and is charged the price of the second highest bidder in the auction. Moreover, not every base station of an operator interferes with each base station of other operators. Therefore, achieving strategy-proof spectrum allocation across the multiple BSs of the coexisting operators using second price auction is not possible. Moreover, it fails to exploit the re-usability of the spectrum which again results in inefficient usage of the spectrum.

VCG mechanism is the first strategy-proof mechanism that always chooses the optimal allocation strategy. VCG mechanism selects the set of participants that maximizes the overall sum of valuation in the auction [8]–[10]. But, determining the optimal allocation and pricing strategy is burdened with the high computational complexity of the auctions. Due to high computational cost, VCG mechanism is not suitable for dynamic spectrum allocation auctions even in wireless networks of moderate size [6]. In general, VCG mechanism is applicable in combinatorial auctions for sealed bid format, where each player submits a bid for the channel without the knowledge of other players' bids in the auction. Unlike second price auctions, VCG is applicable for single parameter environment as well as multi-parameter environment. Next, we describe the VCG mechanism for spectrum allocation.

2) Vickrey-Clarke-Groves Mechanism: We assume that there are n BSs to participate in spectrum auction which leads to 2^n possibilities. Due to the interference across the BSs, all 2^n combinations may not be feasible for spectrum allocation. The BSs which are sufficiently far can be allocated channels simultaneously. Let the binary vector $x = \{x_1, x_2, \ldots, x_n\}$ denote a feasible allocation satisfying all the interference constraints, where $x_i = 1$ if a channel is assigned to the BS i, otherwise $x_i = 0$. Let $\mathcal{X}$ denote the set of feasible allocations. BS i submits a bid b_i based on its valuation. Let $b = \{b_1, b_2, \ldots, b_n\}$. The optimal allocation is given as

$$x^\star = \arg\max_{x \in \mathcal{X}} b \cdot x. \tag{1}$$

Now, a pricing scheme is defined to make the auction strategy-proof. Using a pricing scheme, the players are enforced to submit true valuation of the object to the auctioneer. VCG pricing scheme charges the BSs with the welfare loss inflicted due to the presence of BS i.

Let ρ_i denote the price charged to BS i.

$$\rho_i = \max_{x \in \mathcal{X}} \sum_{j \neq i} x_j \cdot b_j - \sum_{j \neq i} x_j^\star \cdot b_j, \tag{2}$$

where $x^\star$ is the optimal allocation obtained from Equation (1). The price charged using Equation (2) also ensures individual rationality i.e., $0 \leq \rho_i \leq b_i$. In other words, any BS would never be charged more than its submitted bid. The individual rationality reflects that the utility gain at a BS can never be negative if a BS bids at its true value.

Though VCG mechanism achieves the optimal channel allocation for social welfare maximization, it becomes intractable for a large set of BSs. Hence, it is not feasible for practical implementation. Next, we propose strategy-proof mechanisms to maximize the social welfare of the spectrum for various scenarios. The proposed algorithms are also computationally efficient in comparison to VCG. VCG is implemented in two steps: Channel Allocation ($\mathcal{O}(2^n)$) and Price Charging scheme ($\mathcal{O}(2^n)$).

B. Comparison of Proposed Algorithms with others

In this section, we summarize the key features (strategy-proofness and computational complexity) of the proposed mechanisms and the various scenarios in Table I.

TABLE I: Summary

Algorithms	**Scenario**	**Strategy-proof**	**Computational Complexity**
SC-SPAM	Single channel, Uniform demand	Strong	$\mathcal{O}(nm^2)$
NUD-AM	Multi-channel, Non-uniform demand, linear bid	No	$\mathcal{O}(nm^2)$
NUD-WSPAM	Multi-channel, Non-uniform demand, non-linear bid	Weak	$\mathcal{O}(nm^2)$
SPECIAL [28]	Multi-channel, Uniform demand, non-linear bid	Strong	$\mathcal{O}(m^2)$
VCG [6]	Multi-channel, Uniform demand, linear bid	Strong	$\mathcal{O}(2^m)$

In above Table, n is the number of operators, $m = \sum_{i=1}^{n} m_i$ is the total number of BSs across all the operators present in the region. By m_i, we denote the number of base stations associated with operator i. The detailed computation complexity analysis of SC-SPAM is presented in [1]. For multiple channel availability, computational complexity can be obtained using similar analysis as given for SC-SPAM.

C. Notations and Definitions

We introduce the following notations:

- $\mathcal{N} = \{1, 2, \ldots, n\}$ represents the set of operators participating in the spectrum auction in a geographical region.
- m_i represents the number of base stations corresponding to operator i.

- $\mathcal{S}_i = \{S_{i1}, S_{i2}, \dots, S_{im_i}\}$ represents the set of base stations of operator i.
- $\mathcal{V}_i$ denote true valuation of operator i. $\mathcal{V}_i(\ell, j)$ is true value for ℓ^{th} channel at BS j of operator i if $(\ell - 1)$ channels are already assigned. If $\mathcal{V}_i(\ell, j) = 0$, then BS j does not require ℓ^{th} channel.
- $\mathcal{B}_i$ denote bid of operator i. $\mathcal{B}_i(\ell, j)$ is bid for demand ℓ at BS j of operator i if $(\ell - 1)$ channels are already assigned. If $\mathcal{B}_i(\ell, j) = 0$, then BS j does not require channel.
- $\mathcal{N}_i$ represents the set of neighboring base stations which are in conflict with the base stations of operator i (same channel cannot be allocated simultaneously).
- $x_i^f = \sum_{i=1}^{k} x_i^k$, $k = \{1, \dots, K\}$. K is the total number of channels available in spectrum database for auction. By x_{ij}^f, we denote the j^{th} component of final allocation vector x_i^f.
- O_i represents operators that are neighbors of i i.e., $(\{\text{operators } y \mid S_y \bigcap \mathcal{N}_i \neq \phi, y \neq i\})$.
- $d_i = \{d_{i1}, d_{i2}, \dots, d_{im_i}\}$ represents the number of channels required at base stations of operator i.
- $N(\mathcal{G}^{'})$ represents the set of active operators from the conflict graph $\mathcal{G}^{'}$ (operators with non-zero demand).

Definition 1. *An auction is truthful (strategy-proof) if there is no incentive in deviating from the true valuation. Thus, the dominant strategy is to bid at the true valuation no matter what strategy others choose.*

$$\mathcal{U}_i(\mathcal{B}_i, \mathcal{B}_{-i}) \leq \mathcal{U}_i(\mathcal{V}_i, \mathcal{B}_{-i}) \quad \forall \mathcal{B}_i, \forall \mathcal{B}_{-i}. \tag{3}$$

where $\mathcal{V}_i$ and $\mathcal{U}_i$ are true valuation and utility of operator i. Moreover, $\mathcal{B}_i$ is the bid of operator i and $\mathcal{B}_{-i} = (\mathcal{B}_1, \dots, \mathcal{B}_{i-1}, \mathcal{B}_{i+1}, \dots, \mathcal{B}_n)$ represents bid of all operators except operator i.

Definition 2. *Spectrum Utilization is defined as the total number of channels assigned to BSs across all the operators.*

$$U^s = \sum_{i=1}^{n} \sum_{j=1}^{m_i} x_{ij}^f, \tag{4}$$

where x_{ij}^f denotes the number of channels allocated at j^{th} base station of operator i.

Definition 3. *Social Welfare is defined as the aggregate true value of the channels assigned to all BSs across all operators.*

$$W^s = \sum_{i=1}^{\mathcal{N}} \sum_{j=1}^{m_i} \sum_{\ell=1}^{x_{ij}^f} \mathcal{V}_i(\ell, j) \tag{5}$$

III. Strategy-proof auction for unit demand

In this section, we describe our proposed algorithm Single Channel Strategy-proof Auction Mechanism (SC-SPAM) for channel allocation among the base stations of multiple operators. As the name SC-SPAM suggests, we consider only one channel is available for auction i.e., $K = 1$ where K denotes the number of channels. In auctions, the mechanism design has two steps: channel allocation and price charging strategy. In channel allocation phase, the auctioneer decides who should be given the right to use the channel. What price should be charged is decided in the pricing strategy phase. The price charged enforces the operators to declare the true valuations to ensure a strategy-proof auction.

For single channel scenario, the demand at each base station is restricted to one, i.e., $\ell = 1$ and therefore, for simplicity of notation we denote $\mathcal{V}_i(\ell, j) = v_{ij}$, which represents the true valuation of j^{th} base station associated with operator i (i.e., S_{ij}). Furthermore, $\mathcal{V}_i$ reduces to one dimensional vector, which we denote as $v_i = [v_{i1}, v_{i2}, \dots, v_{im_i}]$. Similarly, $\mathcal{B}_i(\ell, j) = b_{ij}$ denote bid at j^{th} base station associated with operator i and $\mathcal{B}_i \simeq b_i = [b_{i1}, b_{i2}, \dots b_{im_i}]$. Next, we define some new terms:

- *True valuation* (σ_i^v) : True valuation σ_i^v of any operator i is defined as the sum of the actual valuations (which are private and not known to the auctioneer) of all the BSs corresponding to operator i.
$$\sigma_i^v = \sum_{j=1}^{m_i} v_{ij}. \tag{6}$$
- *Bidding valuation* (σ_i^b) : Bidding valuation σ_i^b of operator i is defined as the sum of the bids (which may or may not be same as the actual valuation) of all the BSs corresponding to operator i.
$$\sigma_i^b = \sum_{j=1}^{m_i} b_{ij}. \tag{7}$$
- *Price* (p_i): It is defined as the price that an operator i has to pay, in case operator i wins the resources (channels), else it is zero.
- *Operator Utility* $(\mathcal{U}_i)$: Utility of an operator i is the difference between the operator valuation (unknown to the auctioneer) and the price charged on the allocation of the channel. If the operator does not get the channel, the utility is zero. In other words, it represents the overall gain of an operator i if it is allocated a channel.
$$\mathcal{U}_i(\mathcal{B}_i, \mathcal{B}_{-i}) = \begin{cases} \sigma_i^v - p_i, & \text{if the channel is allocated} \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$
where $\mathcal{B}_i$ is the bid of operator i and $\mathcal{B}_{-i}$ represents the bids of all operators except operator i.

Now, we define critical operator which is used later in the price charging strategy.

Definition 4. *A critical operator $C(i)$ of an operator i is defined as the operator in O_i whose sum of the bids of base stations is maximum among all the operators in O_i. The critical operator $C(i)$ is given as any $y \in O_i$ such that*

$$\sum_{k \in \{\mathcal{N}_i \bigcap S_y\}} b_{yk} \geq \sum_{k \in \{\mathcal{N}_i \bigcap S_{y'}\}} b_{y'k}, \quad \forall y' \neq y, \ i \text{ and } y' \in O_i. \tag{9}$$

Let us define a set $\mathcal{L}_y^i = \mathcal{N}_i \cap S_y$, which contains the BSs of operator y in conflict with the BSs of operator i. Let Λ_y^i be the valuation of set $\mathcal{L}_y^i$ which is given as, $\Lambda_y^i = \sum b_{yk} \mathbb{1}_{\{S_{yk} \in \mathcal{L}_y^i\}}$. The critical operator of an operator i can be obtained as

$C(i) = \arg\max_{y \neq i} \Lambda_y^i,\ y \in O_i$ and the critical operator valuation σ_i^c is given as, $\sigma_i^c = \max_{y \neq i} \Lambda_y^i, y \in O_i$.

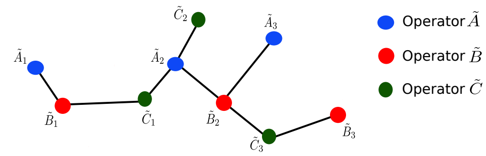
The strategy-proof algorithm proposed is described in Algorithm 1. This algorithm takes conflict graph $\mathcal{G}$ and bid vector corresponding to each operator $\{b_i\}_{\{i \in \mathcal{N}\}}$ as input. Binary channel allocation vector $\{x_i\}_{\{i \in \mathcal{N}\}}$ and payment vector $\{p_i\}_{\{i \in \mathcal{N}\}}$ for all the operators are initialized to zero. Initially, we determine the maximum bidding operator and its critical neighbor $C(i^*) = \arg\max_{y \neq i} \Lambda_y^{i^*},\ y \in O_i*$ (line8). Channel allocation vector, x_i for the maximum bidding operator (winner) is updated to 1 and the payment for the winning operator is updated to the price of the critical neighbor valuation, $\sigma_{i^*}^c$. The conflict graph $\mathcal{G}^{'}$ is updated with the remaining nodes after the removal of the nodes corresponding to the winning operator i^* and its neighboring nodes $\mathcal{N}_{i^*}$. Repeat the process until $\mathcal{G}^{'}$ is NULL (line 13), i.e., no other BSs is present in $\mathcal{G}^{'}$. For single channel auction, final allocation vector $x_i^f = x_i$, which is a binary vector. By x_{ij}^f and x_{ij}, we denote j^{th} element (allocation at j^{th} BS of operator i) in vectors x_i^f and x_i, respectively. However, when multiple channels are available for auction, $x_{ij}^f \in \mathbb{R}_+$. Hence, the final allocation vector $x_i^f \neq x_i$. Next, we explain Algorithm 1 through an example.

Algorithm 1 Single Channel Strategy-proof Auction Mechanism

1: **Input:** Conflict Graph $\mathcal{G}$, bid vector, $\{b_i\}_{\{i \in \mathcal{N}\}}$.
2: **Output:** Binary channel allocation vector $\{x_i\}_{\{i \in \mathcal{N}\}}$, price $\{p_i\}_{\{i \in \mathcal{N}\}}$.
3: Initialize $x_i \leftarrow 0$, $N(\mathcal{G}) = \{1, 2, \ldots, n\}$
4: Initialize $p_i \leftarrow 0$, $\mathcal{G}^{'} \leftarrow \mathcal{G}$, $N(\mathcal{G}^{'}) \leftarrow N(\mathcal{G})$, $FLAG \leftarrow True$.
5: **while** $(FLAG = True)$ **do**
6:     Make $i^* \leftarrow \arg\max_{i \in N(\mathcal{G}')} \sigma_i^b$.
7:     Find $\mathcal{N}_{i^*}$.
8:     Set $C(i^*) \leftarrow \arg\max_{y \neq i^*} \Lambda_y^{i^*},\ y \in O_i*$ and $\sigma_{i^*}^c \leftarrow \max_{y \neq i^*} \Lambda_y^{i^*},\ y \in O_i*$.
9:     Make $p_{i^*} \leftarrow \sigma_{i^*}^c$ and $x_{i^*} \leftarrow 1$.
10:     **if** $(\mathcal{G}^{'} \cap (S_{i^*} \cup \mathcal{N}_{i^*}) = \mathcal{G}^{'})$ **then**
11:         $FLAG \leftarrow False$.
12:     **else**
13:         $\mathcal{G}^{'} \leftarrow \mathcal{G}^{'} \backslash \{S_{i^*} \cup \mathcal{N}_{i^*}\}$.
14:     **end if**
15: **end while**

Example: Consider a network of 3 operators $\tilde{A}, \tilde{B}, \tilde{C}$, where each operator has 3 BSs deployed in the region to provide services to the subscribers. BSs $\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\}$, $\{\tilde{B}_1, \tilde{B}_2, \tilde{B}_3\}$ and $\{\tilde{C}_1, \tilde{C}_2, \tilde{C}_3\}$ correspond to operators $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$, respectively. The conflict graph is illustrated in Fig. 2a based on the interference criteria.

In Fig. 2b, the bid vector of each operator is shown. In the first iteration, Operator $\tilde{A}$ has the highest bid among the operators with a value of $\sigma_{\tilde{A}}^b = 25$. Therefore, Operator $\tilde{A}$ is allocated channel across BSs, and it has to pay the price of its critical operator. As per Definition 4, critical operator for winning operator $\tilde{A}$ is operator $\tilde{C}$ and $p_{\tilde{A}} = \sigma_{\tilde{A}}^c = 18$. Thus, the utility of operator $\tilde{A} = \mathcal{U}_{\tilde{A}} = 7$. We update the conflict graph with the BSs of operators $\tilde{B}$ and $\tilde{C}$ not in conflict with the BSs of operator $\tilde{A}$. In second iteration, the updated $\mathcal{G}$ comprises BSs $\tilde{B}_3$ and $\tilde{C}_3$. Operator $\tilde{B}$ wins the channel and pays the price, $\sigma_{\tilde{B}}^c = 3$. The utility of operator $\tilde{B}$ is 2. Operator $\tilde{C}$ is not allocated channel.



(a)

	1	2	3	Total
operator Ã	8	10	7	25
operator B̃	9	8	5	22
operator C̃	9	9	3	21

(b)

Fig. 2: Network of 3 operators (a) Conflict Graph (b) Bid vector table corresponding to operator $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$.

Now, if operator $\tilde{B}$ tries to increase its utility by deviating from its true valuation $\sigma_{\tilde{B}}^v = 22$ to $\sigma_{\tilde{B}}^b = 28$ by increasing the bid of its BSs, operator $\tilde{B}$ will get channel being the highest bidder among the operators. But, it has to pay the price of its critical operator which is operator $\tilde{A}$ and therefore, pays $\sigma_{\tilde{B}}^c = 25$. This leads to a negative utility -3 for operator $\tilde{B}$. Thus, bidding at the true valuation is the best strategy for an operator in the auction.

Next, we prove that the proposed algorithm follows monotonicity, individual rationality and strategy-proofness.

Lemma 1. *If operator i is allocated a channel by bidding at σ_i^b, it will also be allocated if it bids $\sigma_i^{b'}$, where $\sigma_i^{b'} \geq \sigma_i^b$ provided all the other operators' bids remain unchanged.*

Proof. As stated in Algorithm 1, all the operator bids are arranged in non-increasing order of the bids $\sigma_i^b, \forall i \in \mathcal{N}$. Let us assume in the sorted list (S, say) operator i lies at position k. Now, keeping all the other operator bids unchanged, increase the bid of operator i to $\sigma_i^{b'}$, and again arrange all the operator bids in non-increasing order in another sorted list S'. Let us say, the position of operator i in S' is l, where $l \leq k$. Thus, the operator moves higher in the position which ensures that it still gets the channel. This completes the proof. □

Lemma 2. *Algorithm 1 is individually rational.*

Proof. As stated in the pricing scheme of Algorithm 1, winning operator i is charged price $p_i = \sigma_i^c$. Moreover, we know that the valuation of winning operator i is the highest among all operators.

$$\therefore \quad \sigma_i^b > \sigma_y^b, \ \ \forall\ y \neq i. \tag{10}$$

Using Definition 4, $\sigma_i^c = \max_{y \neq i,\ y \in O_i} \Lambda_y^i$. This implies that

$$\sigma_i^c \leq \max_{y \neq i} \sigma_y^b. \tag{11}$$

From Equations (10) and (11), we get $\sigma_i^c < \sigma_i^b$. Hence, $p_i \leq \sigma_i^b$. This proves individual rationality of the algorithm. □

Theorem 3. *Algorithm 1 is strategy-proof.*

Proof. Refer Appendix A. □

IV. Extension for non uniform demand of channels among the base stations of operators

In this section, we extend SC-SPAM for the case where the demand of channels across the BSs of an operator is not uniform (or same). Instead, the BSs of an operator may have different channel requirements depending on the traffic conditions. Let us define the demand of operator i as $d_i = \{d_{i1}, \ldots, d_{im_i}\}$, where d_{ij} represents the channel demand at j^{th} BS associated with operator i. It is assumed that the operators do not have strict demand, i.e., they are willing to accept any number of channels between 0 to d_{ij} at BS j.

Let $\mathcal{B}_i$ and $\mathcal{V}_i$ denote the bid and the true value of operator i across its base stations. Here, it is assumed that the valuation of the channel increases linearly with the demand at any BS. This implies that the per channel value at a base station is same for every assigned channel. In case, the demand of the channel at any BS is d_{ij}, then valuation at the particular BS gets multiplied by the demand, i.e., $d_{ij} \cdot v_{ij}$. The bid vector, $b_i = \mathcal{B}_i(1,:)$ reflects per channel bid for BSs of an operator.

Let us define,

$$\sigma_i^b(k) = \sum_{j=1}^{m_i} b_{ij} \mathbb{1}_{\{d_{ij}>0\}},\ k = \{1, \ldots K\}$$

for every operator i where b_{ij} is per channel bid value corresponding to j^{th} BS of operator i. $\sigma_i^b(k)$ computes the valuation of each operator corresponding to demand of channel at its BSs for a channel. As stated above, at least one channel is required at all the BSs participating in auction of any operator, therefore, $\sigma_i^b(1) = \sigma_i^b$ (Equation (7)).

We propose Non-uniform Demand Auction Mechanism (NUD-AM) in Algorithm 2 which takes the demand vector $\{d_i\}_{i \in \mathcal{N}}$ as input along with the number of channels for auction. Channel allocation and price computation are performed iteratively for each channel present in the database. For each channel allocation, we compute $\sigma_i^b(k)$, which determines the operator valuation as per the demand at its BSs (line 3). Based on the operator valuation, we determine the channel allocation and the price charged from the operators using SC-SPAM. Then, the demand across BSs is updated based on the allocation vector for every operator (line 5). Next, we update the conflict graph before the next channel allocation. Channels are allocated corresponding to $\sigma_i^b(k)$, to ensure the maximization of the social welfare. The process continues until all the channels are allocated. Next, we describe the operations of NUD-AM with an example.

Algorithm 2 Non-uniform Demand Auction Mechanism (NUD-AM)

Input: Conflict Graph $\mathcal{G}$, K channels, bid vector $\{\mathcal{B}_i\}_{\{i \in \mathcal{N}\}}$; demand vector $\{d_i\}_{\{i \in \mathcal{N}\}}$.
Output: Allocation vector $\{x_i^f\}_{\{i \in \mathcal{N}\}}$, price $\{p_i\}_{\{i \in \mathcal{N}\}}$.

1: Initialize demand vector $d_i' \leftarrow d_i$ for every i, $k = K$, $b_i = \mathcal{B}_i(1,:)\ \forall\ i \in \mathcal{N}$, $x_i^f \leftarrow Null$, $\mathcal{G}' \leftarrow \mathcal{G}$
2: **while** $(k > 0)$ **do**
3: Compute $\sigma_i^b(k) = \sum_{j=1}^{m_i} b_{ij} \mathbb{1}_{\{d_{ij}>0\}}$.
4: Allocate channel and compute price (Algorithm 1).
5: Update $d_i' \leftarrow d_i' - x_i$ for every i
6: Update $x_i^f \leftarrow x_i^f + x_i$
7: **procedure** CONFLICT–GRAPH–UPDATION
8: If $(d_{ij} = 0)$ update $\mathcal{G}' \leftarrow \mathcal{G}' \setminus \{S_{ij}\}$
9: Else $\mathcal{G}' \leftarrow \mathcal{G}'$
10: **end procedure**
11: $k \leftarrow k - 1$.
12: **end while**

A. Example

We consider a wireless network of 3 operators A, B, and C. Each operator has multiple BSs to provide services to the users in a geographical region. As illustrated in Fig.3, operator A, B and C have BSs $\{A_1, A_2, A_3, A_4\}$, $\{B_1, B_2, B_3, B_4\}$ and $\{C_1, C_2\}$, respectively. We consider that the channel demand across the BSs of an operator is not the same, and the valuation at any BS increases linearly with the demand. We consider 2 channels are available for auction. An operator can bid for at most the number of channels available for auction at any of its BSs. Each operator submits a bid vector. As stated above bids are linearly increasing with demand, the bid vector contains bid per channel at each BS.

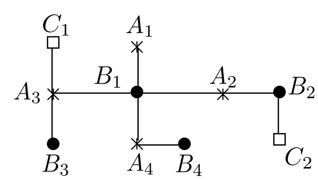


Fig. 3: Conflict graph of the 3 operators.

We consider the demand vectors for the operators A, B and C are given as $d_A = [2\ 1\ 2\ 2]$, $d_B = [2\ 1\ 1\ 2]$ and $d_C = [2\ 1]$, respectively. The bids at the BSs of operators A, B and C are represented as $b_A = [8\ 10\ 7\ 6]$, $b_B = [8\ 9\ 9\ 10]$ and $b_C = [10\ 9]$, respectively. Channel allocation procedure is performed in two iterations.

Case 1 : All operators bid at true value across BSs.

• Iteration 1: First we determine the $\sigma_i^b(1)$, $\forall\ i = \{A, B, C\}$. $\sigma_A^b(1) = 31$, $\sigma_B^b(1) = 36$ and $\sigma_C^b(1) = 19$. Similar to the calculations shown in Section 1, Operators B and C get channel at BSs $\{B_1, B_2, B_3, B_4\}$ and $\{C_1\}$. Now, we obtain the price charged from the winners of the auction using critical operator (Definition 4). The price charged from operator $p_B = 31$ and $p_C = 0$. Next, we update the conflict graph for second channel allocation with non-zero demand across BSs as illustrated in Fig. 4.

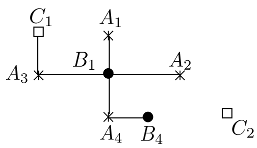


Fig. 4: Updated conflict graph after the first iteration.

• Iteration 2: Again we perform same procedure as described in Iteration 1 on updated $\sigma_A^b(2) = 31$, $\sigma_B^b(2) = 18$ and $\sigma_A^C(2) = 19$. Now, BSs $\{A_1, A_2, A_3, A_4\}$ and $\{C_2\}$ get channel corresponding to operators A and C. The price charged from operators A and C are $p_A = 18$ and $p_C = 0$, respectively.

Case 2 : Except operator B all operators bid at true value. Let Operator B deviates from the true valuation and submits $b_B = (8,\ 6,\ 6,\ 9)$ to the auctioneer.

• Iteration 1: As Operator B deviates from the true value, $\sigma_B^b(1)$ reduces to 29. Channels are allocated at $\{A_1, A_2, A_3, A_4\}$ and $\{C_2\}$ BSs of operators A and C, respectively. The price charged are $p_A = 29$ and $p_C = 0$. Next, update the conflict graph.

• Iteration 2: Channels are allocated on the updated graph

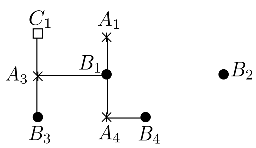


Fig. 5: Updated conflict graph after the first iteration.

shown in Fig. 5 at $\{B_1, B_2, B_3, B_4\}$ and $\{C_1\}$ BSs of operators B and C, respectively. We observe that the demand at BS A_2 is zero, so it is no longer the part of the conflict graph. Therefore, the price charged from the operator B and C are 21 and 0, respectively.

It is observed that operator B gets the same number of channels in both the cases (true valuation and misreporting to lower value). However, the price charged at the true value and the deviated bid value are 31 and 21, respectively for operator B. This clearly shows that the utility gain of operator B is 10. Hence, NUD-AM is not always strategy-proof.

As channel allocation procedure for a channel in NUD-AM is the same as SC-SPAM, therefore, NUD-AM is strategy-proof individually for every iteration. But it may not be strategy-proof as a whole. The reason behind NUD-AM not being strategy-proof is the updation of the conflict graph after each allocation. This results in the removal of BSs where demand is satisfied. This shows that addressing non-uniform demand is challenging. Next, we present another algorithm for this purpose.

V. Weakly Strategy-proof Algorithm for Non-uniform Demand

Algorithm NUD-AM proposed in Section IV considers non-uniform demand across the base stations of an operator where the channel valuation increases linearly with the demand at the base stations. This implies that per channel bid at each base station remains the same with the demand. As NUD-AM charges price sequentially from the BSs in each step, it fails to be strategy-proof in certain cases, e.g. if an operator chooses to bid lower than its true valuation. In this section, we propose Non-uniform Demand Weakly Strategy-proof Auction Mechanism (NUD-WSPAM) which ensures that the operators have no incentive to deviate from the true valuation even if the demand across BSs is non-uniform and per channel bid may differ with the demand at a base station.

We consider that the bids are non-increasing with the demand which means each subsequent channel is valued less than the previous channel - this is akin to the assumption of decreasing marginal utility made commonly in economics. Hence, the operators are required to report the bid corresponding to multiple channel demand at each BS to the auctioneer. We assume that the demand at any BS across the network cannot be greater than the total number of channels available in the spectrum database. Now, each operator reports a bid vector for each BS associated with it. Let $\mathcal{B}_i$ denote the bid for operator i. Here, $\mathcal{B}_i(\ell, j)$ is bid for demand ℓ at BS j of operator i, if $(\ell - 1)$ channels are already assigned. We enforce that the bids submitted by operators are non-increasing i.e.,

$$\mathcal{B}_i(\ell, j) \geq \mathcal{B}_i(\ell + 1, j), \text{ for all } i, j, \ell.$$

The non-increasing marginal true value per channel with the demand is also considered by the authors in [28]. The authors referred the pattern of non-increasing marginal bid with the demand as *flexible bidding*. The authors in [29], have studied the saturated throughput variation with the bandwidth at the base stations. It is observed that the saturated throughput may not increase linearly with the bandwidth.

In Algorithm 3, we present a generalized algorithm NUD-WSPAM. Unlike previous mechanisms, NUD-WSPAM first determines allocation for all the channels present in the spectrum database and then computes the price to be charged. At each iteration, a channel is allocated using Algorithm 1 for every channel available in the spectrum database as mentioned in line 4. To compute the price, we update the conflict graph which comprises the BSs where channel requirement is not satisfied after the allocation is complete. The price is charged based on the critical operator in the final updated graph (line 19).

As described in Algorithm 3, allocation is performed iteratively for each channel and then the bids are updated after each allocation for all the operators. The bid of operator i is $\mathcal{B}_i$, where BS j has multiple bids given as $\{\mathcal{B}_i(\ell, j) | 0 < \ell \leq d_{ij}\}$. Let b_i^r denote the active bids (maximum of bid for the demand that is not satisfied) at BSs of operator i in r^{th} iteration. The bid updation process is described in the Algorithm 3. By b_i^f, we denote the updated bid vector after all the K channels are allocated. Thus, bid vector b_i^f projects the bids at the BSs of operator i for $(K+1)^{\text{th}}$ iteration, where K is the number of channels available. The bid at BS j of operator i in vector b_i^f is given as $b_{ij}^f = \mathcal{B}_i(x_{ij}^f + 1, j)$, where x_{ij}^f is final allocation of operator i at BS j or j^{th} component of x_i^f. The vector b_i^f

Algorithm 3 Non-uniform Demand Weakly Strategy-proof Auction Mechanism (NUD-WSPAM)

Input: Conflict Graph $\mathcal{G}$, K channels, non-increasing bid vector, $\mathcal{B}_{i\{i\in\mathcal{N}\}}$, demand vector $\{d_i\}_{\{i\in\mathcal{N}\}}$.
Output: Final allocation vector $\{x_i^f\}_{\{i\in\mathcal{N}\}}$, price $\{p_i\}_{\{i\in\mathcal{N}\}}$

```
Initialize final allocation vector x_i^f ← 0, G' ← G
Initialize p_i ← 0, b_i = B_i(1,:)   ∀i ∈ N
while (K > 0) do
    Find x_1, ..., x_n using Algorithm 1
    Update x_i^f ← x_i^f + x_i, ∀ i
    Update d_i^f ← d_i^f − x_i^f, ∀ i
    procedure BID–UPDATION
        for i = 1 : n
            for j = 1 : m_i
                b_ij^f = B_i(x_ij^f + 1, j)
            end
        end
    end procedure
    procedure CONFLICT–GRAPH–UPDATION
        see Algorithm 2
    end procedure
    K ← K − 1
end while
Charge price using Equation (14).
```

has the highest bid values corresponding to unsatisfied demand (non-increasing bid assumption) for operator i.

Let d_i^f, $i \in O$ denotes final demand vector of the operator i after the allocation process is complete. Here, $d_{ij}^f = 0$ signifies that the demand is satisfied at j^{th} BS of operator i. Furthermore, the set of BSs where demand is unsatisfied is indicated as S_i^f i.e., $S_i^f = \{j|d_{ij}^f > 0\}$. Based on S_i^f, final conflict graph $\mathcal{G}^f$ is obtained. $\mathcal{G}^f$ has BSs where demand is not satisfied.

Let, $\Gamma_y^i = \mathcal{N}_i \bigcap S_y^f$ denotes the BSs of operator y in $\mathcal{G}^f$ which are in neighborhood of BSs of operator i in initial conflict graph $\mathcal{G}$. We define the critical operator $C(i)$ any $y \in O_i$ such that

$$\sum_{k\in\Gamma_y^i} b_{yk}^f \geq \sum_{k\in\Gamma_{y'}^i} b_{y'k}^f, \quad \forall y' \neq y,\ y' \in O_i. \tag{12}$$

For single channel auction, Equation (12) reduces to Definition 4. The only difference is that the BSs where the demand is zero after the allocation process is no longer part of the conflict graph $\mathcal{G}^f$. We compute the valuation of operator y which is not allocated channel χ_y^i. Critical operator valuation σ_i^c is obtained using Equation (14).

$$\chi_y^i = \sum_{k\in\Gamma_y^i} b_{yk}^f. \tag{13}$$

$$\sigma_i^c = \chi_{C(i)}^i. \tag{14}$$

The price charged from operator i is $p_i = \sigma_i^c$. This price reduces to the earlier critical operator valuation mentioned in Section III for the single-channel scenario.

Next, we define a new concept of *weak strategy-proofness*:

Definition 5. *Let $\mathcal{V}_i$ denote true valuation of operator i. An auction is said to be weakly strategy-proof if an operator does not gain by deviating to $\tilde{\mathcal{B}}_i$ from $\mathcal{V}_i$, where $\tilde{\mathcal{B}}_i$ satisfies either (1) $\exists\ j$ such that $\tilde{\mathcal{B}}_i(\ell,j) > \mathcal{V}_i(\ell,j)$, $\forall \ell$ or (2) $\exists\ j$ such that $\tilde{\mathcal{B}}_i(\ell,j) < \mathcal{V}_i(\ell,j)$, $\forall \ell$. i.e.,*

$$\mathcal{U}_i(\tilde{\mathcal{B}}_i, \mathcal{V}_{-i}) \leq \mathcal{U}_i(\mathcal{V}_i, \mathcal{V}_{-i}) \quad \forall\ \tilde{\mathcal{B}}_i \& \mathcal{V}_{-i}. \tag{15}$$

where, $\tilde{\mathcal{B}}_i$ satisfy conditions (1) or (2) and $\mathcal{V}_{-i} = \{\mathcal{V}_1, \ldots, \mathcal{V}_{i-1}, \mathcal{V}_{i+1}, \ldots, \mathcal{V}_n\}$ is tuple with bid of all other operators except operator i.

A. Example

We revisit the Example IV-A in context of NUD-WSPAM. The wireless network is illustrated in Fig. 3 is same except the channel valuation at a BS is no longer linearly increasing with the demand. As stated earlier, per channel valuation is non-increasing function of demand at any BS. An operator can bid for at most the number of channels available for auction at any BS. We consider demand vectors to be the same as mentioned in the example previously. Let q_{ij} represents the bid vector at BS j of operator i corresponding to its demand. The bid at BSs of operator A are given as $\mathcal{B}_A = [q_{A1}^{\mathrm{T}}\ q_{A2}^{\mathrm{T}}\ q_{A3}^{\mathrm{T}}\ q_{A4}^{\mathrm{T}}]$, where $q_{A1} = [8\ \ 5]$, $q_{A2} = [10\ \ 0]$ and $q_{A3} = [7\ \ 3]$ and $q_{A4} = [6\ \ 3]$. Here, a^{T} indicate the transpose of a. The bid for operator B is $\mathcal{B}_B = [q_{B1}^{\mathrm{T}}\ q_{B2}^{\mathrm{T}}\ q_{B3}^{\mathrm{T}}\ q_{B4}^{\mathrm{T}}]$, where $q_{B1} = [8\ 4]$, $q_{B2} = [9\ 0]$, $q_{B3} = [9\ \ 0]$ and $q_{B4} = [10\ \ 3]$. The bid for operator C is $\mathcal{B}_C = [q_{C1}^{\mathrm{T}}\ q_{C2}^{\mathrm{T}}]$, where $q_{C1} = [10\ 5]$, $q_{C2} = [9\ 0]$.

Case 1: All operators reveal their true valuations

- Iteration 1: From the given bid vectors, we determine the bids of operators, $\sigma_A = 31$, $\sigma_B = 36$ and $\sigma_C = 19$. The channel is allocated at all the BSs of the highest bidding operator. Then channel is allocated to the BSs of the remaining operators in the order of decreasing valuations which do not conflict with the BSs that are already allocated channel. Therefore, the channel is allocated to operator B at $\{B_1, B_2, B_3, B_4\}$ and operator C at $\{C_1\}$ BSs.
- Iteration 2: For second channel allocation, demand and bid vectors are updated depending on the allocation in previous iteration. The updated demand vectors are $d_A = [2\ 1\ 2\ 2]$, $d_B = [1\ 0\ 0\ 1]$ and $d_C = [1\ 1]$. The operators valuation for the iteration is determined from the updated bid $\sigma_A = 31$, $\sigma_B = 7$ and $\sigma_C = 19$. Channel is allocated to operator A at $\{A_1, A_2, A_3, A_4\}$ and operator C at $\{C_2\}$ BSs.

This completes the channel allocation phase. Now, the demand at the operators is $d_A = [1\ 0\ 1\ 1]$, $d_B = [1\ 0\ 0\ 1]$ and $d_C = [1\ 0]$.

Price Charging Step: In Algorithm 3, the price is charged after all the channels are allocated based on the BSs where demand is non-zero. We construct the conflict graph with the BSs having demand greater than zero as illustrated in Fig. 6. Each operator is charged as per their critical operator (see Definition 4). The sum of the highest bids of the BSs $\{B_1, B_4\}$ of operator B for which demand is not satisfied comprise the critical operator of operator A. Similarly, the bids of BSs $\{A_1, A_3, A_4\}$ constitute the critical operator for

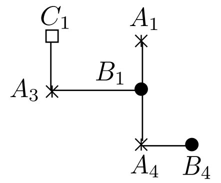


Fig. 6: Updated conflict graph after channel allocation phase is complete.

operator B and the bid at the BS $\{A_3\}$ is critical operator bid for operator C. Thus, the price charged from operator A, B and C is given by $p_A = 7$, $p_B = 11$ and $p_C = 3$.

Case 2: Operator B deviates from true valuation and bids at a lower value

Now, we revisit the wireless network mentioned in Fig. 3, considering that except the operator B others submit bid equal to true value for the associated BSs. The demand vector of all the operators remain unchanged as in the first case. We consider that operator bid is $\mathcal{B}_B^{'} = [q_{B1}^{'}{}^{\mathrm{T}}, q_{B2}^{'}{}^{\mathrm{T}}, q_{B3}^{'}{}^{\mathrm{T}}, q_{B4}^{'}{}^{\mathrm{T}}]$, where $q_{B1}^{'} = (8,\ 4)$, $q_{B2}^{'} = (6,\ 0)$, $q_{B3}^{'} = (6,\ 0)$ and $q_{B4}^{'} = (9,\ 3)$. As described in Case 1, channel is allocated to the operators.

- Iteration 1: Here, the operator bids for channel allocation are $\sigma_A = 31$, $\sigma_B = 29$ and $\sigma_C = 19$. Operators A and C are allocated channel at BSs $\{A_1, A_2, A_3, A_4\}$ and $\{C_2\}$, respectively.
- Iteration 2: Second channel is allocated to BSs $\{B_1, B_2, B_3, B_4\}$ and $\{C_1\}$.

We can see that operator B gets channel at their BSs in iteration 2. Channel allocation remains the same even after deviating from the true valuation. Next, we determine the price charged by the operators.

Price Charging Step: We update the conflict graph based on the remaining channel demand across the BSs of every operator as illustrated in Fig. 6. Then, we determine the price charged from every operator based on the critical operator. The price charged remains the same as it is obtained for Case 1 (operators reveal their true valuations).

From the above example, it is seen that the deviation from true valuation does not provide utility gain. Thus, operators have no incentive in misreporting the true valuation. Hence, Algorithm 3 is strategy-proof.

As we defined earlier, $\sigma_i^b(k) = \sum_j b_{ij}^k$, where $b_i^k = [b_{i1}^k \dots b_{im_i}^k]$ has the bids at which operator i demands channel at its BSs in k^{th} iteration of allocation.

Lemma 4. *Algorithm 3 is individually rational.*

Proof. As per the assumption, marginal bid per channel decreases with the demand ℓ at any BS i.e., $\mathcal{B}_i(\ell, j) \geq \mathcal{B}_i(\ell', j)$ for $\ell < \ell'$ for all operator i, BS j. Therefore, each operator bid is non-increasing sequentially in the allocation process i.e., $\sigma_i^b(k) \geq \sigma_i^b(k')$ for $k < k'$, where k denotes channel allocation iteration.

Operator i with maximum bid gets channel in each iteration. As stated in Algorithm 3, updated graph $\mathcal{G}^f$ comprises BSs $\{s | s \in S_y^f, \forall y\}$. Operator i is charged as $\sigma_i^c = \max_{y \neq i} \chi_y^i$. As proved in Lemma (2), $\sigma_i^c(k) \leq \sigma_i^b(k)$, for all k. But, in Algorithm 3, σ_i^c is determined from $\mathcal{G}^f$. Therefore, $\sigma_i^c \leq \sigma_i^c(k), \forall k$.

Let x_i^f denotes the final allocation vector for operator i. We denote the sum of the channel bids corresponding to allocation vector x_i^f is α_i^b. Thus, $\alpha_i^b = \sum_{j=1}^{m_i} \sum_{\ell=1}^{x_{ij}^f} \mathcal{B}_i(\ell, j)$. Moreover, $\alpha_i^b \geq \sigma_i^b(1)$, where $\sigma_i^b(1)$ is operator i bid for first channel. As we know $\sigma_i^c < \sigma_i^b$, therefore $\sigma_i^c < \alpha_i^b$. Now, the price charged is given by

$$\begin{aligned} p_i &= \alpha_i^b - \sigma_i^c, \\ &\leq \alpha_i^b. \qquad (\because 0 \leq \sigma_i^c \leq \alpha_i^b). \end{aligned}$$

Thus, $0 \leq p_i \leq \alpha_i^b$. This proves that Algorithm 3 is individually rational. □

Lemma 5. *In Algorithm 3, suppose final allocation vectors of operator i are x_i^f and $\tilde{x}_i^f$ at bids $(\mathcal{B}_i, \mathcal{B}_{-i})$ and $(\tilde{\mathcal{B}}_i, \mathcal{B}_{-i})$, respectively. If there exists some BS j such that $\tilde{\mathcal{B}}_i(\ell, j) > \mathcal{B}_i(\ell, j) \quad \forall \ell$, then $\tilde{x}_i^f - x_i^f \geq 0$. This implies that the number of channels allocated across the BSs of operator i at $\tilde{\mathcal{B}}_i$ are atleast equal to the number of channels allocated at $\mathcal{B}_i$.*

Proof. As per the assumption in Section V, $\tilde{\mathcal{B}}_i(\ell, j) \geq \tilde{\mathcal{B}}_i(\ell', j)$ such that $\ell < \ell'$ for all BSs j. Let operator i be allocated channels in k iterations in the allocation process at $\mathcal{B}_i$. As channel allocation is performed greedily based on the bid, with a bid $\tilde{\mathcal{B}}_i \geq \mathcal{B}_i$, it must be allocated at least k iterations. Since $\mathcal{B}_{-i}$ is unchanged, operator i may get a channel in more than k iterations, if increase in bid results in $\sigma_i^b > \sigma_y^b$, for $y \neq i$ in more iterations in the allocation process. □

Theorem 6. *Algorithm 3 is weakly strategy-proof .*

Proof. Refer Appendix B. □

VI. Simulation Results

In this section, we evaluate the performance of the proposed algorithms in multi-operator settings in a wireless network. In the simulations, we consider 3 operators providing services in a region. We model the wireless network by creating conflict graphs $\mathcal{G} = (V, \mathcal{E})$ using the configuration model [30]. To create an overall topology of the wireless network in a given region, we first generate three conflict graphs $\mathcal{G}_{12}$, $\mathcal{G}_{13}$, $\mathcal{G}_{23}$. Here, $\mathcal{G}_{iy}$ represents a conflict graph among the BSs of operators i and y. Using the conflict graphs, we obtain corresponding binary interference matrices $\mathcal{I}_{12}$, $\mathcal{I}_{13}$ and $\mathcal{I}_{23}$, where $\mathcal{I}_{iy}$ represents the interference among the BSs of operator i and operator y. In an interference matrix, 1 indicates interfering pair of BSs. Further, we obtain interference matrices $\mathcal{I}_{yi}$ from the transpose of the matrix $\mathcal{I}_{iy}$. The overall interference matrix $\mathcal{I}$ of wireless access network in the region is obtained using $\mathcal{I}_{12}$, $\mathcal{I}_{13}$, $\mathcal{I}_{23}$, $\mathcal{I}_{21}$, $\mathcal{I}_{31}$, and $\mathcal{I}_{32}$. We perform Monte Carlo simulations for various scenarios. All the results are obtained by averaging over 50 different topologies. The simulations are performed in MATLAB [14]. We evaluate the performance of the algorithms based on Spectrum Utilization and Social Welfare mentioned in Definitions 2 and 3, respectively.

We compare the proposed algorithms with VCG [6], SMALL [11] and SPECIAL [28] mechanisms. As discussed earlier, VCG mechanism chooses an allocation with the highest social welfare (optimal) from the set of all the feasible allocations. SMALL groups the non-conflicting BSs together and determines the group valuation for each group. The group valuation is obtained as the number of BSs with the bid greater than the minimum bid of the group times the minimum bid. Channel is allocated to the highest bidding group and all the BSs except the one with minimum bid are charged with the minimum bid in the group. SPECIAL considers that the demand at each base station is equal to the number of channels available for auction along with the flexible biding at the base stations described in Section V.

A. Performance evaluation for Single Channel

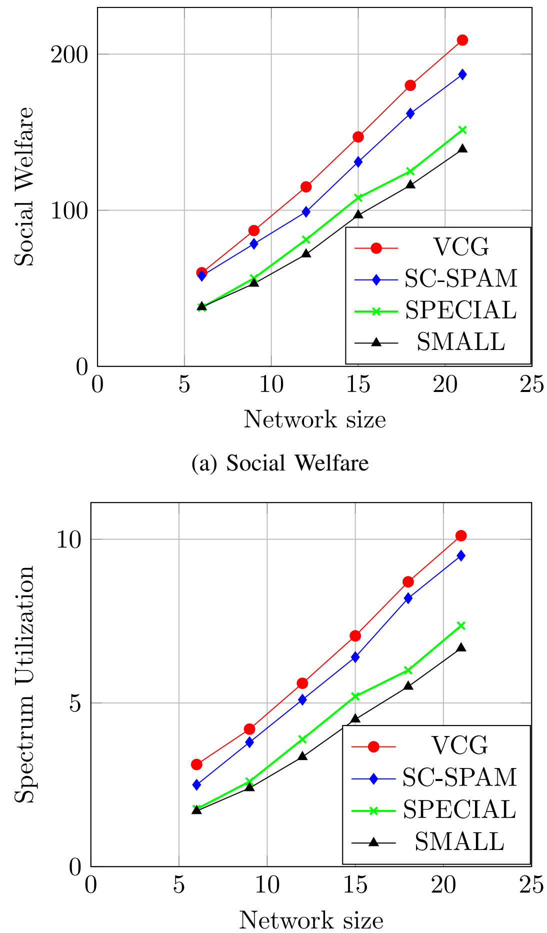
1) Social Welfare and Spectrum Utilization: The bids across the BSs are uniformly distributed in the interval $[15, 25]$ for each operator. As VCG becomes computationally intractable for large networks, we restrict our simulations to small size networks which vary from 6 to 21 BSs. In this case, a single channel is available in the spectrum database. In Fig. 7, we observe that the social welfare and the spectrum utilization of SC-SPAM are close to the optimal obtained from VCG. However, SC-SPAM outperforms SMALL and SPECIAL both in spectrum utilization and social welfare.

2) Execution time: We evaluate the performance of various algorithms based on their required execution times for allocation illustrated in Fig. 8. We observe that the proposed mechanism (SC-SPAM) outperforms significantly. Furthermore, results justify the exponential increase in the execution time of VCG with an increase in the number of BSs. Although VCG provides optimal social welfare, the computational complexity makes it infeasible for resource allocation in moderate size $(10-15$ BSs) network.

Another important observation is that for VCG, SPECIAL and SMALL algorithms execution time varies significantly even with the small increase of base stations in the network. However, the execution time of SC-SPAM does not vary significantly with the increase of base stations in the network. With the least execution time of SC-SPAM among the algorithms, SC-SPAM is the best candidate for resource allocation in real-time implementation.

B. Performance evaluation for Multiple Channels

1) Social Welfare and Spectrum utilization: Next, we compare the performance of the proposed mechanism with SMALL [11] in large networks with the number of BSs ranging from 30 to 300. We consider that 2 channels are available in the spectrum database. Each BS has a demand of 2 channels for all the operators. Each operator submits a per channel bid vector at every base station. The operators choose bids uniformly between $[10, 25]$. From Fig. 9, we observe that the performance of the proposed mechanism for multiple channel allocation is better than that of SMALL and SPECIAL. Here, spectrum utilization is determined as the total number of channels allocated across the BSs of all the operators. The trend observed justifies the following facts: First, SMALL sacrifices the BSs with minimum bid to achieve strategy-proofness, resulting in lower social welfare. Second, BSs only in winning groups are allocated channel, even though there may be some BSs which do not conflict with the winning BSs. Furthermore, it is seen that the performance of SMALL



(a) Social Welfare

(b) Spectrum utilization

Fig. 7: Performance comparison of the VCG, SC-SPAM, SPECIAL and SMALL in three operator scenario.

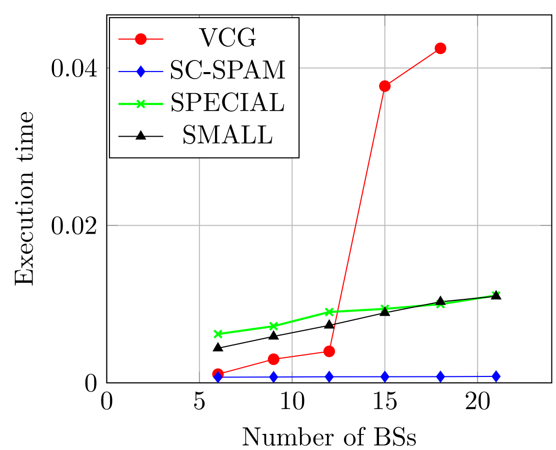
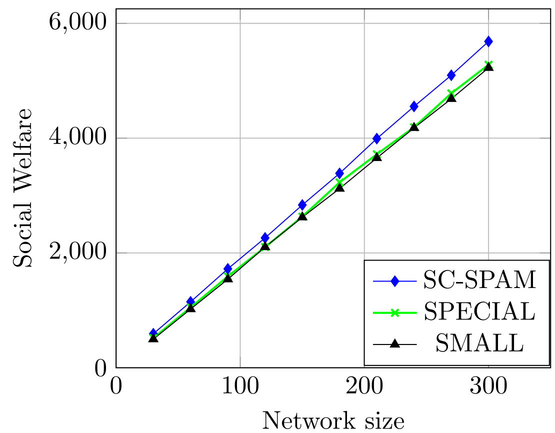
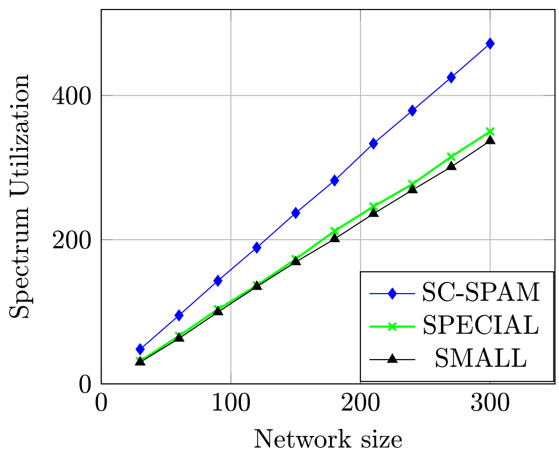


Fig. 8: Comparison of execution times of various algorithms.

and SPECIAL degrades with an increase in the number of BSs in the region.



(a) Social Welfare



(b) Spectrum utilization

Fig. 9: Performance comparison for uniform demand $d = 2$ across the BSs of multiple operators, with linearly increasing bid with demand at each base station.

2) Percentage of BSs allocated resource: In Fig.10, we compare the performance of various algorithms based on the percentage of base stations allocated at least one channel in resource allocation process. The simulations are performed considering both linearly increasing bid per channel and flexible bidding at the base stations. A variation of SC-SPAM with multiple channel availability and uniform demand across base stations is considered in SC-SPAM(NLB). Furthermore, evaluation of SC-SPAM(NLB) is performed considering the flexible bid at base stations. It is observed that the flexible bidding with non-uniform demand across the base station scenario (NUD-WSPAM) outperforms all the other scenarios of resource allocation. The intuition behind the observation is that in NUD-SPAM, some base stations may drop out in subsequent channel allocations when the demand is satisfied.

3) Spectrum Utilization vs. Channels in NUD-WSPAM: We consider channel demand at any BS to be a function of the traffic in the cell. The demand at any BS is uniformly distributed in the interval $[0, 3]$. We perform simulations to evaluate the number of channels required to satisfy the demands across the BSs for all operators in the region. In Fig.11, we observe that the number of channels required to fulfill the demand for all the operators shows a similar trend irrespective of the number of BSs. The number of channels required for the wireless network of 150 BSs remains same as that of 300 BSs. The reason for this behavior is that the degree distribution of BSs does not change with the size of the network (number of BSs).

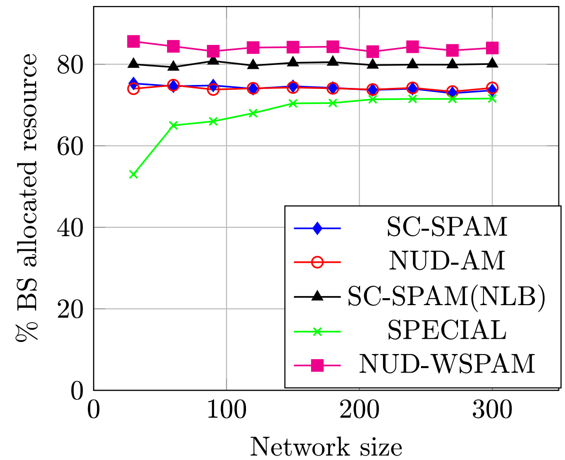


Fig. 10: Comparison of percentage of base stations allocated channel in wireless networks.

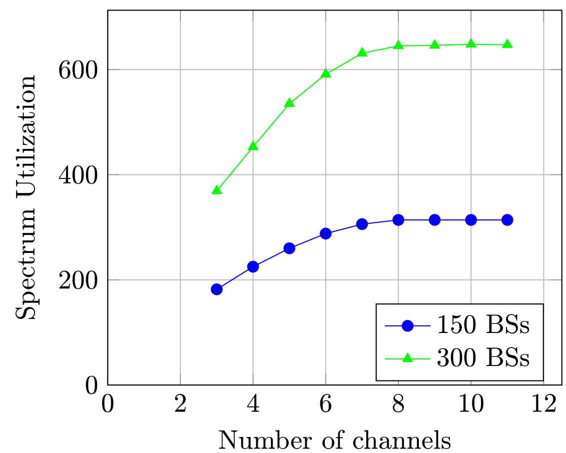


Fig. 11: Comparison of spectrum utilization and number of channels for NUD-WSPAM in large networks.

VII. Conclusions

In this paper, we have investigated the problem of spectrum allocation at the operator level, for multiple existing operators in a region. We consider multiple base stations to be associated with an operator to provide services to the end-users. Therefore, an operator has demand and valuation corresponding to each BS associated with it. To address the issue of multiple valuations at an operator, we have modeled the spectrum allocation problem among non-cooperative operators in a multi-parameter environment to maximize the social welfare of the system. First, we propose a strategy-proof mechanism for single-channel demand across BSs of co-existing operators.

Then we extend it for multiple channels considering non-uniform demand across the BSs of the operators. We prove that the mechanisms SC-SPAM and NUD-WSPAM are guaranteed to be strategy-proof and weakly strategy-proof, respectively. The performances of the proposed algorithms are evaluated using Monte Carlo simulations and compared with those of the other existing mechanisms. The performances of the proposed mechanisms are near-optimal in terms of spectrum utilization and social welfare. Furthermore, the analysis of computational complexity reveals that the proposed mechanisms are implementable in large networks in real-time scenarios. Thus, the proposed mechanisms solve the issue of intractability arising in VCG mechanism.

Appendix

A. Proof of Theorem 3

To show the strategy-proofness of the algorithm, possible scenarios can be divided into two categories:

Scenario 1 : A operator i tries to deviate from truthfulness by bidding greater than the true valuation, i.e., $\sigma_i^b > \sigma_i^v$.

Case (i): Operator i does not win the channel even after bidding untruthfully at σ_i^b, greater than σ_i^v. Hence, it will have utility, $\mathcal{U}_i = 0$.

Case (ii): Operator i wins the channel at its bidding valuation σ_i^b (which is greater than the true valuation) as well as its true valuation σ_i^v. It will have positive utility, $\mathcal{U}_i = \sigma_i^v - p_i$, which is same as in the case operator bids at the true valuation. Thus, bidding at higher valuation does not lead to any extra incentive.

Case (iii): Operator i wins channel at σ_i^b, but looses at σ_i^v. Here, Operator i gets channel on higher bid (by misreporting) which is greater than its critical operator bid (Algorithm 1). But, it has to pay higher price which results in negative utility.

$$\begin{aligned}\mathcal{U}_i &= \sigma_i^v - p_i,\\ &= \sigma_i^v - \sigma_i^c \quad \text{where } p_i = \sigma_i^c,\\ &\leq 0. \quad (\because \sigma_i^v < \sigma_i^c).\end{aligned}$$

Scenario 2 : Operator i tries to deviate from truthfulness by bidding less than the true valuation, i.e., $\sigma_i^b < \sigma_i^v$.

Case (i): Operator i looses the channel at σ_i^b as well as its true valuation, σ_i^v. Thus, it will have $\mathcal{U}_i = 0$.

Case (ii): Operator i wins the channel at σ_i^b as well as its true valuation, σ_i^v which follows from monotonicity. Thus, it will have $\mathcal{U}_i = \sigma_i^v - p_i$.

Case (iii) : Operator i looses at σ_i^b, but wins bidding at σ_i^v. Thus, the operator suffers loss by deviating to untruthful value with zero utility. However, bidding at σ_i^v results in channel allocation to operator i with non-negative utility $\mathcal{U}_i = \sigma_i^v - p_i$.

From the above scenarios, it can be seen that bidding at $\sigma_i^b \neq \sigma_i^v$, does not improve the utility of operator. Thus, $\sigma_i^b = \sigma_i^v$ is the *weakly dominant strategy* for operator i. This completes the proof.

B. Proof of Theorem 6

To prove the strategy-proofness, we are required to show that the deviation from the true valuation for any operator can never increase the utility. We consider two scenarios: (1) if an operator bids at a value higher than the true value, and (2) if an operator bids at a value less than the true valuation.

Let σ_i^{t} denote the sum of the true valuations at the BSs of operator i for the allocated channels.

Let β_i^{t} denote the sum of the bids of the channels allocated across the BSs of operator i.

Critical valuation, utility and final conflict graph at β_i^{t} are denoted as $\tilde{\sigma}_i^c$, $\tilde{U}_i$ and $\tilde{\mathcal{G}}^f$, respectively. Let x_i^f and $\tilde{x_i}^f$ denote the final allocation vector of operator i with bids σ_i^{t} and β_i^{t}, respectively. Further, we define $\tilde{x_i}^f > x_i^f$, if $\exists$ at least a BS ℓ such that $\tilde{x_i}^f(\ell) > x_i^f(\ell)$.

Scenario 1 : The operator bid is more than the true valuation of the channels allocated at its BSs, $\sigma_i^{\mathrm{t}} < \beta_i^{\mathrm{t}}$. Here, again we may have following cases:

Case (i): Final allocation vector for all operators remains unchanged i.e., $\tilde{x_i}^f = x_i^f$, $\forall\ i$. Therefore, $C(i)$ and σ_i^c for operator i remains the same even at β_i^{t}. Hence, operator utility $\mathcal{U}_i$ remains the same.

Case (ii): Operator i is allocated more number channels i.e., $\tilde{x_i}^f > x_i^f$. Since supply is limited, number of channels allocated to some operators other than i decreases i.e., $\tilde{x_y}^f < x_y^f$ such that $y \neq i$. Let us say, operator i is allocated extra channels in iteration k. Then, $\sigma_i^b(k) > \sigma_y^b(k) > \sigma_i^v(k)$ for $y \neq i$. However, at true value unsatisfied BSs of operator i are not allocated channel and are present in $\mathcal{G}^f$. Due to untruthful bidding of operator i, $\tilde{\mathcal{G}}^f$ comprise of the BSs of operator y with higher aggregate true valuation. Therefore, $\tilde{\sigma}_i^c > \sigma_i^c$, this implies $\tilde{\mathcal{U}}_i < \mathcal{U}_i$. Hence, deviation from true value does not increases utility of operator i.

Case (iii): Operator i is allocated less number channels i.e., $\tilde{x_i}^f < x_i^f$. This is not possible due to monotonicity (Lemma 5).

Scenario 2 : The operator bid is less than the true valuation of the channels allocated at its BSs, i.e., $\sigma_i^{\mathrm{t}} > \beta_i^{\mathrm{t}}$.

Case (i): The number of channels allocated across the BSs and the final allocation vector remains unchanged.

With the similar argument as in Case (a) of Scenario 1. The utility of the operator i does not change.

Case (ii): Operator i is allocated more number channels i.e., $\tilde{x_i}^f > x_i^f$. This is not possible due to monotonicity (see Lemma 5).

Case (iii): Operator i is allocated less number channels i.e., $\tilde{x_i}^f < x_i^f$. As the number of channels allocated decrease on deviation from the σ_i^t, operator i suffers loss.

Thus, we establish that the deviation from true valuation does not lead to utility gain. Therefore, the proposed algorithm is weakly strategy-proof.

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