Strategy-Proof Spectrum Auction Mechanism for Time-varying Demands

Indu Yadav*, Prasanna Chaporkar*, and Abhay Karandikar*†

*Department of Electrical Engineering, Indian Institute Technology Bombay, India 400076
Email: {indu, chaporkar, karandi}@ee.iitb.ac.in
†Director and Professor, Indian Institute Technology Kanpur, India 208016
Email: karandi@iitk.ac.in

***Abstract*—To support the exponentially increasing data requirements of users, efficient use of scarce and limited spectrum is paramount. One of the possible solutions is to allocate the spectrum dynamically for short duration with the consideration of temporal variations in the wireless network load. We consider a spectrum allocation for multiple Base Stations (BSs), where their respective resource requirements are time varying in nature. The resource allocation has to be done while maintaining the spectrum reuse constraints for guaranteed Quality of Service (QoS) requirements in the network. We know that the socially optimal resource allocation is NP-hard and hence, computationally infeasible even for modest network sizes. Further, we formulate the problem within the sealed bid auction framework with the aim of maximizing the social welfare. We propose an efficient strategy-proof spectrum allocation mechanism. Moreover, using Monte Carlo simulations, we show that the proposed algorithm can be implemented in large networks. It not only has near-optimal social welfare, but also provides fair allocation in spite of some base stations constantly bidding much higher values than others.**

I. Introduction

Spectrum is a limited and scarce resource. With the surging mobile data traffic, it becomes challenging to support the data requirements of the users in wireless networks [1]. Moreover, the statistics suggest varying traffic density patterns temporally as well as spatially across the network [2]. For instance, while in a regular day residential areas may observe peak traffic density in the evening, business parks or offices experience peak traffic hours around 10-12 a.m. or 2-3 p.m. In the existing network, spectrum allocation is done considering the peak traffic requirements of the network which results in a significant peak to average traffic ratio [3]. During the off-peak hours, the spectrum remains under-utilized, whereas it may not be sufficient at some overloaded base stations (BSs). However, it is observed that the peak traffic density is sporadic and for short intervals across the networks. Fortunately, the hourly traffic is fairly predictable [3], [4]. Note that the static allocation of the spectrum does not address the dynamic nature of traffic across the network [5]. Hence, the static allocation of spectrum is highly inefficient, and it becomes important to find other alternatives.

In this paper, we consider a scenario in which BSs are allocated static resource to support average traffic, and extra resource for supporting peak traffic can be obtained through auctions. The auctioned spectrum can be white spaces, unlicensed band or even certain band reserved for short duration allocation. Our aim here is to propose a strategy-proof spectrum auction that is computationally efficient and has desirable properties like near-optimal social welfare and fairness. Recently, Dynamic Spectrum Allocation (DSA) has emerged as a potential solution for efficient spectrum utilization [6]. The DSA facilitates flexible spectrum allocation using auctions [7]. Most of the mechanisms are designed to fulfill the worst case requirements, i.e., assuming each BS has spectrum demand for the entire duration of the auction.

The practical feasibility of the auctions makes it an attractive choice in devising strategies for distributing spectrum among the base stations (BSs) [7]. However, designing auction mechanisms have their own challenges. In auctions, buyers provide their valuations for the object to the auctioneer. Based on the valuations, auctioneer distributes the resource among the buyers. Since the buyers are selfish and want to maximize their own utility, they tend to manipulate the valuation to seek benefit. Hence, strategy-proofness/truthfulness is a key challenge in auction design [8]. Strategy-proof auctions ensure that all the bidders report their true valuation. For this, a mechanism to be designed such that no bidder has an incentive to misreport the valuation.

Another challenge specific to spectrum auctions is efficient channel utilization. Spatial and temporal re-usability of the channels makes the spectrum auction different from conventional auctions in practice. Due to spatial re-usability, the same spectrum (channel) can be reused for non-overlapping coverage areas. The classical Vickrey-Clarke-Groves (VCG) auction ensures strategy-proof spectrum allocation [9]–[11]. Although VCG provides optimal social welfare, its prohibitive computational complexity leaves it infeasible in real networks. To address this issue, greedy [12] and SMALL [13] which are computationally efficient and strategy-proof spectrum allocation mechanisms, are proposed in the literature. While in greedy allocation some BSs may starve for spectrum due to unfair allocation across time, SMALL suffers from low spectrum utilization and social welfare. In [14] authors propose TRUST, which is a strategy-proof double auction spectrum allocation mechanism. Authors in [15], propose strategy-proof spectrum allocation mechanism, which ensures certain fairness of spectrum allocation among the BSs in the network. DSA auction mechanisms for spectrum allocation are presented in

[16]–[19]. However, [18], [19] do not guarantee to be strategy-proof. Authors in [20], propose strategy-proof spectrum allocation mechanism for adaptive channel width. None of the works mentioned consider temporal spectrum re-usability.

In general, spectrum auction mechanisms are focused on maximizing the social welfare to maximize the benefit of the end users. Auctions are performed repeatedly for dynamic spectrum allocation. Therefore, achieving some fairness is necessary, otherwise continuous starvation of some base stations may lead to monopoly. In the long term, monopoly is undesirable for end users as it removes the competition. We know that determining the optimal spectrum allocation is not only NP-Hard [21], but it is also computationally infeasible. *In this paper, we aim to propose a strategy-proof and computationally efficient auction-based spectrum allocation mechanism considering the time-varying traffic demands at each BS in the duration of the auction. We show that the proposed mechanism achieves efficient spectrum utilization and obtains near-optimal social welfare of the auction, in addition to fair allocation.*

Most of the spectrum auction mechanisms presented in literature focus on maximizing the social welfare of the auction considering only the spatial re-usability of the spectrum [12], [13], [14], [15]. However, in our work, we consider spatial and temporal variations of traffic during the auction. In our mechanism, we incorporate the temporal variations of the spectrum requirement at a BS for short duration instead of the total auction duration. Only mechanism [15] ensures the fairness in channel allocation among the BSs. However, our mechanism ensures fair channel allocation along with maximizing social welfare and spectrum utilization. Our main contributions in the paper are as follows:

- We study the problem of the spectrum allocation among multiple BSs considering spatial and temporal variations in traffic, which is a NP-Hard problem.
- We propose a computationally efficient strategy-proof sealed-bid auction mechanism for spectrum allocation among the BSs.
- Using simulations, we demonstrate that the performance of proposed mechanism is near-optimal in terms of social welfare and spectrum utilization. In comparison to other work, the proposed mechanism guarantees fair spectrum allocation in long term among the base stations.

The rest of the paper is organized as follows. The system model and problem formulation are described in Section II. In Section III, the spectrum allocation mechanism is proposed, and strategy-proofness of the mechanism is illustrated. We provide an illustrative example of the proposed mechanism and other existing mechanisms in Section IV. Simulation results are provided in Section V. In Section VI, we discuss the possible extensions of the problem and finally, we conclude our work in Section VII.

II. System Model

We consider a spectrum allocation framework comprising of an auctioneer, spectrum database and a set of base stations. The auctioneer is responsible for spectrum allocation. The spectrum database contains the information about the spectrum available for allocation. Generally, spectrum is divided into multiple channels. For the sake of simplicity, we assume only one channel is available in the spectrum database. Let $\mathscr{N} = \{1, 2, \ldots, n\}$ denote the set of base stations. We assume that base stations are selfish, rational and do not collude. We consider a spectrum allocation problem where spectrum is allocated dynamically based on the demand. We assume that the sealed-bid spectrum auction is performed for a specific duration. The duration is adjustable depending on the spatio-temporal variations in the traffic of the network.

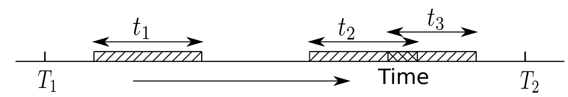


Fig. 1: Illustration of auction duration $[T_1, T_2]$ and demand time intervals t_1, t_2 and t_3 corresponding to base stations 1, 2 and 3, respectively.

We consider that the auctioneer broadcasts the duration $[T_1, T_2]$ for spectrum auction to the base stations. To exploit the re-usability aspect of the spectrum auctions, auctioneer requests the BSs to report both the spectrum demand slot and their valuation. Fig.1 illustrates the auction duration $[T_1, T_2]$ and the demand time intervals t_1, t_2 and t_3 for BSs 1, 2 and 3, respectively. Each BS i communicates the spectrum demand interval, $t_i = [t_{si}, t_{ei}]$, where $t_{si} \leq t_{ei}$; $T_1 \leq t_{si}, t_{ei} \leq T_2$. Here, t_{si} and t_{ei} denote start and end time of the demand interval for user i. Thus, t_i may denote the peak traffic interval for BS i in the interval $[T_1, T_2]$. Let b_i and v_i denote the bid value and the true valuation, respectively, of user i in the demand interval t_i. In sealed-bid auctions, $v_i(t)$ is private information of any BS i. Since BSs are selfish, they may deviate from the true value if doing so results in a utility gain, i.e., b_i need not equal v_i. We assume that a BS i can report any bid $b_i > 0$. The bids in a network consisting of n BSs is denoted as n-tuple, $\boldsymbol{b} = (b_1, \ldots, b_n)$. Therefore, the action space or possible set of bid values is $\boldsymbol{b} \in \mathscr{R}_+^n$. We also define $\boldsymbol{b}_{-i} \in \mathscr{R}_+^{n-1}$ as the bid valuation of all the BSs except i.

If two nearby BSs transmit simultaneously, then they cause interference to each other and this may lead to the unacceptable degradation in the desired quality of service (QoS). Thus, to meet the QoS requirements, spectrum must not be allotted to the BSs in close proximity. We model this through conflict graph $\mathscr{G}' = (V, E')$, where nodes denote BSs and edges denote an interfering pair of BSs in the network. The graph $\mathscr{G}'$ is an undirected graph. Note that if $(i, j) \in E'$, then the same spectrum can not be allocated to the BSs i and j at the same time. Let $\mathscr{S}'_i$ denotes the set of neighbors for node i in $\mathscr{G}'$. Observe that if $j \in \mathscr{S}'_i$, then $i \in \mathscr{S}'_j$. The conflict graph $\mathscr{G}'$ captures the constraints on spectrum allocation. Next, we define the set of feasible allocations.

Let a binary vector $\boldsymbol{x} = (x_1, \ldots, x_n)$ denote allocation vector. Here, $x_i = 1$ signifies base station i is allocated channel in the interval t_i.

Definition 1. *A vector $\boldsymbol{x}$ is a feasible channel allocation if $t_i \cap t_j \neq \phi$ for any $j \in \mathscr{S}_i'$ then $x_i + x_j \leq 1$. Allocation vector $\bar{\boldsymbol{x}}$ is said to be maximal if for every j such that $x_j = 0$, there exists $i \in \mathscr{S}_j'$ satisfying $t_i \cap t_j = \phi$ and $x_i = 1$.*

Note that any feasible $\boldsymbol{x}$ does not allocate spectrum simultaneously to the conflicting BSs. Let $\mathscr{X}$ denote the set of all feasible channel allocations. Now, we define auction based spectrum allocation mechanism.

Definition 2. *An auction based spectrum allocation mechanism π is a map from $\mathscr{R}_+^n$ to $\mathscr{X} \times [0,\infty)^n$, i.e., for given bids $\boldsymbol{b}$, π outputs a feasible allocation $\boldsymbol{x}^\pi(\boldsymbol{b})$ and a price vector $\boldsymbol{p}^\pi(\boldsymbol{b}) = (p_1^\pi(\boldsymbol{b}), \ldots, p_n^\pi(\boldsymbol{b}))$.*

Thus, a spectrum allocation mechanism π outputs a feasible channel allocation for any given bid vector $\boldsymbol{b}$, and also the price that each BS needs to pay for the allocated channel. Let Π denote the set of all auction based allocation policies.

Definition 3. *Social welfare under mechanism π for bid values $\boldsymbol{b}$ is defined as $W_s^\pi(\boldsymbol{b}) = \sum_{i=1}^n v_i x_i^\pi(\boldsymbol{b})$. Moreover, utility for BS i for bids $\boldsymbol{b}$ under π is given as $U_i^\pi(\boldsymbol{b}) = (v_i - p_i^\pi(\boldsymbol{b}))x_i^\pi(\boldsymbol{b})$.*

Note that the social welfare is the sum of true evaluations v_i's, not the bid values b_i's reported by the BSs to mechanism π. Moreover, utility for a BS is the difference between its true evaluation v_i and price p_i charged under the mechanism π. The aim of the spectrum auctioneer is to design a mechanism π that maximizes social welfare, i.e., it needs to evaluate

$$\pi^\star \in \arg\max_{\pi \in \Pi} W_s^\pi(\boldsymbol{b}), \tag{1}$$

while each BS wants to bid so as to maximize its own utility. Note that $\boldsymbol{v}$ is the private information of the base stations, and the spectrum auctioneer may not know it. Therefore, we need to design a mechanism in which a rational BSs do not have any incentive to submit bid other than their true evaluations.

Definition 4. *A mechanism π is truthful (strategy-proof) if*

$$U_i^\pi(v_i, \boldsymbol{b}_{-i}) \geq U_i^\pi(b_i, \boldsymbol{b}_{-i}), \text{ for all } (b_i, \boldsymbol{b}_{-i}).$$

Note that for a strategy-proof mechanism π, a BS has no incentive to bid anything other than its true evaluation. Thus, for a strategy proof mechanism the social welfare is equal to the sum of utilities of individual user utilities. The well known VCG auctions are strategy-proof and maximize the social welfare. However, in the proposed setup, computing VCG allocation is NP-hard. Hence, we need a mechanism that is computationally feasible and achieves near-optimal social welfare. We describe our proposed scheme next.

III. Algorithm

In this section, we propose an efficient, strategy-proof spectrum auction mechanism for time-varying demand intervals/slots during the auction period. The proposed mechanism consists of a channel assignment strategy and a pricing scheme. Each base station is charged based on the pricing scheme to ensure strategy-proofness.

Algorithm 1 Pseudo code for randomized conflict-free grouping

Input: $\mathscr{N}$, $\mathscr{S}_i$ for every $i \in \mathscr{N}$

Output: A conflict-free partition $\{G_1, \ldots, G_\eta\}$

1: Initialize $\mathscr{N}_{\text{temp}} = \mathscr{N}$ and $G_u = \phi$, $\forall\ u = 1, \ldots, \eta$
2: **while** $\mathscr{N}_{\text{temp}} \neq \phi$ **do**
3: Choose i, from $\mathscr{N}_{\text{temp}}$ uniformly w.p. $\frac{1}{|\mathscr{N}_{\text{temp}}|}$
4: Find $u_{\min} = \min\{u : u \in \{1, \ldots, \eta\}$ and $G_u \cap \mathscr{S}_i = \phi\}$
5: $G_{u_{\min}} \leftarrow G_{u_{\min}} \cup \{i\}$
6: $\mathscr{N}_{\text{temp}} \leftarrow \mathscr{N}_{\text{temp}} \setminus \{i\}$
7: **end while**

A. Channel Allocation Strategy

As described in the system model, the BSs may demand the spectrum only for specific time slots during the auction. The channel allocation strategy determines allocation vector $\boldsymbol{x}^\star \in \mathscr{X}$ for the desired goal of the auction. We first, construct the modified interference graph based on the demand slots of the users. The interference graph $\mathscr{G} = (V, E)$ contains $\mathscr{N}$ base stations, but the interfering pair of base stations are determined based on the demand slots of each BS as follows: an undirected edge $(i,j) \in E$ if $(i,j) \in E'$ and $t_i \cap t_j \neq \phi$. Define, $\mathscr{S}_i$ to be the set of neighbors of i in $\mathscr{G}$.

Lemma 1. *If $\boldsymbol{x} \in \mathscr{X}$, then $x_i + x_j \leq 1$ for all i and $j \in \mathscr{S}_i$.*

As discussed above, the channel allocation phase determines which BSs should be allocated the channel, i.e., we select $\boldsymbol{x} \in \mathscr{X}$. Our first step is to partition the set of all BSs $\mathscr{N}$ into at most η non-conflicting groups denoted as $\{G_1, \ldots, G_\eta\}$, where $\eta = \max_{i \in \mathscr{N}} |\mathscr{S}_i| + 1$ [22]. Here, $|\mathscr{A}|$ denotes the cardinality of set $\mathscr{A}$. The partitioning is achieved using an iterative greedy algorithm. In the first iteration a BS is selected at random and put in group G_1. In further iterations, a BS i is picked at random from $\mathscr{N} \setminus \cup_{k=1}^{\eta} G_k$ and placed in the group $G_{u_{\min}}$ such that $u_{\min} = \min\{u : u \in \{1, \ldots, \eta\}$ and $G_u \cap \mathscr{S}_i = \phi\}$. We continue this process until $\cup_{k=1}^{\eta} G_k = \mathscr{N}$. Pseudo code for the randomized conflict-free grouping is provided in Algorithm 1. Following lemma summarizes key properties of the partitioning step.

Lemma 2. *A conflict-free grouping algorithm given in Algorithm 1 outputs a partition $\{G_1, \ldots, G_\eta\}$ of $\mathscr{N}$ such that if $i, j \in G_u$, then $j \notin \mathscr{S}_i$.*

Proof. We need to show that the RHS in Step 4 of the algorithm is a non-empty set in every iteration. Rest follows immediately from the set construction. Required follows from the fact maximum cardinality of any $\mathscr{S}_i$ is $\eta - 1$. Thus, there exist at least one u such that $G_u \cap \mathscr{S}_i = \phi$. ☐

Lemma 2 states that the channel can be allocated to all the members of any group G_u without violating the allocation constraint. Moreover, it is important to note that the grouping does not depend on the bid values $\boldsymbol{b}$.

Now, let Ω_g denote the set of all possible orderings of the sets $\{G_1, \ldots, G_\eta\}$ obtained using conflict-free grouping

Algorithm 2 Pseudo code for channel allocation for given group ordering ω_j

Input: G_{ju} for every $1 \leq u \leq \eta$, $\mathscr{S}_i$ for every $i \in \mathscr{N}$
Output: A channel allocation $\boldsymbol{x}(j)$

```
1: Initialize G_temp = φ, ℓ = 1 and x_i(j) = 0 for all i ∈ 𝒩
2: while ℓ ≤ η do
3:     G_a ← G_jℓ \ (∪_{i∈G_temp} 𝒮_i)
4:     x_i(j) ← 1 for every i ∈ G_a
5:     G_temp ← G_temp ∪ G_a
6:     ℓ ← ℓ + 1
7: end while
```

algorithm. Thus, $|\Omega_g| = \eta!$. Furthermore, let $\omega_j \in \Omega_g$ denote the j^{th} ordering of the groups in the set Ω_g. We denote ω_j by a tuple $(G_{j1}, \ldots, G_{j\eta})$. For example if $\eta = 3$, then there are $|\Omega_g| = 3! = 6$ different orderings. One of the possible 6 group ordering or tuple is $\omega_j = (G_2,\ G_1,\ G_3)$. Thus, $G_{j1} = G_2$, $G_{j2} = G_1$ and $G_{j3} = G_3$. Channel allocation in a given group ordering ω_j is done as follows. We first assign the channel to each BS in G_{j1}, then to all the BSs in $G_{j2} \setminus (\cup_{i \in G_{j1}} \mathscr{S}_i)$, and so on. Pseudo-code to obtain channel allocation corresponding to group ordering ω_j is given in Algorithm 2. Following guarantee can be given about output of the algorithm.

Lemma 3. *The channel allocation vector $\boldsymbol{x}(j)$ given by Algorithm 2 corresponding to any group tuple ω_j is feasible, i.e., $\boldsymbol{x}(j) \in \mathscr{X}$. Moreover, $\boldsymbol{x}(j)$ is a maximal allocation vector for every j.*

Proof. Let $\boldsymbol{x}_\ell(j)$ denote the allocation after ℓ iterations of the algorithm. We first show that $\boldsymbol{x}_\ell(j) \in \mathscr{X}$ for every $1 \leq \ell \leq \eta$. Note that for $\ell = 1$, $x_{\ell i} = 1$ only for $i \in G_{j1}$. From Lemma 2, $\boldsymbol{x}_1(j) \in \mathscr{X}$ follows. Suppose $\boldsymbol{x}_\ell(j) \in \mathscr{X}$ holds for every $1 \leq \ell \leq \ell'$. Consider $(\ell'+1)^{\text{th}}$ iteration of the algorithm. Note that the G_{temp} in every iteration contains BSs to which the channel is allocated until that iteration. Note that in Step 5 of the algorithm the channel is allocated only to BSs in $G_{j(\ell'+1)}$ that do not conflict with the BSs in G_{temp}. This proves that $\boldsymbol{x}_{\ell'+1}(j) \in \mathscr{X}$ and the required follows using induction. Now, we prove that the channel allocation is maximal. Suppose not, then there exist a BS u such that $x_{\ell u}(j) = 0$ in the output of the algorithm, but $\boldsymbol{x}'$ such that $x'_i = x_{\ell i}(j)$ for every $i \neq u$ and $x'_u = 1$ is in $\mathscr{X}$. Since, $(G_{j1}, \ldots, G_{j\eta})$ is a partition of $\mathscr{N}$, u must belong to some $G_{j\ell}$. Also, u must not belong to $\mathscr{S}_i$ for any i which is allocated the channel in first $\ell - 1$ iterations of the algorithm. But, then the algorithm will allocate channel to BS u in ℓ^{th} iteration. Hence, no such BS exists. This proves the required. $\square$

Now define, with a little abuse of notation, the perceived social utility under allocation $\boldsymbol{x}(j)$ as

$$\tilde{U}_j(\boldsymbol{b}) = \sum_{i=1}^{n} b_i x_i(j).$$

Moreover, define $j^\star_{\boldsymbol{b}} = \arg\max_{\{j:\omega_j \in \Omega_g\}} \tilde{U}_j(\boldsymbol{b})$. Thus, $\omega_{j^\star}$ is the group permutation for which perceived utility is maximized among all possible group permutations. We choose allocation $\boldsymbol{x}(j^\star_{\boldsymbol{b}})$. Note that even though the grouping does not depend on the bids $\boldsymbol{b}$, the chosen channel allocation does. Let $\tilde{U}^\star(\boldsymbol{b})$ denote the maximum value of the perceived social utility for the bids $\boldsymbol{b}$. Next, we describe our proposed pricing scheme.

B. Pricing Scheme

After the channel allocation, we propose the appropriate pricing scheme which ensures the strategy-proofness of the proposed algorithm. That is, if any base station tries to deviate from its v_i, it is penalized. Let $(\epsilon, \boldsymbol{b}_{-i})$ denote the bid vector in which the bids of all the BSs except i are same as that in $\boldsymbol{b}$, but the bid of BS i is $\epsilon > 0$. Now, the price charged from the BS i is given as:

$$p_i(\boldsymbol{b}) = \left[\lim_{\epsilon \downarrow 0} \tilde{U}^\star(\epsilon, \boldsymbol{b}_{-i}) - (\tilde{U}^\star(\boldsymbol{b}) - b_i)\right] \times x_i(j^\star_{\boldsymbol{b}}). \quad (2)$$

We state the following straightforward result.

Lemma 4. *Under any bid values $\boldsymbol{b} > 0$, $0 \leq p_i \leq b_i$ for every $i \in \mathscr{N}$.*

Proof. Note that for every $\epsilon > 0$,

$$\tilde{U}^\star(\epsilon, \boldsymbol{b}_{-i})) \geq \tilde{U}^\star(\boldsymbol{b}) - b_i + \epsilon.$$

Thus, the proof follows by taking limit $\epsilon \downarrow 0$ on both sides of the above inequality. $\square$

This lemma clearly shows that for any truthful BS i, utility obtained is non-negative, irrespective of the bids of other BSs.

Note that the optimal group permutation under bid vectors $\boldsymbol{b}$ and $(\epsilon, \boldsymbol{b}_{-i})$ can be different. Unlike VCG, in our pricing scheme we do not completely remove BS i, rather BS i is always present. Only the bid value of BS i goes to zero. This distinction is important as removing a BS changes channel allocation conflicts. As illustration consider a system with five BSs with constraint sets given by $\mathscr{S}_1 = \{3, 4, 5\}$, $\mathscr{S}_2 = \phi$, $\mathscr{S}_i = \{1\}$ for $i = 3, 4, 5$. Suppose grouping given by Algorithm 1 is $G_1 = \{1, 2\}$ and $G_2 = \{3, 4, 5\}$. Consider permutation $\omega_1 = (G_1, G_2)$. Thus, as per Algorithm 2, the channel will be first allocated to all the BSs in G_1 and then to the BSs in G_2 that do not have a conflict with the BSs in G_1. Note that when BS 1 is present in the system, the channel cannot be allocated to any BS in G_2. But, if we remove BS 1 completely, then all the BSs in G_2 can get the channel. Thus, there is a clear difference under our scheme regarding the presence and absence of BS. In our pricing scheme, BS is retained in the system while calculation, unlike VCG. Pseudo code for the proposed algorithm is given in Algorithm 3. Next, we prove the key properties of our proposed algorithm.

Lemma 5. *If a base station i is allocated channels for bids $\boldsymbol{b}$, then it will also be allocated channels for bids $(\epsilon, \boldsymbol{b}_{-i})$ for every $\epsilon > b_i$. Moreover, optimal group permutation under $\boldsymbol{b}$ and $(\epsilon, \boldsymbol{b}_{-i})$ are the same, i.e., $j^\star_{\boldsymbol{b}} = j^\star_{(\epsilon, \boldsymbol{b}_{-i})}$.*

Proof. Without loss of generality, let $\epsilon = b_i + \Delta$ for some $\Delta > 0$. Note that since the bid value of only BS i has changed, we can conclude that

$$\tilde{U}_j(\epsilon, \boldsymbol{b}_{-i}) - \tilde{U}_j(\boldsymbol{b}) \leq \Delta, \quad (3)$$

Algorithm 3 Pseudo code for Proposed mechanism

Input: bid vector $\boldsymbol{b}$, $\mathscr{S}_i$ for every $i \in \mathscr{N}$

Output: Resource allocation $\boldsymbol{x}(\boldsymbol{b})$ and price vector $\boldsymbol{p}(\boldsymbol{b})$

1: Use Algorithm 1 to obtain conflict free grouping $(G_1, \ldots, G_\eta)$
2: **for** $\omega_j \in \Omega_g$ **do**
3: Find allocation $\boldsymbol{x}(j)$ using Algorithm 2
4: Compute $\tilde{U}_j(\boldsymbol{b}) = \sum_{i=1}^{n} b_i x_i(j)$
5: **end for**
6: Find $j^\star_{\boldsymbol{b}} = \arg\max_{\{j:\omega_j \in \Omega_g\}} \tilde{U}_j(\boldsymbol{b})$
7: Choose $\boldsymbol{x}(\boldsymbol{b}) = \boldsymbol{x}(j^\star_{\boldsymbol{b}})$
8: Compute prices using (2)

for every group permutation ω_j. Moreover,

$$\tilde{U}_{j^\star_{\boldsymbol{b}}}(\boldsymbol{b}) + \Delta = \tilde{U}_{j^\star_{\boldsymbol{b}}}(\epsilon, \boldsymbol{b}_{-i}), \tag{4}$$

i.e., the perceived social utilities under group permutation $j^\star_{\boldsymbol{b}}$ for bid vectors $\boldsymbol{b}$ and $(\epsilon, \boldsymbol{b}_{-i})$ differ by amount Δ with latter having the larger value. Thus, we can conclude from (3) and (4) that $j^\star_{\boldsymbol{b}}$ is optimal group permutation for $(\epsilon, \boldsymbol{b}_{-i})$ as well. Now, the required follows from Algorithm 2. □

Lemma 5 implies that if a base station unilaterally increases its bid, then it is more likely to get the channels. Next, we prove that our proposed algorithm is strategy proof.

Theorem 1. *Algorithm 3 is strategy-proof.*

Proof. We prove the required by considering two scenarios.
Scenario 1 : BS i bids more than its true valuation, i.e., $b_i > v_i$. Without loss of generality, $b_i = v_i + \Delta$ for some $\Delta > 0$. Bids of the other BSs can be arbitrary. Thus, we compare two bid vectors, viz. $\boldsymbol{b}$ and $(v_i, \boldsymbol{b}_{-i})$, where latter corresponds to BS i bidding truthfully. This scenario is further bifurcated into three cases.

Case (i): BS i gets channel under both bid vectors $\boldsymbol{b}$ and $(v_i, \boldsymbol{b}_{-i})$. By Lemma 5, it follows that the optimal group permutation remains same for both the bid vectors. It follows that the optimal perceived utility values satisfy $\tilde{U}^\star(\boldsymbol{b}) = \tilde{U}^\star(v_i, \boldsymbol{b}_{-i}) + \Delta$. Now, from (2), it follows that $p_i(\boldsymbol{b}) = p_i(v_i, \boldsymbol{b}_{-i})$. Thus the required holds.
Case (ii): BS i does not get the channel under $(v_i, \boldsymbol{b}_{-i})$, but gets it under $\boldsymbol{b}$. Note that utility for BS i under $(v_i, \boldsymbol{b}_{-i})$ is zero as it does not get the channel. Now, we bound BS i utility under $\boldsymbol{b}$. Since the bid for only BS i is different under two bid vectors, we can conclude that

$$\tilde{U}^\star(\boldsymbol{b}) - \tilde{U}^\star(v_i, \boldsymbol{b}_{-i}) \leq \Delta. \tag{5}$$

Now, from (2), it follows that

$$\begin{aligned} p_i(\boldsymbol{b}) &= \lim_{\epsilon \downarrow 0} \tilde{U}^\star(\boldsymbol{b}) - (\tilde{U}^\star(\boldsymbol{b}) - b_i) \\ &= \lim_{\epsilon \downarrow 0} \tilde{U}^\star(\boldsymbol{b}) - (\tilde{U}^\star(\boldsymbol{b}) - v_i) + \Delta \quad (6) \\ &= (\tilde{U}^\star(v_i, \boldsymbol{b}_{-i}) - \tilde{U}^\star(\boldsymbol{b})) + v_i + \Delta \quad (7) \\ &\geq v_i. \quad (8) \end{aligned}$$

Equality (6) follows as $b_i = v_i + \Delta$. Equality (7) follows by Lemma 5. Note that for every ϵ smaller than v_i BS i can not get channel as it can not get it at bid value v_i. Moreover, since only bid for BS i is changing, the optimal perceived social utility remains unchanged. Hence, the limiting value equals maximum perceived social utility for bid $(v_i, \boldsymbol{b}_{-i})$. Finally, (8) follows from (5). Now, (8) implies that the utility for BS i under $\boldsymbol{b}$ can at most be 0, which is same when it bids true valuation v_i. This proves the required.
Case (iii): The BS i neither gets channel at b_i, nor at $b_i + \Delta$. Here, utility for BS i will remain zero.

Scenario 2 : BS i bids less than its true valuation, i.e., $b_i < v_i$. Without loss of generality, $v_i = b_i + \Delta$ for some $\Delta > 0$. Bids of the other BSs can be arbitrary. Thus, we compare two bid vectors, viz. $\boldsymbol{b}$ and $(v_i, \boldsymbol{b}_{-i})$, where latter corresponds to BS i bidding truthfully. This scenario is further bifurcated into three cases.

Case (i): The BS i is allocated channel under $(v_i, \boldsymbol{b}_{-i})$ and also under $\boldsymbol{b}$. Analysis of this case is similar to that in Case (i) of Scenario 1. Again here, it can be shown that the utility for BS remains unchanged, and hence there is no benefit for deviating from true evaluation.

Case (ii): The BS i is allocated channel under $(v_i, \boldsymbol{b}_{-i})$, but it does not get it under $\boldsymbol{b}$. This implies that the BS i has utility $v_i - p_i(v_i, \boldsymbol{b}_{-i})$ for bid vector $(v_i, \boldsymbol{b}_{-i})$, but on deviation its utility becomes zero. Now, the required follows from Lemma 4.

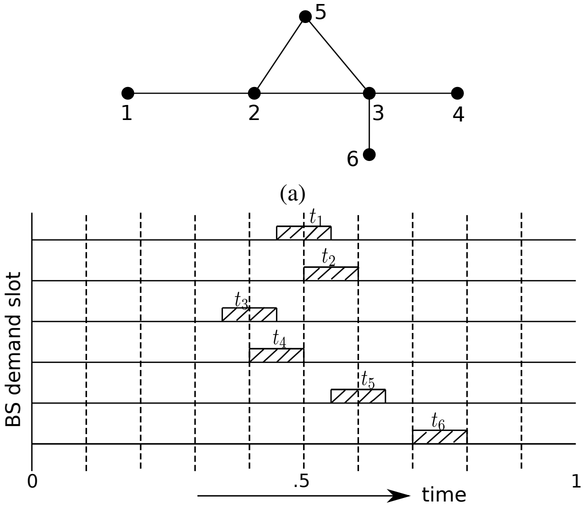
Case (iii): The BS i neither gets a channel at $(v_i, \boldsymbol{b}_{-i})$ nor at $\boldsymbol{b}$. Here, the utility for the BS remains zero. Thus, no incentive on deviation from true value. This completes the proof. □

In the next section, we describe the functioning of the proposed algorithm using example.

IV. Illustrative Example

Example: As illustrated in Fig. 2(a), we consider a wireless network consisting of 6 BSs. The wireless network is represented as a graph. The nodes denote the BSs and edges denote interfering pair of BSs in the graph. We consider that the auctioneer collects the spectrum demand from the BSs apriori for $[T_1, T_2] = [0, 1]$, auction duration. We assume that BSs submit a non-zero bid along with the time slot in the interval $[0, 1]$ for which channel is required, to the auctioneer. Let the bid vector is $\boldsymbol{b} = [9\ \ 10\ \ 8\ \ 7\ \ 5\ \ 7]$. For simplicity of calculations, we assume each BS needs spectrum for $\tau = 0.1$ time unit in the interval of auction. Let the vector $t_{\text{si}} = [0.45\ \ 0.50\ \ 0.35\ \ 0.40\ \ 0.55\ \ 0.70]$ denote the start of the demand time slot corresponding to each BS. In Fig. 2(b), t_i represents the time interval for channel demand corresponding to BS i. For instance, the time interval for BS 1 corresponds to $[0.45,\ 0.55]$.

We know that a channel can be allocated between any pair of interfering BSs in non-overlapping time slots. For efficient spectrum usage, we exploit the temporal variation in the demand across the BSs of the network. In case the



(a)

(b)

Fig. 2: (a) Interference graph (b) Channel demand time slots

channel is required in non-overlapping time slots, the edge joining them can be removed. Therefore, on consideration of the demand time slots, the conflict graph in Fig. 2(a) reduces to conflict graph illustrated in Fig. 3. We briefly describe the channel allocation for the proposed strategy-proof mechanism and compare with the two other strategy-proof mechanisms SMALL [13] and greedy [12].

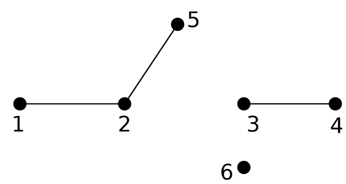


Fig. 3: Re-constructed interference graph considering demand time intervals for each base station.

A. Allocation in Proposed Algorithm

As discussed in the Section III, the proposed algorithm first re-constructs the conflict/interference graph based on the demand time interval of the spectrum for each BS in the network. Then, it performs grouping of the non-conflicting BSs, irrespective of their bids. Let the BSs in the re-constructed interference graph mentioned in Fig. 3 grouped into 2 groups, namely $G_1 = \{1,\ 4,\ 5\}$ and $G_2 = \{2,\ 3,\ 6\}$. These two groups result in two arrangements $[G_1,\ G_2]$ and $[G_2,\ G_1]$.

Based on the bid vector $\boldsymbol{b}$, we determine the social welfare for each arrangement. The social welfare for arrangements $\omega_1 = [G_1, G_2]$ and $\omega_2 = [G_2, G_1]$ are 28 and 26, respectively. Since the arrangement ω_1 has maximum social welfare, the arrangement corresponds to the channel allocation $\boldsymbol{x}^\star(\boldsymbol{b}) = (1, 0, 0, 1, 1, 1)$ (see Algorithm 2). Using the pricing scheme (see 2), the BSs 1, 4, 5 and 6 get channel at the price 6, 4, 2 and 0, respectively. It is important to note that the consideration of demand slots reduces the number of groups to 2, whereas no matter how the BSs are grouped in the network illustrated in Fig. 2(a) the minimum number of groups required is 3. In the example, the number of groups reduction from 3 to 2 decreases the group arrangements from 6 to 2. Thus, the decrease in the number of groups reduces the possible arrangements significantly. In other words, less number of groups will have a smaller set of possible arrangements.

B. Allocation in SMALL

Now, we briefly describe SMALL for the time-varying demand framework. Auction mechanism SMALL also partitions the set of BSs randomly into non-conflicting groups. SMALL determines the group valuation $\sigma(G_j)$ for each group j, where $\sigma(G_j) = (|G_j| - 1) \times \min\{b_j | j \in G_j\}$. Without loss of generality, let the grouping be same. All the BSs of the maximum valuation group are assigned channels except the one with the least bid in the group. The BSs allocated channel pay price equal to the least bid value in the group. In the example considered here, G_1 and G_2 have group valuations 10 and 14, respectively. The BSs in the group G_2 get channel except the one with the least bid value, i.e., BS 6. Hence, the allocation vector and the social welfare comes out to be $\boldsymbol{x}^\star(\boldsymbol{b}) = (0, 1, 1, 0, 0, 0)$ and 14, respectively. The BSs 2 and 3 are charged price 7, individually.

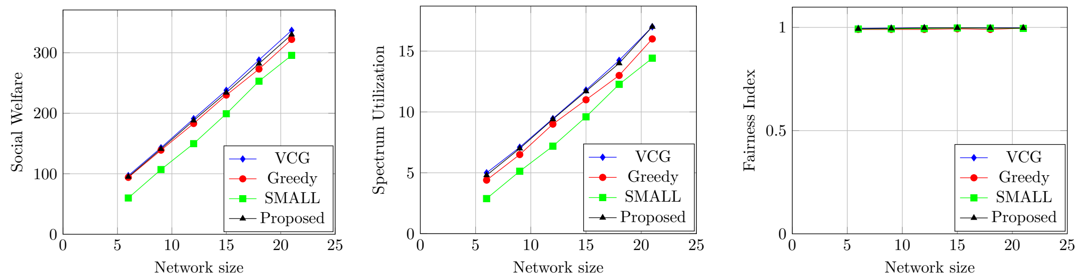
C. Allocation in Greedy

The greedy mechanism allocates channels greedily based on the bids submitted by the BSs. The highest bidding BS is assigned a channel. In the iterative step, the highest bidding user in the set of users which are neither already selected nor are in the constraint set of the selected users is selected. The process continues until no user can be selected. The price for a selected user i is calculated as follows: We remove the user i from the system. Compute the greedy allocation for the remaining $n - 1$ users. Let c be the highest bid value of a selected user in $\mathscr{S}_i$ in the new allocation. Then, the price for i is c. As described in the previous mechanisms, we again re-construct the interference graph based on the demand time intervals. Then, we perform a greedy allocation mechanism as described above. In the example, BSs 2, 3 and 6 are assigned the channel, and the prices charged for these are 9, 7 and 0, respectively. The social welfare achieved is 25.

In the constructed example, the proposed algorithm outperforms SMALL and greedy mechanisms in terms of social welfare. Social welfare of greedy allocation is close to the proposed algorithm, while SMALL achieves much lesser value. Moreover, the proposed algorithm and greedy provide maximal allocation, while SMALL has poor resource utilization. Though the proposed algorithm performs on par with existing schemes in the constructed example, to understand the performance comparison of these schemes we perform Monte Carlo simulations as described in the following section.

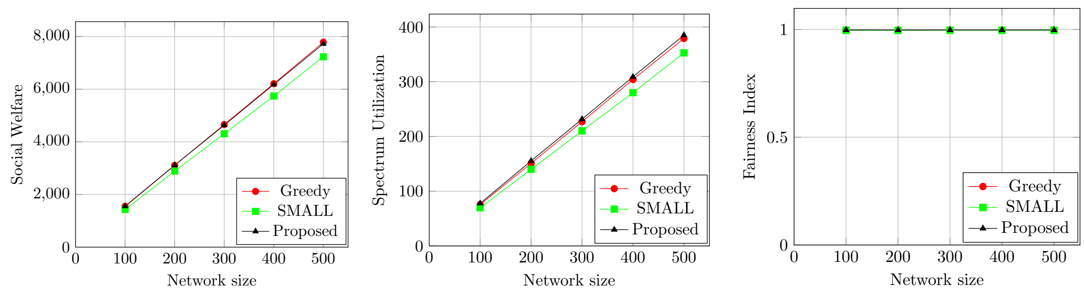
V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed strategy-proof algorithm for spectrum allocation in a wireless network. We also compare the performance of the proposed



(a) Social Welfare (b) Spectrum utilization (c) Fairness Index

Fig. 4: Performance comparison for different algorithms in small network.



(a) Social Welfare (b) Spectrum utilization (c) Fairness Index

Fig. 5: Performance comparison for different algorithms in large network.

algorithm with various other algorithms from the literature. We use undirected graph $\mathscr{G} = (V, E)$ to represent a wireless network. We randomly generate the undirected graphs using configuration model [23] from a given degree distribution of nodes in the graph. The nodes and edges in the graph denote BSs and the pair of interfering BSs, respectively. We perform simulations in MATLAB [24].

In our simulations, we consider uniform degree distribution of BSs in the network, with maximum degree restricted to 5. As discussed in the system model, auctioneer specifies the duration $[T_1, T_2] = [0, 1]$ for which the spectrum auction is to be conducted. Each BS submits the time slot for which the spectrum is required in the specified period of auction along with the bid to the auctioneer. We assume that the BS bids are uniformly distributed. All the results are averaged over 100 iterations for 100 different topologies. We evaluate the mechanism performance for the following parameters:

- Social Welfare: It is defined as the sum of the valuations of the BSs which are allocated channels.
- Spectrum utilization: It is defined as the total number of BSs which are assigned channels.
- Temporal fairness: It quantifies disparity between the average number of times the channel is allocated to BSs.

In Fig. 4, we compare the performance of the proposed algorithm with VCG, SMALL and Greedy mechanisms. Simulations are carried out for small network of maximum size up to 21 BSs (nodes in the graph). Bids generated at each BS are uniformly distributed in the interval $[15, 25]$. We consider each BS has a demand of τ time slot in the period of the auction. The demand slot start t_{si} at each BS is generated uniformly in the interval $[0, 0.90]$. A BS can be allocated channel for duration $[t_{\text{si}}, t_{\text{si}} + \tau]$ in the period of the auction. Note that VCG provides optimal socal welfare, but due to computational complexity it is not practically feasible. It can be observed in Fig. 4, that the social welfare obtained in the proposed algorithm is slightly greater than the greedy mechanism and is close to the optimal. However, the performance of SMALL is worse off the other algorithms. Similar, trends in performance are observed for spectrum utilization. The proposed algorithm outperforms both greedy and SMALL.

Next, we evaluate the performance in large networks. As VCG is computationally intractable in large networks, we compare the performance of the proposed algorithm with SMALL and greedy mechanism. In Fig. 5, social welfare of the proposed algorithm and greedy are approximately same. However, the proposed algorithm shows better spectrum utilization than greedy. The performance of the proposed algorithm is better than SMALL for both social welfare and spectrum utilization. The bid value and demand slot interval at each BS is uniformly distributed in the interval $[15, 25]$ and $[0, 0.90]$, respectively.

Using simulations, we observe the temporal fairness of the algorithms when spectrum auction is performed repeatedly. As explained earlier, we generate a network topology and choose bid values uniformly distributed in the interval $[15, 25]$. For each, we calculate the resource allocation under all the three schemes. Let α_i^π denote the fraction of time BS i is allocated resource under mechanism π. Based on the vector α_i^π we calculate Jain's fairness index [25]. Jain's fairness index is a metric used in networking to determine the share of system

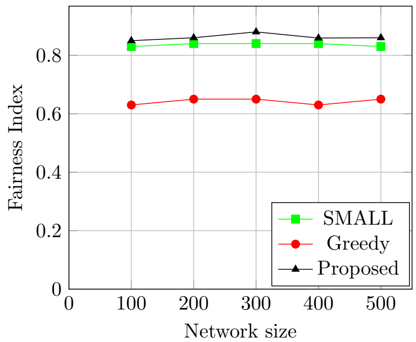


Fig. 6: Comparison of fairness index for different algorithms when some BSs bid at constantly higher values than others.

resources allocated to a user.

Further, we consider a scenario where bid values are distributed around a mean value which is constant across time for users. Let the mean vector $\mu = \{\mu_i | i \in N\}$. The mean value μ_i is uniformly distributed in interval $[15, 35]$ for each user i. Now, we generate bid deviation d_i in the period of auction T for each user i, where d_i is uniformly distributed in the interval $[-1, 2]$. The bid value for each user is given as $b_i = \mu_i + d_i$. Fig. 6 illustrates the fairness index for various mechanisms. Here, we observe that the proposed alogorithm and SMALL significantly outperform the greedy scheme. In the greedy mechanism, mostly the BSs with higher μ_i are allocated channels, while BSs with lower μ_i starve.

VI. Possible Extensions

In this section, we describe the possible extensions of the proposed spectrum allocation mechanism. Although in this work, we proposed a mechanism where a BS demands spectrum only once in the duration of the auction, the mechanism can be extended further when a BS has multiple disjoint demand intervals during $[T_1, T_2]$. Here, the allocation can have two variations depending on the type of demand slot scenarios: (i) spectrum demand is strict i.e., a BS accepts channel only if it gets it in all the requested demand slots. (ii) BS accepts channel for any subset of the requested demand slots. In the first case, the proposed mechanism is applicable without any modification. However, in the second case, conflict graph needs to be augmented with additional nodes to account for the requested slots by a BS, after which the same proposed algorithm can be applied for allocation.

VII. Conclusion

In this paper, we have modeled a spectrum allocation problem with time-varying demand slots and proposed a computationally efficient and strategy-proof mechanism. Using Monte Carlo simulations, we compare the performance of the proposed mechanism with other existing strategy-proof mechanisms. Results show that the performance of the proposed mechanism is close to the optimal obtained using VCG [9]–[11], and it is also feasible to implement in large networks. We observe that the proposed mechanism outperforms other mechanisms in spectrum utilization and social welfare is close to optimal. It also achieves better fairness index in spectrum allocation compared to others.

References

[1] V. N. I. Cisco, "Global Mobile Data Traffic Forecast Update, 2015–2020 White Paper," *Document ID*, vol. 958959758, 2016.

[2] U. Paul, A. P. Subramanian, M. M. Buddhikot, and S. R. Das, "Understanding Traffic Dynamics in Cellular Data Networks," in *IEEE INFOCOM*. IEEE, 2011, pp. 882–890.

[3] H. Wang, J. Ding, Y. Li, P. Hui, J. Yuan, and D. Jin, "Characterizing the Spatio-Temporal Inhomogeneity of Mobile Traffic in Large-scale Cellular Data Networks," in *Workshop on HotPOST*. ACM, 2015, pp. 19–24.

[4] L.-V. Le, D. Sinh, B.-S. P. Lin, and L.-P. Tung, "Applying Big Data, Machine Learning, and SDN/NFV to 5G Traffic Clustering, Forecasting, and Management," in *IEEE NetSoft)*, 2018, pp. 168–176.

[5] R. Murthy, R. Chandra, T. Moscibroda, and P. Bahl, "Senseless: A Database-Driven White Spaces Network," *IEEE Transactions on Mobile Computing*, vol. 11, no. 2, pp. 189–203, 2012.

[6] Q. Zhao and A. Swami, "A Survey of Dynamic Spectrum Access: Signal Processing and Networking Perspectives," in *IEEE ICASSP*, vol. 4, 2007, pp. 1341–1349.

[7] V. Krishna, *Auction Theory*. Academic press, 2009.

[8] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. MIT press, 1994.

[9] W. Vickrey, "Counterspeculation, Auctions, and Competitive Sealed Tenders," *The Journal of finance*, vol. 16, no. 1, pp. 8–37, 1961.

[10] E. H. Clarke, "Multipart Pricing of Public Goods," *Public choice*, vol. 11, no. 1, pp. 17–33, 1971.

[11] T. Groves, "Incentives in Teams," *Econometrica: Journal of the Econometric Society*, vol. 41, no. 4, pp. 617–631, 1973.

[12] X. Zhou, S. Gandhi, S. Suri, and H. Zheng, "eBay in the Sky: Strategy-proof Wireless Spectrum Auctions," in *ACM ICMCN*, 2008, pp. 2–13.

[13] F. Wu and N. Vaidya, "SMALL: A Strategy-proof Mechanism for Radio Spectrum Allocation," in *IEEE INFOCOM*, 2011, pp. 81–85.

[14] X. Zhou and H. Zheng, "TRUST: A General Framework for Truthful Double Spectrum Auctions," in *IEEE INFOCOM*, 2009, pp. 999–1007.

[15] A. Gopinathan, Z. Li, and C. Wu, "Strategyproof Auctions For Balancing Social Welfare and Fairness in Secondary Spectrum Markets," in *IEEE INFOCOM*, 2011, pp. 3020–3028.

[16] G. S. Kasbekar and S. Sarkar, "Spectrum Auction Framework for Access Allocation in Cognitive Radio Networks," *IEEE/ACM Transactions on Networking*, vol. 18, no. 6, pp. 1841–1854, 2010.

[17] Z. Ji and K. R. Liu, "Cognitive Radios for Dynamic Spectrum Access-Dynamic Spectrum Sharing: A Game Theoretical Overview," *IEEE Communications Magazine*, vol. 45, no. 5, 2007.

[18] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "Towards Real-time Dynamic Spectrum Auctions," *Computer Networks*, vol. 52, no. 4, pp. 879–897, 2008.

[19] A. P. Subramanian, M. Al-Ayyoub, H. Gupta, S. R. Das, and M. M. Buddhikot, "Near-Optimal Dynamic Spectrum Allocation in Cellular Networks," in *IEEE Symposium on DySPAN*. IEEE, 2008, pp. 1–11.

[20] F. Wu, T. Zhang, C. Qiao, and G. Chen, "A Strategy-Proof Auction Mechanism for Adaptive-Width Channel Allocation in Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 10, pp. 2678–2689, 2016.

[21] M. R. Garey and D. S. Johnson, *Computers and Intractability : A Guide to the Theory of NP-Completeness*. WH Freeman New York, 2002, vol. 29.

[22] D. B. West *et al.*, *Introduction to Graph Theory*. Prentice hall Upper Saddle River, 2001, vol. 2.

[23] F. Chung and L. Lu, "Connected components in random graphs with given expected degree sequences," *Annals of combinatorics*, vol. 6, no. 2, pp. 125–145, 2002.

[24] "MATLAB Toolbox," https://in.mathworks.com.

[25] R. K. Jain, D. M. W. Chiu, and W. R. Hawe, "A Quantitative Measure of Fairness and Discrimination," *Eastern Research Laboratory, Digital Equipment Corporation, Hudson, MA*, 1984.