

On Efficient Wireless Backhaul Planning for the “Frugal 5G” Network

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Abstract—The technology advances in 5G are urban in nature and have a larger focus on high data rates, very low latency, and very high-speed mobility. Amidst this, the issues such as coverage and affordability are likely to persist which will widen the rural-urban divide even further. The connectivity needs of rural areas call for an affordable broadband network which can be accomplished if we design a low mobility energy-efficient network. We refer to such a network as the *Frugal 5G* network. Deployment of Wireless Local Area Network (WLAN) Access Points (APs) is a cost-effective method to provide high-speed low mobility coverage to rural areas. Backhauling the WLAN APs is a challenging task as fiber backhaul is generally unavailable in rural areas. We suggest a method to design the wireless backhaul network in order to serve the WLAN APs. The design objective is to maximize system performance with respect to network throughput and delay while reducing infrastructure utilization. We propose a backhaul design method based on Simulated Annealing (SA) to achieve our objective. Our analysis shows that the proposed algorithm enhances the backhaul network throughput by 24% while minimizing the infrastructure used.

I. INTRODUCTION

The term *Frugal 5G* refers to the vision of providing an affordable high-speed broadband service to the unserved or under-served rural areas [1]. An essential first step in realizing the Frugal 5G vision is to rethink the connectivity requirements of rural areas. The critical requirement is to reduce the overall cost of the network so as to provide affordable connectivity. There exist various design parameters in rural networks which may be exploited to design a broadband network that is suited to these areas. These include i) limited mobility and delay tolerant services to limit cost and complexity, ii) high-speed broadband access in limited areas (rural populations are sparse and clustered) to reduce infrastructure cost, and iii) relaxed network availability constraints with limited redundancy also reduces capital expenditure. Solving these issues would enable the development of relevant technical standards to provide broadband connectivity in rural areas. The standards development project P2061 under Frugal 5G working group, initiated recently by IEEE, is an important example in this regard. The P2061 project aims to design an architecture for a low mobility energy efficient network for affordable broadband access [2]. Prior work that we have conducted explicitly considers and solves some of the problems related to Frugal 5G systems that are mentioned herein [3]–[6]. The Frugal 5G networks may be viewed as community networks, wherein the characteristics of these networks differ signifi-

cantly from conventional urban networks, thereby requiring different performance and Quality of Service (QoS) metrics.

We now propose a network architecture to realize a rural network with the requirements and constraints outlined above. We assume a standard tiered network architecture comprising of a Core Network (CN) and a heterogeneous Access Network (AN). AN provides last mile connectivity to end-user via Macro Base Station and Wireless Local Area Network (WLAN) Access Points (APs). The Macro BS is located in the vicinity of the Point of Presence (PoP) and provides a carpet coverage in a given geographic area. Since rural population lives in clusters, a high-speed broadband access can be provided by deploying WLAN APs in these clusters. The WLAN APs are generally inexpensive, energy-efficient and suitable for low mobility environments. The critical challenge here is to backhaul the data from the WLAN APs to the Point of Presence (PoP), which may be located a few kilometers away. The expensive and time-consuming process of fiber deployment is unsuitable for rural areas. Therefore, we suggest the use of wireless backhauling of WLAN APs in these areas which is referred to as the *middle mile network*. The middle mile network consists of middle mile Access Point (AP), middle mile client (serves the WLAN APs) and Intermediate Nodes (INs) (similar to middle mile client but does not serve WLAN APs). Efficient planning of the middle mile network is the key to realizing the Frugal 5G network. There are other design challenges in Frugal 5G networks, such as seamless integration of wireless backhaul with the AN, defining the control signalling for the wireless backhaul, and defining the control architecture that is calibrated according to services required in rural areas. These problems will be considered in future work. In this paper, we focus largely on methods to design the wireless backhaul network.

Next generation wireless backhaul networks are being designed to backhaul the small cell BSs deployed in a small area (2 – 3 km²) [7]–[9]. We, however, consider a middle mile network which serves a large rural area (25 – 30 km²). Since we assume a large area, designing a multi-hop backhaul network is essential to improve its performance and coverage. Prior work, in this regard, has been done in WiMAX based networks. Such work has largely focused on situations that concern IEEE 802.16j multi-hop relay networks, including planning models [10], relay selection and placement [11], and enhancement of service area and quality using relays [12]. The

inter-operation of relays with base stations and their optimal deployments have also been studied [13], [14]. In addition, since the prior work largely considers multi-hop networks from a relay perspective, the number of hops considered is also limited. Our work focuses on middle mile clients and INs, which are distinct from relays, in that they do not merely act as amplify or decode and forward nodes, but also function as access nodes which can serve WLAN APs with user traffic. Moreover, we want to investigate the impact of allowing more hops on system performance and the infrastructure utilization. Thus, the design objectives and metrics for deployment of these nodes should consider these additional requirements. In this paper, we propose a method to design a fixed wireless multi-hop middle mile network. The main design challenge is to provide a high quality of service without increasing the infrastructure cost. In addition, the network performance in terms of throughput and delay has to be maximized. To minimize the delay, the number of hops in a multi-hop network should be minimized. We propose a simulated annealing (SA) based algorithm to design such a middle mile network. We also compare the performance of the proposed algorithm with a greedy approach to solving the problem.

The remainder of the paper is structured as follows. In Section II, the network architecture is described in detail. We also present the problem formulation in this section. In Section III, we propose an algorithm based on SA to plan the wireless backhaul network. The performance of the algorithm is discussed in Section IV where we compare its performance with a greedy scheme. In Section V, we conclude the work.

II. NETWORK ARCHITECTURE

A. System Model

We consider a geographic area located in a rural setting that needs to be served via a Frugal 5G network. Since rural populations generally live in clusters, we deploy WLAN APs in these clusters to serve the end-users. These clusters are, in general, uniformly scattered over the given area. The middle mile network enables the connectivity between the WLAN APs and the PoP. We assume that a middle mile AP is located at the center of the given area and has a wired backhaul via the PoP. We further assume that there is one middle mile client situated in each village cluster. Since the middle mile clients are located far away from each other, it is difficult to connect them directly. Thus, we employ Intermediate Nodes (INs) to connect them and form a wireless multi-hop middle mile network. The middle mile clients and INs are identical multi-hop nodes, but differ in the fact that middle mile clients backhaul the WLAN APs, while the INs do not. An IN can be easily used to serve a WLAN AP if demand arises in future. The middle mile clients with the help of INs aggregate the data from the WLAN APs and transport it to the middle mile AP (co-located with the PoP) over a wireless medium. An instance of a middle mile network is shown in Fig. 1. A single middle mile client may serve many WLAN APs and the connectivity between them may be wired or wireless. The manner in which

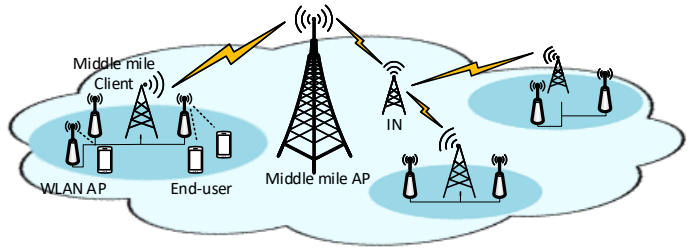


Fig. 1: An instance of the middle mile network.

the middle mile clients connect with the WLAN APs is not in the scope of this work and will not be discussed here.

We model the middle mile network planning as a graph-theoretic problem. The network is modeled as an undirected graph $G(\mathcal{V}, \mathcal{E})$ where \mathcal{V} is the set of nodes in the network and \mathcal{E} is the set of edges between the nodes. The set \mathcal{V} comprises of the middle mile AP, the middle mile clients as well as the INs used to connect them. The set of middle mile clients is denoted by $\mathcal{M} = \{1, 2, 3, \dots, M\}$. The set of INs is denoted by $\mathcal{L} = \{1, 2, 3, \dots, L\}$. The number of hops between the middle mile AP and the middle mile clients \mathcal{M} is denoted by $\mathcal{H} = \{h_1, h_2, h_3, \dots, h_M \mid h_i \in \mathbb{N}\}$. Assume that the maximum range of an IN or a middle mile client to be fixed and is denoted by r m. A wireless link between any two nodes of a middle mile network is a point to point link. The middle mile client and INs are equipped with directional antennas. The middle mile AP has an omnidirectional antenna.

B. Problem Formulation

For planning the middle mile network, our objective is to maximize the network throughput along with minimizing the hops as well as number of INs in a network. The throughput experienced by a middle mile client i is denoted by U_i . The network throughput is defined as the sum throughput of M middle mile clients in the network graph $G(\mathcal{V}, \mathcal{E})$ i.e $\sum_{i=1}^M U_i$. Mathematically, the problem can be written as:

$$G^*(\mathcal{V}^*, \mathcal{E}^*) = \arg \max_{G(\mathcal{V}, \mathcal{E})} \sum_{i=1}^M U_i - f(h_i) - g(L) \quad (1)$$

Here, f and g are exponential penalty functions of \mathcal{H} and \mathcal{L} respectively. Exponential penalty functions are chosen to assign a high cost if the number of hops and INs is large. The function f is an average of hops between middle mile client and AP and is defined as $f = \frac{1}{M} \sum_{i=1}^M e^{a_0 h_i}$. The function g is defined as $g = e^{\frac{b_0 L}{M}}$. The constants a_0 and b_0 in the functions are chosen to match the penalties closely.

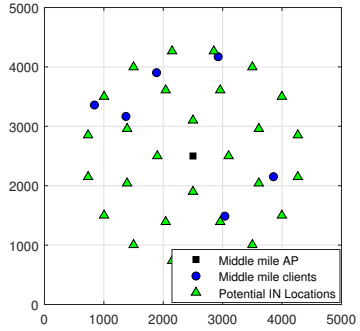
Minimizing the INs in a network by means of an exhaustive search has an exponential complexity in the size of the grid and the number of INs. The problem we consider not only aims to minimize INs but also minimizes the hops in the network along with maximizing the throughput. Therefore, to solve this problem efficiently, we propose a low-complexity heuristic algorithm in the next section.

III. MIDDLE MILE NETWORK PLANNING

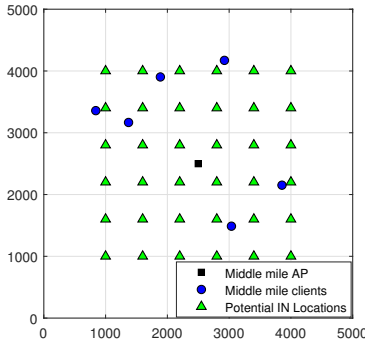
To design a heuristic for the proposed problem in Section II-B, we model it as an IN selection problem. We consider a set of potential IN locations in a given area. Next, we run a selection algorithm over these potential locations so as to attain our objective. The potential IN locations are chosen strategically instead of placing them in a random manner. The arrangement of these IN locations are described next.

A. Arrangement of Potential Intermediate Nodes

We have chosen two possible arrangements of potential IN locations on which we run our selection algorithm. Fig. 2a shows a circular structure of potential IN locations and a set of middle mile clients in a given area. A circular grid is chosen because the contours of constant received power are circular in nature. A square grid may also be considered, as shown in Fig. 2b, for its regular structure. Next, we need to determine the best multi-hop graph that spans over the selected INs and the middle mile nodes.



(a) Circular grid for deploying INs



(b) Square grid for deploying INs

Fig. 2: Potential IN Placement Locations (Distances are in meters).

B. Intermediate Node Selection

We present a Simulated Annealing (SA) based algorithm to solve the IN selection problem. SA, a Markov chain Monte Carlo method, is a meta-heuristic search based technique to estimate the global optimum of a given function. When the search space is very large, such as in our case, SA can be used to estimate the optimal solution. SA is inspired by annealing in metallurgy which involves heating and controlled cooling of a

material. SA basically comprises of two stochastic processes i.e. i) generation of a new solution and ii) acceptance of a new solution. The acceptance of any solution is controlled by a temperature T . It is important to note that we do not necessarily require a polynomial time algorithm as the wireless backhaul planning is done only once before deployment or when a new node is added in the network. However, frequent re-planning with differing sets of conditions would require several time-consuming exhaustive search operations, thus necessitating an efficient solution to this problem.

We construct a Discrete Time Markov Chain (DTMC) on the states of the problem under consideration. Then, we transition on these states to attain the global optimum. A DTMC is characterized by its states, how neighboring states are constructed and the Transition Probability Matrix (TPM). We define these elements next.

Algorithm 1 Proposed Algorithm

Require: $G_i \leftarrow$ Initial solution
 $T \leftarrow T_0$; Calculate $W(G_i)$
while Global loop condition not satisfied **do**
 while Local loop condition not satisfied **do**
 $G_j \leftarrow$ Generate a neighbor graph from G_i
 if $W(G_j) > W(G_i)$ **then**
 $G_i \leftarrow G_j$;
 else
 $r \leftarrow$ Uniform random number between 0 and 1
 $p \leftarrow e^{(W(G_j)-W(G_i))/T}$
 if $r < p$ **then**
 $G_i \leftarrow G_j$;
 end if
 end if
 end while
 $T \leftarrow \alpha T$
end while
Result: G_i

1) *States:* The state refers to the different feasible graphs G_i that can be formed using the middle mile AP, middle mile clients, and INs. A feasible graph is a multi-hop tree with middle mile AP as the root node and only middle mile clients as the leaf nodes. Each state i.e. a feasible graph G_i has an associated weight $W(G_i)$ to it. Let us denote G^* as the global optimum, i.e. the state which has maximum weight $W(G^*)$. The weight function $W(G) = \sum_{i=1}^M U_i - f(h_i) - g(L)$ defined here is the objective of our optimization problem defined in Section II-B.

2) *Neighboring States:* We now explain how the neighboring states are determined in the DTMC. In our scenario, a neighbor graph is generated by selecting one of the two actions i.e. adding an edge or removing an edge. Through this process, the SA algorithm can traverse through all the feasible graphs. Assume a feasible graph G_i . A graph G_j is its neighbor if it is obtained by choosing one of the following two actions.

- *Adding an edge (A_1):* When an edge is added to a graph, a cycle may be formed which renders the graph

infeasible. Therefore, we randomly remove an edge such that the cycle is removed and no middle mile client is disconnected.

- *Removing an edge (A_2):* When an edge is removed from a graph, middle mile clients may be disconnected making the graph invalid. Therefore, we can only remove those edges which do not disconnect the middle mile clients.

3) *Transition Probability Matrix (TPM):* Now, we define the TPM for the DTMC constructed above. Let p_{ij} be the transition probability from state G_i to its neighboring state G_j . The transition happens in the following two steps.

- In the first step, we choose an action i.e. adding or removing an edge with equal probability i.e. $p^{A_1} = p^{A_2} = \frac{1}{2}$. Each action is executed in the following manner:
 - *Add:* Assume that there are E_i number of edges which can be added to the graph G_i . Addition of an edge creates a cycle in the graph. If $L_i (\geq 1)$ edges can be removed from the graph to make it free from cycles, then the probability with which an edge can be added is $\alpha_i^{A_1} = \frac{1}{E_i} \frac{1}{L_i}$. The state remains unchanged if the same edge, which is added to the graph, is removed.
 - *Remove:* Assume that M_i edges can be removed from the graph G_i . If $M_i = 0$, then state remains unchanged. The probability of action A_2 can be written as $\alpha_i^{A_2} = \frac{1}{M_i}$.
- In the second step, the weight of both the graphs, G_i and G_j is compared. If $W(G_j) \geq W(G_i)$, then transition happens with probability 1. If the $W(G_j) < W(G_i)$, then transition happens with probability $e^{(W(G_j) - W(G_i))/T}$, where T is the temperature parameter in the SA algorithm. The probability of this step can be written as $\beta = \min(1, e^{(W(G_j) - W(G_i))/T})$.

Now, the transition probability can be written as,

$$p_{ij} = \begin{cases} p^{A_1} \alpha_i^{A_2} \beta & \text{Action 1} \\ p^{A_2} \alpha_i^{A_2} \beta & \text{Action 2} \\ 0 & \text{when } j \text{ is not a neighbor of } i \end{cases} \quad (2)$$

In the SA algorithm, we simulate the DTMC with the above transition probabilities as explained in algorithm 1. These transition probabilities can be calculated in polynomial time and need not be stored. This ensures the computational viability of our approach. Next, we prove certain important properties of the constructed DTMC which ensures that there exists a unique steady state distribution.

Theorem 1. *The constructed DTMC $\{G_n, n = 0, 1, 2, \dots\}$ is finite, aperiodic and irreducible.*

Proof. Assume that L_0 nodes are selected from L potential nodes. Hence, we have a total of $M + L_0$ nodes in the network. The total possible edges in the network are $E_0 = \binom{M + L_0}{2}$. The number of all possible graphs that can be formed using these edges is 2^{E_0} . Since the total feasible graphs are a subset of all possible graphs, the number of feasible graphs is finite. The DTMC has self-loops as there is a non-zero probability of remaining in the same state. Therefore, the DTMC is aperiodic. There exists a path to go from any graph to any

other graph by dropping all the edges and then adding only the required edges. This proves that DTMC is irreducible. \square

IV. PERFORMANCE ANALYSIS

A. Simulation Framework

We consider an area of 5000×5000 m² in which middle mile clients are deployed uniformly at random. Without loss of generality, we assume that the middle mile AP is located at the center of the given geographic area. We consider the operational frequency to be 3.3 GHz and the bandwidth to be 40 MHz. The transmit power for all the nodes in the network is 20 dBm and the noise level is -100 dBm. The range of middle mile client and IN is approximately 800 m based on link budget analysis for the system. Depending on this range, we design the circular grid and square grid. In general, there are small number of village clusters in a given rural area and therefore the number of middle mile clients are also kept small. In the simulations for the proposed algorithm, $a_0 = 2$ and $b_0 = 3$. The results are averaged over 100 different distributions for a given number of middle mile clients.

Since we assume a fixed wireless network, we only consider the path loss while formulating the problem. The wireless channel effects such as fading and shadowing are not considered. We employ Friis free space path loss model to calculate the attenuation of the signal over a link. The Friis Equation is given as $P_r = P_t \left(\frac{G_r G_t}{d^2 \lambda^2} \right)$. Here, P_r is the received signal strength, P_t is the transmitted signal strength, G_r is received antenna gain, G_t is transmitted antenna gain, d is the distance between the transmitted and received antenna, and λ is the wavelength of the radio frequency used. The capacity of the wireless link can be modeled using the standard Additive White Gaussian Noise (AWGN) channel capacity given by $C = B \log_2(1 + P_r/N)$. Here, C is the capacity in bit/sec, B is the channel bandwidth in Hz, and P_r/N is the received Signal to Noise Ratio (SNR). For the purposes of fading channels, we may assume that this can be converted to an outage based rate computation. Here, the discussion is not specific to particular wireless standard or technology. The proposed algorithm can be easily modified to suit any technology. Therefore, we consider a generalized scenario here.

Before probing into the results, we describe two approaches which can be used as a benchmark for performance comparison.

1) *Exhaustive Search for IN Placement:* Assume that we have L possible locations for placement of the INs. To determine the best INs, we have to search for all the possible combinations of INs which are 2^L . After, selecting INs we need to determine the best possible graphs which can be formed from these INs. It can be clearly seen that the computational complexity is exponential of the possible IN locations. Hence, an exhaustive search cannot be used to compare with the proposed algorithm.

2) *Greedy Placement of INs:* A heuristic method based on a greedy approach can be used to deploy INs in a network, and it serves as a benchmark to evaluate the performance of our

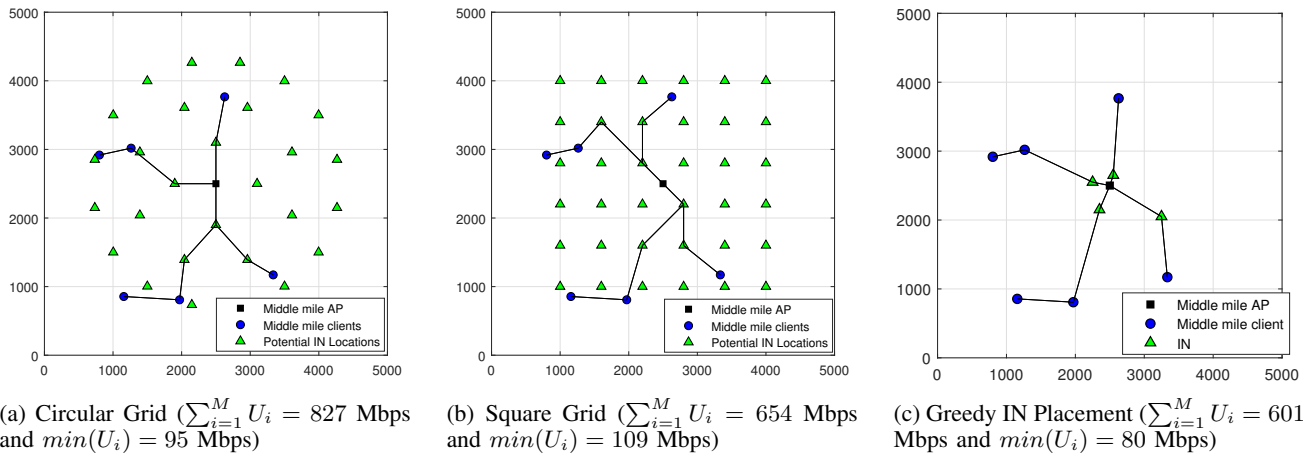


Fig. 3: Comparison of a deployment example between the proposed algorithm for circular and square grid and greedy approach.

algorithm. This is an iterative method in which we add an IN in every iteration. Here, we assume a virtual grid over the given area. An IN can be placed at the center of any square in a grid. After selecting a position to place IN, we form a multi-hop graph using a minimum spanning tree algorithm. The graph which maximizes the weight $W(G)$ as defined above is chosen among all possible locations on the grid. The corresponding IN location is fixed. In the same manner, we deploy the INs iteratively.

B. Results

We now discuss the results obtained from our simulations in MATLAB. As the computational complexity of the exhaustive search approach is exponential, we only compare our algorithm with the greedy approach. Fig. 3 illustrates the deployment results for an example scenario obtained from our algorithm as well as the greedy approach. The network topology as proposed by our algorithm has chosen INs in a uniform fashion over the given area which in turn improves the worst case performance in the network. On the other hand, the greedy algorithm maximizes the throughput by making shorter, high capacity links and compromises the throughput of other links.

Figs. 4a and 4b show the performance comparison of our proposed algorithm for a circular grid, square grid, and the greedy IN placement algorithm. There is a 24% improvement in network throughput of the circular grid based approach as compared to the greedy IN placement. The square grid based approach shows 9% improvement in network throughput as compared to the greedy approach. The same trend follows for the minimum user throughput where an improvement of 25% is observed in case of a circular grid as compared to the greedy approach. This clearly states that a circular grid based approach not only maximizes the global outcome but also maximizes the individual user performance. We observe a saturation in network throughput and decline in minimum user throughput as the number of middle mile clients are increased. This is due to the increased number of branches in the multi-hop network which limits the offered capacity at each middle mile client.

Figs. 4c, 4d and 4e show the comparison of three other parameters in the network. First, we compare the number of INs used for a given distribution of middle mile clients. A similar performance is observed for all the three cases. This ensures that SA based technique is not increasing infrastructure used to improve the performance. We then compare the average number of hops for all the middle mile clients in a given network. Also, the maximum number of hops among all the middle mile client is plotted. It can be clearly seen that though the average number of hops for greedy are almost the same as the other two approaches, the maximum number of hops are highest. The reason for this outcome is that the greedy IN placement does not have a global view. It favors some middle mile clients while the performance for other middle mile clients suffers. Since the contours of constant power are circular, the circular grid is a much more natural option than the square grid. The observation on the performance of the circular grid based approach supports this.

The middle mile network design proposed here incorporates several design parameters for the Frugal 5G backhaul network such as limited mobility support, large operational area, and relaxed network availability. It is clearly observed that with incorporating 4-5 INs and allowing at most 4 hops, the performance of middle mile network is significantly increased with our efficient planning algorithm. These INs are envisioned to be low power nodes and hence can easily run on renewable energy in a cost-effective manner. Such a design is an excellent candidate for the Frugal 5G backhaul network.

V. CONCLUSION AND DISCUSSIONS

In this paper, we have proposed a network architecture for Frugal 5G which suits the requirements of rural areas. WLAN is an effective method to provide last mile connectivity. However, backhauling WLAN APs is a difficult task. To solve this problem, we have proposed a wireless backhaul network i.e. the middle mile network. In order to design the network, we have formulated an optimization problem which aims to maximize network throughput while minimizing the number of hops and INs in the network. Since the proposed problem has

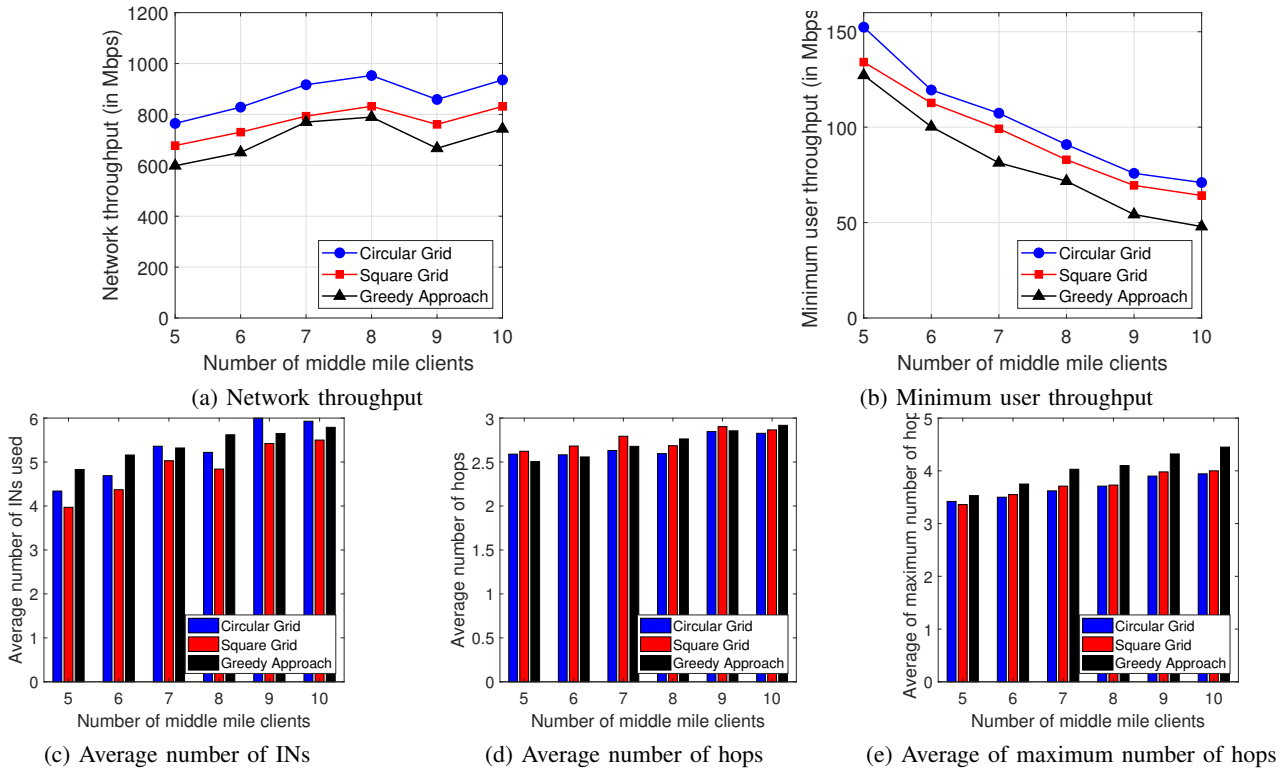


Fig. 4: Comparative analysis of the proposed algorithm for circular and square grid, and the greedy IN placement algorithm.

exponential complexity, we have presented a low-complexity SA based algorithm. The results have revealed that the proposed method performs much better than other approaches, including greedy design, even in the critical parameters such as minimum user throughput and maximum hop count. This can be extended to account for temporal and seasonal variations in the usage pattern to allow for higher redundancy with reasonable energy efficiency.

It must be noted that the work here considers only the topology of the network. However, the protocol design, systems and network aspects of Frugal 5G networks are still under development. Future work would consider the impact of design characteristics such as low mobility, delay tolerances and wide operational areas on the system design. Studies on how design choices for the Frugal 5G system components would impact deployment and operation costs, energy efficiency and quality of service in rural areas are currently underway.

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