Random Access Algorithm with Power Control and Rate Guarantees over a Fading Wireless Channel

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Abstract—**In this paper, we consider uplink transmissions involving multiple users communicating with a base station over a fading channel. We assume that the base station does not coordinate the transmissions of the users and hence the users employ random access communication. The situation is modeled as a non-cooperative repeated game with incomplete information. Each user attempts to minimize its long term power consumption subject to a minimum rate requirement. We propose a three timescale stochastic gradient algorithm (TTSGA) for tuning the users' transmission probabilities. The algorithm includes a 'waterfilling threshold update mechanism' that ensures that the rate constraints are satisfied. We prove that under the algorithm, the users' transmission probabilities converge to a Nash equilibrium. Moreover, we also prove that the rate constraints are satisfied; this is also demonstrated using simulation studies.**

Index Terms—**Power Efficient Scheduling, Random Access Mechanism, Stochastic Approximation, Quality of Service, IEEE 802.11**

I. Introduction

Wireless networks have witnessed large scale proliferation in recent years. Apart from voice applications, data applications such as email, World Wide Web (WWW) etc. are also becoming popular over the wireless network. Different applications have different Quality of Service (QoS) requirements in order to perform satisfactorily. These QoS requirements can be in terms of parameters such as rate, delay, delay jitter etc. In this paper, we consider providing average rate guarantees as QoS to wireless Internet applications. This entails addressing the following important issues:

- Medium access control (MAC): Multiple users need to access the common wireless channel simultaneously in order to communicate with a common receiver such as a base station or an access point. The access mechanism must be so designed that it satisfies the QoS requirements of user applications.
- Challenges offered by the Wireless Medium: Wireless channel is characterized by decay of signal strength due to distance (path loss), obstructions due to objects such as buildings and hills (shadowing), and constructive and destructive interference caused by copies of the same signal received over multiple paths (multipath fading), possibly with time varying path lengths, resulting in a time varying channel condition [1]. These phenomena distort the signal in an unpredictable manner and can cause packet errors at the receiver. The time varying wireless channel poses significant challenges for design of efficient and reliable communication systems.

Various MAC protocols have been proposed in the literature that attempt to satisfy different application requirements based on packet arrival characteristics and QoS attributes. These protocols can be classified into following two types:

- Fixed resource allocation protocols: These protocols assign fixed amount of resources to the users by means of orthogonal or near orthogonal channels. They require a central scheduling entity (like a base station) that performs the channel allocation task. Examples include Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), Code Division Multiple Access (CDMA) and Orthogonal Frequency Division Multiple Access (OFDMA). These have been implemented in cellular systems and wireless local area networks [1].
- Random access protocols: In these protocols, users access the channel randomly. Users vary their channel access probabilities or access times based on limited feedback from the channel. Since the users take transmission decision based on local information available with them, these protocols are suited for distributed implementation. Random access protocols have been implemented in cellular systems, satellite communication systems and multitap bus among others. These have been well studied for the past several decades. [2], [3] provide excellent textbook treatment of random access protocols.

Recent research suggest that significant performance gains can be obtained at the MAC layer by exploiting physical layer information like channel condition [4]. This is termed as *cross layer* approach [5]. In this paper, we consider such a cross layer random access scheme. Specifically, we consider multiuser uplink scenario where the base station does not coordinate the transmission of the users and each user accesses the channel probabilistically. In order to conserve energy, the objective is to devise a random access and power control scheme wherein the long term average power consumption for each user is minimized subject to satisfying the average rate requirement of each user.

There have been several attempts to design channel aware random access schemes in order to improve the performance. Existing work on channel aware random access schemes can be classified into the following categories:

- Signal processing and diversity techniques to correctly

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decode received packets as in [6], [7], [8].

- Work that advocates the adaptation of retransmission probabilities of users either through 'Splitting algorithms' that adapt the set of contending users based on feedback from the channel as in [9], [10] or through 'channel aware ALOHA' schemes as in [11], [12], [13], [14], [15].

Our work falls under the latter category.

A. Related Work

In this section we review representative research that advocates adapting retransmission probabilities of users under channel aware ALOHA schemes.

In [11], [15], the authors attempt to exploit multiuser diversity in a distributed fashion with only local channel information, i.e., each user is aware of its own channel condition only. The authors propose a channel aware ALOHA protocol and provide a throughput analysis of the proposed protocol under an infinitely backlogged model. In [13], the authors consider symmetric as well as asymmetric fading. The authors propose a *binary* scheduling algorithm where users access the channel when the corresponding channel condition is above a certain threshold and prove that it maximizes sum throughput under symmetric fading. Moreover, for asymmetric fading, they prove that binary scheduling maximizes the sum of log of average throughput of the users and is fair in the long run. Furthermore, they also consider channels with memory and provide simple extensions of the binary scheduling algorithm. In [12], the authors study distributed schemes for exploiting multiuser diversity in the uplink (multipoint to point) context. They propose a channel aware ALOHA scheme where the transmission probability is a function of channel state information. They characterize the maximum stable throughput for such a system with both finite as well as infinite user models. In [14], the author defines an interference-dominating wireless network as the one in which a receiver could simultaneously receive a number of packets from a variety of transmitters, as long as the signal-to-interference-plus-noise ratio exceeds a predetermined threshold. The author proposes an analytical approach to derive the exact value of saturation throughput of slotted ALOHA in such an interference-dominating wireless ad-hoc network.

In addition to the above, there has been a lot of interest in power control techniques in random access wireless networks. Game theory [16] serves as a useful tool for designing these power and access control protocols with provision for information exchange (cooperative game) as well as no information exchange (non-cooperative game) between users. Game theoretic models have been applied in [17], [18], [19], [20]. The slotted ALOHA protocol has been modeled both as a non-cooperative game as well as a cooperative game. Various objectives like delay minimization [17], throughput maximization [17], [18], [19] have been considered. Power control coupled with retransmission control has been variously studied in [21], [22], [23]. In [24], the authors analyze the equilibrium points achieved by a non-cooperative group of users that have a certain QoS requirement and willingness to pay. The reader is referred to [22], [25], [26] for further information on applications of game theory for modeling the random access problem.

The model considered in this paper is similar to that in [15]. However, in [15], the authors consider the throughput scaling under long term and short term power constraints, while we consider a different problem of minimizing the long term power expenditure of each user subject to satisfying the rate constraint of each user. Another closely related work is that of [27]. However, in [27] also, the authors do not consider the problem of power control. In [27], the focus is on *analysis* of the Nash Equilibria of the game involving a group of non-cooperative users sharing a channel and desiring certain long term throughput, on the other hand, we adopt a *prescriptive* approach. We focus on algorithm design and show that the proposed Three Timescale Stochastic Gradient Algorithm (TTSGA) converges to the Nash equilibrium of the game under certain conditions.

B. Our Contributions

In this paper, our objective is to design a cross layer random access scheme where each user accesses the fading channel probabilistically so as to minimize its long term power consumption subject to a minimum average rate. We assume that the distribution of the channel fading process is not known to the users. We assume that each user accesses the channel independently of others based on its local Channel State Information (CSI) only. Moreover, the users are not aware of the rate requirements and the channel conditions of the other users. Furthermore, there is no mechanism for information exchange between the users. This situation is modeled as a constrained repeated non-cooperative game, where each user has an objective of minimizing the long term average power expenditure subject to achieving a certain long term average rate. The users modulate their transmission rates based on the CSI fed by the base station. We propose an iterative primal-dual technique for tuning the transmission probabilities of the users and ensuring that the constraints are satisfied. The primal variable is the transmission probability while the dual is the waterfilling threshold that adjusts the average transmission power for ensuring that the rate constraint is satisfied. Our contributions can be summarized as follows:

- We formulate the user problem where the objective is to minimize average power consumption subject to an average rate constraint as a constrained repeated non-cooperative game where each user has knowledge of its own rate requirement and CSI only.
- We propose a Three Timescale Stochastic Gradient Algorithm (TTSGA) for iteratively tuning the transmission probability. This algorithm iterates the transmission probability in the direction of the gradient of the average power consumption. Moreover, the 'waterfilling threshold' is tuned on a faster timescale to ensure that the rate constraints are satisfied.
- We prove that under TTSGA, the transmission probability iterates converge to a Nash equilibrium provided certain conditions are met.

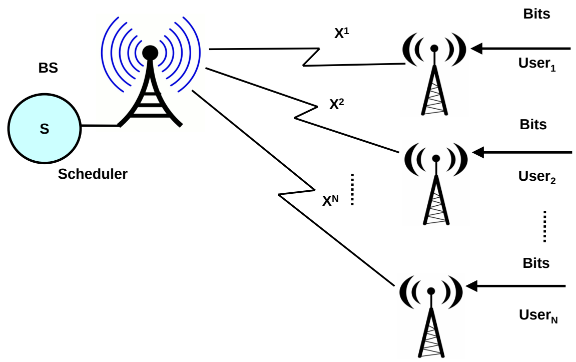

Fig. 1. Uplink transmission scenario

- The approach ensures that the rate constraints are met. This is proved analytically and validated through simulations.

The rest of the paper is organized as follows. Section II provides details regarding the system model. We formulate the problem as a constrained repeated game in Section III. In Section IV, we motivate the solution strategy and propose TTSGA. In Section VI, we analyze certain properties of the algorithm and prove that the transmission strategies of users converge to a Nash equilibrium. Moreover at equilibrium, the rate constraints are satisfied. We present the simulation results in Section VII. In Section VIII, we discuss implementation aspects of the algorithm within IEEE 802.11 framework.

II. System Model

We consider an uplink scenario similar to that in [15] as depicted in Figure 1 where N users communicate with a base station. We consider a time slotted system, i.e., time is divided into slots of equal duration normalized to unity. There can be multiple flows between a user and the base station. However, for the sake of notational simplicity, we assume that only one flow exists between a user and the base station. The analysis can be easily extended to more general cases.

We assume that the system operates in a *distributed* fashion. Hence we assume that the base station does not coordinate the transmissions of the users. In each slot n, user i transmits with a certain probability θ_n^i to the base station. If more than one user transmits in a slot, then all transmissions are unsuccessful, i.e., there is a *collision* at the base station. We assume that each user receives a $(0, 1, e)$ feedback in each slot, where 0 denotes that there is no transmission in the slot, 1 denotes successful transmission and e denotes collision or unsuccessful transmission. We assume that this feedback is immediate and error free. In practice, this information can be conveyed through acknowledgement messages sent by the base station over a feedback channel.

We assume a wireless channel with block fading [28]. Under this model, if χ_n^i is the transmitted signal by user i in slot n, then the signal Y_n^i received by the base station in slot n can be expressed as:

$$Y_n^i = H_n^i \chi_n^i + Z_n, \tag{1}$$

where Z_n is complex Additive White Gaussian Noise (AWGN) at the base station. H_n^i is the channel gain and we denote $X_n^i = |H_n^i|^2 \in \mathbb{X}$ as the channel state for user i in slot n. We assume that the users possess perfect knowledge of channel state X_n^i in each slot [1]. Moreover, the distribution of X_n^i is not known to user i. We discuss possible mechanisms for conveying information such as successful reception of a packet and CSI in Section VIII.

All packets are assumed to be of equal size, say, ℓ bits. We assume a backlogged model, i.e., users always have packets to transmit. Let $\pi^i : \mathbb{X} \rightarrow \mathbb{R}^+$ denote the power allocation policy for user i that determines its transmission power P_n^i in each slot n based on the channel state X_n^i. Let U_n^i denote the number of packets that user i transmits to the base station in slot n. Since the slot duration is normalized to unity, U_n^i can also be considered to be the transmission rate in slot n. U_n^i can be determined using a rate allocation policy $\mu^i : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ that maps the transmission power into an appropriate transmission rate. For most communications system, the power required for transmitting at a given rate is an increasing and convex function of the rate.

III. Problem Formulation

In this section, we formulate the problem as a repeated non-cooperative game. We begin by considering ith user. The long term power consumption for user i can be expressed as:

$$\bar{\mathcal{P}}^i = \limsup_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} \theta_n^i P_n^i = \limsup_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} \theta_n^i \pi^i(X_n^i). \tag{2}$$

where θ_n^i denotes the transmission probability for user i in slot n and π^i denotes the power allocation.

Let β_n^i denote the probability of successful transmission for user i in slot n. The long term throughput or rate achieved by user i can be expressed as:

$$\bar{\mathcal{U}}^i = \liminf_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} \beta_n^i U_n^i = \liminf_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} \beta_n^i \mu^i(P_n^i). \tag{3}$$

Here μ^i denotes the rate allocated for the transmission powerP_n^i. The optimization problem for user i can be expressed as:

$$\text{Minimize } \; \bar{\mathcal{P}}^i \; \text{ subject to } \bar{\mathcal{U}}^i \geq \bar{\rho}^i, \tag{4}$$

where $\bar{\rho}^i$ denotes the long term average rate requirement for user i.

We are interested in determining an 'equilibrium' or 'steady state' transmission probability (say $\theta^{i,*}$ for user i) and the transmission power for each user such that if a user transmits at this probability with the power prescribed by the policy, the average power is minimized and the rate constraint is satisfied. Note that the above problem can be viewed as a rate constrained random access problem where each user i solves a similar optimization problem.

[1]In practice, the users perform channel estimation for downlink transmissions using the pilot signal transmitted by the base station. In a Time Division Duplex (TDD) system because of symmetry, these estimates can be used for uplink transmissions as well.

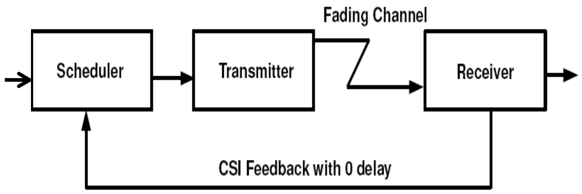

Fig. 2. Single user scenario

Remark 1: Let $\mathcal{P}^i_{\theta^i}$ denote the average power expenditure for user i when the transmission probability is fixed at θ^i. It consists of two parts: 'useful' power, i.e., the power consumed in successful transmissions and 'wasted' power, i.e., the power wasted in collisions. Let $\mathcal{P}^i_{\beta^i}$ and $\mathcal{P}^i_{\varepsilon^i}$ denote the 'useful' power and 'wasted' power respectively when the success and collision probabilities are β^i and ε^i respectively. The problem for each user is to determine an equilibrium transmission probability $\theta^{i,*}$ such that $\mathcal{P}^i_{\theta^{i,*}} = \mathcal{P}^i_{\beta^{i,*}} + \mathcal{P}^i_{\varepsilon^{i,*}}$ is minimum.

The problems being solved by N users are not independent. Transmission probability of one user (say user i) impacts the collision/success probabilities of all other users. This affects their transmission probabilities which in turn impact the transmission probability of user i. Thus, this is a game situation [16]. Each user attempts to minimize its own disutility (power) subject to its rate requirement. We assume that a user is not aware of the rate requirement of the other users. Moreover, since there is no provision for information exchange between the users, the CSI is also localized at the users, i.e., a user is not aware of the CSI of other users. Furthermore, since the users are only provided with a $(0, 1, e)$ feedback by the base station, the users cannot fully observe the actions taken by the other users. We view the situation as a repeated non-cooperative game with incomplete information. The solution concept that we target is that of the Nash equilibrium. In this case, at equilibrium, each user's transmission strategy in the long run can be viewed as a 'best response' to the long term transmission strategies of the rest of the users.

Before proceeding with the multiuser scenario, we analyze the single user scenario that provides us with key insights that aid in designing an efficient multiuser solution in the subsequent section.

IV. Single User Scenario

Consider the single user scenario as depicted in Figure 2. We assume that the user is split into two virtual entities: a transmitter and a scheduler. The user transmits over a block fading channel in a time slotted system. The scheduler has an objective of minimizing the long term average power expenditure subject to an average rate constraint (say $\bar{\rho}$)[2]. To meet this objective, in each slot, the scheduler determines the transmission rate (say U_n) based on the channel state (say X_n) and directs the transmitter to transmit at that rate. However, with a certain probability θ_n, the transmitter proceeds with the transmission whereas with a probability $1-\theta_n$, it is unable to proceed with the transmission. Since there are no collisions in the single user scenario, transmission probability is equal to success probability, i.e., $\theta_n = \beta_n$. In this case, the long term power consumption can be expressed as:

$$\bar{\mathcal{P}} = \limsup_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M} \theta_n P_n = \limsup_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M} \theta_n \pi(X_n),$$

where $\pi(\cdot)$ denotes the power allocation policy. Moreover, the long term throughput achieved by the user can be expressed as:

$$\bar{\mathcal{U}} = \liminf_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M} \beta_n U_n = \liminf_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M} \beta_n \mu(P_n), \quad (5)$$

where $\mu(\cdot)$ denotes the rate allocation policy.

The problem can be precisely expressed as:

$$\text{Minimize } \; \bar{\mathcal{P}} \; \text{ subject to } \bar{\mathcal{U}} \geq \bar{\rho}. \quad (6)$$

Note that the present problem is a generalization of dual of the problem considered in [29]. The analysis for determining the optimal transmission policy proceeds on similar lines as in [29]. The constrained problem in (6) can be converted into an unconstrained problem using the Lagrangian approach [30]. The unconstrained problem can be expressed as:

$$\text{Minimize } \; \bar{\mathcal{P}} - \lambda(\bar{\mathcal{U}} - \bar{\rho}), \quad (7)$$

where $\lambda \geq 0$ is referred to as the Lagrange Multiplier (LM).

It can be verified that for the present problem, the optimal power allocation policy $\pi^*(x)$ for a channel state $X_n = x$ can be expressed as:

$$\pi^*(x) = P_w(x) = (\lambda^* - \frac{N_0}{x}), \quad (8)$$

where N_0 is the power spectral density of the AWGN at the base station. For the given power allocation, the rate U_n can then be determined using the rate allocation policy $\mu(\cdot)$. Note that $P_w(\cdot)$ depends on the optimal LM λ^* which in turn is a function of the transmission probability θ. This optimal LM λ^*, which we also refer to as the 'waterfilling threshold' needs to be determined.

Remark 2: Note that in the single user scenario, the transmitter transmits with a probability θ and remains idle with probability $1-\theta$. The slots where the transmitter does not transmit can be considered to be equivalent to the slots where the channel state is extremely poor. For such slots, the Goldsmith-Varaiya (G-V) waterfilling power allocation scheme [29] does not transmit. It is easy to see that the G-V scheme is optimal for this case also, albeit with an appropriately scaled LM or the 'waterfilling threshold' as compared to the case where the transmitter transmits in each

[2]Since we deal with the single user scenario, in this section, we omit the superscript from the notation for notational simplicity.

slot. Effectively, as θ or equivalently β decreases, the scheduler perceives a progressively poorer channel. This results in a correspondingly higher value for the optimal LM λ^* resulting in higher power allocation in each channel state for satisfying the rate constraint.

Remark 3: It can be argued that for the single user scenario, in the optimal solution, the constraint is met with equality. Note that the power is an increasing convex function of the transmission rate. Now, suppose that the constraint is not met with equality, then a further reduction in power consumption can be achieved by transmitting at lower power in each channel state and thereby lower rate and then meeting the constraint with equality. This implies that if the constraint is not met with equality, the solution is not optimal. Hence, in the optimal solution, the constraint is met with equality.

Remark 4: Note that for the single user problem considered in (6), the waterfilling power allocation scheme (8) is optimal. For different values of transmission probability, the optimal LM λ^* takes different values. Hence in the multiuser situation, we employ (8) for each user i for determining the transmission power at a given channel state. The task that remains is to determine the equilibrium transmission probability $\theta^{i,*}$ (and thereby the success probability $\beta^{i,*}$) and the corresponding constraint satisfying LM $\lambda^{i,*}$ for each user.

V. Iterative Approach for Determining the Equilibrium Transmission Strategy in the Multiuser Scenario

In the previous section, we have discussed the single user scenario. In this section, we focus our attention on the multiuser scenario. In determining the equilibrium transmission probability within the multiuser setting, a user attempts to address the following tradeoff: if it were to transmit with too high probability, there could be too many collisions and wastage of power, on the other hand if it were to transmit with too low probability, it would need to transmit at higher power in each channel state in order to satisfy the rate constraint. The users attempt to achieve a balance between these conflicting objectives for arriving at a solution.

We now suggest a strategy for determining the equilibrium transmission probabilities. The essence of the solution strategy is the following: each user tunes its transmission probability iteratively so as to arrive at a Nash equilibrium. The users tune their transmission probabilities in the direction of the gradient of average power expenditure. Moreover, the waterfilling threshold is also iteratively tuned so as to ensure that the rate constraints are satisfied. Before providing details of the solution strategy, we convert the constrained problem in (4) into an unconstrained problem using the Lagrangian approach [30].

A. Lagrangian Approach

Let λ^i be a Lagrange Multiplier (LM). The unconstrained problem (corresponding to the problem in (4)) can be stated as:

$$\text{Minimize } \mathcal{L}^i(\theta^i, \lambda^i) = \bar{\mathcal{P}}^i - \lambda^i(\bar{U}^i - \bar{\rho}^i). \tag{9}$$

The objective is to determine the saddle point of the Lagrangian $\mathcal{L}^i(\theta^i, \lambda^i)$, i.e., to determine $\theta^{i,*}$ and $\lambda^{i,*}$ such that the following saddle point optimality conditions are satisfied:

$$\mathcal{L}^i(\theta^{i,*}, \lambda^i) \geq \mathcal{L}^i(\theta^{i,*}, \lambda^{i,*}) \geq \mathcal{L}^i(\theta^i, \lambda^{i,*}). \tag{10}$$

Note that the LM also acts as the waterfilling threshold. Hence, determining the equilibrium LM also results in determining the equilibrium waterfilling power allocation.

B. Three Timescale Stochastic Gradient Algorithm (TTSGA)

We now present a stochastic gradient based algorithm for determining transmission strategy. The essence of the algorithm is the following: in each slot a user i determines its transmission power $P_w^i(X_n^i)$ based on its channel state X_n^i using (8). User i transmits with probability θ_n^i that serves as an estimate of the equilibrium transmission probability $\theta^{i,*}$. User i tunes this estimate in the direction of the gradient of the Lagrangian in (9). In Section VI we prove that this strategy leads to a Nash equilibrium.

The Lagrangian in (9) can be expressed as:

$$\begin{aligned} \mathcal{L}^i(\theta^i, \lambda^i) &= \limsup_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} \left(\theta_n^i P_w(X_n^i) - \lambda^i(\beta_n^i U_n^i - \bar{\rho}^i)\right) \\ &= \limsup_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} g(X_n^i, U_n^i, \theta_n^i, \beta_n^i, \lambda_n^i), \end{aligned} \tag{11}$$

where we refer to $g(\cdot,\cdot,\cdot,\cdot,\cdot)$ as the immediate cost function. Let $\nabla^{\theta^i}\mathcal{L}^i(\cdot,\cdot)$ and $\nabla^{\lambda^i}\mathcal{L}^i(\cdot,\cdot)$ denote the partial gradient of $\mathcal{L}^i(\cdot,\cdot)$ w.r.t θ^i and λ^i respectively. At the saddle point, $\theta^{i,*}$ and $\lambda^{i,*}$ satisfy the following conditions:

$$\nabla^{\theta^i}\mathcal{L}^i(\theta^i, \lambda^i)\Big|_{\theta^i=\theta^{i,*}} = 0, \tag{12}$$

$$\nabla^{\lambda^i}\mathcal{L}^i(\theta^i, \lambda)\Big|_{\lambda^i=\lambda^{i,*}} = 0, \tag{13}$$

and the complementary slackness condition,

$$\lambda^{i,*}(\bar{\mathcal{U}}^i - \bar{\rho}^i) = 0. \tag{14}$$

Note that the Lagrangian in (11) is a time average of the immediate cost function that can not be determined a priori in a real time implementation setup. If the equilibrium LM $\lambda^{i,*}$ is known, we can use an iterative method that improves its estimate of *only* the equilibrium θ^i. Since the equilibrium LM $\lambda^{i,*}$ is also not known, we resort to a primal-dual method that determines *both* $\theta^{i,*}$ and $\lambda^{i,*}$ iteratively [30]. In order to ensure convergence of θ^i and λ^i iterates to the equilibrium $\theta^{i,*}$ and $\lambda^{i,*}$, the iterations proceed at different *timescales*, i.e., θ^i and λ^i values are updated at different rates. We iterate θ^i on a slower timescale, and λ^i on a faster timescale. This implies that θ^i is maintained constant for a large number of λ^i iterations. More specifically, as viewed from the λ^i iteration, the θ^i iterates appear to be almost constant while as viewed from the θ^i iteration, the λ^i iterates appear to be converged to the optimal value for the current value of θ^i. The can be done in two ways. The first way is to physically separate the timescales by updating the LM in each time slot and updating the probability after a large number of time slots.

The second way is by updating both quantities in each time slot, but carefully selecting the update sequences employed in the iteration. It can be shown that this has the same effect as that of the physical separation of time scales [31].

Let $\{a_n\}$ and $\{c_n\}$ be two positive sequences that have the following properties:

$$a_n \to 0, c_n \to 0; \ \sum_n a_n = \infty, \ \sum_n c_n = \infty; \\ \sum_k (a_n)^2 < \infty; \ \sum_n (c_n)^2 < \infty. \quad (15)$$

Fix $\theta^i = \theta$. The equilibrium LM for this transmission probability that ensures that the complementary slackness condition (14) is satisfied can be determined using the following iteration carried out in each slot:

$$\lambda^i_{n+1} = \lambda^i_n - a_n(J^i_n U^i_n - \bar{\rho}^i), \ \lambda^i_n \geq 0 \ \forall n, \quad (16)$$

where J^i_n is an indicator variable that is set to 1 if user i transmission is successful in slot n. (16) forms the 'waterfilling threshold update mechanism'. λ^i is the waterfilling threshold for user i, increasing this threshold results in an increase in power consumption, while decreasing this threshold results in a decrease in the average power consumption.

We now describe an approach for iterating the transmission probability θ^i in the direction of the gradient of the average power expenditure. Let $(2n-1)$ and $(2n)$ refer to odd and even numbered slots respectively. User i transmits with probability $(\theta^i_{2n-1} + \delta), \delta > 0, \delta << 1$, and $(\theta^i_{2n-1} - \delta)$ in odd and even numbered slots respectively at powers recommended by the waterfilling power allocation policy $P_w(\cdot)$ in (8). Based on the finite differences method [32], the gradient of the average power expenditure is determined and the transmission probability is updated in the direction of this gradient in odd numbered slots. This update equation can be expressed as:

$$\theta^i_{2n+1} = \pi_1 \left[\theta^i_{2n-1} - c_{2n-1} \left(\frac{\mathcal{P}^i_{2n-1} - \mathcal{P}^i_{2n}}{2\delta} \right) \right], \quad (17)$$

where $\pi_1[\cdot]$ is a projection operator that projects the probability iterates in the interval $[0, \omega]$ for some configuration parameter ω and $\mathcal{P}^i_n$ is an estimate of the average power expenditure for user i in slot n. Note that we do not have access to this estimate of the average power expenditure. In order to address this issue, we carry out simultaneous averaging of the power consumed. $\mathcal{P}^i_n$ is thus computed using the following recursive equation:

$$\mathcal{P}^i_{n+1} = \mathcal{P}^i_n + b_n \theta^i_n P^i_w(X^i_n), \quad (18)$$

where b_n is an update sequence that has the same properties as those of a_n and c_n in (15).

We update $\mathcal{P}^i_n$ on a faster timescale as compared to the probability update timescale. This is done by imposing additional properties on update sequences b_n and c_n explained below. This ensures that as viewed from the average power update timescale, the transmission probability appears to be almost constant; the physical interpretation being that one is computing the average power expenditure for a certain large time interval with fixed value of the transmission probability. Note that in (18), $\theta^i_n = \theta^i$ can be considered to be the current quasi-static value of the transmission probability as seen from the power update timescale. LM is updated at the fastest timescale because it determines the waterfilling threshold. This guarantees that the scheme uses the correct waterfilling threshold and hence the transmission power $P^i_w(\cdot)$ is the correct power which in turn leads to correct average power values $\mathcal{P}^i$.

The different timescales specified above can be realized by imposing the following additional requirements on the update sequences $\{a_n\}, \{b_n\}, \{c_n\}$ [31]:

$$\frac{b_n}{a_n} \to 0; \ \frac{c_n}{b_n} \to 0. \quad (19)$$

Practically, these timescales can be realized by having, e.g., $c_n = \frac{1}{n}, b_n = \frac{1}{n^{0.8}}, a_n = \frac{1}{n^{0.6}}$.

(16), (17) and (18) form the Three Timescale Stochastic Gradient Algorithm (TTSGA). In a nutshell, TTSGA consists of updating three quantities: LM that determines the waterfilling threshold, the average power consumption and transmission probability. These three quantities are updated on different timescales.

In Section VI-A, we show that if each user implements TTSGA then the transmission probability vector converges to a Nash equilibrium. Moreover, the LMs converge to values such that the rate constraints are satisfied.

VI. Analysis of TTSGA

In this section, we comment on several properties of TTSGA (17), (18) and (16) such as convergence and fairness. We begin with convergence analysis.

A. Convergence Analysis

In this section, we prove that:

- The probability iterates under TTSGA converge to a Nash equilibrium under certain conditions.
- Rate constraints are satisfied.

Let $\boldsymbol{\theta}_n = [\theta^1_n, \ldots, \theta^N_n]$ denote the vector of transmission probabilities of the users. For a fixed transmission probability vector $\boldsymbol{\theta} = [\theta^1, \ldots, \theta^N]$, let β^i denote the probability of successful transmission for user i. It can be expressed as:

$$\beta^i = \theta^i \prod_{j \neq i} (1 - \theta^j). \quad (20)$$

In this case, average power and LM update equations can be expressed as:

$$\mathcal{P}^i_{n+1} = \mathcal{P}^i_n + b_n \theta^i P^i_w(X^i_n), \quad (21)$$

and

$$\lambda^i_{n+1} = \lambda^i_n - a_n(\beta^i U^i_n - \bar{\rho}^i), \ \lambda^i_n \geq 0 \ \forall n. \quad (22)$$

Note that as long as $\beta^i > 0$, the stability of λ^i iterates is ensured. This is because there is no restriction on maximum transmission power in a slot, hence by transmitting at appropriate power levels and rates, one can ensure that the rate constraint is satisfied. This ensures that the λ^i iterates are stable. Equation (22) can be rewritten as:

$$\lambda^i_{n+1} = \lambda^i_n - a_n(\beta^i U^i_n - \beta^i \bar{U}^i_n + \beta^i \bar{U}^i_n - \bar{\rho}^i), \ \lambda^i_n \geq 0 \ \forall i, \quad (23)$$

where $\bar{U}_n^i$ is the running average of the transmission rate and $V_n^i \triangleq \beta^i(U_n^i - \bar{U}_n^i)$ forms a martingale difference sequence. We assume that the sequence $\{V_n^i\}$ satisfies:

$$\mathbf{E}[V_{n+1}^i | V_k^i, \lambda_k^i, k \leq n] = 0 \ \ \forall n. \tag{24}$$

Note that the LM determines the waterfilling threshold and thereby the power with which a user transmits in each channel state which in turn determines the average rate that the user would achieve. Hence $\bar{U}_n^i$ can be expressed as:

$$\bar{U}_n^i = F^i(\lambda_n^i), \tag{25}$$

for some continuously differentiable function $F^i(\cdot)$. Using (25), (23) can be expressed as:

$$\lambda_{n+1}^i = \lambda_n^i - a_n(V_n^i + \beta^i F^i(\lambda_n^i) - \bar{\rho}^i), \ \lambda_n^i \geq 0 \ \forall i, \tag{26}$$

Consider a 'fluid approximation' of (23) with the interpretation that we consider smaller and smaller slot lengths, so that power and transmission probability are interpreted as 'per unit time' quantities instead of 'per slot' quantities. Under this approximation, (22) can be considered to be a noisy discretization of the following ordinary differential equation (o.d.e.):

$$\dot{\lambda}^i(t) = -(\beta^i F^i(\lambda^i(t)) - \bar{\rho}^i). \tag{27}$$

The set of equilibria of the o.d.e. in (27) $H \triangleq \{\lambda^{i,*} : \beta^i F^i(\lambda^{i,*}) = \bar{\rho}^i\}$. The stability of the o.d.e. (27) allows us to comment on the convergence of the iterates in (22). Using Theorem 2, Chapter 2 (p. 15) of [33] we have the following:

Lemma 1: For a fixed $\boldsymbol{\theta}$ vector, the LM iterates in (22) converge to the set H of equilibria of (27).

Note that convergence of the LM iterates directly implies that the rate constraint is satisfied with equality.

Consider the transmission probability update equation:

$$\theta_{2n+1}^i = \pi_1 \left[\theta_{2n-1}^i - c_{2n-1} \left(\frac{\mathcal{P}_{2n-1}^i - \mathcal{P}_{2n}^i}{2\delta} \right) \right]. \tag{28}$$

For the sake of analysis, let us drop the projection operator. We will comment on the projection operation later in this section.

Let ∇_{θ^i} denote the gradient w.r.t. θ^i. (28) can be represented in the following form:

$$\theta_{m+1}^i = \theta_m^i - c_{m-1}\left(-\nabla_{\theta^i}\mathcal{P}^i + M_m^i\right), \tag{29}$$

where $m = 2n$ and $M_m^i = [\nabla_{\theta^i}\mathcal{P}^i - \nabla_{\theta^i}\mathcal{P}_{m-1}^i]$ is a martingale sequence that accounts for having access to only an estimate of the gradient of the average power expenditure ($\nabla_{\theta^i}\mathcal{P}_m^i$) and not the actual gradient ($\nabla_{\theta^i}\mathcal{P}^i$) itself. We assume that the sequence $\{M_n^i\}$ satisfies:

$$\mathbf{E}[M_{n+1}^i | M_k^i, \theta_k^i, k \leq n] = 0 \ \ \forall n. \tag{30}$$

Now, consider a limiting o.d.e. corresponding to (29) expressed as:

$$\dot{\theta}^i(t) = -\nabla_{\theta^i}\mathcal{P}^i(t). \tag{31}$$

The system wide vector o.d.e. can be expressed as:

$$\dot{\boldsymbol{\theta}}(t) = -\nabla_{\theta}\mathbf{P}(t), \tag{32}$$

where $\mathbf{P}$ is the power vector $[\mathcal{P}^1, \cdots, \mathcal{P}^N]$. Note that $\nabla_\theta \mathbf{P}(\cdot)$ acts as a Lyapunov function (See [33] Chapter 10, (10.2.1)), i.e.,

$$\frac{d}{dt}\mathbf{P}(t) = -||\nabla_{\boldsymbol{\theta}}\mathbf{P}(t)||^2 < 0 \tag{33}$$

Let $G \triangleq \{\boldsymbol{\theta} : \nabla_{\boldsymbol{\theta}}\mathbf{P} = 0\}$ denote the set of local minima of $\mathbf{P}$, i.e., the equilibrium points for this o.d.e.

We now prove that the Hessian $\mathcal{H}$ of $\mathbf{P}$ at $\boldsymbol{\theta} = \boldsymbol{\theta}^*$ under certain conditions is positive definite. This implies that $\boldsymbol{\theta}^* \in G$ denotes a stable equilibrium point of the o.d.e. (32) [34]. In fact, as argued below, the equilibrium is a Nash equilibrium.

In order to prove the positive definiteness of $\mathcal{H}$ at $\boldsymbol{\theta} = \boldsymbol{\theta}^*$, we show that it is strictly diagonally dominant which is a sufficient condition for positive definiteness ([35], Section 4.2.1).

Recall that $\mathcal{P}_{\theta^i}^i$ denotes the average power consumed by user i when the transmission probability is θ^i. It consists of two components: $\mathcal{P}_{\beta^i}^i$ which corresponds to the power consumed in successful transmissions and $\mathcal{P}_{\varepsilon^i}^i$ which corresponds to the power wasted in collisions. Let $p(x^i)$ denote the probability that user i has channel condition x^i. The average throughput for user i with a success probability of β^i can be expressed as:

$$\beta^i \sum_{x^i} p(x^i) \log_2(1 + \frac{P_{\beta^i}^i(x^i)x^i}{N_0}), \tag{34}$$

where $P_{\beta^i}^i(x^i)$ is the transmission power in channel state x^i when the success probability is β^i. From Remark 3, the constraint is satisfied with equality. Hence, using (14) and (34), we can claim that:

$$\beta^i \sum_{x^i} p(x^i) \log_2(1 + \frac{P_{\beta^i}^i(x^i)x^i}{N_0}) = \bar{\rho}^i$$

$$\implies \sum_{x^i} p(x^i) \log_2(1 + \frac{P_{\beta^i}^i(x^i)x^i}{N_0}) = \frac{\bar{\rho}^i}{\beta^i}. \tag{35}$$

Let $\bar{\rho}^i(x^i)$ denote the rate at which user i transmits in channel state x^i when $\beta^i = 1$. Note that since constraint is satisfied with equality,

$$\bar{\rho}^i = \sum_{x^i} p(x^i)\bar{\rho}^i(x^i) \tag{36}$$

Hence from (35) and (36),

$$\log_2(1 + \frac{P_{\beta^i}^i(x^i)x^i}{N_0}) = \frac{\bar{\rho}^i(x^i)}{\beta^i}$$

$$\implies P_{\beta^i}^i(x^i) = \frac{N_0}{x^i}(2^{\frac{\bar{\rho}^i(x^i)}{\beta^i}} - 1). \tag{37}$$

It can be seen that $P_{\beta^i}^i(x^i)$ is a convex decreasing function of β^i. Now,

$$\mathcal{P}_{\beta^i}^i = \sum_{x^i} p(x^i) P_{\beta^i}^i(x^i) = \sum_{x^i} p(x^i) \frac{N_0}{x^i}(2^{\frac{\bar{\rho}^i(x^i)}{\beta^i}} - 1). \tag{38}$$

Hence $\mathcal{P}_{\beta^i}^i$ is also a convex decreasing function of β^i. $\mathcal{P}_{\theta^i}^i$ can be expressed as:

$$\mathcal{P}_{\theta^i}^i = \frac{\theta^i}{\beta^i}\mathcal{P}_{\beta^i}^i. \tag{39}$$

Lemma 2: The following condition is a sufficient condition for the Hessian H to be positive definite:

$$\frac{1}{\theta^{i,*}} - \sum_{k \neq i} \frac{1}{(1-\theta^{k,*})} > 0. \quad (40)$$

Proof: Note that,

$$\frac{\partial(\mathcal{P}^i_{\theta^i})}{\partial\theta^i} = \sum_{x^i} p(x^i)\Big(\frac{N_0\bar{\rho}^i(x^i)\ln(2)}{(\theta^i)^2(\epsilon^i)^2}\Big)2^{\frac{\bar{\rho}^i(x^i)}{\beta^i}}, \quad (41)$$

where $\epsilon^i = \prod_{j\neq i}(1-\theta^j)$. Moreover for $k \neq i$,

$$\frac{\partial(\mathcal{P}^i_{\theta^i})}{\partial\theta^k} = \sum_{x^i} p(x^i)\Big(\frac{N_0 2^{\frac{\bar{\rho}^i(x^i)}{\beta^i}}(\epsilon^i\theta^i + \ln(2)\bar{\rho}^i(x^i))}{(1-\theta^k)^2(\epsilon^i)^2\theta^i x^i}\Big). \quad (42)$$

Furthermore,

$$\frac{\partial^2(\mathcal{P}^i_{\theta^i})}{\partial(\theta^i)^2} = \sum_{x^i} p(x^i)\Big(\frac{N_0\ln(2)\bar{\rho}^i(x^i)2^{\frac{\bar{\rho}^i(x^i)}{\beta^i}}(2\epsilon^i\theta^i + \ln(2)\bar{\rho}^i(x^i))}{x^i(\theta^i)^4(\epsilon^i)^3}\Big). \quad (43)$$

Finally,

$$|\frac{\partial^2(\mathcal{P}^i_{\theta^i})}{\partial(\theta^i\theta^k)}| = |\frac{\partial^2(\mathcal{P}^i_{\theta^i})}{\partial(\theta^k\theta^i)}| = \sum_{x^i} p(x^i)\Big(\frac{N_0\ln(2)\bar{\rho}^i(x^i)2^{\frac{\bar{\rho}^i(x^i)}{\beta^i}}(2\epsilon^i\theta^i + \ln(2)\bar{\rho}^i(x^i))}{x^i(\theta^i)^3(\epsilon^i)^3(1-\theta^k)}\Big). \quad (44)$$

From [35], Section 3.4.10, the following two conditions are sufficient for strict diagonal dominance of H at $\boldsymbol{\theta} = \boldsymbol{\theta}^*$; first: $|\frac{\partial^2(\mathcal{P}^i_{\theta^i})}{\partial(\theta^i)^2}| > 0 \; \forall i$ which is easily verified from (43); and second:

$$\Bigg[|\frac{\partial^2(\mathcal{P}^i_{\theta^i})}{\partial(\theta^i)^2}| - \sum_{k\neq i}|\frac{\partial^2(\mathcal{P}^i_{\theta^i})}{\partial(\theta^i\theta^k)}|\Bigg]_{\boldsymbol{\theta}=\boldsymbol{\theta}^*} > 0. \quad (45)$$

With some algebraic manipulation on (43) and (44), it can be shown that this condition is equivalent to:

$$\frac{1}{\theta^{i,*}} - \sum_{k \neq i} \frac{1}{(1-\theta^{k,*})} > 0. \quad (46)$$

■

Positive definiteness of Hessian enables us to claim the following:

Theorem 1: If there is a stable equilibrium point $\boldsymbol{\theta}^* \in G$, such that (46) is satisfied, then for any initial transmission probabilities $\boldsymbol{\theta}_0$ the dynamics $\boldsymbol{\theta}(t)$ in (32) converge to $\boldsymbol{\theta}^*$ asymptotically.

Note that so far, we have studied the convergence of the o.d.e. (32). However, we are really concerned about the convergence of the primal TTSGA iterates in (28). Theorem 2, Chapter 2 (p. 15) of [33] allows us to claim the convergence of these iterates. The readers are refered to the Appendix for a statement and discussion of this theorem (Theorem 2, Chapter 2 (p. 15) of [33]) which illustrates the applicability of the theorem in the present context. We thus have:

Theorem 2: If there is a stable equilibrium point $\boldsymbol{\theta}^* \in G$ of the o.d.e. (32), then for any initial transmission probabilities $\boldsymbol{\theta}_0$ the $\boldsymbol{\theta}_n$ iterates in (28) converge to $\boldsymbol{\theta}^*$.

Finally, using Theorem 2, Chapter 6 (p. 66) of [33] we claim that the coupled iterates converge, Again the readers are refered to the Appendix for a statement and discussion of Theorem 2, Chapter 6 (p. 66) of [33]. We thus have:

Theorem 3: The coupled iterates (λ^i, θ^i) in (16) and (28) converge to their respective equilibrium values.

Note that since the θ^i iterates are updated on the slower timescale, these iterates see converged values of LMs at each update instant. Finally, when the $\boldsymbol{\theta}$ iterates converge to the equilibrium, the corresponding LM values while ensuring that rate constraints are satisfied, also determine the correct long term power consumption.

Finally, we had ignored the projection operator $\pi_1(\cdot)$ for the sake of analysis. If the iterates converge to an interior point of $[0, \omega]$, the error introduced by the projection operation is asymptotically negligible and can be ignored.

Remark 5: One way to ensure the condition in (46) is to enforce a limit on the maximum probability with which a user can transmit in a slot. We already have this mechanism in place through the limit ω on transmission probability. The second term in (46) takes its maximum value when $\theta^{k,*} = \omega, \; \forall k$, while the first term takes its minimum value when $\theta^{i,*} = \omega$. For the minimum value of the LHS to be greater than 0 we require that:

$$\frac{1}{\omega} - \frac{N-1}{1-\omega} > 0 \implies \omega < \frac{1}{N}. \quad (47)$$

Condition (47) forces the transmission probability iterates in the interval $[0, \frac{1}{N})$. If there exists an equilibrium point in the interval $[0, \frac{1}{N})$, the iterates converge to such an equilibrium.

B. Equilibrium as a Best Response

Nash equilibrium embodies the notion of best response offered by a player to the strategies of the other players. Unilateral deviation from the equilibrium does not result in an increase in the utility for any player. In the present case also the transmission probability is a best response to the transmission probabilities of the other players. Unilateral deviation from the equilibrium transmission probability does not result in a decrease in the average power consumption for a player. This is because decreasing the transmission probability by, say, user i from its equilibrium value decreases both - its success probability β^i and collision probability ε^i. This increases $\mathcal{P}^i_{\beta^i}$ and reduces $\mathcal{P}^i_{\varepsilon^i}$; the net effect being that there is an increase in the overall power consumption. On the other hand, increasing the transmission probability from its equilibrium value increases β^i but also increases ε^i. This reduces $\mathcal{P}^i_{\beta^i}$ but increases $\mathcal{P}^i_{\varepsilon^i}$; the net effect being that there is again an increase in the overall power consumption.

C. Multiuser Penalty

Let $\mathcal{P}^i_1$ ($\beta^i = 1$) denote the average power consumed by a user i in the single user scenario with successful transmission in every slot. In the multiuser scenario, $\beta^i < 1$. Since the user is able to successfully communicate only during a fraction of the slots, the rate constraint appears to be appropriately scaled, resulting in a corresponding scaling of the LM or the waterfilling threshold. This results in a corresponding increase in the power consumption. This is the penalty that the user pays in operating in a multiuser environment. We term this penalty as the *multiuser penalty*. Note that larger the number of users, potentially larger is the penalty paid by the user for obtaining a certain throughput.

D. Fairness

A user does not have an incentive for arbitrarily increasing the transmission probability, it can be increased only when the reduction in power consumption due to increase in success probability outweighs the corresponding wastage due to increased probability of collision. The useful power consumption depends on two factors: the rate constraint and average channel condition. A user having a high rate constraint would require high rates in each channel state as compared to a user having a lower rate constraint but with same channel condition. A user with higher rate constraint would therefore require more 'useful' power. Increasing the transmission rate increases the success probability and reduces the 'useful' power requirement. Therefore, such a user would have a higher transmission probability but this also results in higher collision rate and higher 'wasted' power. Similar arguments can be made for users with same rate requirement and different average channel conditions. This discussion implies that users having higher rate requirement or poorer channel consume more power thus ensuring fairness.

In the next section, we simulate TTSGA in a discrete event simulator. Our objective is to demonstrate that TTSGA satisfies the rate constraints through simulation studies.

VII. Experimental Evaluation

In this section, we simulate a single cell wireless system where N users communicate with a base station on the uplink. The users require average rate guarantees. Packets are generated at the application layer and are possibly of variable sizes. At the MAC layer, we assume that each MAC fragment is of constant length equal to $\ell = 2000$ bits. We assume that the system has a bandwidth W of 10 MHz. Each user transmits at a constant power of 1 Watt. We assume $\omega = 0.1$.

We simulate a Rayleigh channel for each user. For a Rayleigh model, channel state X^i is an exponentially distributed random variable with mean α^i and probability density function expressed as $f_X(x) = \frac{1}{\alpha^2}\exp\left(\frac{-x^2}{2\alpha^2}\right)$, $x \geq 0$. We discretize the channel into eight equal probability bins, with the boundaries specified by { (-∞, -8.47 dB), [-8.47 dB, -5.41 dB), [-5.41 dB, -3.28 dB), [-3.28 dB, -1.59 dB), [-1.59 dB, -0.08 dB), [-0.08 dB, 1.42 dB), [1.42 dB, 3.18 dB), [3.18 dB, ∞) }. For each bin, we associate a channel state and the state space $\mathcal{X}$ = { -13 dB, -8.47 dB, -5.41 dB, -3.28 dB, -1.59 dB, -0.08 dB, 1.42 dB, 3.18 dB}. This discretization of the state space of X^i has been justified in [36]. For the sake of simplicity, we assume that the transmission rate for user i in slot n can be determined using the following capacity relation:

$$U_n^i = W \times \log_2(1 + \frac{P_w X_n^i}{N_0 W}). \tag{48}$$

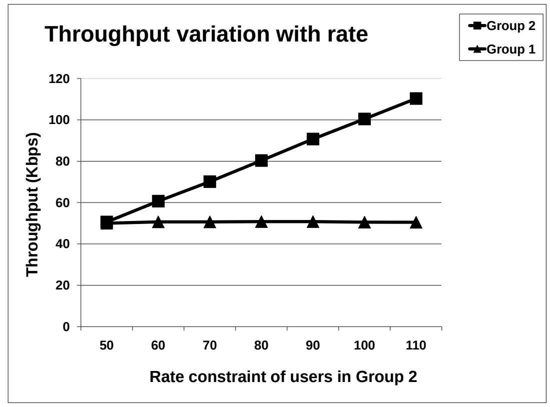

Fig. 3. Throughput for various average rate constraints

We consider a system with $N = 20$ users. We divide the users into 2 groups (Group 1 and Group 2) of 10 users each. We consider two scenarios: rate variation and channel variation. We present the results after averaging over 20 simulation runs each consisting of simulating the algorithm for $100,000$ slots.

Scenario 1: Rate Variation: In this scenario, we vary the average rate constraints for the users in Group 2 in successive experiments while keeping the average rate constraints for the users in Group 1 constant in all the experiments. For Group 2, the rate constraints are varied as 50-110 Kbps in steps of 10 Kbps in successive experiments, while the rate constraints for Group 1 are kept at 50 Kbps in all experiments. The mean channel state α for all the users is kept at -3.28 dB (0.4698) for all the experiments. In each slot, we generate the channel state using the exponential distribution with mean α and subsequently discretize it using the probability bins as mentioned above. Each user makes the transmission decision using its transmission probability θ_n^i in each slot. Based on the feedback received from the base station, it then determines the new value of the transmission probability. We select two users, at random from Group 1 and Group 2. For these users, we determine the power consumed, the stable transmission probabilities and rate achieved within each experiment and plot these in Figures 3, 4 and 5 respectively. It can be seen from Figure 3 that the rate constraint is satisfied. Moreover, from Figures 4, 5 it can be seen that as the rate constraint is increased, the power expended and the transmission probability increase.

Scenario 2: Channel Variation: In this scenario, we vary the average channel state for the users in Group 2 in successive experiments while keeping the average channel state for the users in Group 1 constant in all experiments. For the users in Group 2, the average channel state α is varied as 0.05 (-13 dB), 0.1422 (-8.47 dB), 0.2877 (-5.41 dB), 0.4698 (-3.28 dB), 0.6934 (-1.59 dB), 0.9817 (-0.08 dB), 1.3867 (1.42 dB) in successive experiments, while the av-

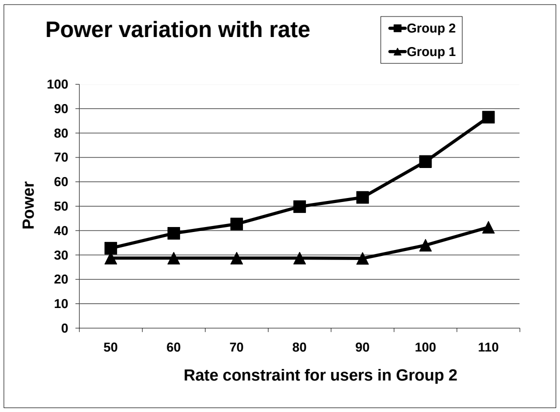

Fig. 4. Power expended for various average rate constraints

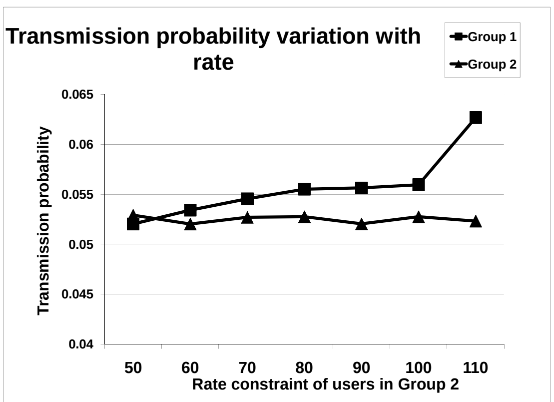

Fig. 5. Transmission probability (TP) for various average rate constraints

erage channel state for the users in Group 1 is kept at 0.4698 $(-3.28\ \text{dB})$ in all experiments. The average rate constraint is kept constant at 50 Kbps for all users in all experiments. In each time slot, we generate the channel state using the exponential distribution with mean α and subsequently discretize it using the probability bins as mentioned above. We select two users at random from Group 1 and Group 2. For these users, we determine the power consumed, the stable transmission probabilities and rate achieved within each experiment and plot these in Figures 6, 7 and 8 respectively. 6 demonstrates that the rate constraints are satisfied. Moreover, it can be seen from Figures 7, 8 that as the average channel state improves, the transmission probability reduces thus reducing the average power expenditure.

VIII. Practical Implementation

In this section, we describe a protocol for practically implementing TTSGA within infrastructure based IEEE 802.11 [37]. IEEE 802.11 does not offer any QoS guarantees in terms of satisfying rate constraints. By implementing the proposed algorithm, we can provide QoS guarantees. The protocol is based on a modification of IEEE 802.11 Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) access framework. In IEEE 802.11 setting, base station is refered to as access point (AP).

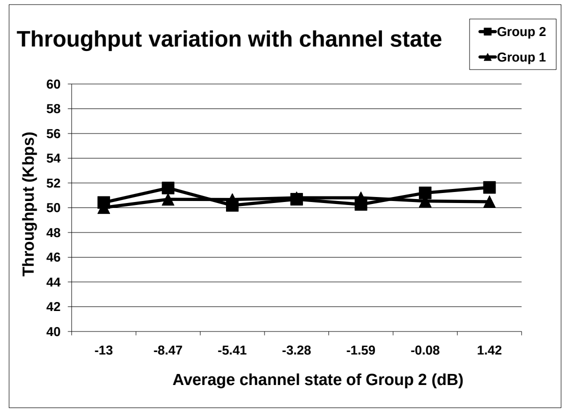

Fig. 6. Throughput for various average channel states

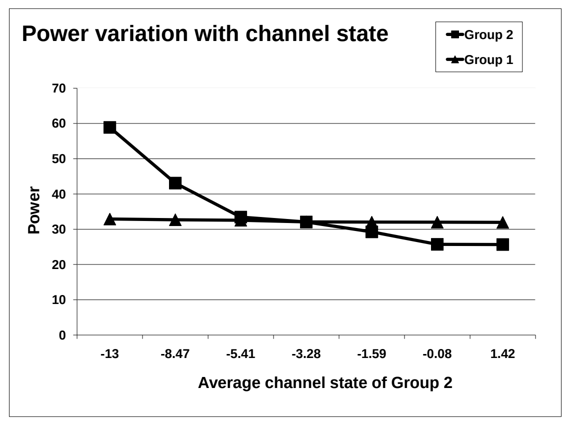

Fig. 7. Power expended for various average channel states

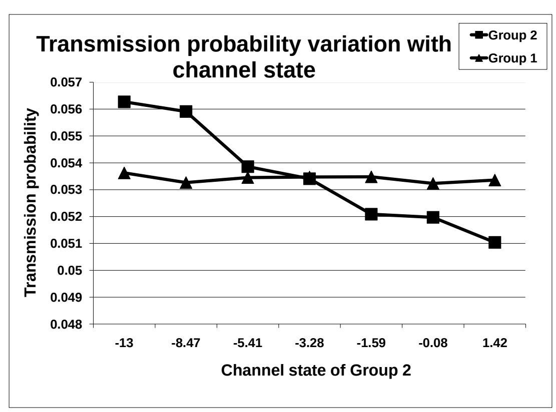

Fig. 8. Transmission probability (TP) for various average channel states

We begin by first discussing in brief the operation of CSMA/CA protocol employed in IEEE 802.11.

A. IEEE 802.11 CSMA/CA Protocol

In CSMA, a user has to first sense the channel for a predetermined amount of time. If the channel is sensed 'idle' then the user proceeds with its transmission. If the channel is sensed 'busy' then the user has to defer its transmission for a random duration of time (backoff). This is done to reduce the collision probability.

Wireless local area networks (WLANs) have the 'hidden terminal' problem, wherein a transmitter T_1 sending data to a receiver R that is not in the transmission range of another transmitter T_2 does not know whether T_2 is transmitting to R. This can lead to collisions at R. This hidden terminal problem is avoided through the Collision Avoidance (CA) mechanism of IEEE 802.11. The CA mechanism involves transmission of Request to Send (RTS) packet by transmitter T_1 to receiver R. The receiver, if idle, sends the Clear to Send (CTS) packet back to T_1. This alerts all the transmitters in the range of R about the data transfer between T_1 and R. The other transmitters then suspend their transmissions for the duration of the data transfer. Transmission of the Acknowledgement (ACK) packet from R to T_1 signals successful transfer.

If T_1 after transmitting RTS does not receive CTS for a certain period of time, it infers that it was involved in a collision. In order to avoid further collisions, it backs off or defers its transmission using an 'exponential backoff' mechanism prescribed by the protocol. This mechanism consists of backing off randomly by selecting a backoff duration from a window $[0, W]$ referred to as the contention window. Each time a transmitter is involved in a collision, it doubles its contention window and chooses a random backoff duration from that window.

B. A Protocol for Practical Implementation of TTSGA

We now describe a protocol for practically implementing TTSGA. This protocol modifies the exponential backoff mechanism of the IEEE 802.11 CSMA/CA protocol. A user first contends for the channel by transmitting a RTS packet based on the current transmission probability computed by TTSGA. The RTS packet is sent at the most robust rate to guard against channel fading. CSI can be estimated by the users based on a pilot transmitted by the access point. In a TDD system, the same CSI can be used by the users for uplink transmissions.

There might be multiple users contending for the channel by transmitting RTS packets to the access point. If there is no collision between these RTS packets, the access point responds with a CTS packet. This packet also contains information regarding rate at which the user should transmit based on the CSI. If the user receives the CTS packet, it utilizes the transmission rate information in the CTS packet and transmits accordingly. If the access point receives the packet correctly, it sends an ACK packet back to the user. The user then enters the contention mode again by transmitting the RTS packet with a recomputed transmission probability based on TTSGA.

When the user receives a CTS packet corresponding to a different user, it suspends its transmission till it receives an ACK packet corresponding to that data transfer. This signals the completion of the transmission between the access point and the associated user. Reception of the appropriate CTS packet allows the user to determine whether it was involved in a collision or not. If an appropriate CTS packet is received, the user determines the number of packets that can be transmitted in the Data part of the slot and transmits those packets. It then recomputes the transmission probability based on whether the transmission was successful or not. Thereafter, the user contends for the channel by transmitting a RTS packet with the newly computed transmission probability.

C. Practical Implementation of TTSGA

Based on online primal-dual computations in (16), (17), (18) and the protocol suggested in previous sub-section, user i implements the access control scheme. The number of packets to be transmitted is determined using (8). User i transmits with probability $\theta^i + \delta$ in odd numbered attempts and with probability $\theta^i - \delta$ in even numbered attempts. The transmission probability is adjusted in odd numbered attempts. If a transmission is successful, U^i packets are received at the access point and an acknowledgement packet is received and the LM λ^i is appropriately updated. The algorithm thus continues. The complete scheme is explained in Algorithm 1.

Initialize the LM $\lambda_0^i \leftarrow 0$, $\theta_0^i \leftarrow \theta_0$, $n \leftarrow 1$, channel state $X_0^i \leftarrow 0$ average power estimate $\mathcal{P}_0^i \leftarrow 0$.
while TRUE **do**
    Transmit RTS packet with probability θ_n^i.
    if CTS received **then**
        Use CSI X_n^i to determine U_n^i.
        Transmit U_n^i packets.
    else
        $U_n^i \leftarrow 0$.
    end if
    if ACK received **then**
        $J_n^i \leftarrow 1$.
    else
        $J_n^i \leftarrow 0$.
    end if
    Update the LM λ_n^i using (16).
    Update transmission probability θ_n^i using (17).
    Update average power estimate $\mathcal{P}_n^i$ using (18).
    $n \leftarrow n + 1$.
end while

Algorithm 1: Three Timescale Stochastic Gradient Algorithm (TTSGA)

Remark 6: Note that in this paper, we assume that the users transmit at a 'reliable' rate. Under this assumption, there are no transmission errors. The purpose of including ACK in the protocol is to indicate to the other users about the end of a transmission so that they can start contending for channel access. The case where transmission errors do occur and

ACK reception (non-reception) is an indication of successful (unsuccessful) transmission forms part of future investigation.

IX. Conclusions

In this paper, we have considered uplink transmissions in a single cell multiuser wireless system. The base station does not coordinate the transmission of the users, hence the users employ random access communication. In each slot, the users obtain a $(0, 1, e)$ feedback from the base station. The users have an objective of minimizing their long term power consumption while achieving certain long term average rates. We have modeled the situation as a constrained repeated non-cooperative game where users have knowledge of their utility function only. We have proposed a three timescale stochastic gradient algorithm (TTSGA) in order to tune their transmission probabilities. The algorithm includes a 'waterfilling threshold update mechanism' that appropriately tunes the threshold for each user and ensures that the rate constraints are satisfied. We have proved that the transmission strategies converge to a Nash equilibrium and that the rate constraints are satisfied. Moreover, our simulation studies have also demonstrated that the rate constraints are satisfied. Finally, we have illustrated how this algorithm can be employed to provide QoS in IEEE 802.11 based wireless network.

Appendix

In this appendix, we discuss certain results in literature that are applicable for proving convergence properties of the proposed algorithm in Section VI-A.

Assume that we are interested in solving an equation of the form:

$$X = h(X), \tag{49}$$

where X is a variable and $h(\cdot)$ is a function from $\mathbb{R}$ onto itself. Now consider the case where $h(\cdot)$ takes the form $h(X) = \nabla f(X) - X$, for some cost function $f(\cdot)$. In that case, (49) takes the form:

$$\nabla f(X) = 0, \tag{50}$$

which is related to the problem of finding a minimum of the function $f(\cdot)$. Assume that the exact functional form of $h(\cdot)$ is not known or that evaluation of $h(\cdot)$ is difficult, but we have an access to $h(X) + M$, M being a random noise term, i.e., we have access to a noise corrupted version of $h(X)$. In that case, the following iterative algorithm can be employed for solving (49):

$$X_{n+1} = X_n + a_n \left(h(X_n) + M_{n+1} \right), \tag{51}$$

The resulting algorithm is called as a Stochastic Approximation (SA) algorithm where the properties in (15) are imposed on the step size sequence $\{a_n\}$.

The o.d.e. method [38], [39] can be used for analyzing the asymptotic properties of SA algorithms such as (51). We assume that the noise sequence $\{M_n\}$ satisfies:

$$\mathbf{E}[M_{n+1} | M_m, X_m, m \leq n] = 0 \;\; \forall n. \tag{52}$$

Let $\sup_n ||X_n|| < \infty$. Consider a well-posed o.d.e.:

$$\dot{x}(t) = h(x(t)), \tag{53}$$

which is assumed to have a globally asymptotically stable attractor set J. The following theorem (Theorem 2, Chapter 2 of [33]) can be used for analyzing the asymptotic behavior of the SA algorithm (51) given that the o.d.e. (53) has a stable attractor set J.

Theorem 4: Almost surely, the sequence $\{X_n\}$ generated by (51) converges to a (possibly sample path dependent) compact connected internally chain transitive invariant set of (53) (i.e., J).

In many situations, one needs to solve two equations that are dependent on each other (coupled), simultaneously. These equations can be of the form:

$$X = h_1(X, Y),$$
$$Y = h_2(X, Y). \tag{54}$$

As before, assume that only noisy measurements of $h_1(\cdot, \cdot)$ and $h_2(\cdot, \cdot)$ are available. In such cases, one can write SA algorithm for each of the equations as:

$$X_{n+1} = X_n + a_n \left(h_1(X_n, Y_n) + M^1_{n+1} \right), \tag{55}$$
$$Y_{n+1} = Y_n + b_n \left(h_2(X_n, Y_n) + M^2_{n+1} \right), \tag{56}$$

where $\{M^1_n\}, \{M^2_n\}$ are noise sequences that have properties specified in (52) and the sequences $\{a_n\}$ and $\{b_n\}$ satisfy properties in (15) and $\frac{a_n}{b_n} \rightarrow 0$. Convergence of coupled sequences as in (55) and (56) has been analyzed in Theorem 2, Chapter 6 (p. 66) of [33]. The line of analysis is the following:

- Assume that the iterates X_n, Y_n have stable equilibrium points.
- Analyze the convergence of o.d.e. corresponding to (55) with fixed value of $Y_n = y$, i.e.
$$\dot{X}(t) = h_1(X(t), y). \tag{57}$$
- Let $B(y)$ characterize the equilibrium of the o.d.e. corresponding to X_n iterates for $Y_n = y$. Now analyze the o.d.e
$$\dot{Y}(t) = h_2(B(y(t)), y(t)) \tag{58}$$
which has a globally asymptotically stable equilibrium y^*.

Theorem 2, Chapter 6 (p. 66) of [33] formalizes the above steps and claims the following:

Theorem 5: $(X_n, Y_n) \rightarrow (B(y^*), y^*)$,

i.e., the coupled iterates converge to their respective equilibrium values.

Note that this scenario is applicable for the algorithm considered in this paper which allows us to apply these results to argue the convergence of the algorithm.

References

[1] T. S. Rappaport, *Wireless Communications Principles and Practice*. Prentice Hall, 2002.

[2] D. P. Bertsekas and R. Gallager, *Data Networks*. Prentice Hall, 1987.

[3] R. Rom and M. Sidi, *Multiple Access Protocols: Performance and analysis*. Springer-Verlag, 1989.

[4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[5] S. Shakkottai, T. S. Rappaport, and P. C. Karlsson, "Cross-Layer Design for Wireless Networks," *IEEE Communications Magazine*, vol. 41, no. 10, pp. 74–80, 2003.

[6] B. Hajek, A. Krishna, and R. O. LaMaire, "On the Capture Probability for a Large Number of Stations," *IEEE Transactions on Communications*, vol. 45, no. 2, pp. 254–260, 1997.

[7] M. K. Tsatsanis, R. Zhang, and S. Banerjee, "Network Assisted Diversity for Random Access Wireless Networks," *IEEE Transactions on Signal Processing*, vol. 48, no. 3, pp. 702–711, 2000.

[8] R. Zhang, N. D. Sidiropoulos, and M. K. Tsatsanis, "Collision Resolution in Packet Radio Networks using Rotational Invariance Techniques," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 146–155, 2002.

[9] X. Qin and R. A. Berry, "Opportunistic Splitting Algorithms for Wireless Networks," in *Proceedings of IEEE INFOCOM*, vol. 3, Mar. 2004, pp. 1662–1672.

[10] B. Tsybakov, "Packet Multiple Access for Channel with Binary Feedback, Capture and Multiple Reception," *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 1073–1085, 2006.

[11] X. Qin and R. A. Berry, "Exploiting Multiuser Diversity for Medium Access Control in Wireless Networks," in *Proceedings of IEEE INFOCOM*, vol. 2, Mar. 2003, pp. 1084–1094.

[12] S. Adiredy and L. Tong, "Exploiting Decentralized Channel State Information for Medium Access Control in Wireless Networks," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 537–561, 2005.

[13] Y. Yu and G. B. Giannakis, "Opportunistic Medium Access for Wireless Networking Adapted to Decentralized CSI," *IEEE Transactions on Wireless Communications*, vol. 5, no. 6, pp. 1445–1455, 2006.

[14] R. H. Gau, "Performance Analysis of Slotted ALOHA in Interference Dominating Wireless Ad Hoc Networks," *IEEE Communication Letters*, vol. 10, no. 5, pp. 402–404, 2006.

[15] X. Qin and R. A. Berry, "Distributed Approaches for Exploiting Multiuser Diversity in Wireless Networks," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 392–413, 2006.

[16] D. Fudenberg and J. Tirole, *Game Theory*. MA: MIT Press, 1995.

[17] E. Altman, R. El-Azouzi, and T. Jimenez, "Slotted Aloha as a Game with Partial Information," *Computer Networks*, no. 45, pp. 701–713, 2004.

[18] A. B. MacKenzie and S. B. Wicker, "Selfish Users in Aloha: A Game-Theoretic Approach," in *Proceedings of IEEE VTC, Fall*, vol. 3, 2001, pp. 1354–1357.

[19] V. S. Borkar and A. A. Kherani, "Random Access in Wireless Ad Hoc Networks as a Distributed Game," in *Proceedings of WiOpt*, 2004.

[20] E. Altman, D. Barman, and R. E.-A. T. Jimenez, "A Game Theoretic Approach for Delay Minimization in Slotted Aloha," in *Proceedings of IEEE ICC*, 2004.

[21] S. Lu, V. Bharghavan, and R. Srikant, "Optimal Power and Retransmission Control Policies for Random Access Systems," *IEEE/ACM Transactions on Networking*, vol. 12, no. 6, pp. 1156–1166, Dec. 2004.

[22] A. B. MacKenzie and S. B. Wicker, "Game Theory in Communications: Motivation, Explanation, and Application to Power Control," in *Proceedings of IEEE GLOBECOM*, vol. 2, 2001, pp. 821–826.

[23] T. Heikkinen, "A Potential Game Approach to Distributed Power Control and Scheduling," *Computer Networks*, vol. 50, no. 13, pp. 2295–2311, 2006.

[24] Y. Jin and G. Kesidis, "Equilibria of Noncooperative Heteregenous Users of an ALOHA Network," *IEEE Communication Letters*, vol. 6, no. 7, pp. 282–284, July 2002.

[25] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A Survey on Networking Games in Telecommunications," *Computers and Operations Research*, vol. 33, no. 2, pp. 286–311, 2004.

[26] A. B. MacKenzie and L. A. DaSilva, *Game Theory for Wireless Engineers*. Morgan and Claypool, 2006.

[27] I. Menache and N. Shimkin, "Efficient Rate-Constrained Nash Equilibrium in Collision Channels with State Information," in *Proceedings of IEEE INFOCOM*, Apr. 2008, pp. 403–411.

[28] R. A. Berry and R. G. Gallager, "Communication over Fading Channels with Delay Constraints," *IEEE Transactions on Information Theory*, vol. 48, no. 5, pp. 1135–1149, 2002.

[29] A. Goldsmith and P. Varaiya, "Capacity of Fading Channels with Channel Side Information," *IEEE Transactions on Information Theory*, vol. 43, no. 6, pp. 1986–1992, 1997.

[30] D. P. Bertsekas, *Nonlinear Programming*. Belmont, MA: Athena Scientific, 1999.

[31] V. S. Borkar, "Stochastic Approximation with Two Time Scales," *Systems and Control Letters*, vol. 29, pp. 291–294, 1996.

[32] J. C. Spall, *Introduction to Stochastic Search and Optimization*. Wiley, 2003.

[33] V. S. Borkar, *Stochastic Approximation - A Dynamical Systems Viewpoint*. Hindustan Publishing Agency and Cambridge University Press, 2008.

[34] J. B. Rosen, "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," *Econometrica*, vol. 33, pp. 520–534, 1965.

[35] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd ed. JHU Press, 1996.

[36] H. Wang and N. Mandayam, "A Simple Packet Transmission Scheme for Wireless Data over Fading Channels," *IEEE Transcations on Communications*, vol. 52, no. 7, pp. 1055–1059, 2004.

[37] LAN/MAN Committee, *IEEE Std 802.11-2007: IEEE Standard for Information Technology - Telecommunications and Information Exchange between Systems - Local and Metropolitan Area Networks - Specific Requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications* . IEEE Computer Society, March 2007.

[38] V. S. Borkar and S. P. Meyn, "The ODE Method for Convergence of Stochastic Approximation and Reinforcement Learning," *SIAM Journal on Control and Optimization*, vol. 38, pp. 447–469, 2000.

[39] H. J. Kushner and G. G. Yin, *Stochastic Approximation Algorithms and Applications*. New York: Springer-Verlag, 1997.