Discrete Rate Scheduling for Power Efficient Transmission over Wireless Link

Premal Shah and Abhay Karandikar
Information Networks Laboratory
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Mumbai 400 076, India

***Abstract*— While scheduling transmissions in a wireless communications system can result in significant power savings, a realistic scheduling scheme must take into acvount practical constraints that apply to wireless transmitters. Here, we examine scheduling actions for power efficiency under the constraints that physical transmission may be carried out at a set of fixed K non-zero rates. For this class of schedulers, we derive the optimal scheduling policy under specified average delay constraints for the simple case when $K = 1$ under constant channel conditions. We also derive bounds on both the achievable delays as well as power performances for a general K-rate scheduler**

***Index Terms*—Power Efficiency, Packet Scheduling, Bulk Queues**

I. Introduction

Power efficiency has always been an important design challenge for wireless networks as it impacts the active battery life of the mobile device, a critical performance metric for any network offering mobility services. Power saving through scheduling in a wireless channel accrues from two different means [1].

- The wireless channel being time-varying, a scheduler can simply defer the transmission of backlogged packets during 'bad' channel states or fades to 'better' channel states. This brings down the average power consumption at the cost of a higher delay for packets arriving during the fades.
- For most transmission schemes, (in wireless as well as wired channels), power (P) increases as a convex function of the physical transmission rate (R). An example is the Shannon's capacity relation for the AWGN channel of unit bandwidth and a noise level of variance σ^2

$$P = \sigma^2(e^R - 1)$$

  The $P - R$ relation is convex even for the practical communication schemes. This allows us to save power simply by appropriately sizing the batches of transmissions at the cost of an increased buffer delay.

Apart from the concept itself, a difference also exists in the implementation and realisation of the delay-power tradeoff from these two means. To tap the gains due to the fading nature of the channel, the transmitter need not change the actual physical transmission rates. Indeed gains, would be possible even if the $P - R$ relation were not convex. This is illustrated in [2], which assumes the $P - R$ relation to be linear. In this paper, we shall focus on the gains that can be achieved due to the latter by suitably varying the transmission rates. For early work on this, refer [1], [3], [4], [5]. In [4], the author provides a comprehensive survey of the problem. Most of these works seek to optimize average power consumption ($\bar{P}$) subject to an average delay ($\bar{D}$). Optimal and close to optimal scheduling techniques towards this end have been proposed.

Although practical systems with variable transmission rates exist and the same may be realized in several ways [6], it is unlikely that transmission rates can be varied in an arbitrary fashion. Not only would this require transmitters capable of varying transmission rates continuously, but also a considerable protocol overhead and receiver complexity. However, in all the previous works related to this problem, the scheduler uses the finest granularity of transmission rate. In this paper, we argue that a realistic scheduling scheme must take into account the practical limitations. Accordingly, we assume that the transmitter can transmit only at a finite set of pre-configured rates denoted by $\mathcal{N}^K : \{0, N_1, N_2, ..., N_K\}$; with K denoting the number of non-zero rates and formulate the problem of power efficient scheduling. The primary contribution of this paper is to derive an optimal scheduling policy for the simple case when $K = 1$ under constant channel conditions. We also provide a lower bound on the power per-

Abhay Karandikar is the corresponding author. His email address is karandi@ee.iitb.ac.in and Fax- +91-22-25723707

formance of a general K-rate scheduler.

II. Augmented Model

We use the same basic model, used in some of the earlier works, notably [1]. This consists of a slotted communication system with a single transmitter and a single receiver or a point-to-point link and is applicable to an uplink channel.

1) In a slot n, a_n denotes the number of packet arrivals and u_n the number of packets transmitted. The packet arrival process is assumed to be iid with Prob$[a_n = i] = p_i$. The average arrival rate $E[a_n]$ is denoted by λ.
2) A packet arrived in slot n is available for transmission only from slot $(n+1)$ onwards. The buffer is assumed to be of infinite size. Buffer occupancy x_n, measured just at the beginning of a slot n then evolves as

$$x_{n+1} \quad = \quad x_n - u_n + a_n \tag{1}$$

3) In general, the channel may be a time-varying one. For the analysis here, we assume the channel to be non-time variant.

The above system can be modeled by a discrete time queue in which, the 'state' of the system can be characterized completely by the buffer occupancy - $\boldsymbol{v}_n = (x_n)$. As stated before, we only investigate transmitters belonging to a certain class $\mathcal{P}$, called the constant power.

Definition 1: A scheduler $S \in \mathcal{P}$ is one characterized by the constraint that it can transmit only at a set of finite pre-configured rates, i.e., $u_n \in \mathcal{N}^K = \{0, N_1, N_2, ..., N_K\}$. A transmitter capable of transmitting at K different non-zero rates is said to belong to the class $\mathcal{P}^K$ of K-rate transmitter.
With the additional assumption imposed on the transmitters in the previous section, a constant power scheduler can be of two types of schedulers.

- *Stuffing Schedulers:* Although the transmitter can transmit only at a fixed rate, a scheduler in principle, can transmit an arbitrary quantum of packet. It can be stuffed with an appropriate amount of dummy packets to form an (the nearest higher) allowable transmissions. However, because of the constraint on transmitter, the $P-R$ relation in effect won't be convex but would rather take a ladder type form as shown in Figure 1. The analysis techniques used in previous works, such as the Dynamic Programming Methods, may then be applied to arrive at an optimal scheduler using this modified cost function. However it is interesting to note that some of the nice properties that the optimal scheduling policies generally have, such as monotonicity of the scheduling action need not hold here.
- *Non-Stuffing Schedulers:* A scheduler on the other hand may only choose to schedule in sizes or rates that are allowed for transmission. Thus the batch size $u_n \in \mathcal{N}^K$. Thus for such a scheduler if $u_n = N_i$, then $x_n \geq N_i$. In particular, it must wait for a number equal to the minimum non-zero transmission rate before dispatching any, i.e., if $u_n > 0$ then $x_n \geq N_1$. An optimal non-stuffing scheduler may also be arrived at using standard Dynamic Programming Methods.

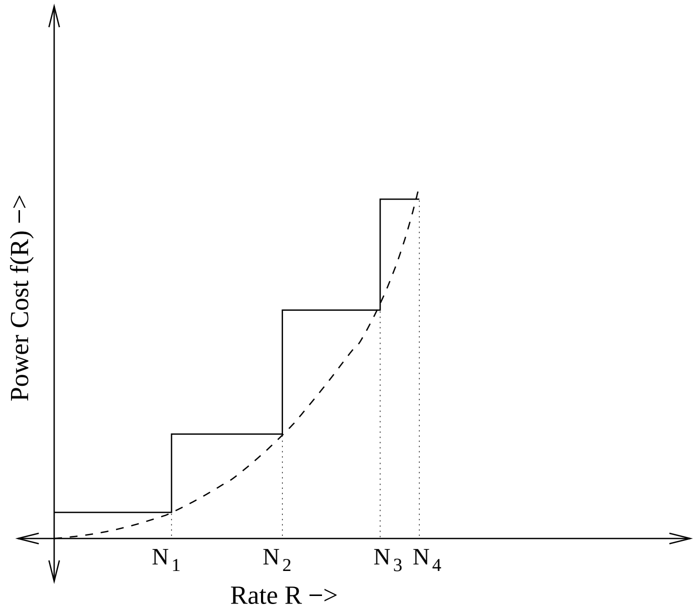

Fig. 1. Equivalent Cost function with stuffing

A non-stuffing scheduler has obvious limitations on the average delay targets it can meet. Even when an average delay target is achievable by the class of non-stuffing schedulers, the optimal non-stuffing scheduler is likely to under perform when compared to an optimal stuffing scheduler in terms of power consumption. On the other hand, non-stuffing scheduling policy might be preferrable from the network perspective as stuffing can limit the effective throughput of the system as a whole especially when the channel medium is shared as in case of a CDMA Network through excessive use of dummy packets which have no useful information content and only add to the interference in the network. Also, when the delay bounds are not very stringent, the scheduling action would naturally tend to favour a non-stuffing technique. In this paper, we consider only the non-stuffing schedulers. The queues associated with special classes of non-stuffing schedulers can be analysed easily using a generating function approach [7]. In the remaining part of this paper we use the words scheduler and transmitter interchangeably and consider them a single entity. We define some more sub-classes of such schedulers.

Definition 2: A deterministic scheduler is the one in which the scheduling action can be specified as a stationary function $u(.)$ on the state $\boldsymbol{v_n}$ i.e. $u_n = u(\boldsymbol{v_n})$.
A stochastic scheduler on the other hand would randomly choose u_n according to a probability distribution which is a function of the system state $\boldsymbol{v_n}$. An important sub-class of deterministic schedulers is the monotone schedulers defined as follows.

Definition 3: A deterministic scheduler S is said to be monotone if the scheduling function $u(.)$ of the buffer-state $\boldsymbol{v} : (x)$ is increasing in x.

Definition 4: A scheduling policy is said to be greedy if, whenever $x_n \geq N_i \in \mathcal{N}^K$, $u_n \geq N_i$.
Thus for any state $\boldsymbol{v_n}$, a greedy (non-stuffing) scheduler transmits at the maximum permissible rate. It follows that among all the schedulers with the same set of allowable rates $\mathcal{N}^K$, the greedy scheduler would give the best average delay performance and sets a bound on the feasible delays.

The problem of power-efficient scheduling may then be stated as follows. Given an iid packet arrival process on a_n, and an average delay constraint $\bar{D}$, design, for a K-rate scheduler using the set of rates $\mathcal{N}^K$, a stationary scheduling policy that would give the optimal power performance. A more challenging problem arises when the set $\mathcal{N}^K$ is also open to design and a joint optimization of the rates as well as policy is sought.

We first solve this problem for the simple possible scenario, i.e., when only one non-zero rate is allowed ($K = 1$).[1]

III. Optimal 1-rate Schedulers

In this section we consider the problem of determining an optimal non-zero rate and the corresponding scheduling policy for 1-rate scheduler under constant channel conditions.

Proposition 1: For the constant channel case, among the class of 1-rate schedulers only, there is a greedy scheduler which gives the optimal performance.
Proof: We first show that any two 1-rate schedulers that use the same rate N will have the same power performance. Let $E(N)$ be the energy required for transmitting N packets in 1 slot and let q be the fraction of slots in which S transmits. If the queue is stable, then we must have $N > \lambda$. Then,

$$\begin{aligned} q &= \frac{\lambda}{N} \\ P(S_N) &= \frac{\lambda}{N} E(N) \end{aligned} \tag{2}$$

where, $P(S_N)$ denotes the average energy per slot or power requirement of the scheduler S using a single rate N. Since the greedy scheduler would give the same energy performance and at least as good a delay performance as any scheduler using that rate; there is a greedy scheduler which gives the optimal performance. ■

Proposition 2: Let $P(N)$ denote the average power requirement of a 1-rate scheduler of rate N. Then if $N' > N, P(N') > P(N)$.
Proof: To prove this result, we will invoke the assumption of the convexity of $E(N)$.

$$\begin{aligned} & P(N') - P(N) \\ = \ & \lambda \left[\frac{E(N')}{N'} - \frac{E(N)}{N} \right] \\ = \ & \frac{\lambda(N'-N)}{N'} \left[\frac{E(N') - E(N)}{N' - N} - \frac{E(N) - E(0)}{N - 0} \right] \\ > \ & 0 \end{aligned} \tag{3}$$

as $N' > N$ and $E(N)$ is convex. ■

From the above results, it follows that, under constant channel conditions, a greedy scheduler transmitting at rate N, denoted by G_N, is an optimal 1-rate scheduler, *iff* N is the smallest integer that satisfies the delay constraint.

Hence the problem of determining the optimal 1-rate scheduler reduces to that of obtaining an expression of the queueing delay as a function of N under G_N for a given arrival process. From this, the optimal N can be computed for a given delay constraint. Note that for $K > 1$, however, a greedy scheduler is not necessarily optimal and determining the structure of optimal policy is in general difficult.

The system considered here can be modeled as discrete time bulk queue with an infinite buffer and determinitstic inter-arrival and service times in which the departure size is restricted to fixed number. We denote the resulting queue as a $D^X/D/\mathcal{N}^1$ queue. We have analyzed such queues for more general arrival process and service sizes in [7]. Here we provide the result only for 1-rate scheduler and geometric arrival process for illustration purposes.

Note that in our model, $u_n = N$ if $x_n \geq N$ and $u_n = 0$ otherwise. Let $\pi_i = \lim_{n\to\infty} \text{Prob}[x_n = i]$. The flow balance equation for the Markov chain on buffer occupancy in terms of the characteristic functions $\boldsymbol{\pi}(z)$ and

[1] [1] uses a 1-rate scheduler for a performance comparison, but does not provide an analysis of arriving at the optimal rates.

$\boldsymbol{p}(z)$ gives

$$\boldsymbol{\pi}(z) = \boldsymbol{p}(z)\boldsymbol{\pi}^{\mathbf{0}}(z) + \boldsymbol{p}(z)\frac{\boldsymbol{\pi}(z) - \boldsymbol{\pi}^{\mathbf{0}}(z)}{z^N}$$

$$where, \tag{4}$$

$$\boldsymbol{p}(z) = \sum_{i=0}^{i=\infty} p_i z^i$$

$$\boldsymbol{\pi}(z) = \sum_{i=0}^{i=\infty} \pi_i z^i$$

$$\boldsymbol{\pi}^{\mathbf{0}}(z) = \sum_{i=0}^{i=N-1} \pi_i z^i \tag{5}$$

Now (4) may be rewritten as

$$\boldsymbol{\pi}(z) = \frac{\boldsymbol{p}(z)\left(z^N - 1\right)}{z^N - \boldsymbol{p}(z)}\boldsymbol{\pi}^{\mathbf{0}}(z) \tag{6}$$

Under the assumption that the queue is stable, i.e., $N > \lambda$ and p_is are geometrically distributed, that is to say,

$$p_i = (1-\alpha)\alpha^i$$

$$\text{i.e., } \boldsymbol{p}(z) = \frac{1-\alpha}{1-\alpha z} \tag{7}$$

It can be easilyshown that (8)

$$\boldsymbol{\pi}(z) = \frac{1-\beta^{-1}}{N}\frac{\sum_{i=0}^{N-1} z^i}{1-\beta^{-1}z} \tag{9}$$

where β is the only zero of $(\alpha[z^{N+1}-1] - [z^N - 1])$ that lies outside the unit circle. This analysis has been explained in detail in [7] for a general arrival process as well. From Little's result, the average delay suffered is given as

$$D_N = \frac{E[x]}{\lambda} = \frac{\boldsymbol{\pi}'(1)}{\lambda}$$

$$= \frac{N-1}{2} + \frac{1}{\beta - 1}$$

$$\Rightarrow D_N = \frac{1-\alpha}{\alpha}\left[\frac{N-1}{2} + \frac{1}{\beta-1}\right] \tag{10}$$

We can plot average delay D_N v/s the rate N of the greedy 1-rate scheduler G_N for various values of N as in Figure 2. From this, given an average delay constraint $\bar{D}$, the optimal rate N may be selected.

When $K > 1$, the problem of determining optimal rates is a much more challenging problem with no apparent straightforwrd solution. In this paper, we characterize the K-rate non-stuffing schedulers in terms of their schedulability extremes, i.e., their best case selay and power-performances. As, noted before, the best delay performance would be given by the greedy scheduler in the class. Using the techniques in [7], it is possible to obtain the steady state solution to queues arriving from such scheduling action for a general arrival process and thus obtain the average queueing delay. This delay serves as the lower bound for the delays feasible to all schedulers using the rates in $\mathcal{N}^K$. Instead of attempting an optimal solution for K-rate scheduler, we characterize the optimal scheduler in the absence of delay constraints. This scheduler can serve as a good beginning point for an iterative scheduler design and provides a lower bound on the mean power consumption for K-rate scheduler.

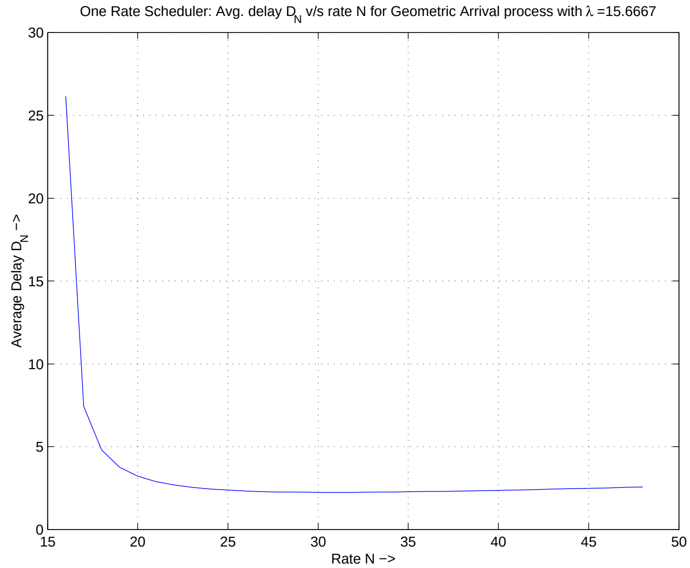

Fig. 2. Average delay D_N v/s rate N for G_N for a Geometric Arrival Process

IV. Minimum Power Requirement

In this section, we discuss the minimum power requirement for a K-rate scheduler, when there is no delay constraint on scheduling. It is interesting to note that the minimal power depends only on the average arrival rate and can be achieved by a corresponding 2-rate scheduler.

Proposition 3: Let $N_p, N_{p+1} \in \mathcal{N}^K$ be such that $N_p < \lambda < N_{p+1}$. Then, the minimum power requirement, that occurs in the absence of delay constraints, i.e., $\bar{D} = \infty$ is given by

$$P_{min} = \frac{N_{p+1} - \lambda}{N_{p+1} - N_p}E(N_p) + \frac{\lambda - N_p}{N_{p+1} - N_p}E(N_{p+1}) \tag{11}$$

Proof: In the absence of delay constraint, the problem of determining minimum power can be formulated as optimization problem. We seek to determine the tuple

$Q^* : (q_1^*, q_2^*, ..., q_K^*)$ that

$$\text{minimizes } P(Q) = \sum_{i=1}^{i=K} q_i E(N_i)$$

subject to,

$$\sum_{i=1}^{i=K} q_i N_i = \lambda \quad (12)$$

$$\sum_{i=1}^{i=K} q_i < 1 \quad (13)$$

Here q_i is the fraction of time for which the scheduler transmits at N_i. The optimal solution, must satisfy (in the limiting sense)

$$\sum_{i=1}^{i=K} q_i^* = 1 \quad (14)$$

This may be seen readily from the following argument. Suppose, $\Delta = 1 - \sum_{i=0}^{i=K} q_i^* > 0$, and let N_h be the highest rate for which $q_h^* > 0$, then consider the tuple Q', which is the same as Q^*, except that $q'_1 = q_1^* + N_h\delta$ and $q'_h = q_h - N_1\delta$; where $\delta = \min\left(\frac{q_h}{N_1}, \frac{\Delta}{N_h - N_1}\right)$. Note that Q' satisfies (12) and (13). Also,

$$\begin{aligned} P(Q^*) - P(Q') &= \delta\left(N_1 E(N_h) - N_h E(N_1)\right) \\ &> 0 \end{aligned}$$

Thus it follows that the optimal tuple must satisfy $\sum_{i=0}^{i=K} q_i^* = 1$. Now to prove the proposition we note that $E(.)$ is a convexly increasing function satisfying $E(0) = 0$. It can be easily seen that $E(.)$ satifies the following property

$$\begin{aligned} &\text{if } A < B < C < D \quad \text{and} \quad a + d = b + c = 1 \\ &\text{and also } aA + dD = bB + cC \\ &\text{then } aE(A) + dE(D) \geq bE(B) + cE(C) \end{aligned}$$

It then follows that a policy transmitting at rates other than N_p, N_{p+1}, the rates closest to λ on either side, will have a higher power requirement than a policy that transmits at only these rates. Hence, the optimal tuple is given by

$$\begin{aligned} q_p^* &= \frac{\lambda - N_p}{N_{p+1} - N_p} \\ q_{p+1}^* &= \frac{N_{p+1} - \lambda}{N_{p+1} - N_p} \\ q_i^* &= 0\,; i \neq p, p+1 \end{aligned}$$

This proves the result. ■

From the above result, it follows that over the K-rate schedulers, the least power will be achieved by the schedulers that use the rates $\lfloor\lambda\rfloor$ and $(\lfloor\lambda\rfloor + 1)$. For these schedulers, $P_{min} = (\lambda - \lfloor\lambda\rfloor)E(\lfloor\lambda\rfloor) + (1 - \lambda + \lfloor\lambda\rfloor)E(\lfloor\lambda\rfloor + 1)$, which is similar to the expression in [1]. Note that P_{min} is achievable in the limit by appropriate scheduling action, which may be non-deterministic in general and provides a lower bound on the mean power consumption for the schedulers that use the set $\mathcal{N}^K$ as the non-zero transmission rates.

V. Conclusions and Future Directions

While scheduling for achieving efficient power performance under delay constraints, the limitations of the transmitter system in varying the physical rate of transmission must be taken into account. The transmitter may be assumed to be capable of transmitting only at a finite set of fixed rates. An optimal scheduler for the simplest case when the channel is non-time varying and only one non-zero rate may be used has been dealt with in this paper. Lower bounds on the power and delay performances of a scheduler using multiple rates under constant channel conditions have also been provided. It would be interesting and challenging to consider the problem of the joint optimal rate-scheduler design for the more complex case of a time varying channel under the constraints of discrete rate scheduling.

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