Dual Degree Dissertation

On Scheduling Transmissions under QoS based Constraints

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Dissertation Approval Certificate

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with regards,

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Synopsis

Modern-day communication networks seek to provide Quality of Service (QoS) guarantees to traffic flows. QoS guarantees are typically in the form of (average or absolute) delays and losses that a flow would experience. A QoS framework places constraints on the traffic inflow, as it would be required to adhere to a pre-negotiated regulating profile. It also constrains the scheduling or servicing manner, as it must deliver the performance metrics as required by the end application and promised by the network.

In this work, three different problems that broadly fall within this framework of scheduling or choosing a transmission strategy that meets the imposed constraints while optimizing on a suitable cost function are separately dealt with.

The first is that of delay-bounded streaming of video under traffic constraints that are specified in terms of a generalized token bucket regulator. The objective is to optimize the distortion and the distortion jitter in the resulting stream. A simple but effective way to capture this objective function is also presented.

The second problem considered is also related to token bucket regulator constrained transmission; the objective function to be optimized here is the entropy of the resultant flow. Such a flow has an additional source of information in terms of the variable sizes of the chunks of transmission.

Finally, the problem of power efficient transmission in wireless networks subject to a constraint on average delay is considered. The property that power increases with the transmission rate in a convex fashion can be exploited to save power consumption. The problem is formulated with an additional practical implementation constraint that transmission may only be carried out at from a set of fixed discrete rates. The queueing theory problem of obtaining the steady state distribution under discrete rate scheduling is addressed for an important class of such schedulers, using the generating function approach.

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Chapter 1

Introduction

1.1 QoS Mechanisms

As communication networks gear to support along with the conventional data communications, a new class of applications such as multimedia streaming, a range of issues related to *Quality of Service (QoS)* comes into play. Broadly, QoS refers to the mechanisms to provide support for performance requirements in packet transfer that will be demanded of the network by the end applications. Data based applications tend to demand high data integrity or low packet loss guarantees. Interactive or multimedia applications demand strict delay guarantees but have relatively less stringent packet loss requirements. Two different frameworks to provide QoS support have been defined - the Integrated Services (*IntServ*) which deals with each traffic flow separately and the Differentiated Services (*DiffServ*) which treats flows in aggregates by classifying them into different classes based on their service requirements. Since the amount of resources with a communication network are limited, to provide QoS guarantees, the network must have an estimate of the incoming flows and then allocate its resources among them accordingly. Also, the network is required to employ mechanisms to control flows. This is achieved through traffic regulation mechanisms, such as the token bucket regulator that are employed at the ingress points to the network. The task of QoS delivery is then of efficiently scheduling resources for the flows which are known to obey the regulator constraints, so as to meet the QoS parameters guaranteed to them. In this work, some problems related to this task are discussed. These problems are briefly outlined in the next section.

1.2 Contributions of the Dissertation

In Chapter 2, the problem of scheduling for distortion optimized streaming of a regulated media source is considered. Here, we consider the case where a media source is required to transmit subject to a strict delay constraint and a token-bucket regulation [1]. To adhere to these, it will have to drop certain packets. The problem dealt with is of selection of the sizes of packets to be dropped in each transmission, so as to minimize the perceptual distortion or irregularities in video quality. A simple model that captures both distortion and quality smoothness is presented. An optimal solution is found to the problem, when the size of the transmissions in the stream is known apriori, i.e., the stream is offline. This is then extended to a useful heuristic scheme when the transmission sizes are not known apriori, i.e., the stream is online. Finally a comparison of different schemes and a study of performance improvements is made.

In Chapter 3, we investigate the utility offered to a customer, who has been given absolute delay guarantees subject to the constraint that the flow passes through a token bucket regulator. For this, we evaluate the entropy or information content that such a flow can potentially contain, in terms of the regulator parameters. We need to take into account, the information that may be conveyed by exploiting the variations in packet lengths that will be allowed by the token bucket regulator.

In Chapter 4, we consider the problem of power-efficient transmission in wireless networks under an average delay constraint. Specifically, we discuss the power saving gains possible over a wireless channel through two different means - its time varying nature and the convex relationship between the transmission rate and power. We focus on the delay-power tradeoff possible due to the latter and which can be realized by appropriately sizing the batches of packets prior to transmission. We argue that variable rate transmissions may be practically feasible only at a small set of discrete rates. Accordingly, we restrict our attention to such schedulers. We then consider the queueing theory problem associated with discrete rate scheduling, that would allow us to evaluate the performance of these schedulers. Using the generating function technique, we obtain the steady distribution of the resulting queues. We begin with the simple case of single-rate scheduling and build upon it to extend the solution to a special class of many-rate schedulers that we call the 'greedy' schedulers. We finally extend the framework to the class of deterministic monotone many-rate schedulers.

We conclude with suggestions of possible extensions and future directions that may be pursued in connection with these problems in Chapter 5.

Chapter 2

Optimal Packet Length Scheduling for Regulated Media Streaming

2.1 Introduction

Packet switched networks that provide *Quality of Service (QoS)* guarantees to traffic flows usually require them to conform to some Traffic Descriptors [1]. Traffic Descriptors form the basis of Service Level Agreement (SLA) [2] between the source and the network which broadly comprise guarantees in terms of loss and delay that the network offers to a traffic flow and the constraints that it would put on the same. Commonly used Traffic Descriptors are Linearly Bounded Arrival Process (LBAP) [2]. An LBAP constraint bounds the maximum number of bits that a source may transmit in a given interval t by a linear function of t. The source employs traffic shapers and the network polices the traffic using Traffic Regulators to ensure that the source adheres to the advertised traffic descriptions. A simple Traffic Regulator for an LBAP descriptor is the Token Bucket (or Leaky Bucket) Regulator which has two parameters - the token generation rate r and the size of the token bucket B.

For Variable Bit Rate (VBR) traffic such as video streams, the selection of appropriate values of the token bucket parameters can be both difficult as well as inefficient in terms of resource utilization [3], [4]. Further, the source would not always be able to procure for itself, its exact requirements from the network. This is because, the network may be offering only a discrete set of combinations of r and B from which the source would be required to choose. Media streaming applications, though delay sensitive, are somewhat flexible in terms of their packet loss requirements and allow for a graceful degradation of quality with loss.

In this chapter, we consider a problem where the network has offered QoS guarantees for a particular set of Token Bucket parameters that the traffic source is required to conform to. Further, by using flexible coding schemes, the source may schedule the size of its packets to adapt to these traffic constraints. We argue that of all the packet length schedules that honor the imposed traffic constraints, the one that minimizes the distortion and quality jitters as perceived by the receiver may be the most appropriate to choose for transmission. We propose that MINMAX loss (in absolute or fractional terms) over sets of contiguous frames spanning disjoint time-intervals may be used as an effective combined measure of distortion and quality smoothness of the stream. Using this criterion , we derive an optimal packet length schedule for an offline (prerecorded) stream and then suggest directions for extending it to the case of online (real time) transmission. Our formulation though is not restricted to video or media transmission and can be applied to any LBAP traffic that exhibits properties of loss tolerance.

In Section 2.2, we formalize the model and the notion of distortion and smoothness used in this chapter. In Section 2.3 we present the distortion model used in this dissertation. In Section 2.4, we present an algorithm to determine the optimal packet length schedule for an offline stream. In Section 2.6 we consider the online scheduling problem. We conclude with Section 2.7 where we present results of our simulation exercises and end with a brief conclusion on the same in Section 2.8.

Related Work

Previously, [4] and [5] have considered the problem of Token Bucket constrained transmission to achieve loss-free or distortion-free streaming and suggested algorithms for selection of token bucket parameters for offline streams. In [6], the authors have considered the problem of optimizing distortion for constrained VBR streaming in ATM networks. The authors have proposed a complex Viterbi algorithm for quantizer selection at encoder based on apriori knowledge of distortions associated with each choice.

For unconstrained streaming, the problem of joint optimization of average bandwidth consumption and distortion or that of rate-distortion optimized streaming of video has been dealt with in many works. [7] has considered a lossy best-effort network and presented transmission schemes for rate distortion optimized streaming of packetized media in such networks. The authors have used an MPEG-type data dependency model of frames for modeling distortion. However only the mean or expected values of rate and distortion were considered in their analysis. In [8], a similar problem pertaining to streaming of scalable media consisting of independent frames in best effort networks has been dealt with. In [9], the authors have considered the problem of delay constrained transmission of layer-encoded multimedia presentation in a network with limited constant bandwidth. They have also used a MINMAX measure of distortion for evaluating the presentation quality.

Our work differs in the respect that we consider a network model which is based on the QoS framework rather than the best effort model and we seek a packet length scheduling policy under stated constraints on both rate and burstiness. Further, while choosing a scheduling policy we also take into account the consistency and smoothness of stream quality over time.

Unlike some of the aforementioned work, our distortion model does not take into account temporal dependencies that may be present in the data units sent in different time-intervals. Such dependencies, which may be represented by an acyclic graph are characteristic of video coding schemes and can result in error propagation over finite durations. This simplification, applicable under reasonable assumptions, allows us to present a distortion model that is simple to analyze and at the same time not tied to any specific coding scheme.

2.2 The Model

We consider a source that transmits for a finite duration N, at discrete times $n = 1, 2, ..., N$. The source is constrained by an augmented variant of the Token Bucket Regulator, henceforth denoted by $TBR(\boldsymbol{r}, \boldsymbol{B}, B_0)$. This representation is more general as we allow the token generation rate and the maximum number of residual tokens to vary with time n and denote them as vectors $\boldsymbol{r} : (r_1, r_2, ..., r_N)$ and $\boldsymbol{B} : (B_1, B_2, ..., B_N)$ respectively. B_0 is the initial token grant. The IETF Token Bucket Regulator commonly employed is subsumed by the special case, $r_n = r$, $B_n = B \;\; \forall n : 1 \leq n \leq N$; r, B being constants. We denote this regulator as simply $TBR(r, B, B_0)$. We do not assume the actual time-interval between two successive time indices to be constant. In case of non-constant time intervals, i.e., when packet arrival is not at equal time intervals, each refill component r_n that represents the tokens gained in the interval from $n - 1$ to n will be appropriately weighted according to the length of the interval.

The source is characterized by a requirement schedule vector $\boldsymbol{y} : (y_1, y_2, ..., y_N)$. y_n denotes the number of bits[1] required by the source to code the media content between times $n - 1$ and n, which we shall refer to as interval n, to a desired level of quality. The transmission is termed 'offline' if all y_ns are known apriori, e.g., streaming of a prerecorded video. We consider the 'offline' case first. This also gives, as discussed in Section 2.6, a considerable insight for dealing with the 'online' transmission problem,

[1]here 'bits' are used in an Information Theoretic context, i.e., as a measure of information

where all y_ns are not known apriori. x_n in the vector $\boldsymbol{x} : (x_1, x_2, ..., x_N)$ denotes the number of bits transmitted at time n. $TBR(\boldsymbol{r}, \boldsymbol{B}, B_0)$ constrains x_n so that

$$x_n \leq t_{n-1} + r_n \;;\forall n : 1 \leq n \leq N \tag{2.1}$$

t_n in $\boldsymbol{t} : (t_1, t_2, ..., t_N)$ is the number of tokens left in the bucket just after the n^{th} transmission. If all packet sizes x_n satisfy (2.1), i.e., the source is conforming, then t_n will evolve as

$$\begin{aligned} t_n &= \min(t_{n-1} + r_n - x_n, B_n) \;; \\ t_0 &= B_0 \end{aligned} \tag{2.2}$$

We assume a zero effective playback buffer/delay scenario, i.e., the information pertaining to the interval n i.e., from times $n-1$ to n, is useful only if transmitted by the source at n. A conformant transmission is assumed to be error-free and at a constant delay. This amounts to an assumption that the playback buffer available at the receiver is just enough to absorb delay jitters caused by the network, a likely case for low-memory devices like handhelds. In Section 2.5 we relax this constraint and extend the analysis to allow for non-zero absolute delay bounds.

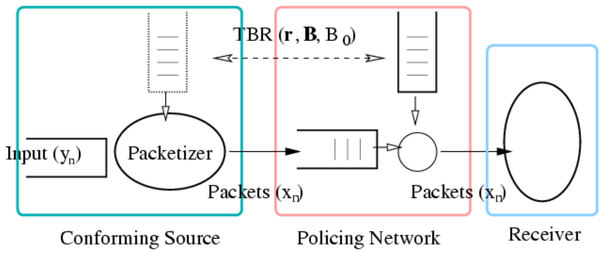

Figure 2.1: Network Model

2.3 Stream Distortion

Distortion measures used in literature are based on Peak Signal to Noise Ratio (PSNR) or mean squared error between the actual and reconstructed frames [7, 8]. The problem of distortion modeling that relates well to a perceptual experience is a difficult one and estimation based on these models can often be computationally expensive. For some examples of methods used for distortion modeling of media streams refer [10] or [11].

Further, these models take into account only the overall distortion for the stream. Apart from overall distortion itself, we believe distortion jitters; a term we use to refer to fluctuations in quality of different frames of the same stream, can also lead to an undesirable viewing experience. To take care of this, for our analysis, we use a model which is simple and amenable to analysis while at the same time is not restricted to specific media or coding schemes. We do not assume a specific form for the distortion function but only make a few assumptions on the same, namely

i.) distortion d_n for any interval n can be expressed as a function of what we term as absolute loss α_n or fractional loss β_n defined as follows

$$\alpha_n \triangleq \max(y_n - u_n, 0) \quad (2.3)$$

$$\beta_n \triangleq \max\left(\frac{y_n - u_n}{y_n}, 0\right) \quad (2.4)$$

As stated before y_n represented the number of bits required to code the content to a quality level assumed to be entirely satisfactory, while u_n is the number of bits actually utilized for the purpose. This assumption requires that distortion (or additional distortion) can be related to the bit deficit in coding, measured in an absolute (or fractional) sense through a function $d_\alpha(.)$ (or $d_\beta(.)$) that remains invariant with time index n. This assumes the stream to have a stationary distribution.

ii.) $d_\alpha(.)$ and $d_\beta(.)$ are increasing and convex functions of their arguments. Convexity is a plausible assumption because of the nature of rate-distortion curves. For example for a Gaussian distributed continuous random variable, the rate-distortion function is of the form

$$D(R) = K2^{-2R} \quad (2.5)$$

If the distortion corresponding to a rate R_0; i.e., $D(R_0)$ is treated as acceptable, then the additional distortion incurred as a result of using a rate R' which is less than R_0 by say ΔR will be given by

$$\Delta D(\Delta R) = D(R_0)\left(2^{2\Delta R} - 1\right) \quad (2.6)$$

which is a convex function of the 'loss' ΔR.

iii.) distortion across data sets is additive, i.e., the distortion across the different non-overlapping time intervals add up to give the overall distortion for the stream. This essentially neglects any temporal dependencies that may have been exploited in the coding scheme. In a practical coding scheme, time dependencies are present over disjoint code blocks, each of which constitutes, what is termed as a Group of Pictures (GOP). Thus, the stated assumption will be valid if the time intervals of consideration are larger than the GOP interval or losses are so less that they are completely absorbed by spatial enhancement frames and do not affect the base frame representations which may have been used in the encoding of subsequent frames.

If these assumptions hold then a reasonable estimate of the overall distortion may be provided by an expression that would take one of the following forms.

$$D_\alpha = \sum_{n=1}^{n=N} d_\alpha(\alpha_n) \tag{2.7}$$

$$D_\beta = \sum_{n=1}^{n=N} d_\beta(\beta_n) \tag{2.8}$$

$$D'_\beta = \sum_{n=1}^{n=N} y_n.d_\beta(\beta_n) \tag{2.9}$$

We consider a hypothetical example to make things clearer. We assume that a VBR scheme is used to code a video stream and the time intervals correspond to a GOP interval. Thus data units belonging to different intervals are independently coded. We assume that the coding is in form of real-valued mutually uncorrelated 'objects' (e.g., DCT coefficients). Thus during intervals that are 'rich' in information content, a good representation of the stream would require a greater number of these 'objects'. We make a simplifying assumption that these 'objects' have similar distributions and need to be coded using the same number of bits, say M. Thus, a coding requirement of y_n for an interval n may be interpreted as a presence of $\frac{y_n}{M}$ objects in the media representation. If only u_n bits are available to code these objects; then the coder can choose either to code only a few objects using M bits each or code all of them using less than M bits each or any intermediate approach. The latter two would result in distortion in each one of the objects. Again if we assume that all the objects are of equal importance, then, because of the convex nature of the rate-distortion curves, a good-coder will code all the objects using same number of bits, evenly distributing the performance degradation due to the bit deficit across all objects. Thus each object would be coded using $M\frac{u_n}{y_n}$ bits. Assuming a Gaussian model for the distribution of 'objects', the distortion e (mean squared error sense) in the coding of each object will be then given by

$$e = K\left(2^{2M\left(1-\frac{u_n}{y_n}\right)} - 1\right) \quad or$$

$$e = K\left(2^{2M\beta_n} - 1\right) \tag{2.10}$$

There being $\frac{y_n}{M}$ objects to be coded, the distortion d_n for the interval n will be given by

$$d_n = \frac{y_n}{M} K \left(2^{2M\beta_n} - 1\right) \tag{2.11}$$

Because the coding across intervals is assumed to be independent, the overall distortion D will be simply given as

$$D = \frac{K}{M} \sum_{n=1}^{n=N} y_n \left(2^{2M\beta_n} - 1\right) \tag{2.12}$$

Normalizing the proportionality constant to 1, the expression is similar to that of (2.9) with $d_\beta(.)$ given by $d_\beta(\beta) = 2^{2M\beta} - 1$ which is indeed a convex function of β.

For the subsequent analysis then, we shall assume that the overall distortion may be modeled with sufficient accuracy by an expression of one of the three types- (2.7), (2.8) or (2.9). Along with overall distortion, we also want to minimize the 'distortion jitter'. The distortion jitter may be characterized by the variance of distortions in individual intervals or maximum distortion overshoot from the mean. We avoid such a characterization here, but would return to these metrics for performance evaluation. For the zero-delay case, $u_n = x_n$, i.e., against a requirement of y_n bits, the number of bits actually used is the same as that transmitted at time n, i.e., x_n. With our assumptions, the resulting perceptual distortion can be completely characterized in terms of the allocation vector $\boldsymbol{x}$ and the requirement vector $\boldsymbol{y}$. We now define a MINMAX kind measure of distortion.

Definition 2.1. $\boldsymbol{\alpha'}, \boldsymbol{\beta'}$ are ordered loss vectors of absolute and fractional losses respectively, obtained by sorting the losses for different intervals in a decreasing order.

Thus for fractional loss,

$$\beta'_1 \geq \beta'_2 \ \ldots \ \geq \ \ldots \ \beta'_N$$
$$\text{in particular,}$$
$$\beta'_1 = \max_{1 \leq n \leq N} \beta_n \tag{2.13}$$

In our analysis we would attempt to find a packet length schedule $\boldsymbol{x}^*$ that gives the minimum $\boldsymbol{\beta'}$ (or $\boldsymbol{\alpha'}$), comparisons made in lexicographic order, for a given requirement schedule $\boldsymbol{y}$ and $TBR(\boldsymbol{r}, \boldsymbol{B}, B_0)$ constraints. Our motivation arises from the fact that such a packet length schedule will also have optimal MINMAX distortion and hence maximum quality smoothness. Further it can be shown that if the overall distortion can be modeled by (2.7) or (2.9), then the optimal offline solution w.r.t. the MINMAX criterion based on $\boldsymbol{\alpha'}$ or $\boldsymbol{\beta'}$ respectively, will also be optimal in terms of overall distortion.

(Refer Appendix A for a proof.) Even when the overall distortion is of the nature of (2.8), for highly convex functions $d_\beta(.)$ or $d_\alpha(.)$, the MINMAX solution will be close to optimal in terms of overall distortion. In the next section, we present an algorithm that gives the optimal $\boldsymbol{\beta'}$ for the offline case. Algorithm that would give the optimal $\boldsymbol{\alpha'}$ is similar with minor modifications. For mathematical convenience, it is assumed that $\boldsymbol{r, B, x, y, t}$ as well as $\boldsymbol{\alpha, \beta} \in \{\mathcal{R}^+ \cup \{0\}\}^N$ rather than $\{\mathcal{Z}^+ \cup \{0\}\}^N$.

2.4 Optimal Packet Length Scheduling

Given the regulator $TBR(\boldsymbol{r}, \boldsymbol{B}, B_0)$, a packet length schedule $\boldsymbol{x}$ is conformant *iff* the packet lengths satisfy the following set of constraints.

$$\sum_{n=i}^{n=j} x_n \leq B_{i-1} + \sum_{n=i}^{n=j} r_n \quad \forall (i,j) : 1 \leq i \leq j \leq N \tag{2.14}$$

Also since the delay bound is zero, we consider only those packet length schedules $\boldsymbol{x}$ that satisfy

$$x_i \leq y_i \quad \forall i : 1 \leq i \leq N \tag{2.15}$$

The set of all allocation vectors $\boldsymbol{x}$ satisfying these constraints (2.14) and (2.15) are said to be admissible and the corresponding loss vectors $\boldsymbol{\beta}$ (or $\boldsymbol{\alpha}$) are said to be feasible. We seek to minimize $\boldsymbol{\beta'}$. Define

$$\gamma_{ij} \triangleq \max\left(\frac{\sum_{n=i}^{n=j} [y_n - r_n] - B_{i-1}}{\sum_{n=i}^{n=j} y_n}, 0\right) \tag{2.16}$$

$$\gamma^* \triangleq \max_{(i,j):1\leq i\leq j\leq N} \{\gamma_{ij}\} \tag{2.17}$$

$$(i^*, j^*) \triangleq \arg \max_{(i,j):1\leq i\leq j\leq N} \{\gamma_{ij}\} \tag{2.18}$$

Lemma 2.1. *For an admissible allocation,*

$$\beta'_1 \geq \gamma^* \tag{2.19}$$

Proof. We will prove this by contradiction. Assume that (2.19) does not hold then

$$\begin{aligned}
\beta'_1 &< \gamma_{ij} \;\; for \;\; some \;\; (i,j) : 1 \leq i \leq j \leq N \\
as \;\; \beta_n &\leq \beta'_1 \;\; \forall n : 1 \leq n \leq N \\
\Rightarrow \sum_{n=i}^{n=j} \beta_n . y_n &< \sum_{n=i}^{n=j} [y_n - r_n] - B_{i-1} \\
\Rightarrow \sum_{n=i}^{n=j} x_n &> B_{i-1} + \sum_{n=i}^{n=j} r_n
\end{aligned}$$

which would contradict (2.14). Hence the proof. □

Lemma 2.2. *If* $\boldsymbol{x^*}$ *and* $\boldsymbol{\beta'^*}$ *denote the optimal packet length schedule and ordered fractional loss vectors respectively, then*

$$\beta_1'^* = \gamma^* \tag{2.20}$$

$$x_n^* = y_n(1-\gamma^*) \quad \forall n : i^* \leq n \leq j^* \tag{2.21}$$

Proof. From Lemma 2.1, Lemma 2.2 would follow if (2.20) were feasible and (2.21) were a necessary condition for the same. The feasibility of (2.20) may be verified by considering the loss vector given by $\beta_i = \gamma^* \ \ \forall i : 1 \leq i \leq N$ and using (2.18) and the admissibility constraints (2.14) and (2.15). Claim (2.21) is equivalent to

$$\beta_n^* = \beta_1'^* = \gamma^* \ \ \forall n: \ i^* \leq n \leq j^*$$

indeed, if this is not so, then

$$\beta_n^* < \beta_1'^* = \gamma^* \ \ for \ \ some \ \ n: \ i^* \leq n \leq j^*$$

$$\Rightarrow \sum_{n=i^*}^{n=j^*} \beta_n^*.y_n < \sum_{n=i^*}^{n=j^*} [y_n - r_n] - B_{i^*-1}$$

Once again, this contradicts (2.14). Hence the proof. □

From the lemmas, the recursive Algorithm 1 that gives the optimal packet length schedule to minimize $\boldsymbol{\beta'}$ can be easily worked out.

The algorithm follows a divide and conquer approach. If, in the notation used in the algorithm, $i^* \neq 1$, the limit on residual tokens, i.e., $\boldsymbol{B}$ is acting as a bottleneck and the allocation uses the maximum number of tokens that can be stored. Hence in the next recursion the same number of tokens are decremented from the appropriate component of the refill vector $\boldsymbol{r}$. The same idea can also be expressed in terms of a state notion where apart from an initial token grant there is also specified a token balance that the allocation scheme is required to leave. In the recursions of the Algorithm 1 described above, this token balance will either be zero or the maximum allowed i.e., the algorithm branches at the points of zero or maximum allowed token content.

2.5 Absolute Delay Bounded Scheduling

In this section, we consider the case where the media content for interval n is not required to be transmitted entirely in the n^{th} transmission but may be transmitted within an absolute delay of D. This relaxation helps the source achieve better distortion

Inputs: vectors $\boldsymbol{y} := y_1^N, \boldsymbol{r} := r_1^N, \boldsymbol{B} := B_1^N; B_0 := B_0$

(a_p^q denotes the vector $(a_p, a_{p+1}, ..., a_q)$)

Output: $\boldsymbol{x}^*$

Compute γ^* and (i^*, j^*)

if $\gamma^* = 0$ **then**

    for all $n : 1 \leq n \leq N$ **do**

        $x_n := y_n$

    end for

    return

end if

for all $n : i^* \leq n \leq j^*$ **do**

    $x_n^* := (1 - \gamma^*) y_n$

end for

if $i^* > 1$ **then**

    $r_{i^*-1} := r_{i^*-1} - B_{i^*-1}$

end if

Apply Algorithm 1 with inputs $(y_1^{i^*-1}, r_1^{i^*-1}, B_1^{i^*-1}; B_0)$

Apply Algorithm 1 with inputs $(y_{j^*+1}^N, r_{j^*+1}^N, B_{j^*+1}^N; 0)$

Algorithm 1: Algorithm for Offline Packet Length Scheduling performance at the cost of increased delay. As we shall show, the solution for this case is not much different from that of the zero delay case. Most of the analysis in fact carries through, with the change being that it applies to u_n, i.e., the total number of bits transmitted for representing the media content for interval n, rather than x_n, i.e, the number of bits transmitted in the transmission n itself. For the zero delay bound case, the two are equal, i.e., $u_n = x_n$. With a delay bound of D, these u_n bits may have been split across some or all of the transmissions that occur at $n, n+1, ..., n+D$.

The constraints on the allocation schedule $\boldsymbol{u}$ are now two-fold. One arises from the Token Bucket Regulation Constraint and the second from the transmission delay bound constraint. The necessary and sufficient conditions for a packet length schedule $\boldsymbol{u}$ to be admissible may now be modified as follows

$$\begin{aligned} u_i &\leq y_i \\ \sum_{n=i}^{n=j} u_n &\leq B_{i-1} + \sum_{n=i}^{\min(j+D,N)} r_n \quad \forall (i,j) : 1 \leq i \leq j \end{aligned} \tag{2.22}$$

Condition (2.14) has been relaxed to (2.22) to incorporate the non-zero transmission delay. Using these, the definitions of parameters γ_{ij}, γ^* and $(i^*, j*)$ are appropriately modified as

$$\gamma_{ij} = \max\left(\frac{\sum_{n=i}^{n=j} y_n - \sum_{n=i}^{\min(j+D,N)} r_n - B_{i-1}}{\sum_{n=i}^{n=j} y_n}, 0\right) \tag{2.23}$$

$$\gamma^* = \max_{(i,j):1\leq i\leq j\leq N} \{\gamma_{ij}\} \tag{2.24}$$

$$(i^*, j^*) = \arg \max_{(i,j):1\leq i\leq j\leq N} \{\gamma_{ij}\} \tag{2.25}$$

It may be shown that the results of Lemma 2.1 and 2.2 are applicable to these modified definitions and with x_n replaced by u_n. We shall skip the proofs of the same here and proceed to present the modified allocation algorithm (Algorithm 2) that gives the optimal schedule $\boldsymbol{u^*}$, directly. It is a simple matter to obtain the schedule $\boldsymbol{x^*}$ from the schedule $\boldsymbol{u^*}$

$$x_n^* = \min\left(t_{n-1}^* + r_n, \sum_{k=1}^{k=n} u_k^* - \sum_{k=1}^{k=n-1} x_k^*\right) \tag{2.26}$$

For any transmission, contents from multiple intervals might be pending and a priority is set in the order of the intervals itself, with the most delayed content having the highest priority. We neglect any fragmentation overheads.

As the case with finite delay is quite similar to that of zero delay, we shall henceforth consider only the zero-delay transmission case.

Inputs: vectors $\boldsymbol{y} := y_1^N, \boldsymbol{r} := r_1^N, \boldsymbol{B} := B_1^N; B_0 := B_0, D := D$

(a_p^q denotes the vector $(a_p, a_{p+1}, ..., a_q)$)

Output: $\boldsymbol{u}^*$

Compute γ^* and (i^*, j^*)
if $\gamma^* = 0$ **then**
  for all $n : 1 \leq n \leq N$ **do**
    $u_n := y_n$
  end for
  return
end if
for all $n : i^* \leq n \leq j^*$ **do**
  $u_n^* := (1 - \gamma^*) y_n$
end for
if $i^* > 1$ **then**
  $r_{i^*-1} := r_{i^*-1} - B_{i^*-1}$
end if
if $j^* < N$ **then**
  for all $n : j^* + 1 \leq n \leq \min(j^* + D, N)$ **do**
    $r_n = 0$
  end for
end if
Apply Algorithm 2 with inputs $(y_1^{i^*-1}, r_1^{i^*-1}, B_1^{i^*-1}; B_0; D)$
Apply Algorithm 2 with inputs $(y_{j^*+1}^N, r_{j^*+1}^N, B_{j^*+1}^N; 0; D)$

Algorithm 2: Algorithm for Offline Packet Length Scheduling - Finite Absolute Delay Case

2.6 Online Packet Scheduling Policy

We now consider the online video streaming case where the decision about the length x_n must be based only on previous inputs and apriori knowledge about the statistics of the input stream. If the particular form of distortion function is available, the problem may be formulated and solved within a Dynamic Programming Framework. The overall distortion will be replaced by the expected value of a 'henceforth' distortion (a sum of current distortion and expected future distortion) and this would comprise the cost function that is to be minimized. As the actual distortion will be replaced by its expected values, in general, the solution using an expected value of $\boldsymbol{\beta'}$ as an objective function, will not be identical to those using (2.7) or (2.9) as the objective function.

2.6.1 Dynamic Programming Formulation

Let $J_n(t, y, Y^n)$ denote the minimum expected cost when n transmissions including the current one are to be made. There are t residual tokens at this stage and the requirement for the current transmission is y. Y^n denotes the history of the input process that is assumed to be stationary. If, for example, $d_\beta(.)$ is the function that gives the distortion based on fractional loss β and the overall distortion can be modeled by (2.8), then

$$J_n(t, y, Y^n) = \min_{u:\, 0 \le u \le y, t+r} \left\{ d_\beta\left(1 - \frac{u}{y}\right) + E_{y'}\left[J_{n-1}\left(\min(t + r - u, B), y', Y^{n-1}\right)|Y^n\right] \right\} \quad (2.27)$$

The minimizing u is the optimal packet length and is denoted by $\Pi_n(t, y, Y^n)$. It follows,

$$\begin{aligned} if \;\; (t + r &\ge B + y), \\ \Pi_n(t, y, Y^n) &= y \;\text{ and} \\ J_n(t, y, Y^n) &= E_{y'}\left[J_{n-1}(B, y', Y^{n-1})|Y^n\right] \quad (2.28) \\ else&, \\ J_n(t, y, Y^n) &= \min_{u:\, 0 \le u \le t+r, y} \left\{ d_\beta\left(1 - \frac{u}{y}\right) + E_{y'}\left[J_{n-1}(t + r - u, y', Y^{n-1})|Y^n\right] \right\} \quad (2.29) \end{aligned}$$

$\Pi_n(t, y, Y^n)$ is then the minimizing u in (2.29). For a stationary first order Markov Process, some simplifications are possible as the process would depend only on the most recent requirement y and the dependence on Y^n will vanish. Specifically,

$$\begin{aligned} if \;\; (t + r &\ge B + y), \\ \Pi_n(t, y) &= y \quad \text{and} \\ J_n(t, y) &= E_{y'}\left[J_{n-1}(B, y')|y\right] \quad (2.30) \\ else&, \\ J_n(t, y) &= \min_{u:\, 0 \le u \le t+r, y} \left\{ d_\beta\left(1 - \frac{u}{y}\right) + E_{y'}\left[J_{n-1}(t + r - u, y')|y\right] \right\} \quad (2.31) \end{aligned}$$

2.6.2 A Heuristic Algorithm

The dynamic programming solution presented in the earlier section will have a high space/time complexity. Further, it requires a knowledge of the explicit nature of the distortion function. Here, we present an alternative method which is a heuristic method based on the offline streaming algorithm presented earlier.

In the offline algorithm, for a simple Token Bucket Regulator with fixed r, B, i.e., $TBR(r, B, B_0)$, we observe at time i

$$\beta_i \leq \left[\max_{j:i\leq j\leq N} \left\{ 1 - \frac{t_{i-1} + (j-i+1)r}{\sum_{n=i}^{n=j} y_n} \right\} \right]^+ \tag{2.32}$$

where, z^+ denotes $\max(z, 0)$.

Equality would hold in (2.32) for large B. If strict inequality applies in (2.32) then

$$if\ \ j^* = \arg \max_{j:i\leq j\leq N} \left\{ 1 - \frac{t_{i-1} + (j-i+1)r}{\sum_{n=i}^{n=j} y_n} \right\}; \tag{2.33}$$

$$\beta_i = \min_{k:i\leq k<j^*} \left\{ 1 - \frac{t_{i-1} + (k-i+1)r - B}{\sum_{n=i}^{n=k} y_n} \right\} \tag{2.34}$$

To compute β_i in the online case, the unknowns may be replaced by their expectations conditioned on the history denoted by Y^i.

$$R_{ij}(y_i) = \frac{1}{y_i + \sum_{n=i+1}^{n=j} y_n} \quad \forall i \leq j \leq N \tag{2.35}$$

$$e_{ij}(t, y_i) = \min \left(\min_{k:i\leq k<j} \{1 - [t + (k-i+1)r - B]\, R_{ik}(y_i)\}, 1 - [t + (j-i+1)r]\, R_{ij}(y_i) \right)$$

Then the recommended loss at time i,

$$\beta_i(t, y_i, Y^i) \triangleq \left[E\left[\max_{q:i\leq q\leq N} \{e_{iq}(t, y_i)\} | Y^i \right] \right]^+ \tag{2.36}$$

If the approximations: i) the input process is stationary First Order Markov and ii) the order of E and min or max operators may be interchanged; are valid, then the algorithm can be simplified and would require only statistics based on reciprocals of sums of packet lengths, i.e., $E[R_{ij}(y_i)|Y^i] = E[R_{ij}(y_i)]$ which may be derived analytically or obtained through simulations. The recommended loss β_i to be undergone will then be given by

$$\beta_i(t, y_i) = \left[\max_{q:0\leq k\leq N-i} \{E\left[e_{0q}(y_i, t)\right]\} \right]^+ \tag{2.37}$$

For large B, i.e., $B = \infty$ this further simplifies to

$$\beta_i(t, y) = \left[\max_{q:0\leq q\leq N-i} \{1 - [t + (q+1)r]E[R_{0q}(y)]\} \right]^+ \tag{2.38}$$

2.7 Simulation Study

The optimal offline and the heuristic online algorithms were compared with a 'naive' algorithm in a simple simulation setup consisting of the standard IETF Token Bucket Regulator $TBR(r, B, B_0)$ with $B_0 = B$. In the 'naive' algorithm, the packet allocation is online allowing a transmission to take as many tokens as available, i.e.,

$$x_n = \min(y_n, t_{n-1} + r_n) \tag{2.39}$$

For the simulations, the video traces were obtained from http://www-tkn.ee.tu-berlin.de/research/trace/ltvt.html. Specifically, we have used high quality MPEG4 streams and VBR H.263 coded streams to derive the requirement schedule $\boldsymbol{y}$. The entries or components of vector $\boldsymbol{y}$ were averages of 12 consecutive frames which roughly corresponded to 1 GOP. We truncated the vectors to size 500 (or 500x12 frames of the original stream). The heuristic online algorithm was slightly modified to simplify implementation and suit the finite buffer case. We considered the sum of reciprocals prediction $E[R_{0k}]$ for $k \leq 100$ only, irrespective of the number of transmissions remaining. This was done so that the estimates used were reliable (the estimates were used using the same input stream). Instead of a 1^{st} order Markov process, for simplicity, the input process was assumed to be stationary iid. Finally, since the bucket limit was finite the online algorithm was augmented to utilize any tokens for current transmission if they could not be stored. Specifically if β_n^{ON} denoted the online estimate for the loss, then the allocation x_n was calculated as follows.

$$\begin{aligned}
\beta_n^{ON} &= \left[\max_{0 \leq q \leq \min(N-n, 100)} \{1 - (t_{n-1} + (q+1)r)R_{0q}\} \right]^+ \\
x_n &:= y_n(1 - \beta_n^{ON}) \\
x_n &:= x_n + \min(y_n - x_n, \max(0, (t_{n-1} + r - x_n - B))) \\
t_n &= \min(t_{n-1} + r - x_n, B)
\end{aligned} \tag{2.40}$$

Figures 2.2 and 2.3 show the allocations and fractional losses according to the different schemes applied to the Bean MPEG4 stream.

The results on some of the traces are summarized in Table 2.1. The token rate r was chosen to be approximately 0.8 times the mean rate while the bucket depth B was chosen to be approximately 4 times the standard deviation. The distortion measure used was that of (2.12) with $M = 4$ and the distortion and deviations have been normalized to that given by the optimal offline algorithm.

The results show that the offline allocation scheme and the modified online scheme perform consistently better than the naive scheme. The performance improvement over

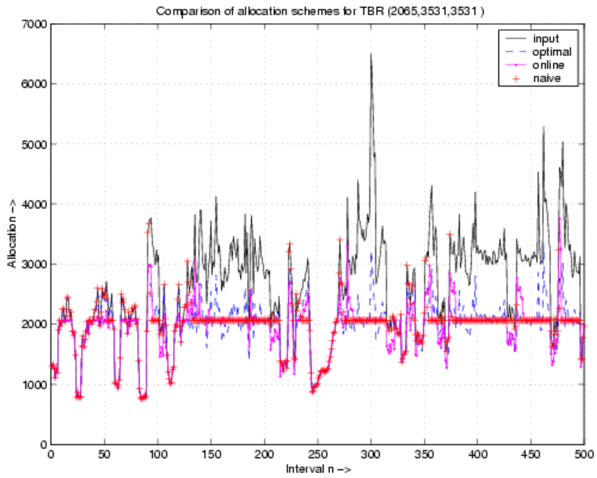

Figure 2.2: Different allocation schemes for the Bean MPEG4 stream

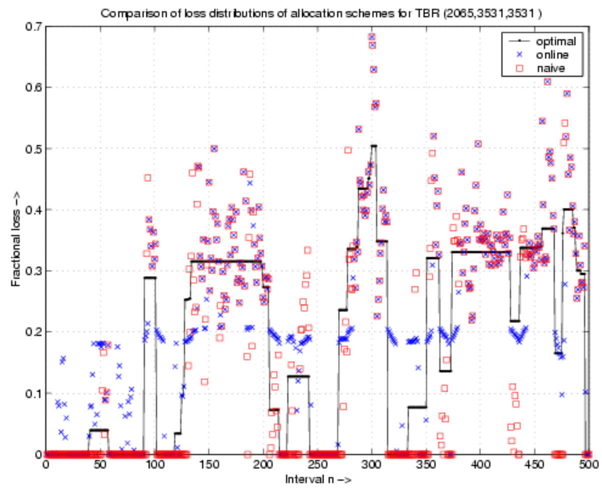

Figure 2.3: Fractional losses as per different allocation schemes for the Bean MPEG4 stream

the naive scheme, in terms of both, overall distortion as well as deviation in distortion of individual frames, is remarkable for the offline scheme. The improvements using the modified online heuristic algorithm are less significant for most of the cases. We also observe that, on the whole, improvements are more marked in the H.263 VBR streams.

To study the effect of the variation of token bucket parameters, we have applied the three algorithms to the Bean MPEG4 and Bean H.263 VBR streams for various values of r and B. Figures 2.4, 2.5, 2.6 and 2.7 show the variation of distortion and distortion deviation with refill rate r on these streams. The value of B was held constant at 4 times the standard deviation while r was made to vary from 0.5 to 1.2 times the mean value of the requirement schedule. We observe that the performance differences are more

Table 2.1: Comparison of Distortion performance of Various Algorithms (Weighted Fractional Measure)

Trace	Mean	S.D.	Peak	r	$B = B_0$	Distortion			S.D. in Distortion		
						Opt.	Naive	Online	Opt.	Naive	Online
bean(MPEG4)	2582	883	3922	2065	3531	1	1.3030	1.2535	1	2.0349	1.9547
bean(var)	2277	859	4267	1821	3436	1	1.6788	1.4679	1	2.6964	2.4908
formula(MPEG4)	4213	1121	4952	3370	4485	1	1.2646	1.1441	1	1.5179	1.2939
formula(var)	3869	1549	6088	3095	6194	1	1.5377	1.3353	1	2.0724	1.8620
Jurassic(MPEG4)	3619	2042	7160	2895	8166	1	1.3188	1.2823	1	1.6777	1.6284
Jurassic(var)	3870	2306	7984	3095	9223	1	1.4428	1.3666	1	2.0107	1.9346
RobinHood(MPEG4)	3007	1394	6877	2405	5575	1	1.3136	1.2543	1	1.4906	1.4787
RobinHood(var)	3161	1560	7357	2528	6240	1	1.5094	1.3880	1	1.9205	1.8787
StarWarsIV(MPEG4)	1665	1059	3484	1332	4237	1	1.2691	1.1846	1	1.4458	1.3182
StarWarsIV(var)	1475	1007	4013	1179	4028	1	1.2400	1.1801	1	1.2668	1.2601

pronounced at lower values of r. As r increases, the naive and the online algorithm give almost identical results.

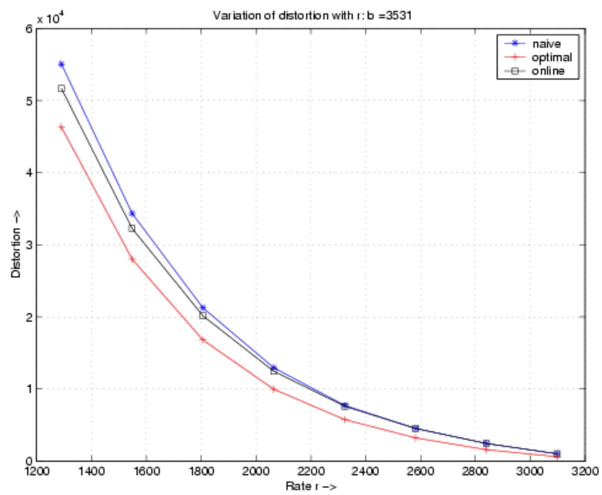

Figure 2.4: Variation of Distortion with r for the Bean MPEG4 stream

A similar study of dependence of distortion and distortion deviation on the Bucket size B revealed that the differences are more pronounced for higher values of B. The value of r was held constant at 0.8 times the mean requirement while B was varied from 0 to 4 times the standard deviation in the original stream. Figures 2.8, 2.9, 2.10 and 2.11 show the results of these simulations.

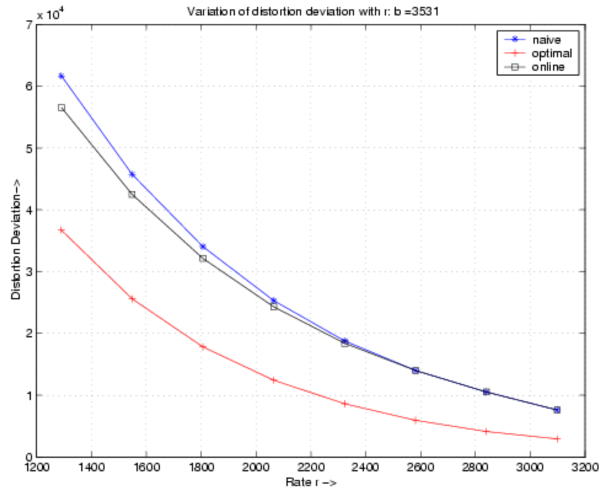

Figure 2.5: Variation of Distortion Deviation with r for the Bean MPEG4 stream

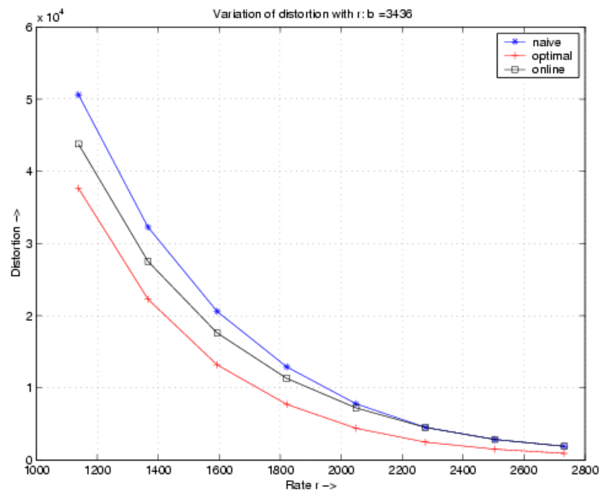

Figure 2.6: Variation of Distortion with r for the Bean VBR stream

2.8 Conclusions

Packet length scheduling when applied to media streams constrained by a Token Bucket Regulator gives performance improvements in terms of distortion and distortion variability of the resultant stream. The improvements are particularly significant for low values of r and large values of B.

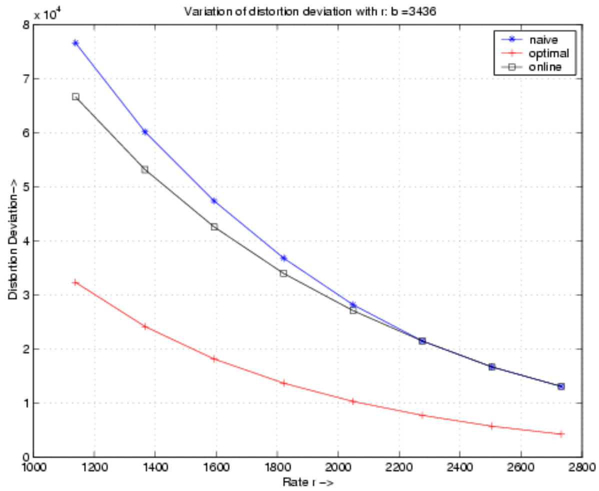

Figure 2.7: Variation of Distortion Deviation with r for the Bean VBR stream

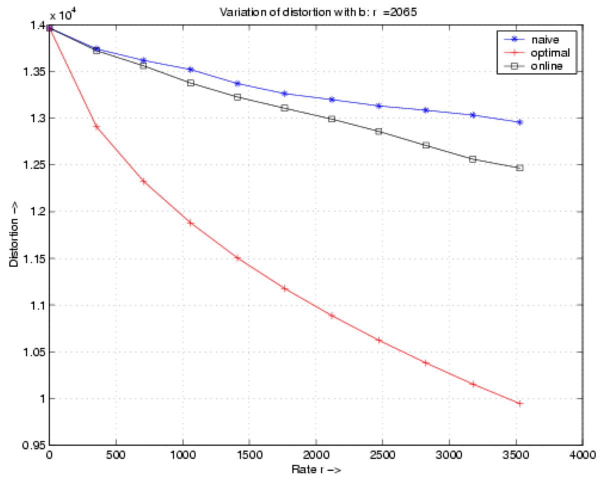

Figure 2.8: Variation of Distortion with B for the Bean MPEG4 stream

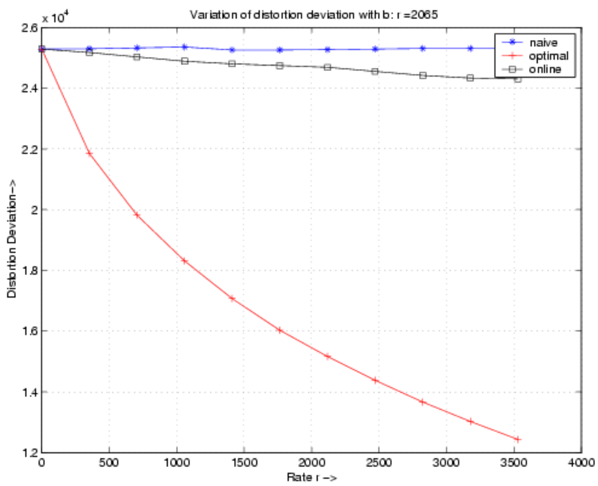

Figure 2.9: Variation of Distortion Deviation with B for the Bean MPEG4 stream

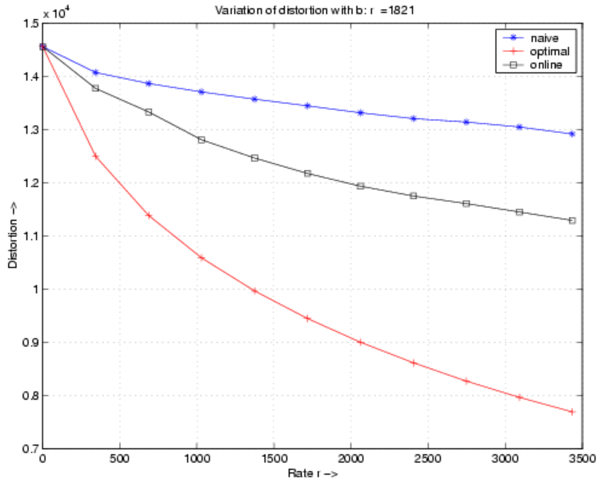

Figure 2.10: Variation of Distortion with B for the Bean VBR stream

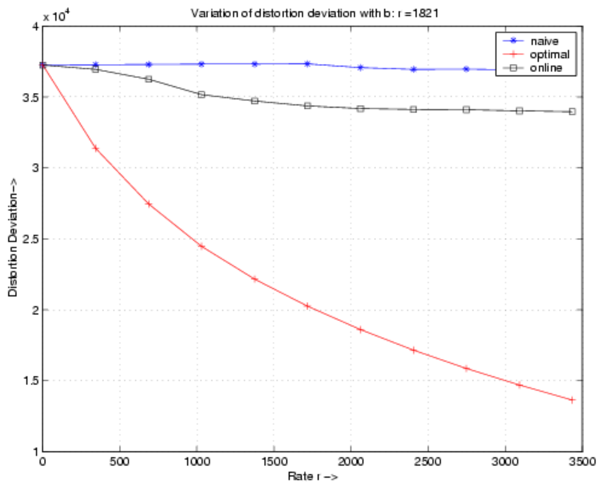

Figure 2.11: Variation of Distortion Deviation with B for the Bean VBR stream

Chapter 3

Information Utility of a Token Bucket Regulator

3.1 Introduction

In this chapter, we investigate the information theoretic utility provided to a flow that has been assured of loss and delay performance guarantees but is required to conform to a Token Bucket Regulator. We take into account the side information, that is present in the form of variable lengths of packets. The idea of using indirect means to convey information or of 'side information channel' has been investigated earlier, by Gallager in his pioneering paper [12]. More recently, [13] gives a detailed exposition to this idea which considers information that can be conveyed through means other than the packet contents themselves, for example, by encoding it into the timing of packets. The network, though, could mask or distort this covert channel by randomly delaying the packets. For the case considered in this chapter, however, the channel becomes distortion free as long as the flow conforms to the regulator, because of the guarantees provided by the network on loss and delay.

In this chapter, we analyze such a 'side information' channel for QoS Networks. Specifically, we derive the maximum amount of information that a traffic flow can convey on an average (or the entropy) using both the packet contents and the packet lengths during a finite transmission interval while still conforming to the Token Bucket Regulator. We call this maximum flow entropy, the information utility of the regulator. This entropy may be achieved by the source by selecting packet lengths according to a state based probability schedule. The maximum entropy gives the maximum information theoretic utility a source can derive, when regulated by a Token Bucket Regulator and would thus have a bearing on a pricing policy that is based on Token Bucket parameters.

3.2 Information Utility

We consider a discrete time model, where the source transmits packets of variable lengths $x_1, x_2, ..., x_N$ at discrete times $1, 2, ..., N$ respectively conforming to the negotiated Token Bucket Regulator, with bucket depth B and token refill rate r. We also take as a regulator parameter, B_0, the initial token count for the bucket and denote the augmented Token Bucket Regulator by $TBR(r, B, B_0)$. Let t_j denote the number of tokens in the bucket just after the j^{th} packet transmission. Note that $t_0 = B_0$. Recall that the constraint imposed by $TBR(r, B, B_0)$ is

$$x_j \leq t_{j-1} + r \ ;\forall j : 1 \leq j \leq N \tag{3.1}$$

If all the packet lengths x_js are conforming, i.e., satisfy (3.1) then the number of residual tokens will evolve as

$$\begin{aligned} t_j &= \min(t_{j-1} + r - x_j, B) \ ; \\ t_0 &= B_0 \end{aligned} \tag{3.2}$$

We seek to maximize the average information that the source may convey in N transmissions or the entropy for a flow of duration N. We denote the flow entropy for a particular source by $\mathcal{E}(B_0, N)$ and the maximum achievable flow entropy by $\mathcal{E}^*(B_0, N)$. The maximum flow entropy is defined to be the information limit of the regulator. As will be shown later, a source may achieve the maximum entropy by following an optimal schedule. The dependence on the Token Bucket Regulator parameters - r and B is to be understood and will not be stated explicitly. We argue that the source has two ways of conveying information to the receiver.

1. At time j, the source transmits a packet of length x_j. It can thus contain x_j bits or $x_j \times ln2$ nats of information.

2. An indirect way of conveying information is the length x_j of the packet selected, which can form an independent alphabet. This arises because the source can transmit packets of any length so long as it conforms to the token bucket constraint, i.e., $0 \leq x_j \leq t_{j-1} + r$.

To find the maximum information that the source may convey on an average in N transmissions, we consider an intermediate stage where the source has n more transmissions to make, i.e., just before the $(N - n + 1)^{th}$ transmission. Let there be b tokens in the bucket, i.e., $t_{N-n} = b$. We assume that the source chooses to transmit a packet of length i with probability $p_i(b, n)$. As before $\mathcal{E}(b, n)$ denotes the entropy in nats for n

slots, with b tokens to begin with (i.e., subject to the $TBR(r, B, b)$ constraint). Then the following recursive equation must hold.

$$\mathcal{E}(b,n) = \sum_{i=0}^{i=b+r} p_i(b,n)\left[i\ln 2 - \ln p_i(b,n) + \mathcal{E}(\min(b+r-i,B),n-1)\right] \quad (3.3)$$

This equation indicates that the flow entropy for duration n is a sum of information contained in the packet length and packet contents of the first of the n transmissions, and the entropy of the remaining flow consisting of $n-1$ transmissions. To simplify notation, we define $\mathcal{E}(b,n)$ to be

$$\begin{aligned} \mathcal{E}(b,n) &= \mathcal{E}(B,n) \;\; ;\forall b > B \\ \mathcal{E}(b,n) &= \sum_{i=0}^{i=b+r} p_i(b,n)\left[i\ln 2 - \ln p_i(b,n) + \mathcal{E}(b+r-i,n-1)\right] \;\; ;\forall b \le B \quad (3.4) \end{aligned}$$

We seek to find the information limit of the regulator, i.e., the maximum possible flow entropy. We observe that the only manner in which the prior transmissions can constrain the rest of the flow is through the number of residual tokens left. Hence to maximize entropy in a flow of duration n, the source would follow a policy that yields maximum entropy for a flow of duration $n-1$, for each of the possible residual token states that it may reach, and this optimal policy would be independent of the probabilities of packet length selection in previous transmissions, i.e., $p_i(b,n)$. This gives for maximum entropy,

$$\mathcal{E}^*(b,n) = \sum_{i=0}^{i=b+r} p_i(b,n)\left[i\ln 2 - \ln p_i(b,n) + \mathcal{E}^*(b+r-i,n-1)\right] \;\; ; \;\; (b \le B) \quad (3.5)$$

For maximum entropy, the probabilities $p_i(b,n)$ are chosen to be those that maximize (3.5) subject to

$$\sum_{i=0}^{i=b+r} p_i(b,n) = 1 \quad (3.6)$$

Using the Lagrange-multiplier method, the optimal $p_i^*(b,n)$ may be obtained by solving the following equation for each $p_i^*(b,n)$.

$$\frac{\partial}{\partial p_i(b,n)}\left\{\left[\sum_{i=0}^{i=b+r} p_i(b,n)\left[i\ln 2 - \ln p_i(b,n) + \mathcal{E}^*(b+r-i,n-1)\right]\right] + \lambda\left[\sum_{i=0}^{i=b+r} p_i(b,n) - 1\right]\right\} = 0$$
$$\text{at } p_i(b,n) = p_i^*(b,n) \quad (3.7)$$

This gives,

$$p_i^*(b,n) = e^{i\ln 2 - 1 + \mathcal{E}^*(b+r-i,n-1)+\lambda} \;, i.e.,$$
$$p_i^*(b,n) \propto e^{i\ln 2 + \mathcal{E}^*(b+r-i,n-1)} \tag{3.8}$$

The constant of proportionality may be evaluated using (3.6). Also,

$$\mathcal{E}^*(b,0) = 0; \quad \forall b$$
$$p_i^*(b,1) \propto 2^i \tag{3.9}$$

Starting with (3.9), and using (3.5) and (3.8) recursively for $n = 1$ to $n = N$, one can compute the values of the optimal probability schedule and the corresponding entropy for a flow of duration N subject to $TBR(r, B, B_0)$. Numerical computations reveal that the dependence of the optimal probabilities $p_i^*(b,n)$ on n vanishes rapidly and the probability schedule does not vary much after $n > 10$. Also for large n, the packet lengths over a wide range are chosen with almost equal probabilities for a given state b, indicating a reduced dependence on i (Figure 3.1). This suggests that a source that is not particularly selective in choosing packet lengths may also achieve close to optimal flow entropy. Further with sufficiently large N, the entropy increases almost linearly with r while logarithmically with B. This is illustrated in Figures 3.2 and 3.3. Also asymptotically, the average entropy per transmission $\left(\frac{\mathcal{E}^*(b,N)}{N}\right)$ and the marginal increase in entropy $(\mathcal{E}^*(b,N) - \mathcal{E}^*(b,N-1))$ approach constant values as illustrated in Figure 3.4.

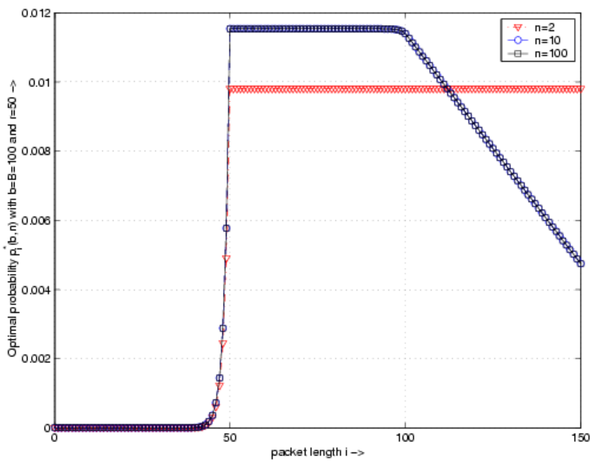

Figure 3.1: Convergence of the optimal Probability Schedule for $r = 50$; $b = B = 100$

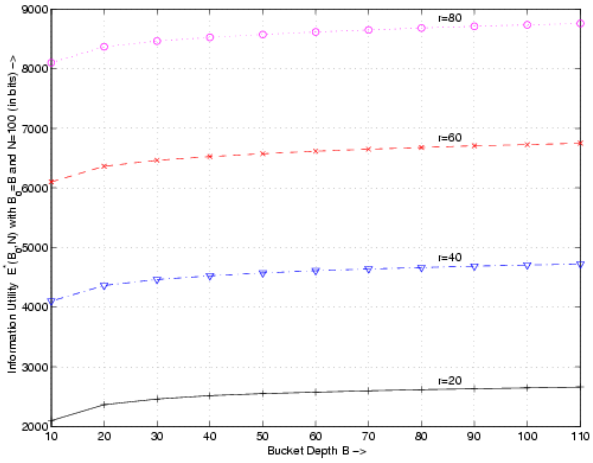

Figure 3.2: Variation of the Information Utility with B for various $r(B_0 = B;\ N = 100)$

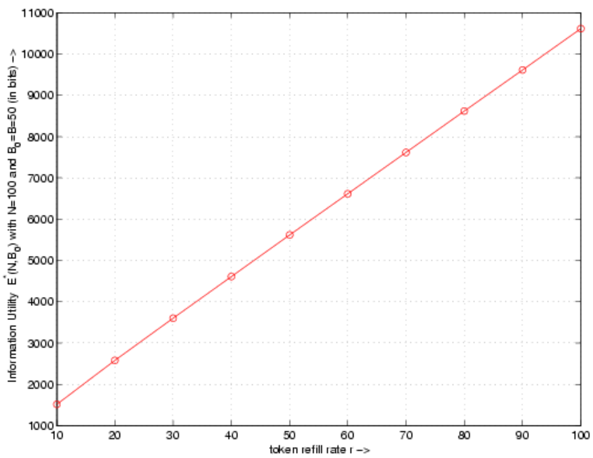

Figure 3.3: Variation of the Information Utility with r: $(B_0 = B = 100;\ N = 100)$

3.3 A Remark on Pricing based on Token Bucket Parameters

Given assured performance guarantees, pricing of services would be a function of the regulator parameters and the flow duration. Our analysis gives the information utility offered to a consumer as a function of these parameters. It has been argued that a sustainable pricing policy must be a linear function of both the regulator parameters r and B, [5]. This is because, these parameters translate linearly to the amount of bandwidth and buffer space required in the network. A function that is non-linear would allow entities to make profits by buying in bulk and selling in small chunks or vice-versa. For example, if the prices were to increase sub-linearly (say, logarithmically) with buffer space then a broker may make profits by buying a large amount of buffer

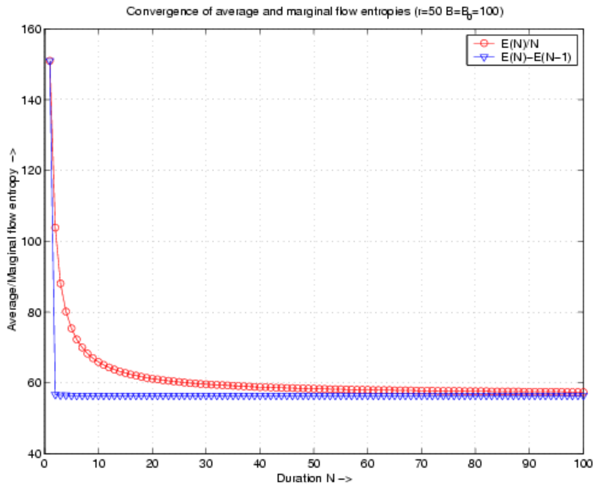

Figure 3.4: Convergence of average and marginal flow entropies for $TBR(50, 100, 100)$

space and selling it in small fragments.

The information theoretic utility, as we show, however is not a linear function of B. It increases much slowly with B. This makes for an interesting case for a consumer seeking to maximize his utility for a given price.

3.4 Conclusions

In this chapter, we have evaluated the maximum average information that can be transmitted by a flow regulated by a Token Bucket Regulator. The maximum entropy of the flow is a function of the negotiated regulator parameters and the flow duration and defines an information utility for the regulator, i.e., it is indicative of the maximum benefits a flow can derive even as it obeys the traffic regulator constraints. Thus it would influence the selection of token bucket parameters by a user and their pricing by the network.

Chapter 4

Discrete Rate Scheduling

4.1 Introduction

The next generation wireless networks seek to evolve to a unified/integrated communication infrastructure to provide a variety of voice, data and multimedia services to the customer. These services will be required to be delivered with widely varying QoS guarantees in terms of performance metrics such as the delay and losses that may be experienced by the associated traffic. The multitude of services that will be offered will not only vary considerably in their performance requirements but also in the statistical properties of the traffic that they will generate. Bandwidth and power are the two primary resources available to any communication system and hence they must be allocated through proper scheduling to meet the services' demands. Further in case of wireless networks, where both bandwidth and power are inherently limited in availability and must be used in the most efficient manner, the task of scheduling becomes even more important. The future communication networks such as the 4G therefore are likely to focus on using the available resources more efficiently, even though this would have to be achieved at a cost of some increase in system complexity, as practical implementations become facilitated by the continued advances in VLSI technology.

Power efficiency has always been an important design challenge for wireless networks, as the user terminals such as the mobile phones, handhelds, notebooks are battery operated. In such a scenario, it is the power consumption or the rate at which the batteries need to be replaced or recharged that would effectively determine the mobility in the network, which is a strong Universal Selling Point (USP) of any wireless network. Further, in certain cases such as the sensor networks, where the sensors may not be charged once their energy has drained, power consumption may in effect determine the lifetime of the network. One of the principal power consuming part of a mobile device is the transmitter of the RF circuitry, which may account for upto 60% of the total power consumed in some systems [14]. Hence any savings in transmission power would translate to substantial improvements in the overall power performance of the system.

While there are avenues for power saving in transmitters at a variety of implementation stages including the R-F circuitry, communication technology and protocols, in this paper we concern ourselves with the power savings that can be achieved in perhaps the simplest possible way, that is through packet scheduling; where the transmitter gains by simply transmitting packets at a more opportune time or in a more opportune fashion. Since these gains would always come at cost of an increased transmission delay, they have to be constrained by the QoS requirements of the traffic in terms of delay and will be limited by the same.

In a wireless network, the tradeoff between power consumption and delay is made possible in two different ways. First, the wireless channel is an erratic one and is characterized by periods of deep fades, in which, transmission at any given rate would require a substantially larger power than what would be required when the channel state is 'good'. If the scheduler can differ the transmission of packets that arrive in a 'bad' channel state to the times when the channel state is 'good', it can bring down the average power consumption of the system. The cost, of course, is the greater delay that would be experienced by packets arriving in 'bad' states and this would tend to increase the overall average delay.

Another means of power-savings arises due to a more fundamental relationship between the power and the rate. Interestingly, this leads to a case where a transmitter may require different amounts of power to serve two sources even if they have the same average packet arrival rate. This occurs because power increases as a convex function of the transmission rate. Specifically, if we consider the Shannon's capacity relations for an Additive White Gaussian Noise (AWGN) channel, then it follows that to transmit at a rate R, in the presence of a noise with variance σ^2, the minimum power requirement for error-free transmission is

$$P = \sigma^2(e^R - 1) \tag{4.1}$$

The corresponding Energy(E) requirement is then

$$E = \frac{\sigma^2(e^R - 1)}{R} \tag{4.2}$$

The convex nature of the relationship between E and R in (4.2) allows one to save energy by choosing the rate R at various stages in transmission in an appropriate fashion i.e., to transmit the data in opportunistic chunks. This again, however, would lead to an increase in buffered data and hence, the average delay. As an example consider a slotted transmission system serving a single application that generates r number of packets of equal sizes in every alternate slot. Consider the performance of the following two transmission schemes.

- *Scheme A* : The transmitter transmits the packets in the same slot as they arrive. The packets then do not experience any buffering delays. However, following (4.1) where we take the slots to be of a unit length and $\sigma = 1$ for convenience, the average power consumed by this transmitter will be $P_A = \frac{e^r - 1}{2}$.
- *Scheme B* : A scheduler sends half the packets i.e $\frac{r}{2}$ packets for transmission in the same slot as they arrive and schedules the remaining for the transmission in the next slot that will have no arrivals. In this scheme, a packet would experience a delay of $\frac{1}{2}$ slot on an average. The average power requirement, however, is now $P_B = e^{\frac{r}{2}} - 1$ which can be a substantial improvement over scheme A depending on the size r.

The above example illustrates the gains that can be realized through simple rearrangement of transmission batches at the cost of an increased delay. The extent of the gains would however depend on the delay budget of the system as well as the nature of the packet arrival process.

Remark 4.1. The relation (4.2) actually gives an asymptotic limit on the power and would not hold when there are delay constraints on the delivery of data. However, the relation between energy (or power) and rate for practical robust transmission schemes is also convex. This convex nature is evident in non-asymptotic cases when error exponents or finite length codes are considered. The error exponent for Gaussian channels, for example, varies logarithmically in power and linearly in rate. i.e., schemes with acceptable rates of error, also show a convex $P - R$ relation.

Remark 4.2. As an example of practical schemes, consider a simple uncoded system using Q-ary QAM modulation [14]. For large ($Q \geq 16$) rectangular QAM constellations, the average power required to achieve a certain minimum distance between constellation points is only marginally greater than the average power required to achieve the same minimum distance using the best known QAM. The constellation points are given by the two-tuples $((2q_1 - 1 - Q)d, (2q_2 - 1 - 1Q)d)$, for $q_1, q_2 = 1, 2, ..., Q$. The transmission data rate is $R = 2\log_2 Q$ bits/symbol and the average transmit power is $P = \frac{2d^2(Q+1)(Q-1)}{3Q}$. The minimum distance between signal points is given by $2d$ for all Q , hence these constellations achieve approximately the same Bit Error Rate (BER). Thus the $P - R$ relation, i.e. the relationship between power and rate at an almost constant BER is given by

$$P = \frac{2d^2(2^R - 1)}{3(2^{\frac{R}{2}})} \tag{4.3}$$

Thus, the uncoded QAM system also exhibits a near exponential $P - R$ relationship.

The convex nature of the $P-R$ relationship thus offers an opportunity to trade delay with power. To realize this, however the system must have the capability of transmitting at different rates.

In practice, a scheduler would exploit power savings that can be achieved due to both the varying states of a wireless channel as well as the convex $P - R$ relationship. However in this work, we shall largely focus on gains that can be reaped through the latter, by suitably varying the transmission rates. Moreover since the $P - R$ relation holds for any communication system and not just wireless systems, it is possible to apply these ideas to wired systems as well. We consider a slotted point-to-point wireless transmission system. This model is believed to be directly applicable to the uplink (mobile station to base station) transmissions in wireless networks (both GSM and CDMA based networks) or certain wired systems as the DOCSIS (cable data transmission system, refer [15] for details) where power savings are of special importance. We fill focus on a class of scheduling actions that may be realized with only a reasonable implementation complexity.

4.1.1 Related Work

The possibility of realizing power gains through varying transmission rates was first recognized in [16]. This paper has considered a hypothetical problem where a set of packets arriving at different times in an interval $[0, T]$ have all to be transmitted on a point-to-point link by a deadline T in the most energy efficient manner. The authors have presented a solution to this problem, i.e., the optimal transmission times leading to the least energy consumption for the 'offline' case where the actual arrival times of individual packets are known apriori and then used this solution to develop a heuristic schedule for an 'online' transmission case. The work was later extended to consider a point-to-multipoint link in [17]. In [14], the authors have considered the problem of power-efficient scheduling under average and absolute delay constraints. They have used a slotted arrival system and considered an arbitrary iid packet arrival process. A characterization of the optimal scheduler has been provided in terms of a smaller class of deterministic schedulers. Though [14] does recognize the potential gains offered by the time-varying nature of wireless channels, [14] and [16] are largely concerned with profitably exploiting the convex $P-R$ relationship. [18] on the other hand has dealt with energy-efficient scheduling under an average delay constraint for time varying channels. However, here the authors assume a linear relationship between P and R and thus fail to take into account the gains that can be realized through varying the transmission rates. More recently, the work in [19] has considered the comprehensive problem of energy-efficient scheduling taking into account both the fluctuating channel conditions and the convex $P - R$ relationship.

A common feature of all these works however, is that while designing the scheduling action, it has been implicitly assumed that transmissions rates (or times in case of [16, 17]) can be varied arbitrarily. This is a rather unrealistic assumption because the rate referred to in (4.2), for which power becomes a convex function, is the actual physical rate of communication. Hence, except for [18], which in fact doesn't tap the gains due to $P - R$ relationship, it would be practically very difficult to have an actual physical system in which the rates can be varied as required by the scheduler. Not only would this require transmitters capable of varying transmission rates continuously, but also a considerable protocol overhead and receiver complexity for demodulation of signals so transmitted. Hence, in our work, we consider only the class of schedulers that we believe would lead to practically realizable systems. Specifically, we consider only a class $\mathcal{P}$ of schedulers, called the constant power schedulers, characterized by the practical constraint that they can transmit at only a finite discrete set of pre-determined rates. We seek to find the most energy efficient scheduler that belongs to $\mathcal{P}$ for a given discrete arrival process under an average delay constraint.

4.2 Model and Terminology

The model used here is similar to that of [14] and to some extent that of [19]. This consists of a single transmitter, single receiver system or a point-to-point link.

1. A slotted transmission system is considered. The slot is the smallest duration of scheduling packets for transmission. All packets are assumed to be of a fixed size.

2. In a slot n, a_n denotes the number of packet arrivals and u_n the number of packets transmitted. The packet arrival process is assumed to be stationary iid with p_i being the probability of i packet arrivals in any slot. The average arrival rate is denoted by $\lambda = E[a_n]$.

3. A packet arrived in slot n is available for transmission only from slot $(n+1)$ onwards (using this convention the minimum achievable delay is of 1 slot for any policy). The buffer is assumed to be of infinite size. Buffer occupancy x_n, measured just at the beginning of a slot n then evolves as

$$x_{n+1} = x_n - u_n + a_n \tag{4.4}$$

4. In general, the channel may be a time-varying one. We assume that the channel state remains unchanged over an interval corresponding to a slot length and denote it by y_n for slot n. We further assume for each n, y_n takes a unique value from a finite discrete set of channel states $\boldsymbol{B} : \{1, 2, ..., M\}$.

5. Each channel state has a corresponding cost of transmission associated with it, denoted by $\{c_1, c_2, ..., c_M\}$. The cost c_j indicates that when the channel is in state j, energy requirement for any rate R is c_j times the minimum requirement for that rate R, which occurs when the channel is in an 'ideal' state. Without loss of generality, the 'ideal' state is labeled as 1 so that $c_1 = 1$ and we enforce the strict ordering $c_1 < c_2 < ... < c_M$. y_n may evolve according to a known memoryless or first order Markov process. The scheduler is assumed to be able to 'sense' the state in each slot, before transmission, and adapt the rate and the power accordingly.

The above system can be modeled by a discrete time queue. Further, the 'state' of the system then can be characterized completely by a two-tuple comprised of the buffer occupancy and the channel state - $\boldsymbol{v}_n = (x_n, y_n)$. Here, we only investigate schedulers belonging to a certain class $\mathcal{P}$, called the constant power.

Definition 4.1. A scheduler $S \in \mathcal{P}$ is one characterized by the constraint that it can transmit only at a set of finite pre-configured rates, i.e., $u_n \in \mathcal{N}^K = \{0, N_1, N_2, ..., N_K\}$. A transmitter capable of transmitting at K different non-zero rates is said to belong to the class $\mathcal{P}^{\mathcal{K}}$ of K-rate schedulers.

Definition 4.2. If in $\mathcal{N}^K$, $N_i = iM$ for some M, then the scheduler is said to be a $(M\text{x}K)$-rate scheduler.

We define and classify a few more sub-classes of the constant power schedulers on the basis of the scheduling policy chosen. We will consider only stationary scheduling policies, as they lead to an equilibrium state distribution.

Definition 4.3. A deterministic scheduler is the one in which the scheduling action can be specified as a stationary function $u(.)$ on the state v_n i.e. $u_n = u(\boldsymbol{v_n})$.

A stochastic scheduler on the other hand would randomly choose u_n according to a probability distribution which is a function of the system state $\boldsymbol{v_n}$. A sub-class of the deterministic schedulers is the greedy schedulers defined as follows.

Definition 4.4. A scheduling policy is said to be greedy if, whenever $x_n \geq N_i \in \mathcal{N}^K$, $u_n \geq N_i$.

Thus for any state $\boldsymbol{v_n}$, a greedy scheduler transmits at the maximum permissible rate. It follows that among all schedulers with the same set of allowable rates $\mathcal{N}^K$, the greedy scheduler would give the best average delay performance. An important sub-class of deterministic schedulers is the monotone schedulers defined as follows.

Definition 4.5. A deterministic scheduler S is said to be monotone if the scheduling function $u(.)$ of the buffer-state $\boldsymbol{v} : (x, y)$ is increasing in x for a fixed value of y.

The problem of power-efficient scheduling then maybe stated as:
Given an iid packet arrival process on a_n, a channel state evolution process on y_n and an average delay constraint $\bar{D}$, design for a K-rate scheduler, the set of rates $\mathcal{N}^K$, and the corresponding stationary scheduling policy that would give the optimal power performance.

Here, we consider a very simplified version of the general problem stated above. The channel state will be assumed to be constant. We refer to this as the constant channel case ($M = 1$). The state $\boldsymbol{v_n}$ then collapses to just the buffer occupancy x_n. Further, rather than finding the optimal scheduler for a given arrival process and delay constraints, we would be more concerned with analyzing the queue associated with a given scheduler action and an arrival process. Specifically, we illustrate a method to obtain the equilibrium or stationary buffer occupancy distribution given the arrival process and the scheduler action. This allows us to evaluate the performance metrics such as the delay and average power requirements of a scheduler for a given arrival process. We first analyze the greedy class of schedulers and then extend the method to monotone schedulers. For the constant channel case, we begin with analyzing the performance of the simplest possible scheduler, a 1-rate scheduler, i.e., a scheduler that can transmit only at 1 constant rate N in a slot (or not transmit at all) in Section 4.4 and then extend the analysis to multirate greedy schedulers and finally monotone schedulers in Sections 4.5 and 4.6 respectively.

4.2.1 Bulk Queues

We model the single link transmission system described in the previous section as a discrete time queue in which there could be both multiple arrivals and multiple departures in a single instant. The associated queueing problem thus falls within the framework of bulk (arrival and service) queues. Transform methods have been profitably used to solve bulk queue problems and we follow a similar approach here. In the queues that we consider, both the inter-arrival and departure times can be taken to be deterministic and of a unit slot length.

Most of the subsequent work is devoted to the problem of finding the steady state distribution for a given arrival process under the action of certain classes of schedulers. These schedulers that we consider are a deterministic sub-class of the class $\mathcal{P}$ schedulers described earlier - schedulers that allow departures to occur only in batches of some fixed sizes. The simplest case arises when only one batch size is allowed and the queueing system that we encounter is similar to that considered in [20, 21]. However, this is not quite the $D^X/D^m/1$ queue described therein. While in the $D^X/D^m/1$ queue, departures occur in any sizes upto an integer m, in the queues that we consider, departure batch sizes is restricted to one or more integers. In fact, to the best of our knowledge, some of the queues solved in this work have not been analyzed before. Besides the power efficient scheduling problem, this queue analysis may have applications in other domains such as ATM networks, transport networks or problems in management and decision sciences. We use the following notation for the queues that we counter.

Definition 4.6. We use the notation $D^X/D/\mathcal{N}^K$ to denote the infinite buffer queue resulting out of a deterministic arrival process and a K-rate greedy scheduling action.

4.3 1-rate Scheduler

In this section we consider the problem of finding the optimal 1-rate scheduler.

Proposition 4.1. *For the constant channel ($M = 1$) case, among the class of 1-rate schedulers only, there is a greedy scheduler which gives the optimal performance.*

Proof. The proof is quite easy. We first show that any two 1-rate schedulers that use the same rate N will have the same power performance. Let $E(N)$ be the energy required for transmitting N packets in 1 slot and let q be the fraction of slots in which S transmits. If the queue is stable, then we must have $N > \lambda$. Then,

$$\begin{aligned} q &= \frac{\lambda}{N} \\ C(S_N) &= \frac{\lambda}{N}E(N) \end{aligned} \tag{4.5}$$

where, $C(S_N)$ denotes the average energy per slot or power requirement of the scheduler S using a single rate N. Thus, as the greedy scheduler would give the same energy performance and at least as good a delay performance as any scheduler using that rate; only greedy schedulers need be considered for this case. □

Proposition 4.2. *Let $C(N)$ denote the average power requirement of a 1-rate scheduler of rate N. Then if $N' > N$, $C(N') > C(N)$.*

Proof. This follows from the convexity of $E(N)$.

$$\begin{aligned}
& C(N') - C(N) \\
= \; & \lambda \left[\frac{E(N')}{N'} - \frac{E(N)}{N} \right] \\
= \; & \frac{\lambda}{N'N} \left[N(E(N') - E(N)) - E(N)(N' - N) \right] \\
= \; & \frac{\lambda(N' - N)}{N'} \left[\frac{E(N') - E(N)}{N' - N} - \frac{E(N) - E(0)}{N - 0} \right] \\
> \; & 0 \qquad (4.6)
\end{aligned}$$

as $N' > N$ and $E(N)$ is convex. □

From the above results, it follows that, under constant channel conditions ($M = 1$), a greedy scheduler transmitting at rate N, denoted by G_N, is an optimal 1-rate scheduler, *iff* N is the smallest integer that satisfies the delay constraint.

4.4 Steady State Queue Analysis of G_N

Let $t_{ij} = P[x_{n+1} = j | x_n = i]$. Let π_i be the steady state probability of the buffer occupancy being i. The scheduler action of G_N on buffer state x is defined as

$$\begin{aligned}
u(x) &= 0 \;\; if \;\; x < N \\
u(x) &= N \;\; else \qquad (4.7)
\end{aligned}$$

It then follows,

$$\begin{aligned}
t_{ij} &= p_{j-i} \;\; if \;\; i < N \\
&= p_{j-i+N} \;\; if \;\; i \geq N \qquad (4.8)
\end{aligned}$$

Then for the steady state distribution for the Markov chain on buffer occupancy x_n, i.e.; the $D^X/D/\mathcal{N}^1$ queue, the flow equations are

$$\begin{aligned}
\pi_i &= \sum_{j=0}^{j=N-1} \pi_j p_{i-j} + \sum_{j=N}^{j=\infty} \pi_j p_{i-j+N} \\
&= \sum_{j=0}^{j=N-1} \pi_j p_{i-j} + \sum_{j=0}^{j=\infty} \pi_{j+N} p_{i-j} \qquad (4.9)
\end{aligned}$$

where, the definitions of p_i have been extended so that $p_i = 0;\ \forall i < 0$. It may be verified that if (4.9) written in terms of the characteristic functions $\boldsymbol{\pi}(z)$ and $\boldsymbol{p}(z)$ gives

$$\begin{aligned} \boldsymbol{\pi}(z) &= \boldsymbol{p}(z)\boldsymbol{\pi}^{\mathbf{0}}(z) + \boldsymbol{p}(z)\frac{\boldsymbol{\pi}(z) - \boldsymbol{\pi}^{\mathbf{0}}(z)}{z^N} \\ where, & \qquad\qquad (4.10) \\ \boldsymbol{p}(z) &= \sum_{i=0}^{i=\infty} p_i z^i \\ \boldsymbol{\pi}(z) &= \sum_{i=0}^{i=\infty} \pi_i z^i \\ \boldsymbol{\pi}^{\mathbf{0}}(z) &= \sum_{i=0}^{i=N-1} \pi_i z^i \qquad (4.11) \end{aligned}$$

Note that, $\boldsymbol{\pi}^{\mathbf{0}}(z)$ is a $(N-1)$ degree polynomial. Now (4.10) may be rewritten as

$$\boldsymbol{\pi}(z) = \frac{\boldsymbol{p}(z)\left(z^N - 1\right)}{z^N - \boldsymbol{p}(z)}\boldsymbol{\pi}^{\mathbf{0}}(z) \qquad (4.12)$$

This expression is like an eigenvalue relationship in so much as the terms in $\boldsymbol{\pi}^{\mathbf{0}}(z)$ also occur in $\boldsymbol{\pi}(z)$[1]. To simplify, define

$$\begin{aligned} \boldsymbol{\chi}(z) &= \frac{\boldsymbol{\pi}(z) - \boldsymbol{\pi}^{\mathbf{0}}(z)}{z^N} \\ \Rightarrow \boldsymbol{\pi}(z) &= \boldsymbol{\pi}^{\mathbf{0}}(z) + z^N\boldsymbol{\chi}(z) \qquad (4.13) \end{aligned}$$

Note that $\boldsymbol{\pi}^{\mathbf{0}}(z)$ and $\boldsymbol{\chi}(z)$ are now decoupled, in the sense they have no implicit relationship other than that demanded by (4.10) which may be now written as

$$\frac{\boldsymbol{\pi}^{\mathbf{0}}(z)}{\boldsymbol{\chi}(z)} = \frac{z^N - \boldsymbol{p}(z)}{\boldsymbol{p}(z) - 1} \qquad (4.14)$$

Solutions to $\boldsymbol{\pi}^{\mathbf{0}}(z)$ and $\boldsymbol{\chi}(z)$ that satisfy (4.14) and also satisfy the constraints of being partial characteristic functions of a probability distribution, i.e., the coefficients are non-negative and the functions converge in some region, on appropriate normalization, would give a possible solution to the distribution $\boldsymbol{\pi}(z)$.

We seek methods that give such functions from (4.14). In particular, we exploit the fact that $\boldsymbol{\pi}^{\mathbf{0}}(z)$ is an $(N-1)$ degree polynomial. The numerator function $\left(z^N - \boldsymbol{p}(z)\right)$ may itself be reduced to an $(N-1)$ degree polynomial by transferring appropriate zeros in it as poles to the denominator function $(\boldsymbol{p}(z) - 1)$. If the resulting numerator and denominator functions, thus obtained, have positive and a convergent power series expansion; then upon normalization this gives a solution for $\boldsymbol{\pi}(z)$ or the steady state distribution. This is indeed possible. This method is best illustrated by taking an example.

[1] This expression is somewhat similar to that derived for a $D^X/D^m/1$ queue in [20].

4.4.1 Example 1: Geometric Arrival Process

Consider the Geometric Arrival Process[2] characterized by

$$\begin{aligned} p_i &= (1-\alpha)\alpha^i \qquad (4.15) \\ \text{then, } \lambda &= \frac{\alpha}{1-\alpha} \\ \text{and } \boldsymbol{p}(z) &= \frac{1-\alpha}{1-\alpha z} \qquad (4.16) \end{aligned}$$

Then from (4.14),

$$\begin{aligned} \frac{\boldsymbol{\pi^0}(z)}{\boldsymbol{\chi}(z)} &= \frac{z^N - \frac{1-\alpha}{1-\alpha z}}{\frac{1-\alpha}{1-\alpha z} - 1} \\ &= \frac{\left(z^N - 1\right) - \alpha\left(z^{N+1} - 1\right)}{\alpha(z-1)} \\ \text{Let, } \boldsymbol{Q}(z) &= \left(z^N - 1\right) - \alpha\left(z^{N+1} - 1\right) \qquad (4.17) \end{aligned}$$

$\boldsymbol{Q}(z)$ here is a polynomial of degree $(N+1)$. It has a zero at $(z=1)$ which cancels with the denominator. It can be shown that $\boldsymbol{Q}(z)$ has one more positive real zero, say β, and that $(\beta > 1)$ for $N > \frac{\alpha}{1-\alpha} = \lambda$; which is the condition for the queue to be stable. For this case, consider the functions

$$\begin{aligned} \boldsymbol{a}(z) &\triangleq \frac{\left(z^N - 1\right) - \alpha\left(z^{N+1} - 1\right)}{(z-1)\left(1-\beta^{-1}z\right)} \\ \boldsymbol{b}(z) &\triangleq \alpha\left(1-\beta^{-1}z\right)^{-1} \qquad (4.18) \end{aligned}$$

$\boldsymbol{a}(z)$ is a polynomial of degree $(N-1)$. It may be shown that $\boldsymbol{a}(z)$ has positive coefficients (as all other roots of $\boldsymbol{Q}(z)$ other than β have negative real parts). Also $\boldsymbol{b}(z)$ has positive coefficients in its power series expansion and the same converges for $z < \beta$. Then a solution to $\boldsymbol{\pi}(z)$ is

$$\boldsymbol{\pi}(z) = \frac{1}{a(1)+b(1)}\left[\boldsymbol{a}(z) + z^N \boldsymbol{b}(z)\right] \qquad (4.19)$$

Clearly $\boldsymbol{\pi}(z)$ satisfies (4.10) and $(\boldsymbol{\pi}(1) = 1)$. Moreover, because there is but one such zero β outside the unit circle, this solution is unique. The constant $(\boldsymbol{a}(1) + \boldsymbol{b}(1))$ can be evaluated to be

$$\boldsymbol{a}(1) + \boldsymbol{b}(1) = N\frac{1-\alpha}{1-\beta^{-1}}$$

[2]Note that, in this chapter, by 'Geometric Arrival Process', we refer to a deterministic bulk arrival process whose batch size is geometrically distributed. This is not to be confused with an arrival process, where the inter-arrival times are geometrically distributed, which also is sometimes referred to as a Geometric Arrival Process.

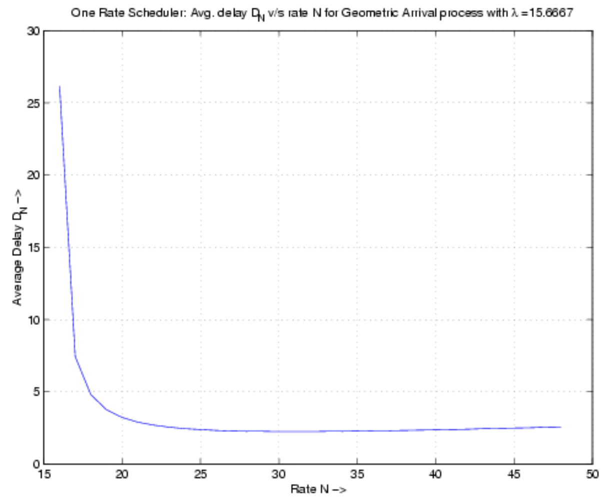

Figure 4.1: Average delay D_N v/s rate N for G_N for a Geometric Arrival Process

$$\begin{aligned} \boldsymbol{\pi}(z) &= \frac{1-\beta^{-1}}{N(1-\alpha)} \frac{\sum_{i=0}^{i=N-1} z^i - \alpha \sum_{i=0}^{i=N} z^i + \alpha z^N}{1-\beta^{-1}z} \\ &= \frac{1-\beta^{-1}}{N} \frac{\sum_{i=0}^{N-1} z^i}{1-\beta^{-1}z} \end{aligned} \tag{4.20}$$

The average delay suffered is given by Little's theorem as

$$\begin{aligned} D_N &= \frac{E[x]}{\lambda} \\ E[x] &= \boldsymbol{\pi}'(1) \\ &= \frac{N-1}{2} + \frac{1}{\beta-1} \\ \Rightarrow D_N &= \frac{1-\alpha}{\alpha}\left[\frac{N-1}{2} + \frac{1}{\beta-1}\right] \end{aligned} \tag{4.21}$$

Figure 4.1 shows a plot of average delay D_N v/s the rate N of the greedy 1-rate scheduler G_N for a geometric arrival process with $\alpha = 0.94$. From this, given an average delay constraint $\bar{D}$, the optimal rate N may be selected.

4.4.2 Example 2: Polynomial Arrival Processes

We now consider the case where the arrival process has a finite support, i.e., $\exists$ a finite R such that $p_i = 0 \;\; \forall i > R$. WLOG, assume $p_R > 0$. We also assume that $p_0 > 0$.[3] It is clear that the arrival transform function $\boldsymbol{p}(z)$ will be a polynomial of degree R. Consider (4.14), rewritten for convenience as

$$\boldsymbol{\chi}(z) = \frac{\boldsymbol{p}(z) - 1}{z^N - \boldsymbol{p}(z)} \boldsymbol{\pi}^{\mathbf{0}}(z) \tag{4.22}$$

[3]The case where $p_0 = 0$ may be tackled similarly by first eliminating the states that are non-recurrent.

We assume that $N \leq R$. In that case the denominator function $\boldsymbol{D}(z) = z^N - \boldsymbol{p}(z)$ in (4.22) is a polynomial of degree R. Note that $z = 1$ is a a zero of $\boldsymbol{D}(z)$. Then we have the following result.

Proposition 4.3. *Suppose, the arrival process $\boldsymbol{p}(z)$ to a $D^x/D/\mathcal{N}^1$ queue is a polynomial of degree $R \geq N$ and $\boldsymbol{p}(0) \neq 0$. Let $c(z-1)\boldsymbol{g}(z)\boldsymbol{h}(z)$ be the factorization of $\boldsymbol{D}(z) = z^N - \boldsymbol{p}(z)$, such that $\boldsymbol{g}(z)$ and $\boldsymbol{h}(z)$ are monic polynomials with the property that $\boldsymbol{g}(z)$ has zeros only on or within the unit circle and $\boldsymbol{h}(z)$ has zeros only outside the unit circle. If, the condition for the queue stability;*

$$\begin{aligned} N &> \lambda \quad \textit{i.e.,} \\ N &> \boldsymbol{p}'(1) \end{aligned} \tag{4.23}$$

is satisfied, an explicit solution to the generating function for the steady state distribution of $D^X/D/\mathcal{N}^1$ is given by

$$\begin{aligned} \boldsymbol{\pi}^{\mathbf{0}}(z) &= \frac{c\boldsymbol{h}(1)}{N}\boldsymbol{g}(z) \\ \boldsymbol{\chi}(z) &= \frac{\boldsymbol{h}(1)}{N}\frac{\boldsymbol{p}(z)-1}{(z-1)\boldsymbol{h}(z)} \\ \boldsymbol{\pi}(z) &= \frac{\boldsymbol{h}(1)}{N}\frac{\boldsymbol{p}(z)\sum_{i=0}^{i=N-1} z^i}{\boldsymbol{h}(z)} \end{aligned} \tag{4.24}$$

Proof. It can be shown using Rouche's theorem, that under the conditions of the problem, $g(z)$ is of degree $N-1$ and that all its zeros lie inside the unit circle. A detailed proof of the same is provided in the Appendix B. Thus $\boldsymbol{D}(z)$ has one zero at $z = 1$ and exactly $N - 1$ zeros inside the unit circle.

Now $\boldsymbol{\chi}(z)$ has the rational form as given by (4.14). For $\boldsymbol{\pi}(z)$, to be a convergent series, the partial generating function $\boldsymbol{\chi}(z)$ must also converge. Hence, it cannot have any poles on or inside the unit circle. The N zeros of $\boldsymbol{D}(z)$ that lie on or within the unit circle, i.e., the zeros of $\boldsymbol{g}(z)$ and the zero $z = 1$ then must also occur in the numerator polynomial. It can be verified through direct substitution that $\left(z^N - \boldsymbol{p}(z)\right)$ and $(1 - \boldsymbol{p}(z))$ have only one zero, viz., $z = 1$ in common for $N > 1$.

Hence, the $N-1$ zeros of $\boldsymbol{D}(z)$ that lie within $|z| = 1$ must occur in $\boldsymbol{\pi}^{\mathbf{0}}(z)$. Note that since $\boldsymbol{\pi}^{\mathbf{0}}(z)$ is a polynomial of degree $N-1$, this uniquely determines $\boldsymbol{\pi}^{\mathbf{0}}(z)$ to a scalar multiplicative constant A as $\boldsymbol{\pi}^{\mathbf{0}}(z) = A\boldsymbol{g}(z)$. From the relation (4.22) and the normalization constraint $\boldsymbol{\pi}(1) = 1$, i.e., $\boldsymbol{\pi}^{\mathbf{0}}(1) + \boldsymbol{\chi}(1) = 1$, it then follows that $\boldsymbol{\chi}(z)$ and $\boldsymbol{\pi}(z)$ as defined in (4.24) satisfy (4.14). Further, as $\boldsymbol{\pi}^{\mathbf{0}}(z)$ is of degree $N-1$, this is the only convergent function that satisfies (4.10). This in turn uniquely fixes $\boldsymbol{\chi}(z)$. Since, an ergodic chain must have a stable solution, and moreover, as this solution is

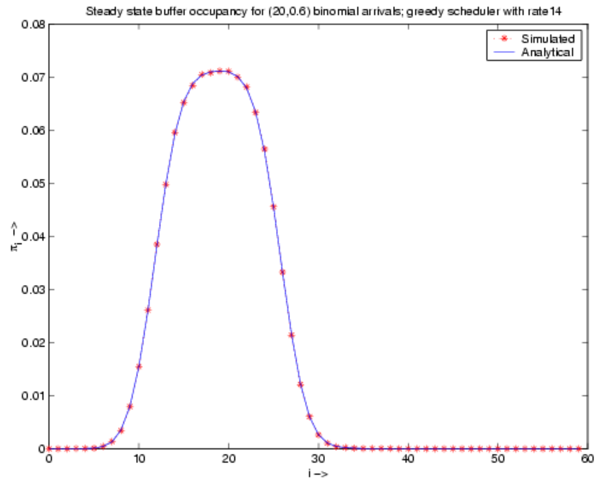

Figure 4.2: Comparison of steady state distributions of buffer occupancy obtained analytically and through simulation for a 1-rate greedy scheduler

unique, it follows that $\boldsymbol{\pi}(z)$ as given in 4.24 is the generating function of the steady state distribution of the concerned queue. □

It can be verified that the average power will be given by $\frac{\lambda}{N}E(N)$ and the average delay suffered by

$$D_N = 1 + \frac{N-1}{2\lambda} - \frac{\boldsymbol{h}'(1)}{\boldsymbol{h}(1)\lambda}. \tag{4.25}$$

Figure 4.2 shows a validation of the expressions in (4.24) when a one-rate scheduler with $N = 15$ schedules packets from a binomial arrival process. This process is simulated as 20 packet producing applications independently generating a packet in a transmission slot with probability 0.6. The corresponding $\boldsymbol{p}(z)$ is described by (4.26)). The analytically predicted steady state buffer occupancy distribution was compared with one obtained through simulations.

$$\boldsymbol{p}(z) = ((1-0.6)+0.6z)^{20} \tag{4.26}$$

Similar results may be obtained when the arrival process is a rational function of the form $\boldsymbol{p}(z) = \frac{\boldsymbol{N}(z)}{\boldsymbol{D}(z)}$ when $\boldsymbol{N}(z)$ and $\boldsymbol{D}(z)$[4] are relatively prime. Specifically, if $c(z-1)\boldsymbol{h}(z)\boldsymbol{g}(z)$ is the factorization of $z^N\boldsymbol{D}(z) - \boldsymbol{N}(z)$ as in (4.24), then

$$\begin{aligned}\boldsymbol{\pi}(z) &= \frac{\boldsymbol{h}(1)}{\boldsymbol{N}(1)N}\frac{\boldsymbol{N}(z)\sum_{i=0}^{i=N-1} z^i}{\boldsymbol{h}(z)} \\ D_N &= \frac{\boldsymbol{N}'(1)}{\boldsymbol{N}(1)\lambda} + \frac{N-1}{2\lambda} - \frac{\boldsymbol{h}'(1)}{\boldsymbol{h}(1)\lambda}. \end{aligned} \tag{4.27}$$

[4]Note that $\boldsymbol{D}(z)$ must have zeros that lie outside the unit circle.

4.5 Multirate Scheduling

In this section, we first consider the problem of obtaining the steady state distribution for a 2-rate greedy scheduler, capable of transmitting at the non-zero rates N_1, N_2 (i.e., the $D^X/D/\mathcal{N}^2$ queue), and then extend the analysis for a K-rate scheduler where the transmitter can transmit at any rate from the set $\mathcal{N}^K : \{0, N_1, N_2, ..., N_K\}$ (i.e., the $D^X/D/\mathcal{N}^K$ queue). The transition probabilities for a K-rate scheduler may be easily identified as

$$\begin{aligned}
t_{ij} &= p_{j-i} \quad i < N_1 \\
&= p_{j-i+N_1} \quad N_1 \leq i < N_2 \\
&\vdots \\
&= p_{j-i+N_p} \quad N_{p-1} \leq iN_p \\
&\vdots \\
&= p_{j-i+N_K} \quad i \geq N_K
\end{aligned}$$

Using these, the flow equation may be written succinctly in terms of the generating functions in a form analogous to (4.10).

$$\sum_{i=0}^{i=K-1} z^{N_i}\boldsymbol{\pi}^{\boldsymbol{i}}(z) + z^{N_K}\boldsymbol{\chi}(z) = \boldsymbol{p}(z)\left[\sum_{i=0}^{i=K-1}\boldsymbol{\pi}^{\boldsymbol{i}}(z) + \boldsymbol{\chi}(z)\right] \tag{4.28}$$

$$\Rightarrow \sum_{i=0}^{i=K-1}\left(\boldsymbol{p}(z) - z^{N_i}\right)\left(\boldsymbol{\pi}^{\boldsymbol{i}}(z)\right) = \left(z^{N_K} - \boldsymbol{p}(z)\right)\boldsymbol{\chi}(z)$$

$$\boldsymbol{\chi}(z) = \frac{\sum_{i=0}^{i=K-1}(\boldsymbol{p}(z) - z^{N_i})\boldsymbol{\pi}^{\boldsymbol{i}}(z)}{z^{N_K} - \boldsymbol{p}(z)} \tag{4.29}$$

note that, $N_0 = 0$.

As before, the partial generating functions $\boldsymbol{\pi}^{\boldsymbol{i}}(z)$and $\boldsymbol{\chi}(z)$ are defined as

$$\begin{aligned}
\boldsymbol{\pi}^{\boldsymbol{i}}(z) &= \sum_{j=N_i}^{j=N_{i+1}-1} \pi_j z^{j-N_i} \\
\boldsymbol{\chi}(z) &= \sum_{j=N_K}^{j=\infty} \pi_j z^{j-N_K} \\
\boldsymbol{\pi}(z) &= \sum_{i=0}^{i=K-1} z^{N_i}\boldsymbol{\pi}^{\boldsymbol{i}}(z) + z^{N_K}\boldsymbol{\chi}(z)
\end{aligned} \tag{4.30}$$

Note that $\boldsymbol{\pi}^{\boldsymbol{i}}(z)$ is a polynomial of degree $N_{i+1} - N_i - 1$. Also the stability condition for multi-rate scheduling requires only the highest rate to be more than the average arrival rate, i.e., $N_K > \lambda$.

As before, we first consider the simple case of geometric arrival process.

4.5.1 Geometric Arrival Process

Consider the action of a greedy 2-rate scheduler capable of transmitting at rates N_1, N_2 on the geometric arrival process described in (4.15). (4.29) then gives

$$\begin{aligned}\boldsymbol{\chi}(z) &= \frac{(\boldsymbol{p}(z)-1)\,\boldsymbol{\pi^0}(z) + \left(\boldsymbol{p}(z) - z_1^N\right)\boldsymbol{\pi^1}(z)}{z^{N_2} - \boldsymbol{p}(z)} \\ &= \frac{\alpha(z-1)\boldsymbol{\pi^0}(z) + \left[\alpha\left(z^{N_1+1}-1\right) - \left(z^{N_1}-1\right)\right]\boldsymbol{\pi^1}(z)}{z^{N_2} - 1 - \alpha\left(z^{N_2+1}-1\right)} \qquad (4.31)\end{aligned}$$

Following a reasoning similar to that in previous sections, the denominator function $\boldsymbol{D}(z) = z^{N_2} - 1 - \alpha\left(z^{N_2+1}-1\right)$ in the above expression is a polynomial of degree N_2+1. It has a zero of multiplicity 1 at $z=1$. If the stability condition, $N_2 > \lambda$ is satisfied then it has one more real zero $\beta > 1$. Thus $\boldsymbol{D}(z)$ has exactly N_2-1 zeros inside the unit circle and one zero at $z=1$ which must cancel with those in the numerator polynomial function $\boldsymbol{N}(z) = \alpha(z-1)\boldsymbol{\pi^0}(z) + \left[\alpha\left(z^{N_1+1}-1\right) - \left(z^{N_1}-1\right)\right]\boldsymbol{\pi^1}(z)$ for $\boldsymbol{\chi}(z)$ to be convergent. Note that the numerator function is a polynomial of degree N_2 with a zero at $z=1$. Then, the N_2-1 zeros of $\boldsymbol{D}(z)$ that lie inside the unit circle uniquely determine $\boldsymbol{N}(z)$ to a multiplicative scalar constant. Consider the factorization $\boldsymbol{D}(z) = -\alpha(z-\beta)(z-1)\boldsymbol{g}(z)$. Then, from the preceding argument, we must have

$$\begin{aligned}\boldsymbol{\chi}(z) &= -\frac{A}{z-\beta} \\ \alpha(z-1)\boldsymbol{\pi^0}(z) + \left[\alpha\left(z^{N_1+1}-1\right) - \left(z^{N_1}-1\right)\right]\boldsymbol{\pi^1}(z) &= A\alpha(z-1)\boldsymbol{g}(z) \\ \Rightarrow \boldsymbol{\pi^0}(z) + \left[\sum_{i=0}^{i=N_1} z^i - \frac{1}{\alpha}\sum_{i=0}^{i=N_1-1} z^i\right]\boldsymbol{\pi^1}(z) &= A\boldsymbol{g}(z) \qquad (4.32)\end{aligned}$$

There are unique polynomials $\boldsymbol{\pi^0}(z)$ and $\boldsymbol{\pi^1}(z)$ that satisfy (4.32) and they are obtained respectively, as the remainder and the quotient in the Euclidean division of $A\boldsymbol{g}(z)$, a polynomial of degree N_2-1, by $\left(\sum_{i=0}^{i=N_1} z^i - \frac{1}{\alpha}\sum_{i=0}^{i=N_1-1} z^i\right)$, a polynomial of degree N_1. The constant A can be evaluated through the normalization constraint $\boldsymbol{\pi}(1) = 1$. Further, this set of values of $\boldsymbol{\chi}(z), \boldsymbol{\pi^0}(z)$ and $\boldsymbol{\pi^1}(z)$, is the solution to the queueing problem, as this is the only assignment leading to a convergent generating function for $\boldsymbol{\chi}(z)$ and because an ergodic chain has exactly one steady state probability distribution.

The extension to multirate scheduling - the greedy K-rate scheduler is straightforward. Proceeding similarly, as in the case of 2-rate scheduler, we obtain

$$\boldsymbol{\chi}(z) = -\frac{A}{z-\beta}$$

$$\boldsymbol{\pi}^{\mathbf{0}}(z)+\sum_{j=1}^{j=K-1}\boldsymbol{\pi}^{\boldsymbol{j}}(z)\left[\sum_{i=0}^{i=N_j}z^i-\frac{1}{\alpha}\sum_{i=0}^{i=N_j-1}z^i\right] \quad = \quad A\boldsymbol{g}(z) \tag{4.33}$$

where again $\left(z^{N_K}-\boldsymbol{p}(z)\right)$ has been factored as $-\alpha(z-1)(z-\beta)\boldsymbol{g}(z)$. Once again $\boldsymbol{\pi}^{\boldsymbol{j}}(z)$ can be determined from (4.33) as quotients of repeated Euclidean division of $\boldsymbol{g}(z)$ by the corresponding multiplicands in (4.33), beginning from the evaluation of $\boldsymbol{\pi}^{\boldsymbol{K-1}}(z)$. A is once again evaluated using the normalization constraint $\boldsymbol{\pi}(1)=1$.

4.5.2 Polynomial Arrival Processes

We now consider the case when the arrival process can be described by $\boldsymbol{p}(z)$; a polynomial of degree R as in Section 4.4.2. Consider first, a 2-rate scheduler with the non-zero transmission rates N_1 and N_2. To recapitulate, according to (4.29), the steady state distribution has a generating function governed by

$$\boldsymbol{\chi}(z) \quad = \quad \frac{(\boldsymbol{p}(z)-1)\boldsymbol{\pi}^{\mathbf{0}}(z)+\left(\boldsymbol{p}(z)-z_1^N\right)\boldsymbol{\pi}^{\mathbf{1}}(z)}{z^{N_2}-\boldsymbol{p}(z)} \tag{4.34}$$

As before we assume $N_2 \leq R$. Then, again using the argument based on the Rouche's theorem, the denominator function $D(z)=z^{N_2}-\boldsymbol{p}(z)$ has N_2-1 zeros inside the unit circle and 1 on the unit circle at $z=1$ which cancels with the numerator. Following the reasoning and the notation in Section 4.4.2 with $z^{N_2}-\boldsymbol{p}(z)=c(z-1)\boldsymbol{g}(z)\boldsymbol{h}(z)$, the numerator polynomial $\boldsymbol{N}(z)$ must divide $\boldsymbol{g}(z)$, which is of degree N_2-1. However, here we note that $\boldsymbol{N}(z)$ is a polynomial of degree $(R-1+\max(N_1,N_2-N_1))$. Let

$$N(z) \quad = \quad (\boldsymbol{p}(z)-1)\boldsymbol{\pi}^{\mathbf{0}}(z)+\left(\boldsymbol{p}(z)-z_1^N\right)\boldsymbol{\pi}^{\mathbf{1}}(z)=(z-1)\boldsymbol{t}(z)\boldsymbol{g}(z) \tag{4.35}$$

where, $\boldsymbol{t}(z)$ is an unknown polynomial of degree $(R-N_2-1+\max(N_1,N_2-N_1))$. This then immediately gives

$$\boldsymbol{\chi}(z)=\frac{\boldsymbol{t}(z)}{c\boldsymbol{h}(z)} \tag{4.36}$$

The equilibrium distribution thus can be evaluated if the polynomial $\boldsymbol{t}(z)$ can be determined. It can be verified that (4.35) actually represents a system of $(R+\max(N_1,N_2-N_1)-1)$ independent linear equations with $(R+\max(N_1,N_2-N_1))$ unknowns. Hence, together with the normalization constraint $\boldsymbol{\pi}(1)=1$, in principle, this allows us to obtain the $(R+\max(N_1,N_2-N_1))$ unknowns and thereby $\boldsymbol{\pi}(z)$ uniquely, by solving a finite system of linear equations.

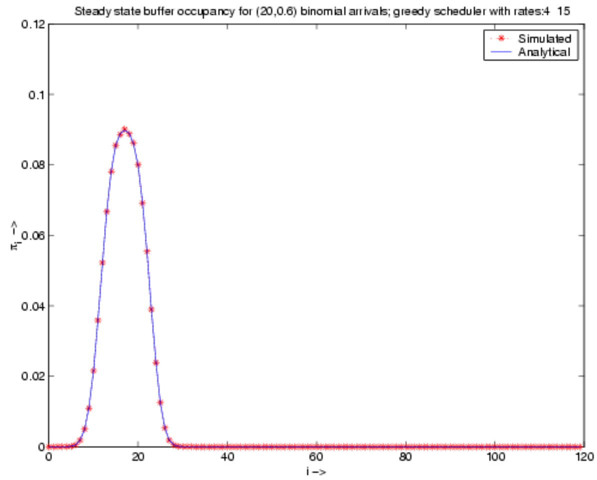

Figure 4.3: Comparison of steady state distributions of buffer occupancy obtained analytically and through simulation for a 2-rate greedy scheduler

In Appendix C, we describe a method to reduce the order of this system to $\max(N_1, N_2 - N_1)$.

For a 2-rate scheduler transmitting at rates 4 and 15, we verify the steady state buffer occupancy distribution obtained through the method described in Appendix C with that obtained through simulation for the binomial arrival process described by (4.26) (Figure 4.3).

The extension to the greedy multi-rate scheduling case is straightforward and most of the analysis carries through with the obvious modifications. The details are given in Appendix C. The same relations (C.7) and (C.3) in Appendix C can also be used to obtain $\boldsymbol{\pi}(z)$. We solved the steady state distribution for the arrival process of (4.26) for a 3-rate greedy scheduler with rates $4, 7$ and 15 and compared with a distribution obtained through simulations. (Figure 4.4).

With some additional effort, this method can also be used to solve for the case where the characteristic function for the arrival process has a rational polynomial form.

4.6 Monotone Schedulers

In this section, we briefly consider the problem of a special non-greedy deterministic multi-rate scheduler, the monotone scheduler. As before let $\mathcal{N}^K : \{0, N_1, N_2, ..., N_K\}$ be the set of discrete rates at which the transmitter can transmit. Further, let $\mathcal{T}^K : \{0, t_1, t_2, ..., t_K\}$ be the set of discrete thresholds such that the transmitter transmits at a rate $u = N_i$ when $x \in [t_i, t_{i+1})$; where we assume $t_i \geq N_i$. We denote the resulting queue

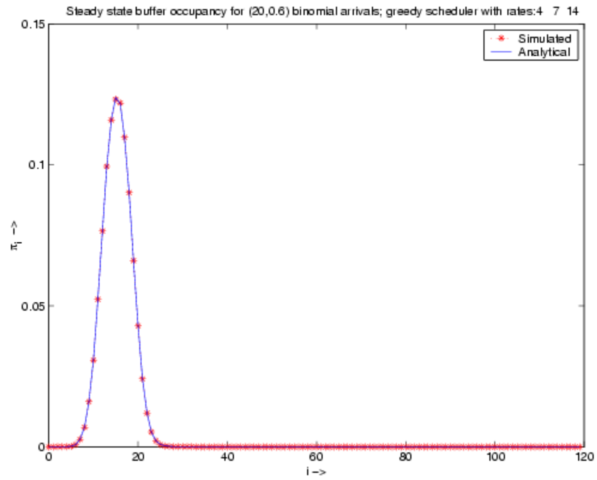

Figure 4.4: Comparison of steady state distributions of buffer occupancy obtained analytically and through simulation for a 3-rate greedy scheduler

(infinite buffer) by $D^X/D/[\mathcal{N}^K, \mathcal{T}^K]$. We assume that the arrival process is polynomial, as in Section 4.4.2 It maybe verified that the characteristic equation in this case will be given by an expression similar to (4.29)

$$\boldsymbol{\pi}(z) \quad = \quad \boldsymbol{p}(z)\left[\sum_{i=0}^{i=K-1} z^{t_i - N_i}\boldsymbol{\pi^i}(z) + z^{t_K - N_K}\boldsymbol{\chi}(z)\right] \tag{4.37}$$

where we define,

$$\begin{aligned} \boldsymbol{\pi^i}(z) &= \sum_{j=t_{i-1}}^{j=t_i} \pi_j z^{j-N_{i-1}} \\ \boldsymbol{\chi}(z) &= \sum_{j=t_K}^{j=\infty} \pi_j z^{j-N_K} \end{aligned} \tag{4.38}$$

Rearranging terms in (4.37), we get

$$\boldsymbol{\chi}(z) \quad = \quad \frac{\boldsymbol{p}(z)\sum_{i=0}^{i=K-1} z^{t_i-N_i}\boldsymbol{\pi^i}(z) - \sum_{i=0}^{i=K-1} z^{t_i}\boldsymbol{\pi^i}(z)}{z^{t_K-N_K}\left(z^{N_k} - \boldsymbol{p}(z)\right)} \tag{4.39}$$

Again, define

$$\begin{aligned} \boldsymbol{\pi_l}(z) &= \sum_{i=0}^{i=K-1} z^{t_i}\boldsymbol{\pi^i}(z) \\ \widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) &= \sum_{i=0}^{i=K-1} z^{t_i-N_i}\boldsymbol{\pi^i}(z) \\ \Rightarrow \boldsymbol{\chi}(z) &= \frac{\boldsymbol{p}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) - \boldsymbol{\pi_l}(z)}{z^{t_K-N_K}\left(z^{N_K} - \boldsymbol{p}(z)\right)} \end{aligned} \tag{4.40}$$

Again, reasoning as before, if $N_K > \lambda$, and $z_{N_K} - \boldsymbol{p}(z) = c(z-1)\boldsymbol{g}(z)\boldsymbol{h}(z)$, then convergence requires

$$\boldsymbol{p}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) - \boldsymbol{\pi}_{\boldsymbol{l}}(z) = z^{t_K - N_K}(z-1)\boldsymbol{g}(z)\boldsymbol{t}(z) \tag{4.41}$$

where $\boldsymbol{t}(z)$ is a polynomial of appropriate degree, $R+v-t_k-1$, where $v = \deg(\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z))+1$. Again, coefficients of $\boldsymbol{t}(z)$ may be obtained by solving the system of linear equations obtained by comparing coefficients in (4.41) which together with the normalization constraint ($\boldsymbol{\pi}(1) = 1$) gives a set of $(R+v)$ independent linear equations in as many unknowns. As

in Section 4.5.2, this order can be reduced. In Appendix D, we describe a method to do so to v.

4.7 Minimum Power Requirement

In this section, we discuss the minimum power requirement for a K-rate scheduler, when there is no delay constraint on scheduling. It is interesting to note that the minimal power depends only on the average arrival rate and can be achieved by a corresponding 2-rate scheduler.

Proposition 4.4. *Let $N_p, N_{p+1} \in \mathcal{N}^K$ be such that $N_p < \lambda < N_{p+1}$. Then, the minimum power requirement, that occurs in the absence of delay constraints, i.e., $\bar{D} = \infty$ is given by*

$$P_{min} = \frac{N_{p+1}-\lambda}{N_{p+1}-N_p}E(N_p) + \frac{\lambda - N_p}{N_{p+1}-N_p}E(N_{p+1}) \tag{4.42}$$

Proof. In the absence of the delay constraint, this can be posed as the optimization problem.

$$\begin{aligned} \text{minimize } P &= \sum_{i=1}^{i=K} q_i E(N_i) \\ \text{subject to,} & \\ \sum_{i=1}^{i=K} q_i N_i &= \lambda \\ \sum_{i=1}^{i=K} q_i &< 1 \end{aligned} \tag{4.43}$$

Here, q_i, is the fractional time for which, the scheduler transmits at N_i. The result (4.42) then follows from the convexity of $E(.)$. Details are given in Appendix E. □

From the above result, it follows that over the K-rate schedulers, the least power will be achieved by the schedulers that use the rates $\lfloor \lambda \rfloor$ and $(\lfloor \lambda \rfloor + 1)$. For these schedulers, $P_{min} = (\lambda - \lfloor \lambda \rfloor)E(\lfloor \lambda \rfloor) + (1 - \lambda + \lfloor \lambda \rfloor)E(\lfloor \lambda \rfloor + 1)$, which is similar to the expression in [14]. Note that P_{min} is achievable in the limit by appropriate scheduling action.

4.8 Summary

In this chapter, the problem of power efficient scheduling subject to an average delay constraint has been considered. For implementation reasons, it has been reasoned that a practical scheduler must consider a transmitter allowed to transmit only at discrete rates. The problem of power-efficient scheduling for a K-rate scheduler and a first order Markovian channel fading process has been formulated. For the simple case of a non-time varying channel and when transmission at only a single discrete rate is allowed, the method to arrive at the optimal scheduler has been discussed.

In the subsequent sections, the queues arising out of the action of a deterministic subclass of discrete rate schedulers (called the greedy schedulers) and a given arrival process have been discussed and analyzed. The analysis has then been extended to include a class of non-greedy schedulers as well. It may be pointed out that the non-greedy scheduler as characterized in Section 4.6 encompass the class of all monotone deterministic schedulers. The methods described in this chapter allow analysis of such monotone schedulers and evaluate their performance. This provides a useful tool in scheduler design, besides addressing an interesting problem in its own right. This may have applications in other fields such as transport problems, management science etc.

Chapter 5

Conclusions and Future Directions

In this dissertation, some aspects of Quality of Service (QoS) scheduling have been considered. Although, the problems considered fall within different application domains, there is a common underlying structure uniting them. Broadly, a QoS scheduling mechanism involves allocation of available resources to meet certain constraints and achieve efficient utilization. Many of these problems can be posed as constrained optimization of convex cost functions. The constraints that arise are two-fold - first, there are traffic constraints that arise because of the restriction that the network places on the flows and second, there are those due to the performance guarantees in terms of delay etc, that the network has promised to the flows. For example, in Chapter 2 the problem of delay and Token Bucket Regulator constrained streaming of video to optimize distortion that has been considered, is a convex optimization problem. The problem considered in Chapter 3, although of a slightly different nature, in that the cost function sought to be optimized was not known in nature, can also be cast in such a framework. The problem of delay constrained scheduling for wireless transmitters to optimize power, considered in Chapter 4, again belongs to this class. It then, perhaps, would help to consider a unified framework for tackling this class of problems, where a single cost function is desired to be optimized, under the commonly arising constraints such as those of traffic regulation, like the Token Bucket Regulation and average or absolute scheduling delays.

Apart from this, significant refinements and extensions can be made to the individual problems considered in this dissertation. In Chapter 2, for example, a simplified distortion function has been considered. It would be interesting to consider a simple but effective distortion model that would allow for data dependencies in some manner. Also, it would be useful if a simple optimal or close to optimal solution to the problem of online scheduling for video streaming could be found or the analysis extended to more elaborate models of both the arrival as well as distortion processes. Finally, it may be worthwhile considering the problem of scheduling vis-a-vis rate allocation or selection of token bucket parameters to optimize a joint cost function comprising pricing and a distortion measure. This would be especially useful in cases where the token bucket or the concerned traffic regulator parameters can be frequently renegotiated during the duration of the flow.

In Chapter 3, the entropy associated with a Token Bucket Regulator has been evaluated. Formulation of traffic regulation in terms of entropy or information utility allows us to compare otherwise different regulators. These regulators may differ in multiple parameters and may even be based on different mechanisms altogether. Characterization in terms of information utility would provide a common criterion for determining pricing. Moreover, it should be possible to factor in the QoS performance guarantees such as delay or loss in entropy evaluation. If done, this would provide a useful common platform to set or evaluate pricing and consumer utility for different services that may be offered by the network, in a consistent manner.

In Chapter 4, the problem of power efficient transmission scheduling has been considered. Towards this end, the domain of scheduling action was restricted to the practically implementable class of discrete rate schedulers. For the non-time varying channel case, the optimal one rate scheduler has been found. Tools have also been developed for analysis of monotone deterministic schedulers for this case. This is important because most of the optimal deterministic policies in convex function optimization turn out to be monotone in nature. Moreover, the optimal stochastic schedulers too, involve only a few randomizations and may be obtained readily from a combination of deterministic schedulers. In this context, the task of determining the optimal scheduling policy, given the allowed discrete rates remains to be addressed. This can of-course be easily done through standard dynamic programming approaches. However, it might be possible to exploit the simplified nature of the problem in case of discrete rate scheduling to come up with simpler strategies for determining the optimal scheduler action. A more difficult problem, would be the joint optimization of both the scheduling rates and scheduling policies. Finally, the effect of channel fading needs to be considered in scheduling for different channel fading models. For time-varying channels, that are characterized with deep fades, an average delay constraint may not be sufficient, as a scheduling policy designed on the basis of only a mean delay constraint is likely to lead to large variances in delays suffered by the packets. It is likely to be particularly severe on packets arriving in a 'bad' channel state, if these states are persistent.

Finally, practical schemes and constraints on the selection of discrete rates should be factored in. In this context, it is easy to see that not all average delays are feasible

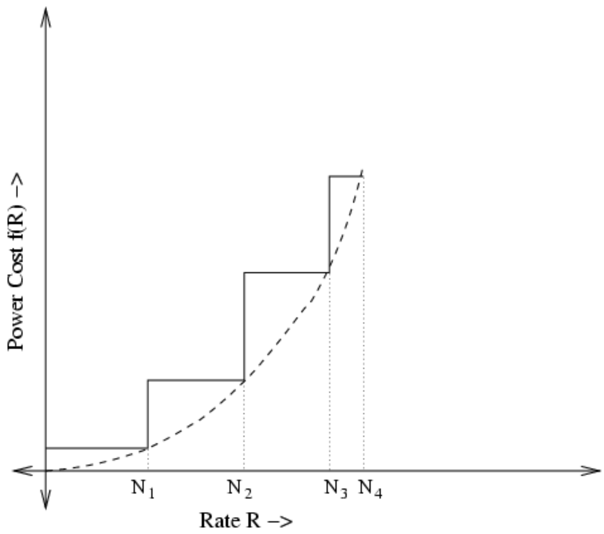

Figure 5.1: Equivalent Cost function with stuffing

for a given set of discrete rates and an arrival process. Specifically, the delay achieved by the greedy schedulers is the least achievable delay and this sets a lower bound on delays that can be achieved. To circumvent this, stuffing as in flushing packets, even before a number corresponding to an available rate has accumulated, by adding dummy packets might have to be done. The dummy packets will be discarded at the receiver. The problem then becomes akin to one where scheduling at arbitrary rates is allowed, but the cost function takes a ladder form with the jumps corresponding to the set of allowed rates (Figure 5.1).

If however, the set delay target is feasible without stuffing, i.e., if $\bar{D}$ is greater than the greedy scheduler delay, then the optimal scheduler would not make use of stuffing (for non-time varying channels). It would, then be simpler to restrict oneself to the smaller class of discrete-rate schedulers for determining the optimal scheduling policy. These issues could be investigated in future studies.

Appendix A

Optimization of distortion cost functions through α', β'

Here, we show that an allocation scheme that is optimal w.r.t. $\boldsymbol{\alpha'}$ or $\boldsymbol{\beta'}$ is also optimal w.r.t. the cost functions (2.7) and (2.9) respectively.

We shall first prove the following result.

Proposition A.1. *If an allocation $\boldsymbol{x^*}$ achieves optimal $\boldsymbol{\alpha'}$ (or $\boldsymbol{\beta'}$), then it is also optimal w.r.t. the objective function $X = \sum_{n=1}^{n=N} x_n$.*

Proof. Let, $\boldsymbol{t^*}$, be the corresponding residual token vector. We first show by induction, that $\boldsymbol{x^*}$, achieves optimal values of $T_n = \sum_{i=1}^{i=n} x_i + t_n \ \forall \ \ n : 1 \leq n \leq N$. It is easy to see that this must be true for $n = 1$. Assume this is true upto some $n = k$, $k \leq N - 1$, i.e., $\boldsymbol{x^*}$, achieves optimal value of T_k. To establish a contradiction, suppose this is not true for $n = k + 1$, i.e., $\exists$ a scheme $\bar{\boldsymbol{x}}$, such that

$$\sum_{i=1}^{i=k+1} x_i^* + t_{k+1}^* \ < \ \sum_{i=1}^{i=k+1} \bar{x}_i + \bar{t}_{k+1} \tag{A.1}$$

From the induction hypothesis,

$$\sum_{i=1}^{i=k} x_i^* + t_k^* \ \geq \ \sum_{i=1}^{i=k} \bar{x}_i + \bar{t}_k \tag{A.2}$$

As $T_k(\boldsymbol{x^*})$ is optimal, (A.1) can occur only if $T_{k+1}(\boldsymbol{x^*}) - T_k(\boldsymbol{x^*}) < r_{k+1}$, i.e., $t_k^* + r_{k+1} - y_{k+1} > B_{k+1}$. This in turn means $x_{k+1}^* = y_{k+1}$ and $t_{k+1}^* = B_{k+1}$. As, these are the maximum values that x_{k+1} and y_{k+1} can respectively take, from (A.1), we have

$$\sum_{i=1}^{i=k} x_i^* \ < \ \sum_{i=1}^{i=k} \bar{x}_i \tag{A.3}$$

$$t_k^* \ > \ \bar{t}_k \tag{A.4}$$

If $x_k^* < y_k$, it follows that, then $\boldsymbol{x^*}$ cannot be optimal, as one may increase x_k^* by upto $\min(t_k^*, t_k^* + r_{k+1} - y_{k+1} - B_{k+1})$, without affecting allocations at other instants and thus it may be possible to bring down $\boldsymbol{\alpha^*}$ or $\boldsymbol{\beta^*}$. If $x_k^* = y_k$, then because of induction hypothesis, (A.3) and (A.4) will hold with k replaced by $k-1$. Let v be the greatest index less than k for which $x_v^* < y_v^*$. Then, x_v^*, may be increased by at least $\min(t_v^*, t_{v+1}^*, ..., t_k^*, t_k^* + r_{k+1} - y_{k+1} - B_{k+1})$, a quantity that is strictly positive due to (A.4), without affecting any other allocations[1]. Then $\boldsymbol{x^*}$ cannot be optimal w.r.t. $\boldsymbol{\alpha'}$ or $\boldsymbol{\beta'}$. Thus, we conclude that $\boldsymbol{x^*}$ achieves the optimal value of $T_N = X + t_N$.

Now, if $t_N^* = 0$, then $\boldsymbol{x^*}$ clearly also achieves the optimal X. Else, i.e., if $t_N^* > 0$, then we must have $x_N^* = y_N$. It then follows easily from an argument similar to the one used to prove the induction claim, that $\boldsymbol{x^*}$ cannot be optimal unless it achieves the optimal X. Hence, $\boldsymbol{x^*}$ must also optimize the value of $X = \sum_{n=0}^{n=N} x_n$. □

An immediate consequence of Proposition A.1 is that a schedule that optimizes $\boldsymbol{\alpha'}$ achieves the least value of

$$l_\alpha = \sum_{n=0}^{n=N} \alpha_n$$

and the one that optimizes $\boldsymbol{\beta'}$ achieves the least value of

$$l_\beta = \sum_{n=0}^{n=N} y_n \beta_n$$

We now proceed to show optimality w.r.t. (2.7) and (2.9). For this, we need the following property of convex functions

Lemma A.1. *If $f(.)$ is a convexly increasing function with $f(0) = 0$ and a, b, c, d and p, q, r, s are positive real nos. such that*

$$\begin{aligned} p + s = q + r &= 1 \\ pa + sd = qb + rc &= m \ \ \textit{and} \\ a \le b &\le m \le c \le d \\ \textit{then,} & \\ pf(a) + sf(d) &\ge qf(c) + rf(d) \end{aligned}$$

[1]Note that, $x*_v < y_v$, for some v else $T_{k+1}(\boldsymbol{x^*}) = \sum_{i=1}^{i=k+1} y_i + B_{k+1}$, which is the maximum possible value for T_{k+1} and would contradict (A.1).

Proof. Note that $p = \frac{d-m}{d-a}$, $s = \frac{m-a}{d-a}$, $q = \frac{c-m}{c-b}$, $r = \frac{m-b}{c-b}$. then,

$$\begin{aligned} pf(a) + sf(d) &= \frac{c-m}{c-b}\left[\frac{d-b}{d-a}f(a) + \frac{b-a}{d-a}f(d)\right] + \frac{m-b}{c-b}\left[\frac{d-c}{d-a}f(a) + \frac{c-a}{d-a}f(d)\right] \\ &\geq \frac{c-m}{c-b}f(b) + \frac{m-b}{c-b}f(c) \qquad \text{(A.5)} \\ &\geq qf(c) + rf(d) \end{aligned}$$

where in (A.5), we make use of the following property of $f(.)$

$$pf(x) + qf(y) \geq f(px + qy) \;\; p, q > 0 \;\; \textit{and} \;\; p + q = 1.$$

□

Proposition A.2. *If an allocation $\boldsymbol{x^*}$ achieves optimal $\boldsymbol{\alpha'}$, then it is also optimal w.r.t. the cost function $D(\boldsymbol{\alpha}) = \sum_{i=0}^{i=N} d(\alpha_i)$, given that $d(.)$ is a convexly increasing function with $d(0) = 0$.*

Proof. Let, if possible, there be a different allocation leading to a different ordered loss vector $\bar{\boldsymbol{\alpha}}'$ such that $D(\bar{\boldsymbol{\alpha}}) < D(\boldsymbol{\alpha^*})$. From Proposition A.1, $\boldsymbol{\alpha^*}$ achieves the optimal l_α. Also $\bar{\boldsymbol{\alpha}}' > \boldsymbol{\alpha'^*}$. It then follows from a straightforward application of Lemma A.1 that $D(\bar{\boldsymbol{\alpha}}) \geq D(\boldsymbol{\alpha^*})$, a contradiction. Thus $\boldsymbol{x^*}$ must achieve optimal $D(.)$. □

Proposition A.3. *If an allocation $\boldsymbol{x^*}$ achieves optimal $\boldsymbol{\beta'}$, then it is also optimal w.r.t. the cost function $D(\boldsymbol{\beta}) = \sum_{i=0}^{i=N} y_i d(\beta_i)$, given that $d(.)$ is a convexly increasing function with $d(0) = 0$.*

Proof. Proposition A.1, shows that $\boldsymbol{\beta^*}$ achieves optimal l_β. Here, we show that if $\bar{\boldsymbol{\beta}}$ is another feasible vector, then $D(\bar{\boldsymbol{\beta}}) \geq D(\boldsymbol{\beta^*})$. We consider the case when $\boldsymbol{\beta^*}$ and $\bar{\boldsymbol{\beta}}$ differ in only 2 positions - i and j. Suppose $\beta_i^* > \beta_j^*$, then as $\boldsymbol{\beta'^*} > \bar{\boldsymbol{\beta}}'$ and $\boldsymbol{\beta^*}$ achieves the least l_β, we must have

$$\begin{aligned} \bar{\beta}_i &= \beta_i^* + y_j\delta + \Delta \\ \bar{\beta}_i &= \beta_i^* - y_i\delta \end{aligned}$$

where, $\delta, \Delta > 0$. Then,

$$\begin{aligned} D(\bar{\boldsymbol{\beta}}) - D(\boldsymbol{\beta^*}) &= y_i\left[d(\beta_i^* + y_j\delta + \Delta) - d(\beta_i^*)\right] - y_j\left[d(\beta_j^*) - d(\beta_i^* - y_i\delta)\right] \\ &\geq y_i\left[d(\beta_i^* + y_j\delta) - d(\beta_i^*)\right] - y_j\left[d(\beta_j^*) - d(\beta_i^* - y_i\delta)\right] \\ &\geq \left[y_i d(\beta_i^* + y_j\delta) + y_j d(\beta_j^* - y_i\delta)\right] - \left[y_i d(\beta_i^*) + y_j(d(\beta_j^*)\right] \\ &\geq 0 \end{aligned}$$

where, in the last inequality, we again use Lemma A.1. It is easy to see that this relation, i.e., $D(\bar{\boldsymbol{\beta}}) \geq D(\boldsymbol{\beta}^*)$ will, in fact, hold for any $\bar{\boldsymbol{\beta}} \neq \boldsymbol{\beta}^*$. Hence, $\boldsymbol{x}^*$ is optimal w.r.t. $D(.)$.

$\square$

Appendix B

Proof of Proposition 4.3

Here, we give a detailed proof of the claim that $g(z)$ is of degree $N-1$ and consists of zeros inside the unit circle only. Recall that, $\boldsymbol{g}(z)$ was obtained from the factorization of $\boldsymbol{D}(z) = z^N - \boldsymbol{p}(z)$ as $c(z-1)\boldsymbol{g}(z)\boldsymbol{h}(z)$, so that $\boldsymbol{h}(z)$ had zeros only outside the unit circle and $\boldsymbol{g}(z)$, only on or inside the unit circle. The proof makes use of the Rouche's theorem. Rouche's theorem states that if two functions $\boldsymbol{a}(.)$ and $\boldsymbol{b}(.)$ are regular inside, and continuous on, a closed contour $\boldsymbol{\Gamma}$, and if they satisfy the strict inequality

$$|\boldsymbol{a}(z)| \quad > \quad |\boldsymbol{b}(z)| \quad (z \in \boldsymbol{\Gamma}) \tag{B.1}$$

then the functions $\boldsymbol{a}(z)$ and $\boldsymbol{a}(z)-\boldsymbol{b}(z)$ have the same number of zeros inside $\boldsymbol{\Gamma}$. For our purpose, we take $\boldsymbol{a}(z) = z^{N-1}$ and $\boldsymbol{b}(z) = \boldsymbol{p}(z)$ and show that they satisfy the conditions of the Rouche's theorem for a contour (circle) $\boldsymbol{\Gamma}$ of the form $|z| = r$ for all r satisfying $1 < r < 1+\delta$, for an appropriately chosen $\delta > 0$. $\boldsymbol{a}(z)$ and $\boldsymbol{b}(z)$, as defined above, being polynomials, are regular inside and on $\boldsymbol{\Gamma}$. Now, since $\boldsymbol{p}(z)$ is uniformly analytic in any bounded domain $\mathcal{D}$, given any $\epsilon > 0$, it is possible to find a $\delta > 0$, sufficiently small, so that

$$\left|\frac{\boldsymbol{p}(re^{i\theta}) - \boldsymbol{p}(e^{i\theta})}{(r-1)e^{i\theta}} - \boldsymbol{p}'(e^{i\theta})\right| < \epsilon \tag{B.2}$$

is satisfied $\forall \;\; r : 1 < r < 1+\delta$. We choose δ such that (B.2) is satisfied for $\epsilon = N - \lambda$ which is strictly positive. From (B.2), we then obtain

$$\begin{aligned}
\left|\boldsymbol{p}(re^{i\theta})\right| &< \left|\boldsymbol{p}(e^{i\theta})\right| + (r-1)\left|\boldsymbol{p}'(e^{i\theta})\right| + (r-1)(N-\lambda) \\
&\leq \sum_{j=0}^{j=R} \left|p_j e^{ji\theta}\right| + (r-1)\left[\sum_{j=0}^{j=R} \left|jp_j e^{(j-1)i\theta}\right| + (N-\lambda)\right] \\
&= \sum_{j=0}^{j=R} p_j + (r-1)\left[\sum_{j=0}^{j=R} jp_j + (N-\lambda)\right]
\end{aligned}$$

$$
\begin{aligned}
&= 1 + (r-1)\left[\lambda + (N-\lambda)\right] \\
&= 1 + (r-1)N \\
&\leq r^N \\
&= \left|(re^{i\theta})^N\right|
\end{aligned}
$$

Thus $\boldsymbol{a}(z)$, $\boldsymbol{b}(z)$ satisfy (B.1) for $\boldsymbol{\Gamma} : |z| = r$. Hence $\exists\ \delta > 0$, such that $D(z) = z^N - \boldsymbol{p}(z)$ has exactly N zeros within the circle $|z| = 1 + \delta$. Since δ can be made arbitrarily small, the function $\boldsymbol{D}(z)$ has exactly N zeros on or within the unit circle. One of these is $z = 1$. As $z = 1$ satisfies $\boldsymbol{D}(z) = 0$ but not $\boldsymbol{D}'(z) = 0$, this zero has multiplicity 1. Also, since $p_0, p_R > 0$, the function $\boldsymbol{D}(z)$ does not have any other zero on $|z| = 1$. Hence we conclude that $\boldsymbol{D}(z) = z^N - \boldsymbol{p}(z)$ has exactly $N - 1$ zeros inside the unit circle. Thus $\boldsymbol{g}(z)$ is of degree $N - 1$. Further, all its zeros lie within the unit circle.

Appendix C

A note on obtaining the $D^X/D/\mathcal{N}^K$ queue distribution for polynomial arrival processes

In Section 4.5.2, we saw that the equilibrium distribution of a $D^X/D/\mathcal{N}^2$ may be obtained, by solving a system of $R+\max(N_1, N_2-N_1)$ linear equations, where N_1, N_2 are the non-zero transmission rates and R is the degree of the polynomial arrival process. Here, we illustrate, how this system may be reduced to a system of only $\max(N_1, N_2-N_1)$ linear equations.

We may rewrite (4.35) of Section 4.5.2 as

$$\begin{aligned}
\boldsymbol{p}(z)\left(\boldsymbol{\pi}^{\mathbf{0}}(z)+\boldsymbol{\pi}^{\mathbf{1}}(z)\right)-\left(\boldsymbol{\pi}^{\mathbf{0}}(z)+z^{N_1}\boldsymbol{\pi}^{\mathbf{1}}(z)\right) &= (z-1)\boldsymbol{t}(z)\boldsymbol{g}(z)\\
\text{define,} & \\
\boldsymbol{\pi_l}(z) &= \boldsymbol{\pi}^{\mathbf{0}}(z)+z^{N_1}\boldsymbol{\pi}^{\mathbf{1}}(z)\\
\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) &= \boldsymbol{\pi}^{\mathbf{0}}(z)+\boldsymbol{\pi}^{\mathbf{1}}(z)\\
\text{thus (4.35) becomes,} & \\
\boldsymbol{p}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z)-\boldsymbol{\pi_l}(z) &= (z-1)\boldsymbol{t}(z)\boldsymbol{g}(z)\\
\left(z^{N_2}+c(z-1)\boldsymbol{g}(z)\boldsymbol{h}(z)\right)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z)-\boldsymbol{\pi_l}(z) &= (z-1)\boldsymbol{t}(z)\boldsymbol{g}(z)\\
z^{N_2}\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z)-\boldsymbol{\pi_l}(z) &= (z-1)\left(\boldsymbol{t}(z)-c\boldsymbol{h}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z)\right)\boldsymbol{g}(z)\\
z^{N_2}\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z)-\boldsymbol{\pi_l}(z) &= (z-1)\boldsymbol{u}(z)\boldsymbol{g}(z) \qquad \text{(C.1)}
\end{aligned}$$

Here, $\boldsymbol{u}(z)$ is now an appropriate polynomial of degree $v-1$, where $v=\max(N_1, N_2-N_1)-1$. Further, (C.1) represents a system of (N_2+v) unknowns and (N_2+v-1) independent equations. These equations along with the normalization constraint

($\boldsymbol{\pi}(1) = 1$), allow us to solve the system uniquely. Also note that

$$\begin{aligned}
\boldsymbol{\pi}(z) &= \boldsymbol{\pi_l}(z) + z^{N_2}\boldsymbol{\chi}(z) \\
&= \boldsymbol{\pi_l}(z) + z^{N_2}\frac{\boldsymbol{p}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) - \boldsymbol{\pi_l}(z)}{z^{N_2} - \boldsymbol{p}(z)} \\
&= \boldsymbol{p}(z)\frac{z^{N_2}\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) - \boldsymbol{\pi_l}(z)}{z^{N_2} - \boldsymbol{p}(z)} \qquad \text{(C.2)} \\
&= \boldsymbol{p}(z)\frac{\boldsymbol{u}(z)}{c\boldsymbol{h}(z)} \qquad \text{(C.3)}
\end{aligned}$$

Then using $\boldsymbol{\pi}(1) = 1$, we get $\boldsymbol{u}(1) = c\boldsymbol{h}(1)$. (C.2) is somewhat similar to the expression obtained in [21] for a $D^X/D^m/1$ queue.

We now write the linear equations obtained by comparing coefficients of different powers of z in (C.1) in a matrix form. Let

$$(z-1)\boldsymbol{g}(z) = z^{N_2} + g_{N_2-1}z^{N_2-1} + ... + g_1 z + g_0 \qquad \text{(C.4)}$$

Then, from (C.1), we have

$$\left[\begin{array}{ccccc|ccccc}
1 & 0 & 0 & .... & 0 & g_0 & 0 & 0 & ... & 0 \\
0 & 1 & 0 & .... & 0 & g_1 & g_0 & 0 & ... & 0 \\
: & : & : & :::: & : & : & : & : & ::: & : \\
0 & 0 & 0 & .... & 1 & g_{N_2-1} & g_{N_2-2} & g_{N_2-3} & \cdots & g_{N_2-v} \\
\hline
-1 & 0 & 0 & .... & 0 & 1 & g_{N_2-1} & g_{N_2-2} & \cdots & g_{N_2-v+1} \\
0 & -1 & 0 & .... & 0 & 0 & 1 & g_{N_2-1} & \cdots & g_{N_2-v+2} \\
: & : & : & :::: & : & : & : & : & ::: & : \\
0 & 0 & 0 & .... & 0 & 0 & 0 & 0 & ... & 1
\end{array}\right]
\left[\begin{array}{c}
\pi_0 \\ \pi_1 \\ : \\ \pi_{N_2-1} \\ \hline u_0 \\ u_1 \\ : \\ u_{v-1}
\end{array}\right] = 0$$

or

$$\left[\begin{array}{c|c} \boldsymbol{I} & \boldsymbol{G} \\ \hline -\boldsymbol{Y} & \boldsymbol{G^v} \end{array}\right]\left[\begin{array}{c} \boldsymbol{\pi} \\ \hline \boldsymbol{u} \end{array}\right] = 0 \qquad \text{(C.5)}$$

where $\boldsymbol{I}$ is the identity matrix of rank N_2, $\boldsymbol{G}$ is a N_2 x v lower triangular matrix of coefficients of $(z-1)\boldsymbol{g}(z)$ given by $[\boldsymbol{G}]_{ij} = g_{i-j}$, $\boldsymbol{G^v}$ is a v x v upper triangular matrix given by $[\boldsymbol{G^v}]_{ij} = g_{N_2+i-j}$ and Y is a v x N_2 matrix obtained from the juxtaposition $\boldsymbol{Y} = [\boldsymbol{I_{v,N_1}I_{v,N_2-N_1}}]$ where $\boldsymbol{I_{m,n}}$, $m \geq n$ is a diagonal matrix with entries 1.

Eliminating $\boldsymbol{\pi}$ from (C.5), we get,

$$(\boldsymbol{YG} + \boldsymbol{G^v})\boldsymbol{u} = 0. \qquad \text{(C.6)}$$

The matrix $\boldsymbol{YG} + \boldsymbol{G^v}$ is of rank at most $v-1$. It is easy to verify that the sum of entries in any column of $\boldsymbol{YG}+\boldsymbol{G^v}$ is $\sum_{i=0}^{i=N_2} g_i = 0$, following (C.4). Let $\boldsymbol{T}$ be the matrix obtained from $\boldsymbol{YG} + \boldsymbol{G^v}$, by replacing its first row by a row of all 1s. Then assuming $\boldsymbol{T^{-1}}$ exists, the coefficients of $\boldsymbol{u}(z)$ are obtained as

$$\boldsymbol{u} = \boldsymbol{T^{-1}} \begin{bmatrix} c\boldsymbol{h}(1) \\ 0 \\ : \\ 0 \end{bmatrix} \quad \text{(C.7)}$$

From $\boldsymbol{u}(z)$, it is straightforward to obtain the transfer function $\boldsymbol{\pi}(z)$ using (C.3). Note that $\boldsymbol{\pi_l}(z)$ is given by $(z-1)\boldsymbol{u}(z)\boldsymbol{g}(z)$ modulo z^{N_2}. Also note that $\boldsymbol{u}(z)$ is constrained to be determined uniquely for $\boldsymbol{g}(z)$ of degree $N_2 - 1$. This means, if $\boldsymbol{g}(z)$ has a degree more than $N_2 - 1$, which would happen if the stability condition, viz., $(N_2 > \lambda)$ is violated, the system of equations in (C.6) becomes overdetermined and there is no solution for $\boldsymbol{u}(z)$. Thus, the stability condition ensures and is required for the steady state distribution to exist.

The problem for the generic greedy multi-rate scheduling under polynomial arrival case can now be extended in a straightforward manner. In fact, the analysis for the 2-rate scheduler carries forward with the following changes in definitions.

$$c(z-1)\boldsymbol{g}(z)\boldsymbol{h}(z) = z^{N_K} - \boldsymbol{p}(z) \quad \text{(C.8)}$$

$$\boldsymbol{\pi_l}(z) = \sum_{i=0}^{i=K-1} z^{N_i}\boldsymbol{\pi^i}(z) \quad \text{(C.9)}$$

$$\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) = \sum_{i=0}^{i=K-1} \boldsymbol{\pi^i}(z) \quad \text{(C.10)}$$

$$v = \max_{1\le i\le K}\{N_i - N_{i-1}\} \quad \text{(C.11)}$$

$$\boldsymbol{Y} = [\boldsymbol{I_{v,N_1}} \boldsymbol{I_{v,N_2-N_1}} \ldots \boldsymbol{I_{v,N_K-N_{K-1}}}] \quad \text{(C.12)}$$

Appendix D

A note on obtaining the $D^X/D/[\mathcal{N}^K, \mathcal{T}^K]$ queue distribution for polynomial arrival processes

The approach of obtaining the distribution by solving an alternate equation, akin to (C.1) of the $D^X/D/\mathcal{N}^K$ queue, which involves a smaller number of unknowns, may also be used here. However this requires a small modification. The relation that is similar to (C.1) for a monotone scheduler is

$$z^{N_K}\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) - \boldsymbol{\pi}_{\boldsymbol{l}}(z) = (z-1)\boldsymbol{u}(z)\boldsymbol{g}(z) \tag{D.1}$$

This may be satisfied with a $\boldsymbol{u}(z)$ of degree $v-1$. Recall that $v = \deg(\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z)) + 1$. This gives us a system with $t_K + v$ unknowns while (D.1) with the normalization constraint, together provide only $N_k + v$ constraints. Hence the system remains under-determined when $t_k > N_k$. The additional $(t_K - N_K)$ constraints arise as follows. Note that for general monotone scheduling, $\boldsymbol{t}(z)$ and $\boldsymbol{u}(z)$ are related by

$$\boldsymbol{u}(z) \quad = \quad z^{t_K - N_K}\boldsymbol{t}(z) - c\boldsymbol{h}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z) \tag{D.2}$$

It follows from (D.2) that the polynomial $(\boldsymbol{u}(z) + \boldsymbol{h}(z)\widetilde{\boldsymbol{\pi}}_{\boldsymbol{l}}(z))$ must divide $z^{t_K - N_K}$, giving us the additional requisite $(t_K - N_K)$ constraints. However, unlike the greedy scheduling case, the polynomial $\boldsymbol{u}(z)$ now depends on the factor $\boldsymbol{h}(z)$ of $\left(z^{N_K} - \boldsymbol{p}(z)\right)$. Finally, it may be noted that the condition for stability is independent of $\mathcal{T}^K$, and as expected remains $N_K > \lambda$.

Appendix E

Proof of Proposition 4.4

We seek to find the optimal tuple $Q^* : (q_1^*, q_2^*, ..., q_K^*)$ to optimize

$$\begin{aligned} \text{minimize } P(Q) &= \sum_{i=1}^{i=K} q_i E(N_i) \\ \text{subject to,} & \\ \sum_{i=1}^{i=K} q_i N_i &= \lambda \qquad \text{(E.1)} \\ \sum_{i=1}^{i=K} q_i &< 1 \qquad \text{(E.2)} \end{aligned}$$

for Proposition 4.4. The problem as posed is a linear programming problem, and the optimal solution must occur on the boundary. Hence, the optimal solution, must satisfy (in the limiting sense)

$$\sum_{i=1}^{i=K} q_i = 1 \qquad \text{(E.3)}$$

This may also be seen readily from the following argument. Suppose, $\Delta = 1 - \sum_{i=0}^{i=K} q_i^* > 0$, and let N_h be the highest rate for which $q_h^* > 0$, then consider the tuple Q', which is the same as Q^*, except that $q_1' = q_1^* + N_h\delta$ and $q_h' = q_h - N_1\delta$; where $\delta = \min\left(\frac{q_h}{N_1}, \frac{\Delta}{N_h - N_1}\right)$. Note that Q' satisfies (E.1) and (E.2). Also,

$$\begin{aligned} P(Q^*) - P(Q') &= \delta\left(N_1 E(N_h) - N_h E(N_1)\right) \\ &> 0 \end{aligned}$$

Thus it follows that the optimal tuple must satisfy $\sum_{i=0}^{i=K} q_i^* = 1$..

Now, to prove Proposition 4.4, we note that $E(.)$ is a convexly increasing function satisfying $E(0) = 0$. From the Lemma A.1 proved in Appendix A, it then follows that a policy Q, transmitting at rates other than N_p, N_{p+1}, the rates closest to λ on either side, will have a higher power requirement than a policy that transmits at only these rates. Hence, as in Proposition 4.4, the optimal tuple is given by

$$
\begin{aligned}
q_p^* &= \frac{\lambda - N_p}{N_{p+1} - N_p} \\
q_{p+1}^* &= \frac{N_{p+1} - \lambda}{N_{p+1} - N_p} \\
q_i^* &= 0\,; i \neq p, p+1
\end{aligned}
$$

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