EE101: Bode plots

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* Bel turned out to be too large in practice $\rightarrow$ deciBel (i.e., one tenth of a Bel).

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* The voltage gain of an amplifier is
  A_V in dB $= 20\, \log\,(V_o/V_i)$,
  with V_i serving as the reference voltage.

Example

Given $V_i = 2.5\,\text{mV}$ and $A_V = 36.3\,\text{dB}$,
compute V_o in dBm and in mV.

(V_i and V_o are peak input and peak output voltages, respectively).

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$$\rightarrow V_o = 162.5\,mV.$$

$$\mathrm{V_i} \longrightarrow \boxed{\text{Amplifier}} \longrightarrow \mathrm{V_o}$$

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$$V_o = A_V \times V_i = 65 \times 2.5\,\mathrm{m}V = 162.5\,\mathrm{m}V.$$

* When sound intensity is specified in dB, the reference pressure is $P_{\text{ref}} = 20\,\mu Pa$ (our hearing threshold).

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loud thunder	110 dB

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loudest sound human ear can tolerate	120 dB

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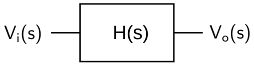
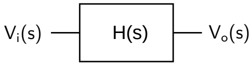
windows break	163 dB


$V_i(s)$
H(s)
$V_o(s)$

Bode plots



* The transfer function of a circuit such as an amplifier or a filter is given by, $H(s) = V_o(s)/V_i(s), \quad s = j\omega$.

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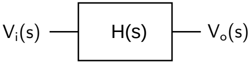

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* $H(j\omega)$ is a complex number, and a complete description of $H(j\omega)$ involves
  (a) a plot of $|H(j\omega)|$ versus ω.
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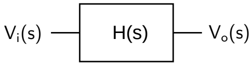

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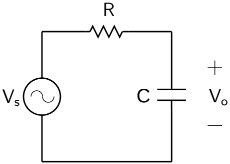
  (a) a plot of $|H(j\omega)|$ versus ω.
  (b) a plot of $\angle H(j\omega)$ versus ω.
* Bode gave simple rules which allow construction of the above "Bode plots" in an approximate (asymptotic) manner.

A simple transfer function



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

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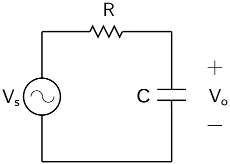

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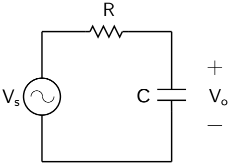

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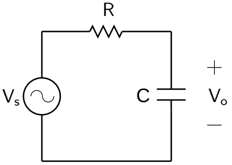

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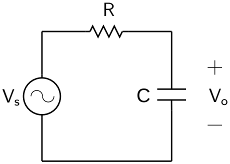

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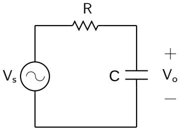
* We are generally interested in a large variation in ω (several orders), and its effect on $|H|$ and $\angle H$.
* The magnitude ($|H|$) varies by orders of magnitude as well.
  The phase ($\angle H$) varies from 0 (for $\omega \ll \omega_0$) to $-\pi/2$ (for $\omega \gg \omega_0$).

A simple transfer function: magnitude



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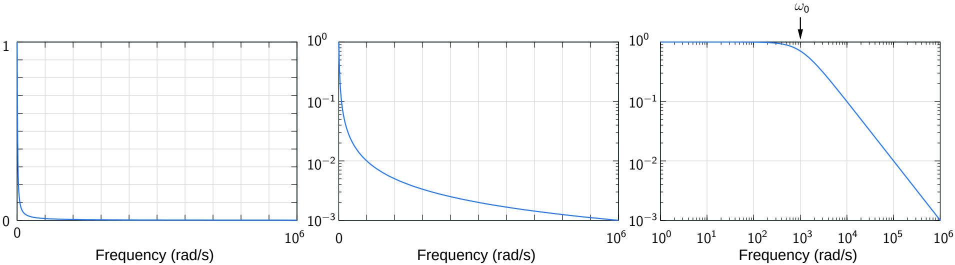

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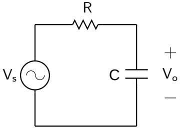

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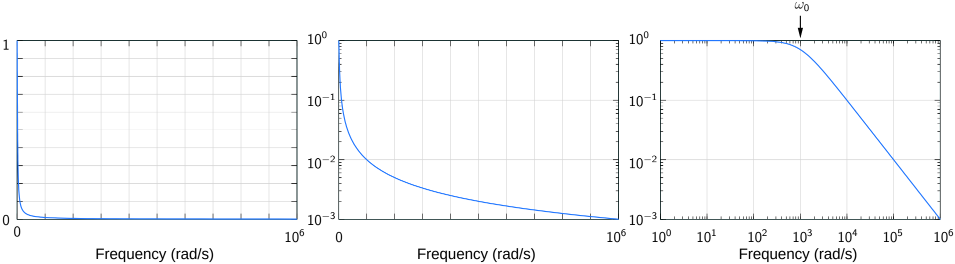

Since ω and $|H(j\,\omega)|$ vary by several orders of magnitude, a linear ω- or $|H|$-axis is not appropriate $\rightarrow$ $\log|H|$ is plotted against $\log\omega$.

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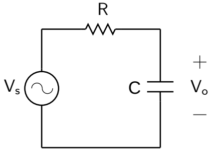

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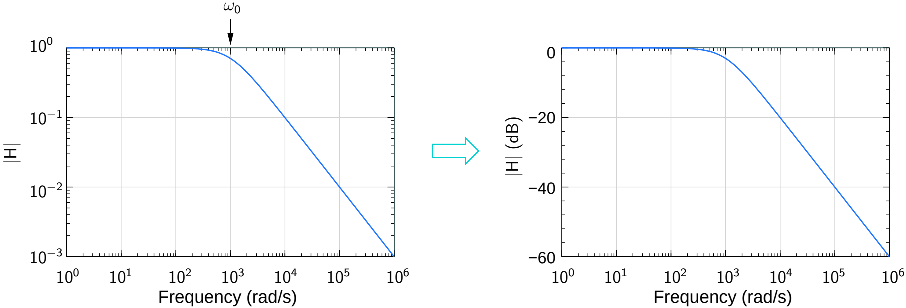

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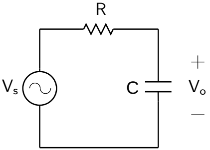

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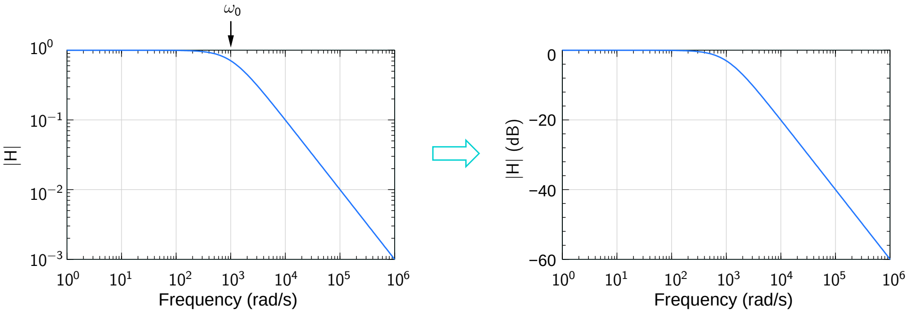

Note that the *shape* of the plot does not change.

$|H|$ (dB) $= 20 \log |H|$ is simply a scaled version of $\log |H|$.

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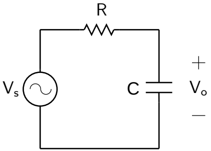

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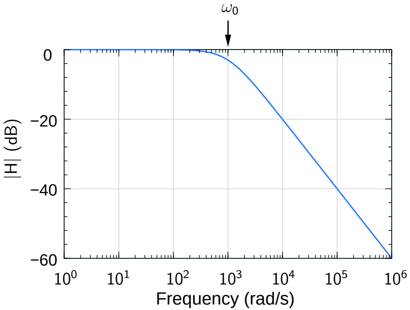
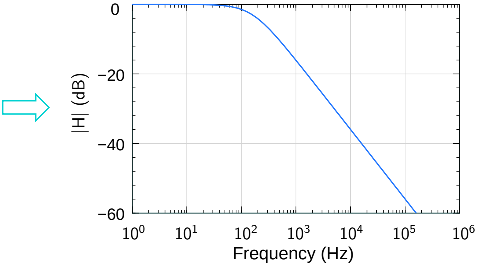
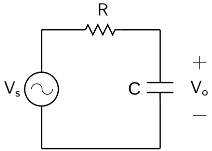




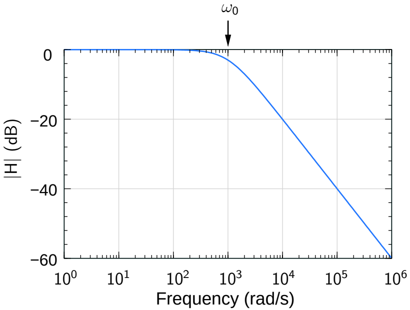
A simple transfer function: magnitude

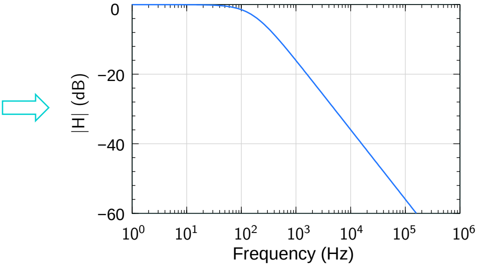


$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\, V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1 + s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

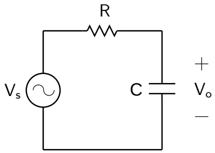
$$\omega_0 = \frac{1}{RC}\,.$$





Since $\omega = 2\pi\, f$, the *shape* of the plot does not change.

A simple transfer function: phase



$$V_o = \frac{(1/sC)}{R + (1/sC)}\, V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

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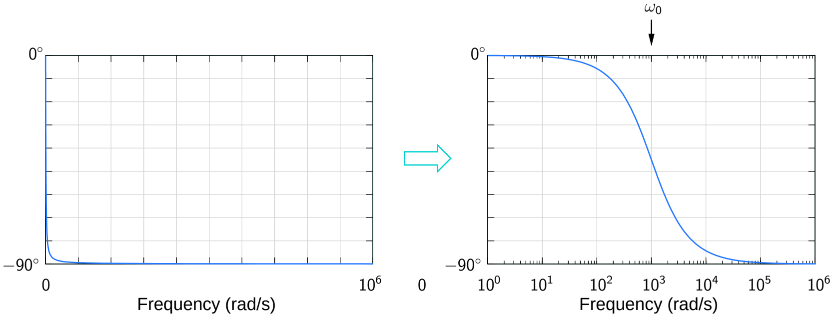
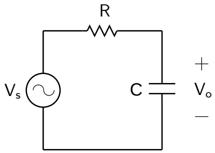


A simple transfer function: phase



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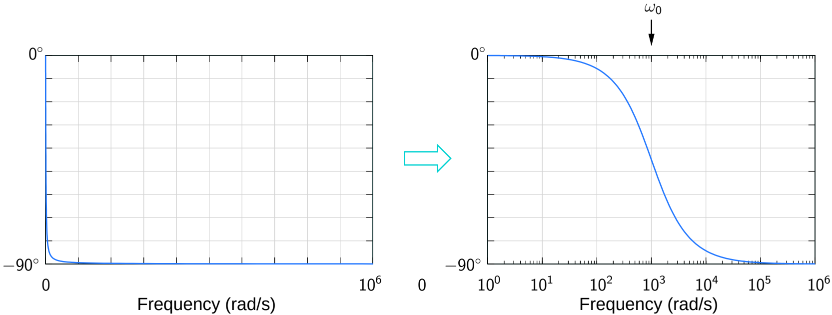
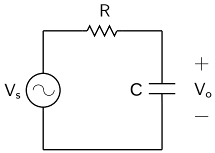


* Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to −90° in this example), a linear axis is appropriate for $\angle H$.

A simple transfer function: phase



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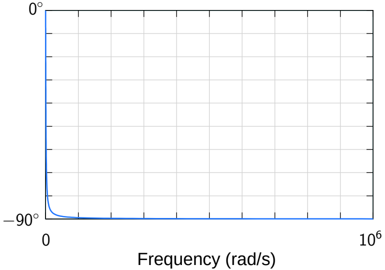

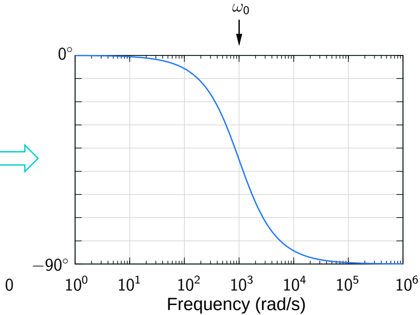

* Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to −90° in this example), a linear axis is appropriate for $\angle H$.
* As in the magnitude plot, we use a log axis for ω, since we are interested in a wide range of ω.

Consider $H(s) = \dfrac{K\,(1+s/z_1)(1+s/z_2)\cdots(1+s/z_M)}{(1+s/p_1)(1+s/p_2)\cdots(1+s/p_N)}$.

Consider $H(s) = \dfrac{K\,(1+s/z_1)(1+s/z_2)\cdots(1+s/z_M)}{(1+s/p_1)(1+s/p_2)\cdots(1+s/p_N)}$.

$-z_1,\ -z_2, \cdots$ are called the "zeros" of $H(s)$.

$-p_1,\ -p_2, \cdots$ are called the "poles" of $H(s)$.

(In addition, there could be terms like s, $s^2, \cdots$ in the numerator.)

We will assume, for simplicity, that the zeros (and poles) are real and distinct.

Construction of Bode plots involves

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Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function ω.

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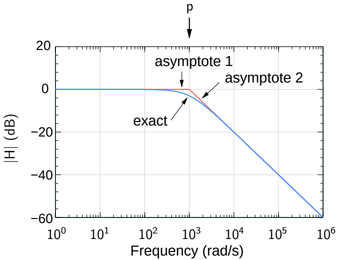
We will assume, for simplicity, that the zeros (and poles) are real and distinct.

Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function ω.

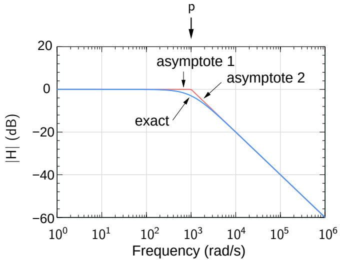
(b) combining the various contributions to obtain $|H|$ and $\angle H$ versus ω.

Contribution of a pole: magnitude



Consider $H(s) = \dfrac{1}{1 + s/p} \rightarrow H(j\omega) = \dfrac{1}{1 + j\,(\omega/p)}\,,\, |H(j\omega)| = \dfrac{1}{\sqrt{1 + (\omega/p)^2}}\,.$

Contribution of a pole: magnitude



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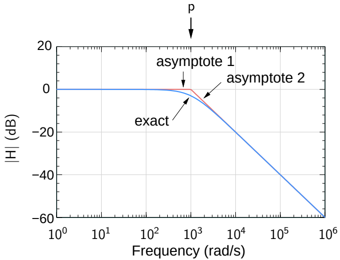
Asymptote 1: $\quad \omega \ll p$: $|H| \rightarrow 1,\;\; 20\log|H| = 0\,\text{dB}.$



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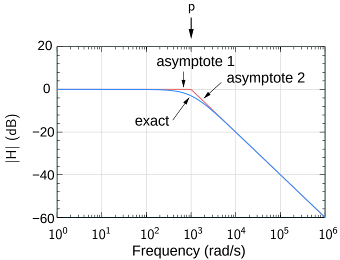
Asymptote 2: $\omega \gg p$: $|H| \rightarrow \dfrac{1}{\omega/p} = \dfrac{p}{\omega} \rightarrow |H| = 20\ \log\ p - 20\ \log \omega\ \text{(dB)}$



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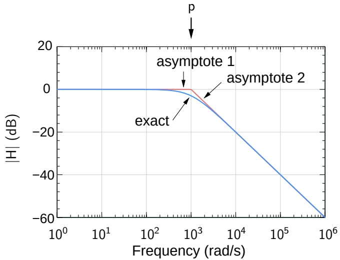
Consider two values of ω: ω_1 and $10\,\omega_1$.

$|H|_1 = 20\,\log\,p - 20\,\log\omega_1$ (dB)

$|H|_2 = 20\,\log\,p - 20\,\log(10\,\omega_1)$ (dB)

Contribution of a pole: magnitude



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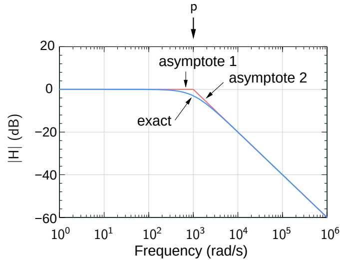
$|H|_1 = 20\;\log\;p - 20\;\log\omega_1\;\;(\text{dB})$

$|H|_2 = 20\;\log\;p - 20\;\log\,(10\,\omega_1)\;\;(\text{dB})$

$|H|_1 - |H|_2 = -20\;\log\;\dfrac{\omega_1}{10\,\omega_1} = 20\;\text{dB}.$

Contribution of a pole: magnitude



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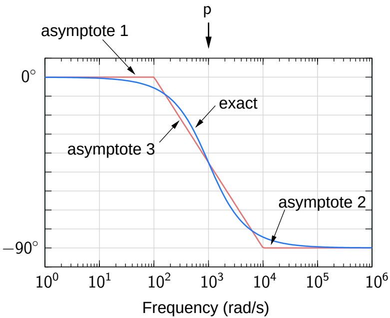
$|H|_1 - |H|_2 = -20\ \log\ \dfrac{\omega_1}{10\,\omega_1} = 20\ \text{dB}.$

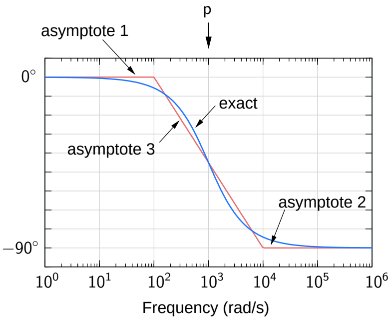
$\rightarrow$ $|H|$ versus ω has a slope of $-20\,\text{dB}/\text{decade}$.

Note that, at $\omega = p$, the actual value of $|H|$ is $1/\sqrt{2}$ (i.e., $-3\,\text{dB}$).



Consider $H(s) = \dfrac{1}{1+s/p} = \dfrac{1}{1+j\,(\omega/p)} \rightarrow \angle H = -\tan^{-1}\left(\dfrac{\omega}{p}\right)$

Contribution of a pole: phase



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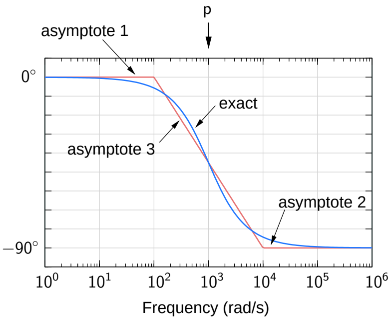
Asymptote 1: $\omega \ll p$ (say, $\omega < p/10$): $\angle H = 0$.

Contribution of a pole: phase



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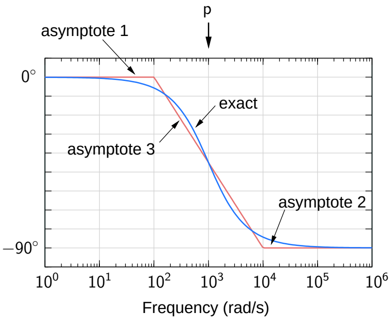

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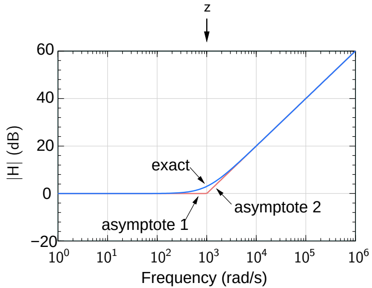
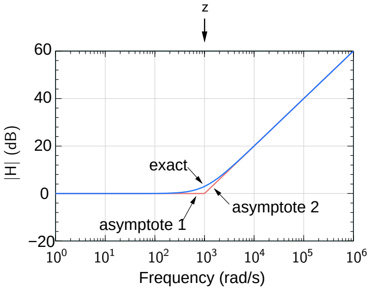


Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j\,(\omega/z)\,, |H(j\omega)| = \sqrt{1 + (\omega/z)^2}\,.$

Contribution of a zero: magnitude



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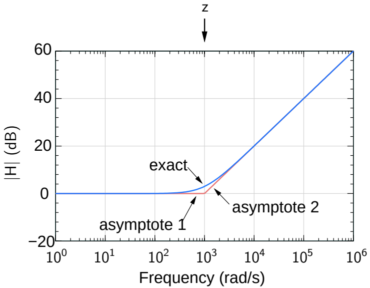
Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1,\;\; 20\log|H| = 0\,\text{dB}.$

Contribution of a zero: magnitude



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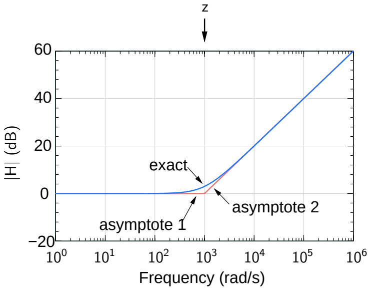
Asymptote 2: $\omega \gg p$: $|H| \rightarrow \dfrac{\omega}{z} \rightarrow |H| = 20\ \log \omega - 20\ \log\ z\ (\mathrm{dB})$

Contribution of a zero: magnitude



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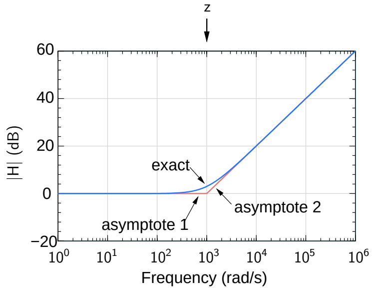
Consider two values of ω: ω_1 and $10\,\omega_1$.

$|H|_1 = 20\ \log\omega_1 - 20\ \log\ z\ (\mathrm{dB})$

$|H|_2 = 20\ \log(10\,\omega_1) - 20\ \log\ z\ (\mathrm{dB})$



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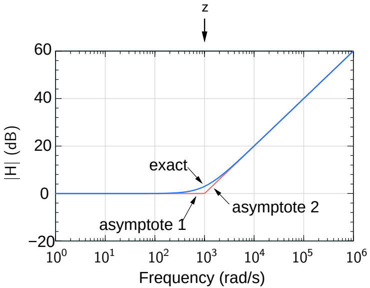
Contribution of a zero: magnitude



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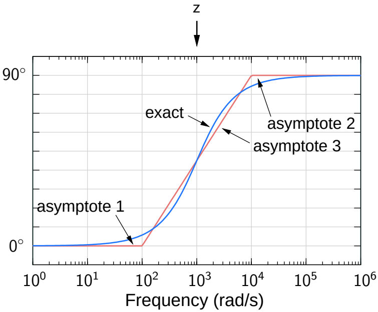
$\rightarrow |H|$ versus ω has a slope of $+20\,\text{dB/decade}$.

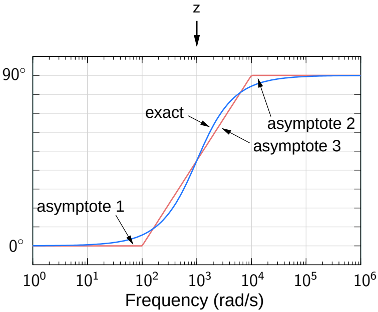
Note that, at $\omega = z$, the actual value of $|H|$ is $\sqrt{2}$ (i.e., $3\,\text{dB}$).

Contribution of a zero: phase



Consider $H(s) = 1 + s/z = 1 + j\,(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\dfrac{\omega}{z}\right)$

Contribution of a zero: phase



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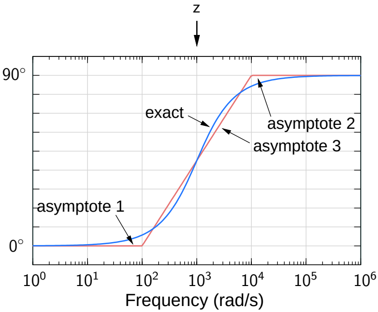
Asymptote 1: $\quad \omega \ll z$ (say, $\omega < z/10$): $\angle H = 0$.

Contribution of a zero: phase



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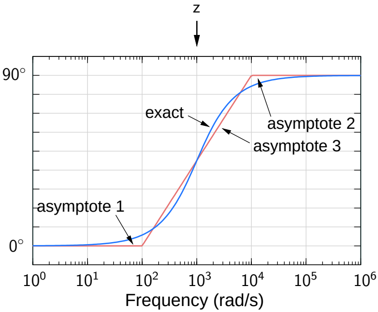
Contribution of a zero: phase



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$\rightarrow$ at $\omega = z$, $\angle H = \pi/4$ (which is also the actual value of $\angle H$).

Contribution of K (constant), s, and s^2

For $H(s) = K$, $20\ \log|H| = 20\ \log\ K$ (a constant), and $\angle H = 0$.

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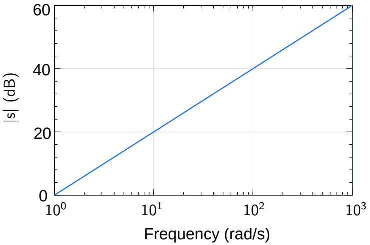
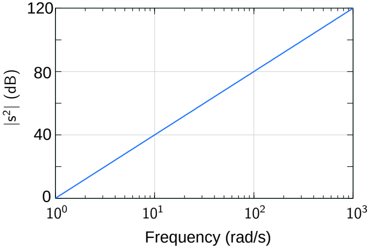
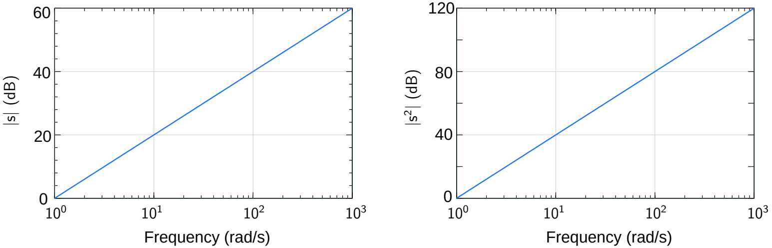




Contribution of K (constant), s, and s^2

For $H(s) = K$, $20 \log |H| = 20 \log K$ (a constant), and $\angle H = 0$.



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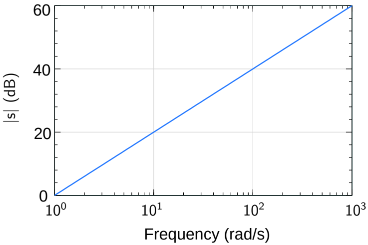

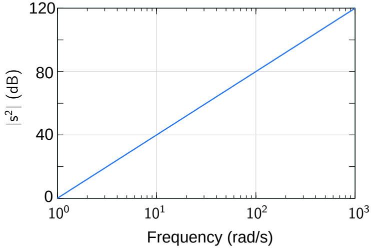

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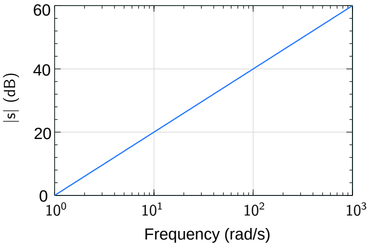

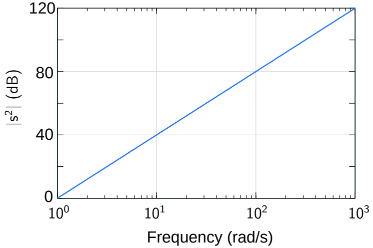

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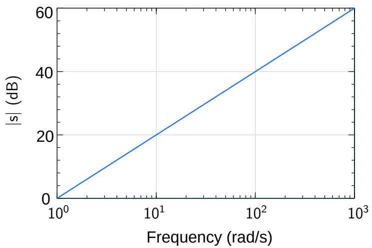

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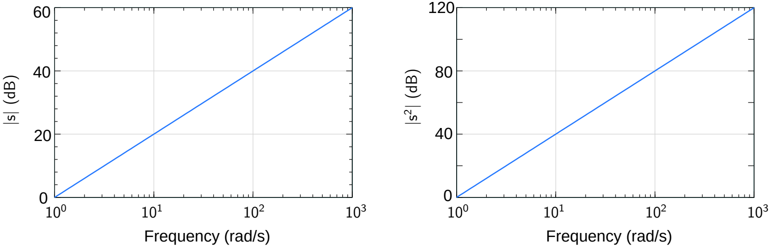

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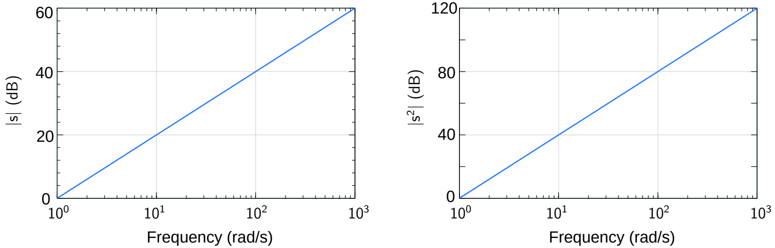

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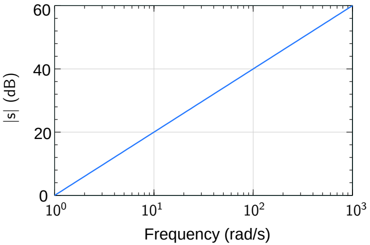

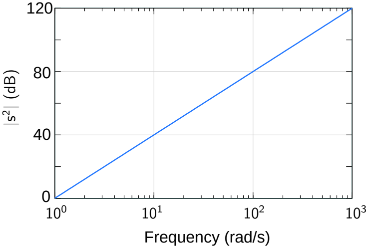

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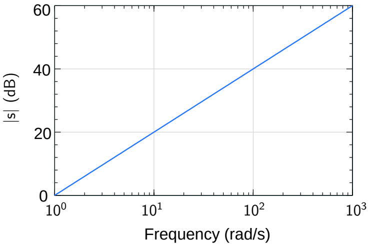

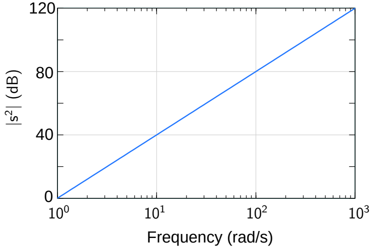

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Phase:

$H_1(j\omega)$ and $H_2(j\omega)$ are complex numbers.

At a given ω, let $H_1 = K_1\angle\alpha = K_1\,e^{j\alpha}$, and $H_2 = K_2\angle\beta = K_2\,e^{j\beta}$.

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In the Bode phase plot, the contributions due to H_1 and H_2 also get added.

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In the Bode phase plot, the contributions due to H_1 and H_2 also get added.

The same reasoning applies to more than two terms as well.

Combining different terms: example

Consider $H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$.

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Let $H(s) = H_1(s)\,H_2(s)\,H_3(s)\,H_4(s)$, where

$H_1(s) = 10$,

$H_2(s) = s$,

$H_3(s) = \dfrac{1}{1+s/p_1}$, $p_1 = 10^2$ rad/s,

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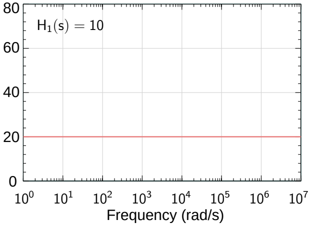
We can now plot the magnitude and phase of H_1, H_2, H_3, H_4 *individually* versus ω and then simply add them to obtain $|H|$ and $\angle H$.

Magnitude plot ($|H|$ in dB)



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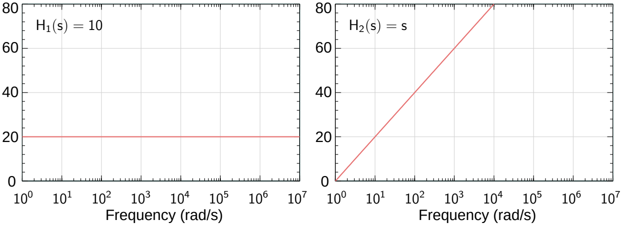

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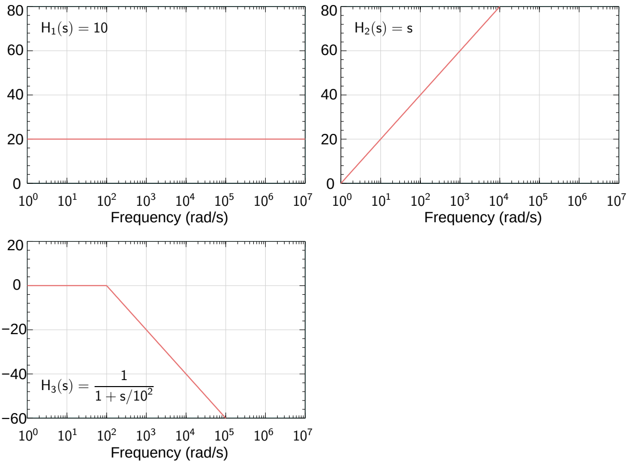

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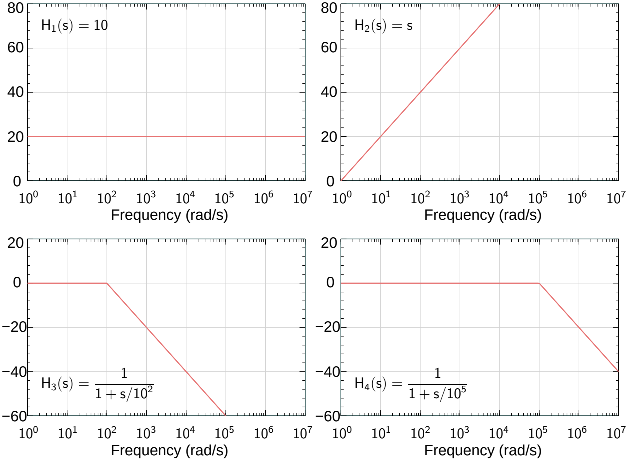

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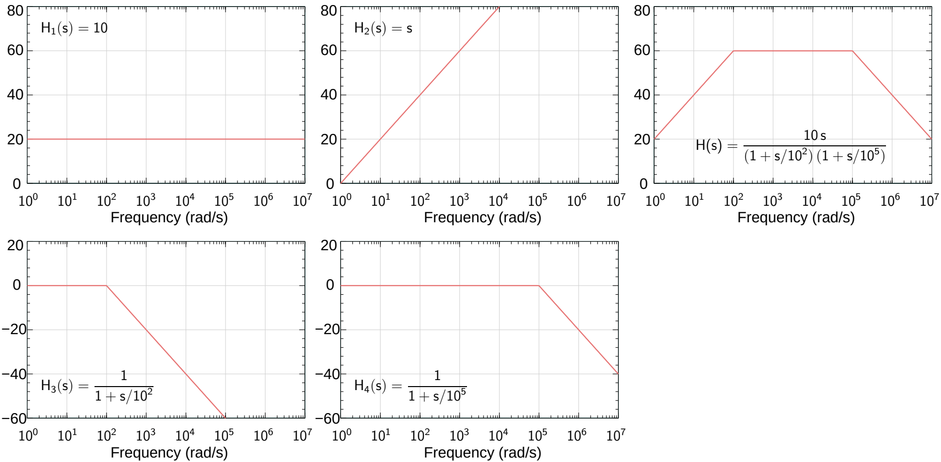

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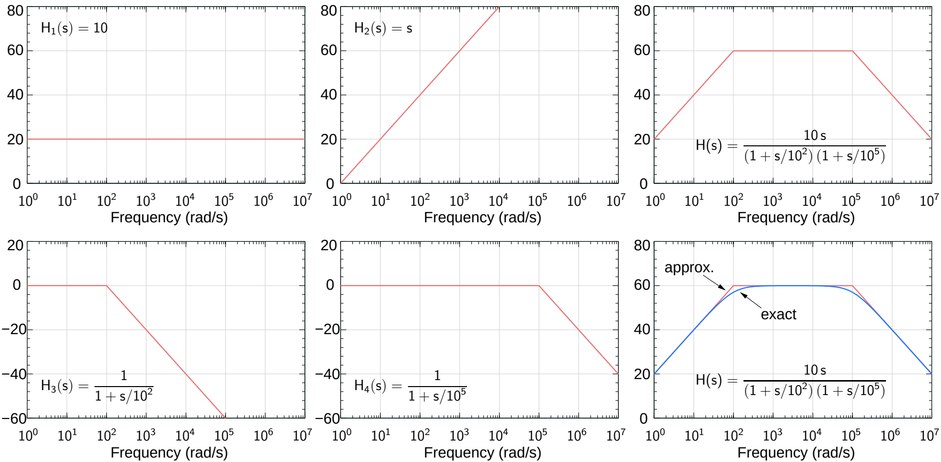
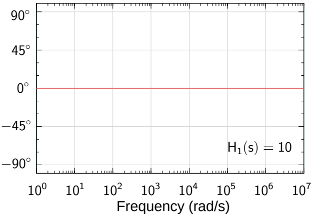
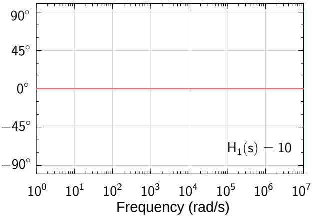


Phase plot



90°
45°
0°
−45°
−90°
$H_1(s) = 10$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



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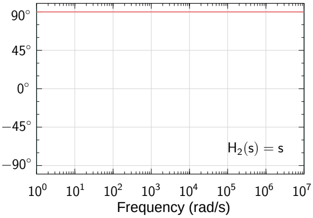
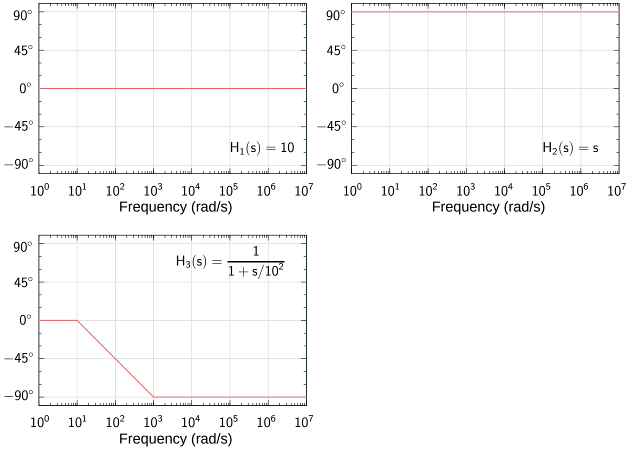


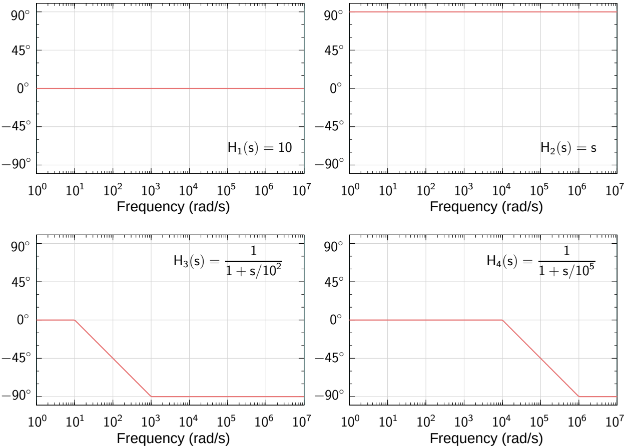
90°
45°
0°
−45°
−90°
$H_2(s) = s$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



90°
45°
0°
−45°
−90°
$H_1(s) = 10$
$H_2(s) = s$
$H_3(s) = \frac{1}{1 + s/10^2}$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot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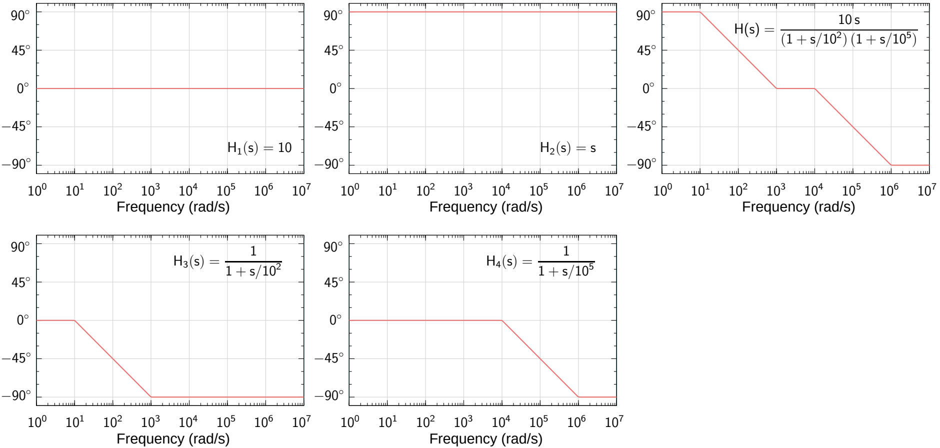

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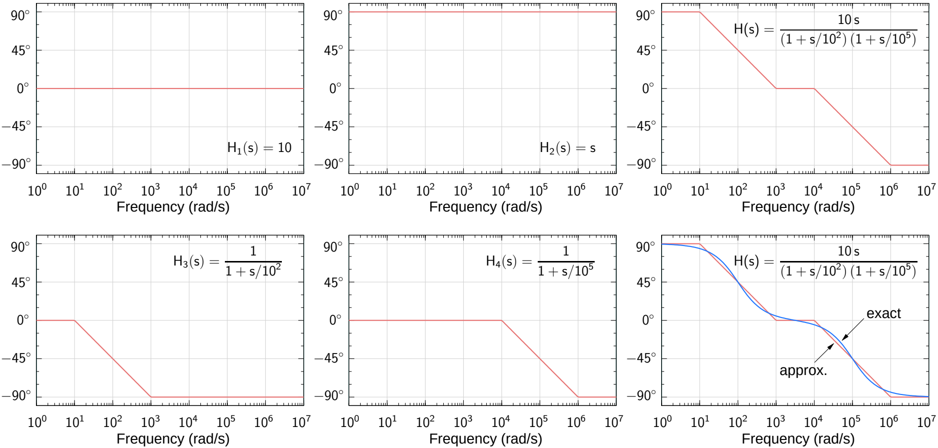

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exact
approx.
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

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* If two poles p_1 and p_2 are close to each other (say, separated by less than a decade in ω), the error becomes larger (next slide).
* When the poles and zeros are not sufficiently separated, the Bode approximation should be used only for a rough estimate, follwed by a numerical calculation. However, even in such cases, it does give a good idea of the *asymptotic* magnitude and phase plots, which is valuable in amplifier design.

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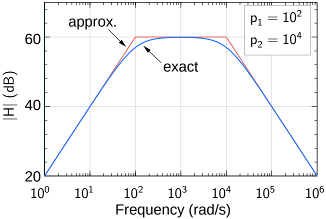
Consider $H(s) = \dfrac{10\,s}{(1 + s/p_1)\,(1 + s/p_2)}$.



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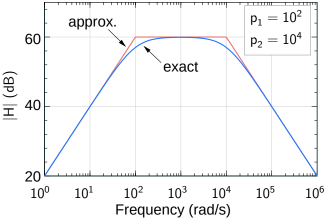

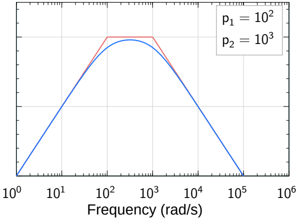

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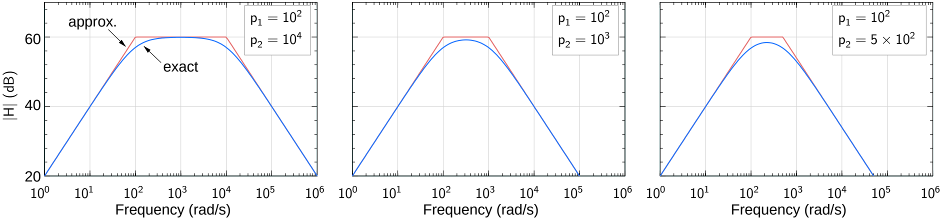

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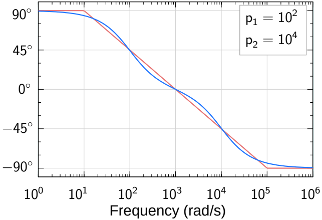

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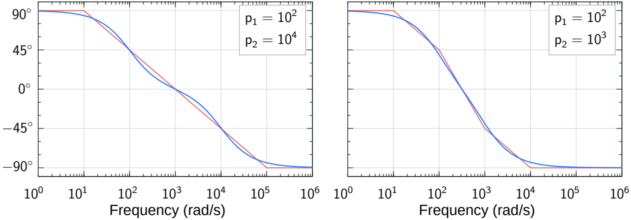

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