

EE101: RLC Circuits (with DC sources)



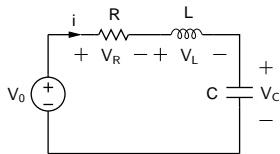
M. B. Patil

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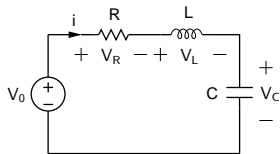
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Department of Electrical Engineering
Indian Institute of Technology Bombay

Series RLC circuit

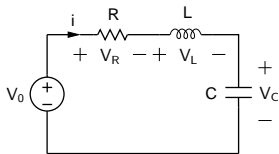


Series RLC circuit



$$\text{KVL: } V_R + V_L + V_C = V_0 \Rightarrow iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = V_0$$

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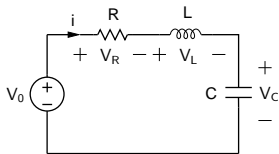


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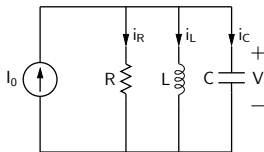
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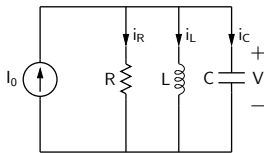
$$\text{i.e., } \frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0,$$

a second-order ODE with constant coefficients.

Parallel RLC circuit

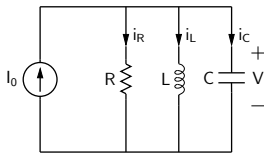


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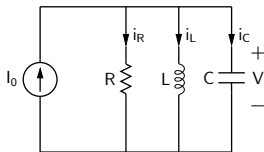


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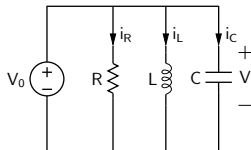
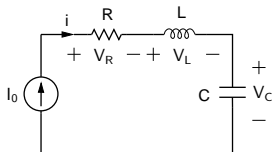
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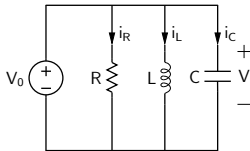
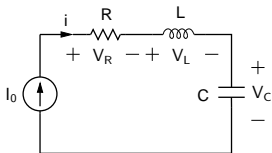
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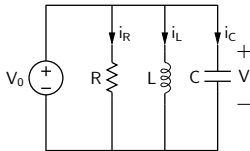
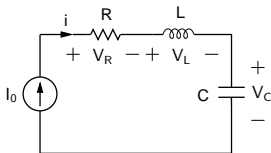


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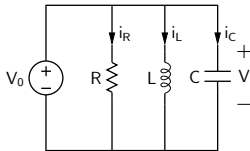
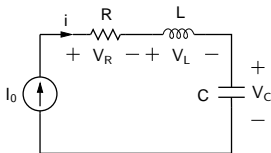
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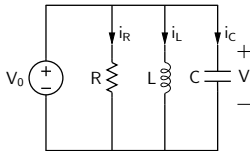
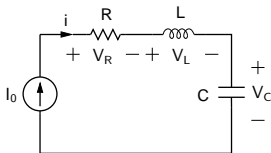


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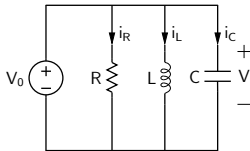
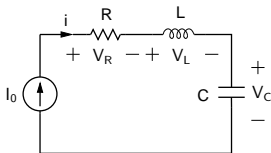
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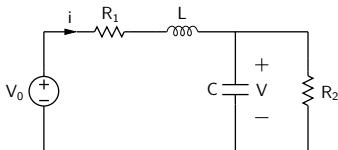
- * The above equations hold even if the applied voltage or current is not constant, and the variables of interest can still be easily obtained without solving a differential equation.

Series/Parallel RLC circuits

A general RLC circuit (with one inductor and one capacitor) also leads to a second-order ODE. As an example, consider the following circuit:

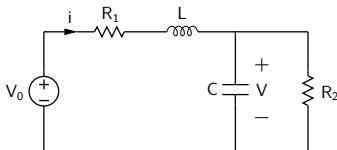
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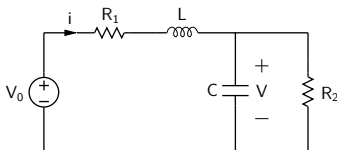


$$V_0 = R_1 i + L \frac{di}{dt} + V \quad (1)$$

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Substituting (2) in (1), we get

$$V_0 = R_1 [CV' + V/R_2] + L [CV'' + V'/R_2] + V, \quad (3)$$

$$V'' [LC] + V' [R_1 C + L/R_2] + V [1 + R_1/R_2] = V_0. \quad (4)$$

General solution

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$$y(t) = y^{(h)}(t) + y^{(p)}(t),$$

where $y^{(h)}(t)$ is the solution of the homogeneous equation,

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In the context of *RLC* circuits, $y^{(p)}(t)$ is the steady-state value of the variable of interest, i.e.,

$$y^{(p)} = \lim_{t \rightarrow \infty} y(t),$$

which can be often found by inspection.

General solution

For the homogeneous equation,

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we first find the roots of the associated *characteristic equation*,

$$r^2 + a r + b = 0.$$

Let the roots be r_1 and r_2 . We have the following possibilities:

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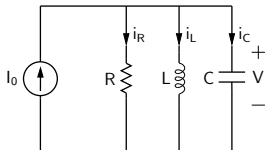
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Parallel RLC circuit



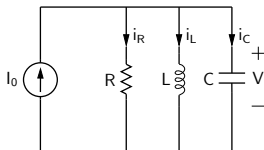
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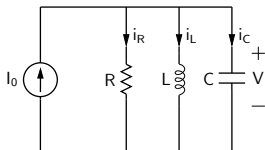
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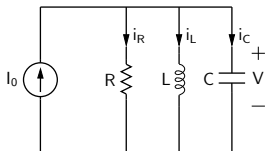
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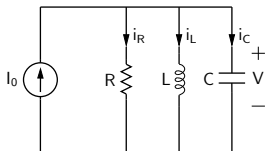
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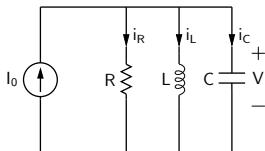
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$$\text{where } \tau_1 = -1/r_1 = 15.4\,\mu\text{s}, \quad \tau_2 = -1/r_2 = 28.6\,\mu\text{s}.$$

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From (1) and (2), we get the values of A and B , and

$$V(t) = -3.3 [\exp(-t/\tau_1) - \exp(-t/\tau_2)] \text{ V}. \quad (3)$$

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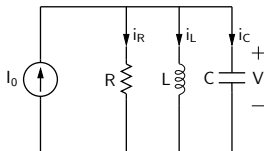
$$i_L(0^+) = I_0 - \frac{1}{R} V(0^+) - C \frac{dV}{dt}(0^+) = 0 \text{ A, which gives}$$

$$(A/\tau_1) + (B/\tau_2) = -I_0/C. \quad (2)$$

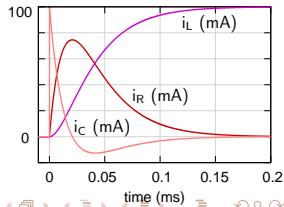
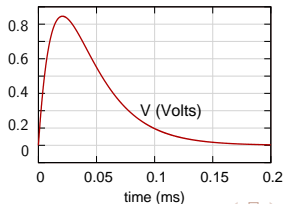
From (1) and (2), we get the values of A and B , and

$$V(t) = -3.3 [\exp(-t/\tau_1) - \exp(-t/\tau_2)] \text{ V}. \quad (3)$$

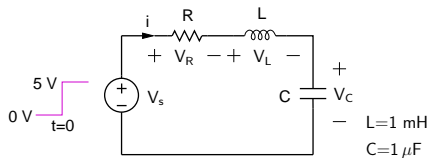
(SEQUEL file: ee101_r1c-1.sqproj)



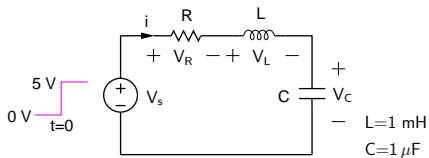
$$\begin{aligned} R &= 10 \Omega \\ C &= 1 \mu\text{F} \\ L &= 0.44 \text{ mH} \\ I_0 &= 100 \text{ mA} \end{aligned}$$



Series RLC circuit: home work

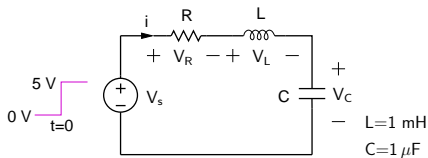


Series RLC circuit: home work



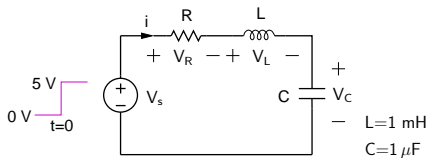
(a) Show that the condition for critically damped response is $R = 63.2 \Omega$.

Series RLC circuit: home work



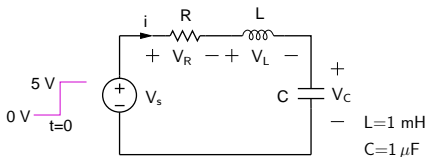
- (a) Show that the condition for critically damped response is $R = 63.2 \Omega$.
- (b) For $R = 20 \Omega$, derive expressions for $i(t)$ and $V_L(t)$ for $t > 0$ (Assume that $V_C(0^-) = 0 \text{ V}$ and $i_L(0^-) = 0 \text{ A}$). Plot them versus time.

Series RLC circuit: home work



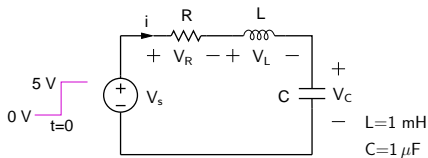
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- (c) Repeat (b) for $R = 100\text{ }\Omega$.

Series RLC circuit: home work



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- (b) For $R = 20 \Omega$, derive expressions for $i(t)$ and $V_L(t)$ for $t > 0$ (Assume that $V_C(0^-) = 0 \text{ V}$ and $i_L(0^-) = 0 \text{ A}$). Plot them versus time.
- (c) Repeat (b) for $R = 100 \Omega$.
- (d) Compare your results with the following plots.
(SEQUEL file: ee101_r1c_2.sqproj)

Series RLC circuit: home work



- Show that the condition for critically damped response is $R = 63.2 \Omega$.
- For $R = 20 \Omega$, derive expressions for $i(t)$ and $V_L(t)$ for $t > 0$ (Assume that $V_C(0^-) = 0 \text{ V}$ and $i_L(0^-) = 0 \text{ A}$). Plot them versus time.
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