EE101: Digital circuits (Part 2)

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* A Karnaugh map ("K map") is a representation of the truth table of a logical function.
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* A "minimal" expression has a minimum number of terms, each with a minimum number of variables. (For some functions, it is possible to have more than one minimal expressions, i.e., more than one expressions with the same complexity.)
* A minimal expression can be implemented with fewer gates.

K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

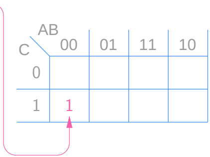
K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0				
1				

K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1



K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
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1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		
1	1			

K maps

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0	0	0	0
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0	1	1	0
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1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1			

K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

⇒

C \ AB	00	01	11	10
0	0	1	0	X
1	1	0	1	0

K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

$\Rightarrow$

C \ AB	00	01	11	10
0	0	1	0	X
1	1	0	1	0

* A K map is the same as the truth table of a function except for the way the entries are arranged.

K maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

$\Rightarrow$

C \ AB	00	01	11	10
0	0	1	0	X
1	1	0	1	0

* A K map is the same as the truth table of a function except for the way the entries are arranged.
* In a K map, the adjacent rows or columns differ only in *one* variable. For example, in going from the column $AB = 01$ to $AB = 11$, there is only one change, viz., $A = 0 \rightarrow A = 1$.

K maps: example with four variables

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	X
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	X
11	1	0	1	1
10	1	0	0	0

Consider the following functions (written in the standard sum-of-products form):

$X_1(A) = A + \overline{A}.$

$X_2(A, B) = A\,B + A\,\overline{B} + \overline{A}\,B + \overline{A}\,\overline{B}.$

$X_3(A, B, C) = A\,B\,C + A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,\overline{B}\,\overline{C}.$

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By using theorems and identities seen earlier, we can show that $X_1 = X_2 = X_3 = 1$.

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By using theorems and identities seen earlier, we can show that $X_1 = X_2 = X_3 = 1$.

Another way to explain why X_1, X_2, X_3 are each equal to 1 is the following.

For example, consider $X_2 \equiv Y_1 + Y_2 + Y_3 + Y_4 + = A\,B + A\,\overline{B} + \overline{A}\,B + \overline{A}\,\overline{B}$.

A	B	Y_1	Y_2	Y_3	Y_4	X_2
0	0	0	0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

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A	B	Y_1	Y_2	Y_3	Y_4	X_2
0	0	0	0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

From the truth table, it is clear that X_2 is equal to 1 because it includes *all* possible minterms that we can make with two variables.

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0	0	0	0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

From the truth table, it is clear that X_2 is equal to 1 because it includes *all* possible minterms that we can make with two variables.

For any combination of A and B, one of the minterms is $1 \Rightarrow X_2 = 1$.

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$X_3(A, B, C) = A\,B\,C + A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,\overline{B}\,\overline{C}$.

By using theorems and identities seen earlier, we can show that $X_1 = X_2 = X_3 = 1$.

Another way to explain why X_1, X_2, X_3 are each equal to 1 is the following.

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A	B	Y_1	Y_2	Y_3	Y_4	X_2
0	0	0	0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

From the truth table, it is clear that X_2 is equal to 1 because it includes *all* possible minterms that we can make with two variables.

For any combination of A and B, one of the minterms is $1 \Rightarrow X_2 = 1$.

Conclusion: "1" can be replaced by a suitable expansion in 1, 2, 3 (or more) variables. We will find this useful in understanding K maps.

Consider a function X_1 of three variables $(A,\ B,\ C)$: $X_1 = A\,\overline{B}$.

A	B	C	X_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$\Rightarrow$

C \ AB	00	01	11	10
0	0	0	0	1
1	0	0	0	1

K maps

Consider a function X_1 of three variables $(A,\ B,\ C)$: $X_1 = A\,\overline{B}$.

A	B	C	X_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

C \ AB	00	01	11	10
0	0	0	0	1
1	0	0	0	1

$$X_1 = A\,\overline{B} = (A\,\overline{B}) \cdot 1 = (A\,\overline{B}) \cdot (C + \overline{C}) = A\,\overline{B}\,C + A\,\overline{B}\,\overline{C}.$$

K maps

Consider a function X_1 of three variables $(A,\ B,\ C)$: $X_1 = A\,\overline{B}$.

A	B	C	X_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

C \ AB	00	01	11	10
0	0	0	0	1
1	0	0	0	1

$X_1 = A\,\overline{B} = (A\,\overline{B}) \cdot 1 = (A\,\overline{B}) \cdot (C + \overline{C}) = A\,\overline{B}\,C + A\,\overline{B}\,\overline{C}$.

$\rightarrow$ X_1 is composed of 2^1 minterms $\rightarrow$ two 1s in the K map.

K maps

Consider a function X_1 of three variables $(A,\ B,\ C)$: $X_1 = A\,\overline{B}$.

A	B	C	X_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

C \ AB	00	01	11	10
0	0	0	0	1
1	0	0	0	1

$X_1 = A\,\overline{B} = (A\,\overline{B}) \cdot 1 = (A\,\overline{B}) \cdot (C + \overline{C}) = A\,\overline{B}\,C + A\,\overline{B}\,\overline{C}$.

$\rightarrow$ X_1 is composed of 2^1 minterms $\rightarrow$ two 1s in the K map.

Further, these 2^1 minterms appear in adjacent boxes, *making up a rectangle*.

Consider a function X_2 of three variables (A, B, C): $X_2 = A$.

A	B	C	X_2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0	0	0	1	1
1	0	0	1	1

K maps

Consider a function X_2 of three variables $(A,\ B,\ C)$: $X_2 = A$.

A	B	C	X_2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$$X_2 = A = A \cdot 1 = A \cdot (B\,C + B\,\overline{C} + \overline{B}\,C + \overline{B}\,\overline{C}) = A\,B\,C + A\,B\,\overline{C} + A\,\overline{B}\,C + A\,\overline{B}\,\overline{C}.$$

Consider a function X_2 of three variables $(A,\ B,\ C)$: $X_2 = A$.

A	B	C	X_2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
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C \ AB	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$X_2 = A = A \cdot 1 = A \cdot (B\,C + B\,\overline{C} + \overline{B}\,C + \overline{B}\,\overline{C}) = A\,B\,C + A\,B\,\overline{C} + A\,\overline{B}\,C + A\,\overline{B}\,\overline{C}$.

$\rightarrow$ X_2 is composed of 2^2 minterms $\rightarrow$ four 1s in the K map.

Consider a function X_2 of three variables $(A,\ B,\ C)$: $X_2 = A$.

A	B	C	X_2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$X_2 = A = A \cdot 1 = A \cdot (B\,C + B\,\overline{C} + \overline{B}\,C + \overline{B}\,\overline{C}) = A\,B\,C + A\,B\,\overline{C} + A\,\overline{B}\,C + A\,\overline{B}\,\overline{C}$.

$\rightarrow$ X_2 is composed of 2^2 minterms $\rightarrow$ four 1s in the K map.

Further, these 2^2 minterms appear in adjacent boxes, *making up a rectangle*.

Let us now look at the *reverse* problem: Given a rectangle of 2^N 1s in the K map, what is the corresponding function (call it Y)?

Consider the following example:

C \ AB	00	01	11	10
0	0	1	1	0
1	0	1	1	0

We note that Y is independent of C (since the $C=0$ and $C=1$ boxes are identical).

Y is also independent of A (since the $A=0$ and $A=1$ boxes are identical).

$\rightarrow$ $Y=B$ or $Y=\overline{B}$.

By inspection, $Y=B$ (since Y is 1 in the $B=1$ boxes).

K maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

K maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$$X_1 = B\,\overline{C}$$

K maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$\Rightarrow$ $X_1 = B\,\overline{C}$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

K maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$$X_1 = B\,\overline{C}$$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$$X_2 = \overline{A}$$

K maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$\Rightarrow$ $X_1 = B\,\overline{C}$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$\Rightarrow$ $X_2 = \overline{A}$

* It should be now clear why we must have no more than one variable changing between adjacent columns (or rows).

K maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$\Rightarrow \quad X_1 = B\,\overline{C}$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$\Rightarrow \quad X_2 = \overline{A}$

* It should be now clear why we must have no more than one variable changing between adjacent columns (or rows).
* If this format is followed, terms that can be combined appear in rectangles of 2^1, 2^2, 2^3, $\cdots$ and can be easily combined by inspection.

C \ AB	00	01	11	10
0	0	0	1	0
1	0	0	0	1

Can the 1s shown in the K map be combined?

C \ AB	00	01	11	10
0	0	0	1	0
1	0	0	0	1

Can the 1s shown in the K map be combined?

Although the number of 1s is a power of 2 (2^1), they cannot be combined because they are not adjacent (i.e., they do not form a rectangle).

C \ AB	00	01	11	10
0	0	0	1	0
1	0	0	0	1

Can the 1s shown in the K map be combined?

Although the number of 1s is a power of 2 (2^1), they cannot be combined because they are not adjacent (i.e., they do not form a rectangle).

$\rightarrow$ the function $(A\,B\,\overline{C} + A\,\overline{B}\,C)$ cannot be minimized.

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

Can the 1s shown in the K map be combined?

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

Can the 1s shown in the K map be combined?

Let us redraw the K map by changing the order of the columns cyclically.

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1s shown in the K map be combined?

Let us redraw the K map by changing the order of the columns cyclically.

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1s shown in the K map be combined?

Let us redraw the K map by changing the order of the columns cyclically.

The two 1s are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1s shown in the K map be combined?

Let us redraw the K map by changing the order of the columns cyclically.

The two 1s are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1s shown in the K map be combined?

Let us redraw the K map by changing the order of the columns cyclically.

The two 1s are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

$\rightarrow$ Columns $A\,B = 00$ and $A\,B = 10$ in the K map on the left are indeed "logically adjacent" (although they are not geometrically adjacent) since they differ only in one variable (A).

We could have therefore combined the 1s without actually redrawing the K map.

K maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

$\Rightarrow$

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1s shown in the K map be combined?

Let us redraw the K map by changing the order of the columns cyclically.

The two 1s are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

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CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

K maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$\Rightarrow \quad X_1 = \overline{B}\,\overline{D}$$

K maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$X_1 = \overline{B}\,\overline{D}$$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

K maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$\Rightarrow$ $X_1 = \overline{B}\,\overline{D}$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

K maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$\Rightarrow$ $X_1 = \overline{B}\,\overline{D}$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$\Rightarrow$ $X_2 = \overline{B}\,\overline{C}$

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$X_1 = \underline{A\,B\,\overline{C}\,\overline{D}} + \underline{A\,B\,\overline{C}\,D} + \underline{\overline{A}\,B\,\overline{C}\,D}$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

K maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$$X_1 = \underline{A\,B\,\overline{C}\,\overline{D}} + \underline{A\,B\,\overline{C}\,D} + \underline{\overline{A}\,B\,\overline{C}\,D}$$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D \qquad \text{(using Y=Y+Y)} \\ &= A\,B\,\overline{C}\,(\overline{D} + D) + B\,\overline{C}\,D\,(A + \overline{A}) \\ &= A\,B\,\overline{C} + B\,\overline{C}\,D \end{aligned}$$

K maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$X_1 = \underline{A\,B\,\overline{C}\,\overline{D}} + \underline{A\,B\,\overline{C}\,D} + \underline{\overline{A}\,B\,\overline{C}\,D}$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D \qquad \text{(using Y=Y+Y)} \\ &= A\,B\,\overline{C}\,(\overline{D} + D) + B\,\overline{C}\,D\,(A + \overline{A}) \\ &= A\,B\,\overline{C} + B\,\overline{C}\,D \end{aligned}$$

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$$X_1 = A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D$$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D \qquad \text{(using Y=Y+Y)} \\ &= A\,B\,\overline{C}\,(\overline{D} + D) + B\,\overline{C}\,D\,(A + \overline{A}) \\ &= A\,B\,\overline{C} + B\,\overline{C}\,D \end{aligned}$$

* A minterm can be combined with others more than once.

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

$$X_1 = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,B\,C\,D + A\,\overline{B}\,\overline{D}$$

K maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

K maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

Since X represents a “don’t care” condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.

K maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

Since X represents a “don’t care” condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.

K maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

Since X represents a “don’t care” condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.

K maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

$$Z = C\,\overline{D} + \overline{A}\,\overline{C}\,D$$

Since X represents a "don't care" condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.