EE101: Basics
KCL, KVL, power, Thevenin's theorem

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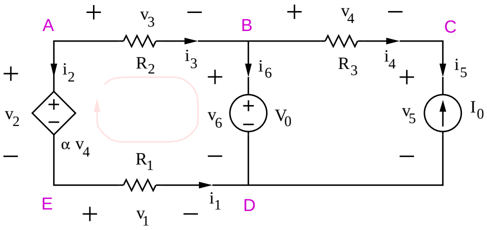
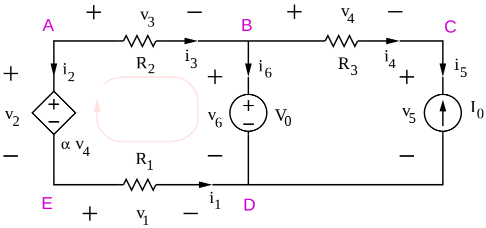

A
B
C
D
E
+ v3 −
+ v4 −
R2
R3
R1
i3
i4
i2
i6
i5
i1
+ v2 −
+ v6 −
+ v5 −
α v4
V0
I0
+ v1 −



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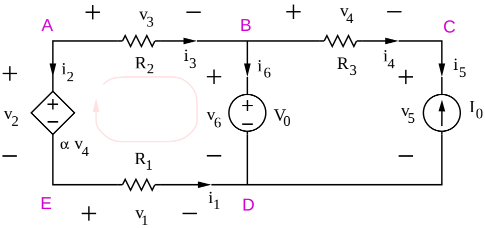
  $\sum i_k = 0$ at each node.



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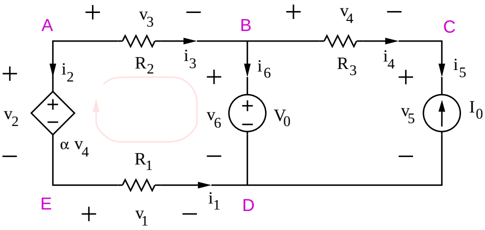

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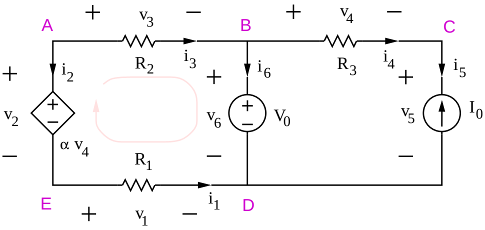

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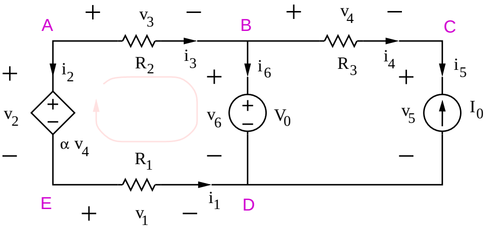

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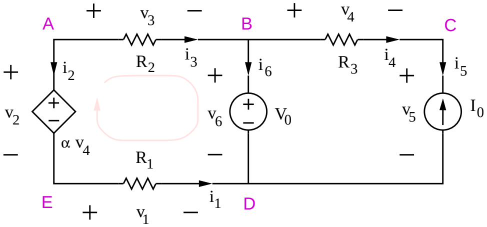

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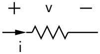
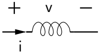
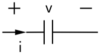
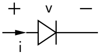
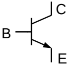
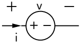
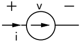
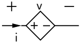
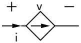
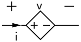
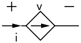
  (We have followed the convention that voltage *drop* across a branch is positive.)

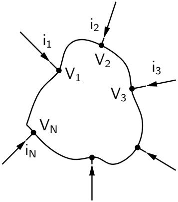
Circuit elements

Element	Symbol	Equation
Resistor		$v = R\,i$
Inductor		$v = L\,\dfrac{di}{dt}$
Capacitor		$i = C\,\dfrac{dv}{dt}$
Diode		to be discussed
BJT		to be discussed

Sources

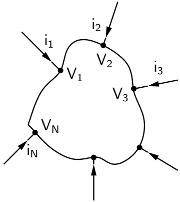
	Element	Symbol	Equation
Independent	Voltage source		$v(t) = v_s(t)$
	Current source		$i(t) = i_s(t)$
Dependent	VCVS		$v(t) = \alpha\, v_c(t)$
	VCCS		$i(t) = g\, v_c(t)$
	CCVS		$v(t) = r\, i_c(t)$
	CCCS		$i(t) = \beta\, i_c(t)$

* α, β: dimensionless, r: Ω, g: Ω^{-1} or ℧ ("mho")
* The subscript 'c' denotes the controlling voltage or current.



$$P(t) = V_1(t)\, i_1(t) + V_2(t)\, i_2(t) + \cdots + V_N(t)\, i_N(t)\,,$$

where V_1, V_2, etc. are "node voltages" (measured with respect to a reference node).



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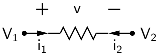
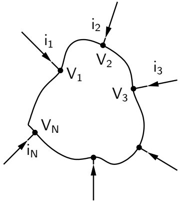


$$\begin{aligned} P &= V_1\, i_1 + V_2\, i_2 \\ &= V_1\, i_1 + V_2\,(-i_1) \\ &= [V_1 - V_2]\; i_1 = v\, i_1 \end{aligned}$$



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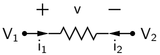

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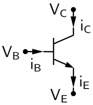

$$\begin{aligned} P &= V_B\, i_B + V_C\, i_C + V_E\,(-i_E) \\ &= V_B\, i_B + V_C\, i_C - V_E\,(i_B + i_C) \\ &= (V_B - V_E)\, i_B + (V_C - V_E)\, i_C \\ &= V_{BE}\, i_B + V_{CE}\, i_E \end{aligned}$$

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Instantaneous power

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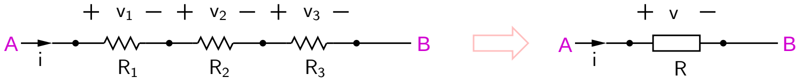
* A capacitor can absorb or deliver power. When it is absorbing power, its charge builds up. Similarly, an inductor can store energy (in the form of magnetic flux).

Resistors in series



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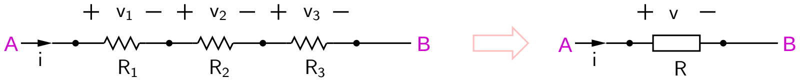

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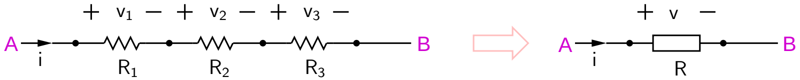

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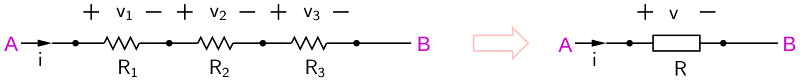

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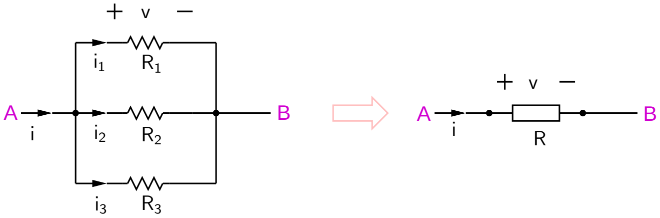
* The voltage drop across R_k is $v \times \dfrac{R_k}{R_{eq}}$.

Resistors in parallel



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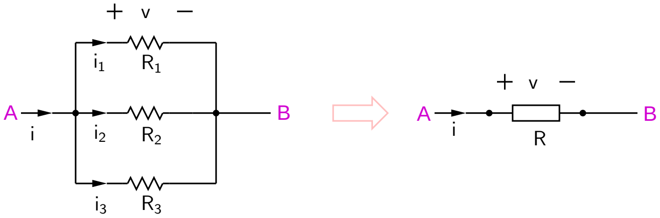

$i_1 = G_1\, v,\ i_2 = G_2\, v,\ i_3 = G_3\, v$, where $G_1 = 1/R_1$, etc.
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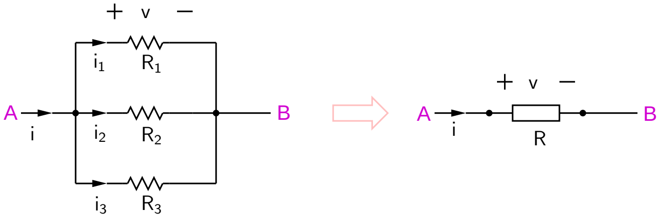

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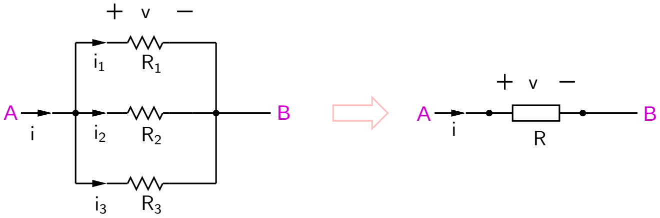

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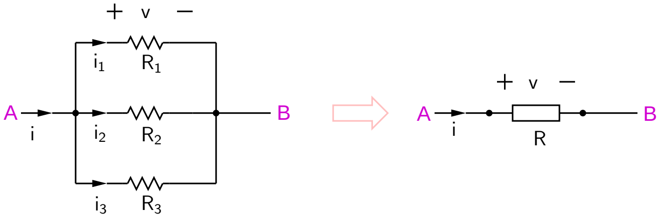

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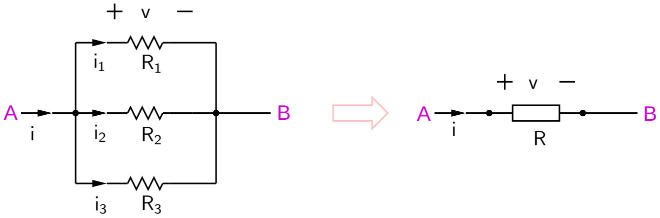
* If $N = 2$, we have

$$R_{eq} = \frac{R_1\, R_2}{R_1 + R_2},\ i_1 = i \times \frac{R_2}{R_1 + R_2},\ i_2 = i \times \frac{R_1}{R_1 + R_2}.$$



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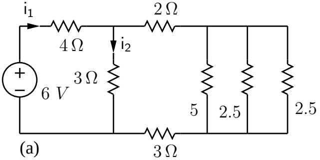
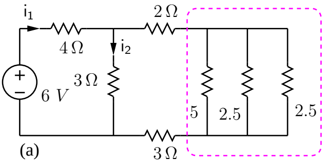
* If $R_k = 0$, all of the current will go through R_k.

Example



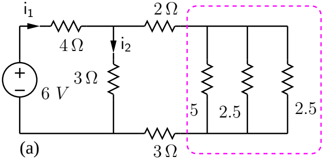
(a)

Example

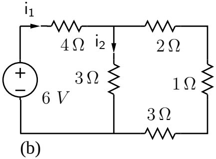


(a)

Example

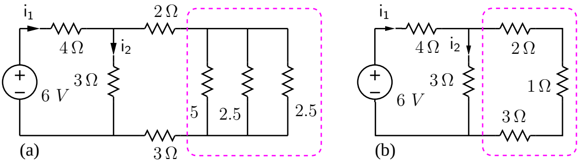


i_1
$4\,\Omega$
i_2
$2\,\Omega$
$3\,\Omega$
$6\ V$
5
2.5
2.5
$3\,\Omega$
(a)

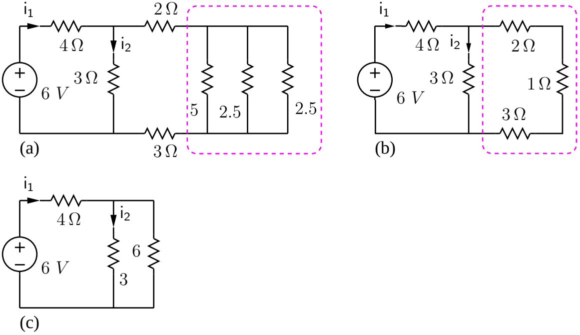


i_1
$4\,\Omega$
i_2
$2\,\Omega$
$3\,\Omega$
$1\,\Omega$
$6\ V$
$3\,\Omega$
(b)

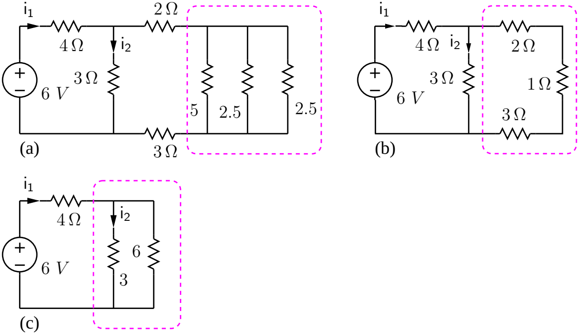
Example



Example

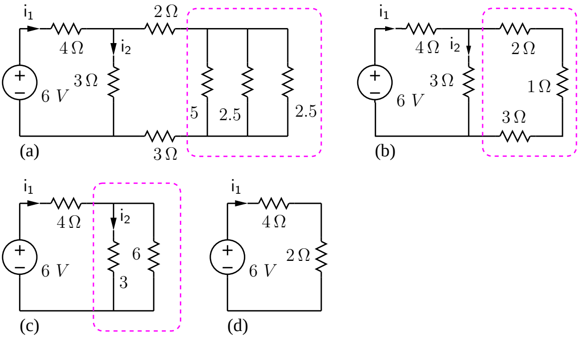


Example



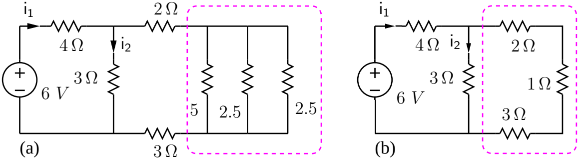
i_1
$4\,\Omega$
$2\,\Omega$
i_2
$3\,\Omega$
$6\ V$
5
2.5
2.5
$3\,\Omega$
(a)
i_1
$4\,\Omega$
i_2
$2\,\Omega$
$3\,\Omega$
$1\,\Omega$
$6\ V$
$3\,\Omega$
(b)
i_1
$4\,\Omega$
i_2
6
$6\ V$
3
(c)

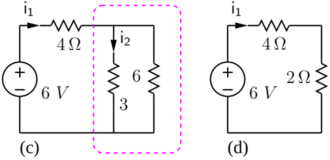
Example



i1
4 Ω
2 Ω
i2
3 Ω
6 V
5
2.5
2.5
3 Ω
(a)
i1
4 Ω
i2
2 Ω
3 Ω
1 Ω
6 V
3 Ω
(b)
i1
4 Ω
i2
6
3
6 V
(c)
i1
4 Ω
2 Ω
6 V
(d)

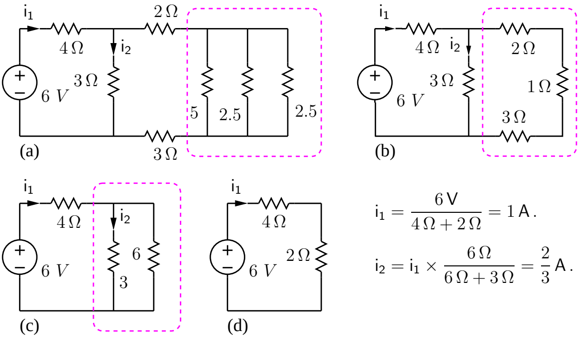
Example





$$i_1 = \frac{6\,\mathrm{V}}{4\,\Omega + 2\,\Omega} = 1\,\mathrm{A}\,.$$

Example



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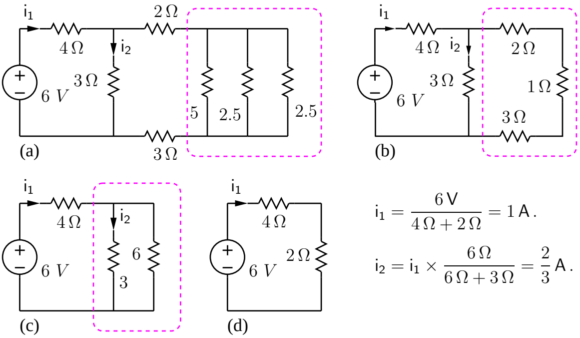

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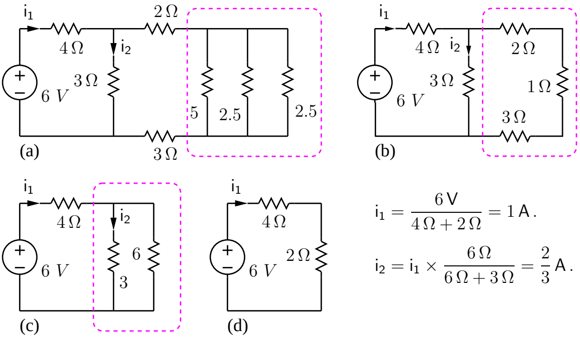

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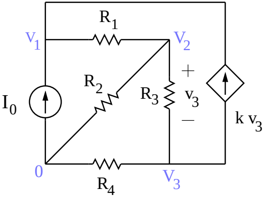
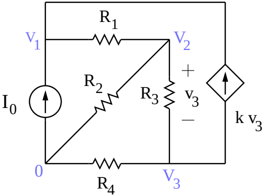
* Verify that KCL and KVL are satisfied for each node/loop.
* Verify that the total power absorbed by the resistors is equal to the power supplied by the source.


R1
V1
V2
R2
R3
v3
I0
k v3
0
R4
V3



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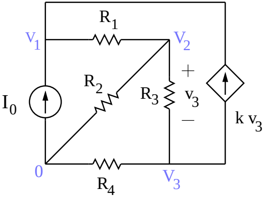

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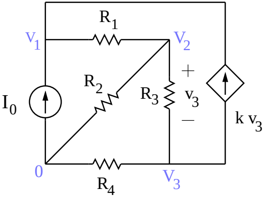

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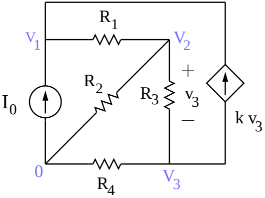

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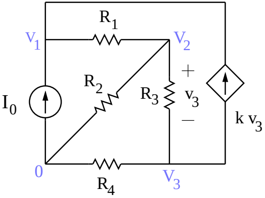

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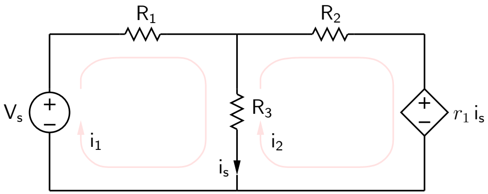
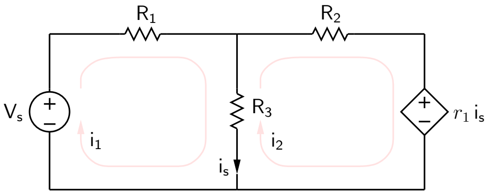
* Solve for the node voltages → branch voltages and currents.

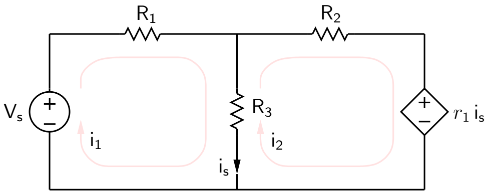
* Remark: Nodal analysis needs to be modified if there are voltage sources.

Mesh analysis





* Write KVL for each loop in terms of the "mesh currents" i_1 and i_2. Use a fixed convention, e.g., voltage drop is positive. (Note that $i_s = i_1 - i_2$.)



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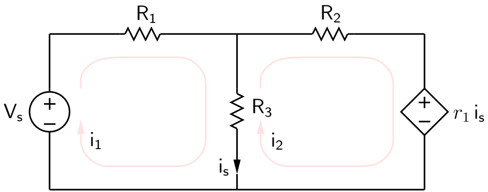

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* Solve for i_1 and i_2 → compute other quantities of interest (branch currents and branch voltages).

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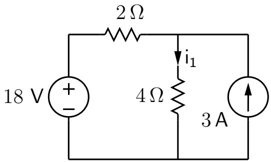
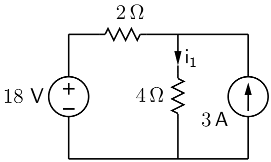
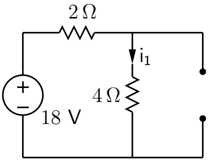
* Deactivating a voltage source $\Rightarrow v_s = 0$, i.e., replace the voltage source with a short circuit.

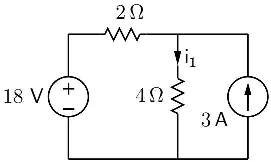
Example





Case 1: Keep V_s, deactivate I_s.





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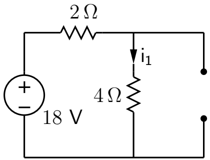
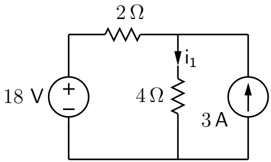


$i_1^{(1)} = 3$ A

Example



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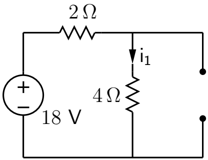

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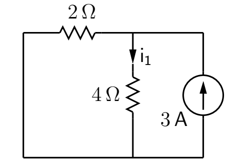

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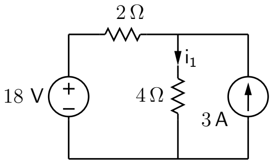

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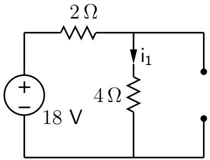

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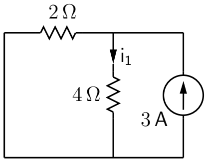
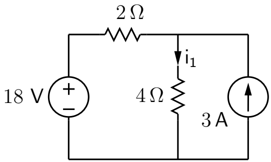


$$i_1^{(2)} = 3\,\text{A} \times \frac{2\,\Omega}{2\,\Omega + 4\,\Omega} = 1\,\text{A}$$

Example



$$i_1^{\text{net}} = i_1^{(1)} + i_1^{(2)} = 3 + 1 = 4\,\text{A}$$

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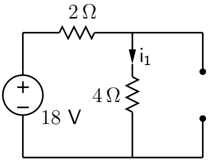

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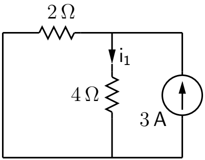

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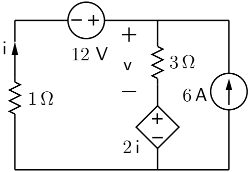

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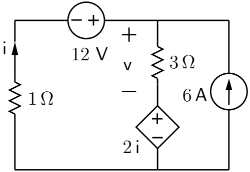

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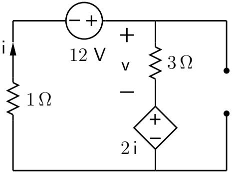

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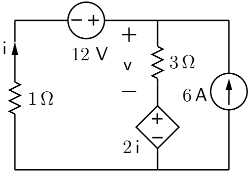

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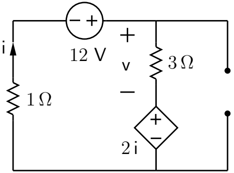

KVL: $-12 + 3\,i + 2\,i + i = 0$

$\Rightarrow i = 2\,A\,, v^{(1)} = 6\,V\,.$

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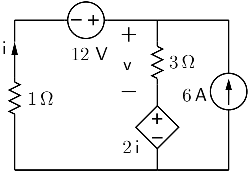

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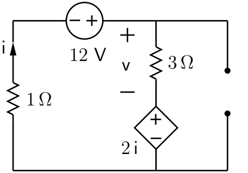

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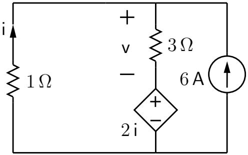

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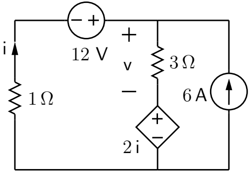

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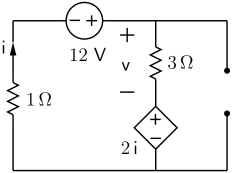

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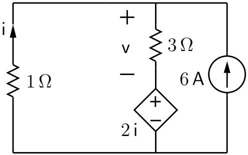
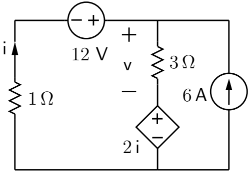


KVL: $\mathrm{i} + (6 + \mathrm{i})\,3 + 2\,\mathrm{i} = 0$

$\Rightarrow \mathrm{i} = -3\,\mathrm{A}\,, \mathrm{v}^{(2)} = (-3 + 6) \times 3 = 9\,\mathrm{V}\,.$

Example



$$v^{net} = v^{(1)} + v^{(2)} = 6 + 9 = 15\,V$$

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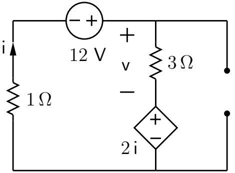

KVL: $-12 + 3\,i + 2\,i + i = 0$

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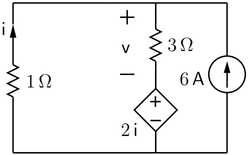

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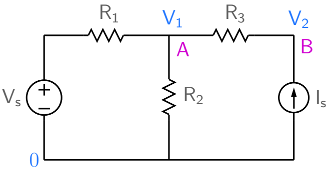
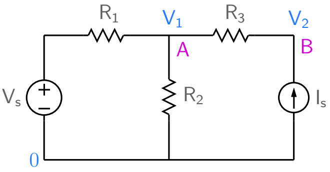
Superposition: Why does it work?





KCL at nodes A and B:

$$\begin{aligned}\frac{1}{R_1}(V_1 - V_s) + \frac{1}{R_2}V_1 + \frac{1}{R_3}(V_1 - V_2) &= 0\,,\\ -I_s + \frac{1}{R_3}(V_2 - V_1) &= 0\,.\end{aligned}$$

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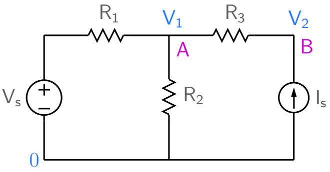

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Writing in a matrix form, we get (using $G_1 = 1/R_1$, etc.),

$$\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}$$

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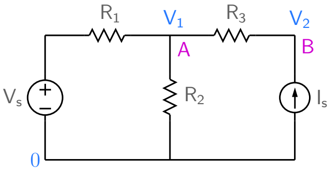

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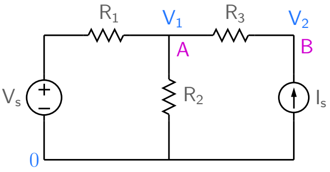
$$-I_s + \frac{1}{R_3}(V_2 - V_1) = 0\,.$$

Writing in a matrix form, we get (using $G_1 = 1/R_1$, etc.),

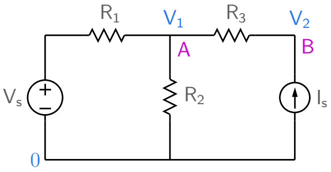
$$\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}$$

i.e., $\mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$

Superposition: Why does it work?



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$$



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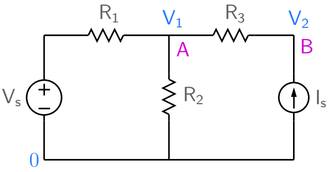
We are now in a position to see why superposition works.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_s \end{bmatrix} \equiv \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix}.$$



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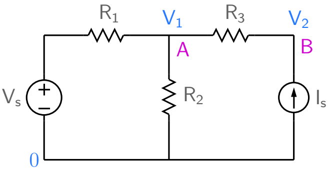
The first vector is the response due to V_s alone (and I_s deactivated).

The second vector is the response due to I_s alone (and V_s deactivated).

Superposition: Why does it work?



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$$

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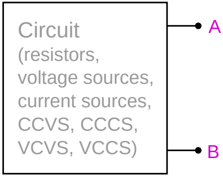
The first vector is the response due to V_s alone (and I_s deactivated).

The second vector is the response due to I_s alone (and V_s deactivated).

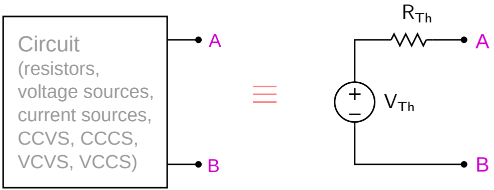
All other currents and voltages are linearly related to V_1 and V_2

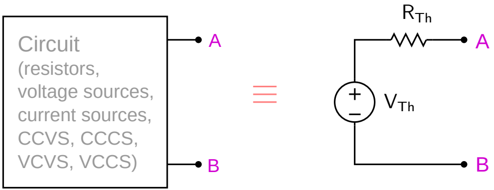
$\Rightarrow$ Any voltage (node voltage or branch voltage) or current can also be computed using superposition.

Thevenin's theorem

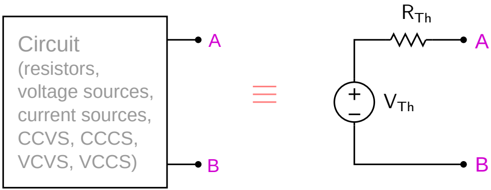


Thevenin's theorem



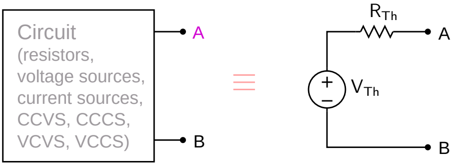


* V_{Th} is simply V_{AB} when nothing is connected on the other side, i.e., $V_{Th} = V_{oc}$.



* V_{Th} is simply V_{AB} when nothing is connected on the other side, i.e., $V_{Th} = V_{oc}$.
* R_{Th} can be found by different methods.

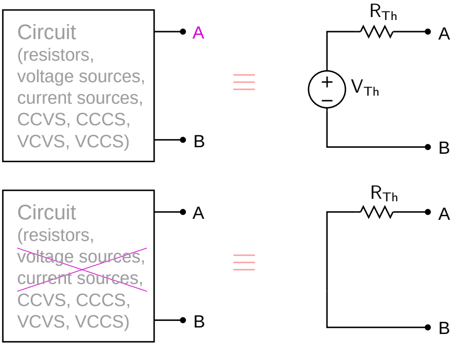
Method 1:



∗ Deactivate all *independent* sources.

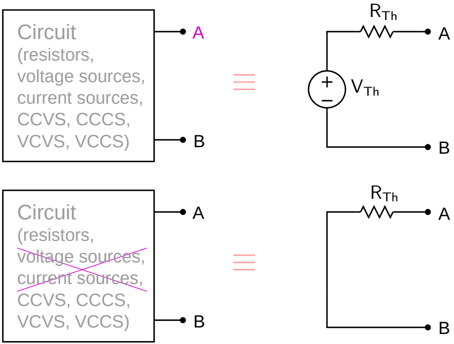
Thevenin's theorem: R_{Th}

Method 1:



* Deactivate all *independent* sources.

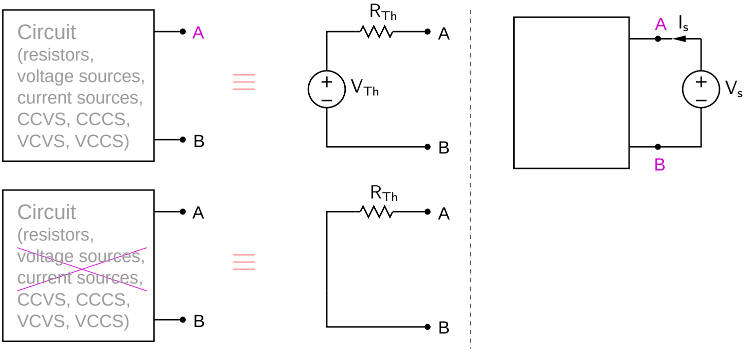
Method 1:



* Deactivate all *independent* sources.
* R_{Th} can often be found by inspection.

Thevenin's theorem: R_{Th}

Method 1:



- ∗ Deactivate all *independent* sources.
- ∗ R_{Th} can often be found by inspection.
- ∗ R_{Th} may be found by connecting a *test* source.

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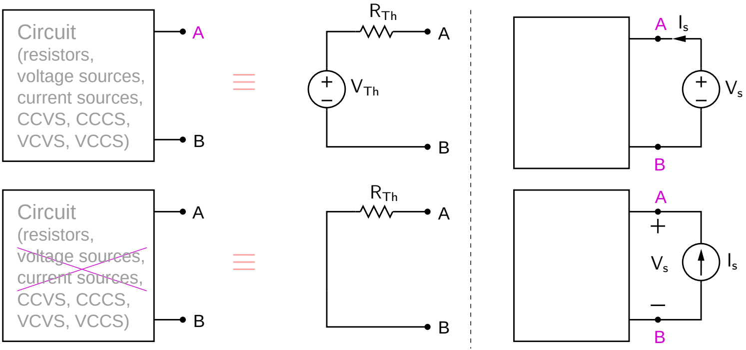

* Deactivate all *independent* sources.
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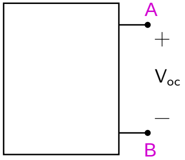
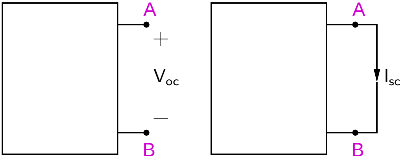
* R_{Th} may be found by connecting a *test* source.

Method 2:



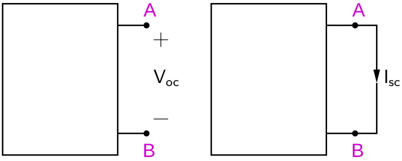
* Find V_{oc}.

Method 2:



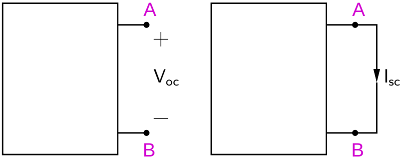
* Find V_{oc}.
* Find I_{sc}.

Method 2:



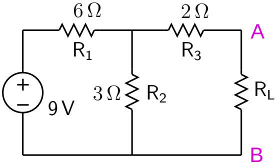
* Find V_{oc}.
* Find I_{sc}.
* $R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.

Method 2:

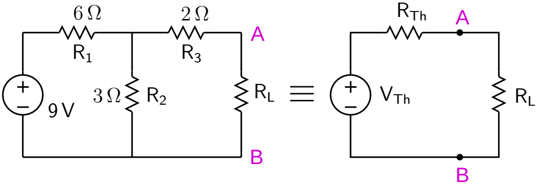


* Find V_{oc}.
* Find I_{sc}.
* $R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.
* Note: Sources are *not* deactivated.

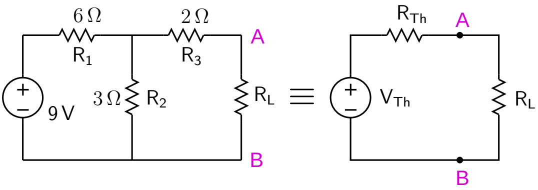
Thevenin's theorem: example

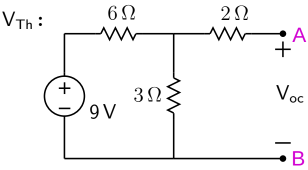


Thevenin's theorem: example

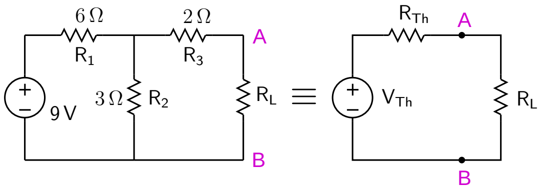


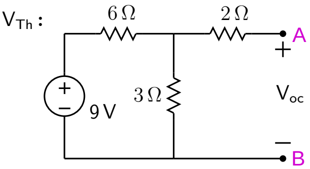
Thevenin's theorem: example





Thevenin's theorem: example

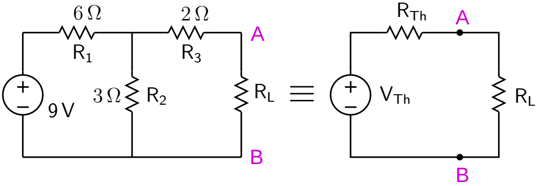




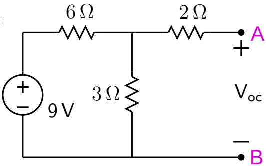
$$V_{oc} = 9\,\mathrm{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

$$= 9\,\mathrm{V} \times \frac{1}{3} = 3\,\mathrm{V}$$

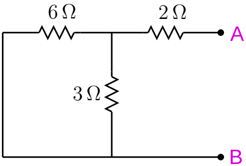
Thevenin's theorem: example



V_{Th} :



R_{Th} :



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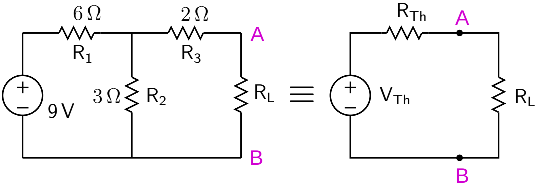
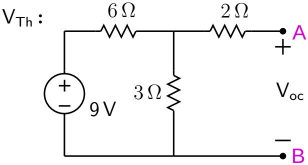
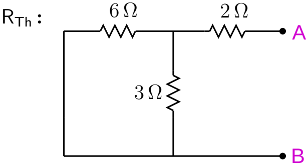
Thevenin's theorem: example





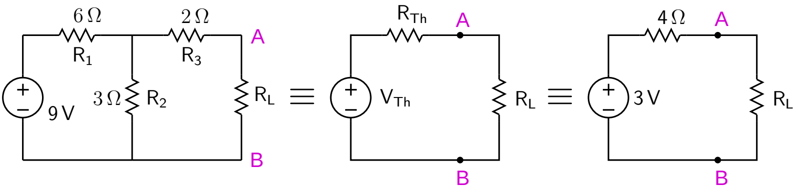


$$V_{oc} = 9\,\mathrm{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

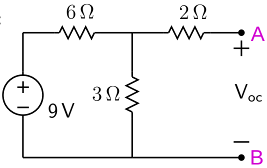
$$= 9\,\mathrm{V} \times \frac{1}{3} = 3\,\mathrm{V}$$

$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$



V_{Th} :



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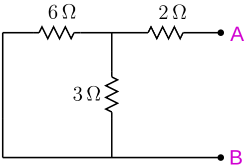

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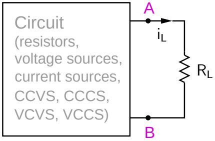
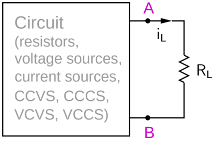
Maximum power transfer

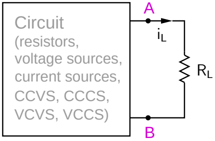


Maximum power transfer



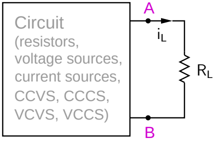
* Power "transferred" to load is,
  $P_L = i_L^2\, R_L$.

Maximum power transfer



- Power "transferred" to load is, $P_L = i_L^2\, R_L$.
- For a given black box, what is the value of R_L for which P_L is maximum?

Maximum power transfer



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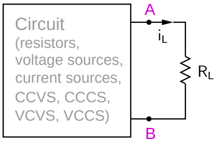
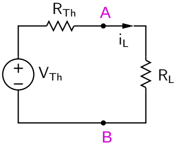
* Replace the black box with its Thevenin equivalent.





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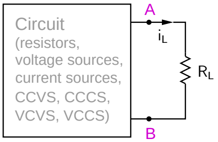

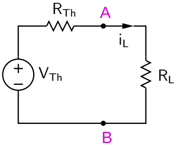

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- $i_L = \dfrac{V_{Th}}{R_{Th} + R_L}$,

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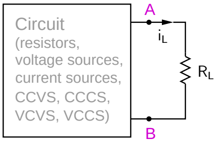
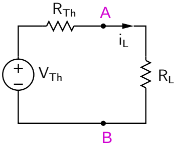
Maximum power transfer





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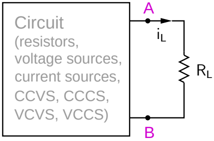
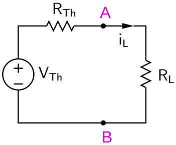
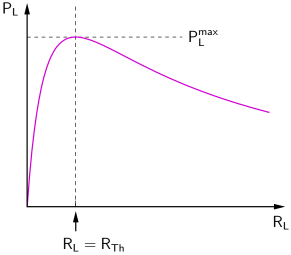
  $$\frac{(R_{Th} + R_L)^2 - R_L \times 2\,(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0\,,$$

  i.e., $R_{Th} + R_L = 2\,R_L \Rightarrow R_L = R_{Th}$.







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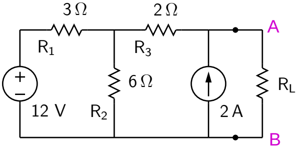
Maximum power transfer: example

Find R_L for which P_L is maximum.



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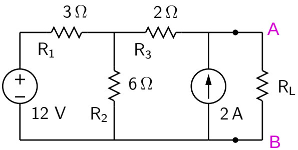
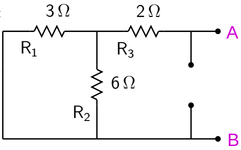


R_{Th}:



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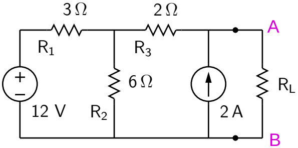
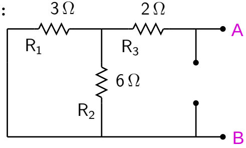


R_{Th}:

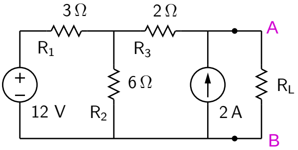


$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

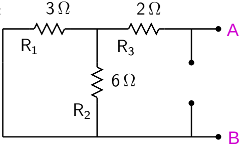
$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

Maximum power transfer: example

Find R_L for which P_L is maximum.



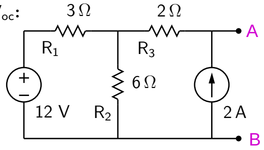
R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2} \right) + 2 = 4\,\Omega$$

V_{oc}:



Maximum power transfer: example

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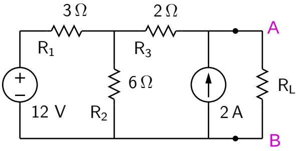
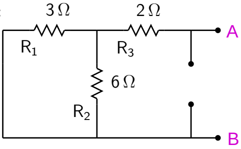


R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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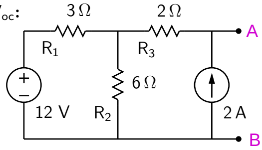
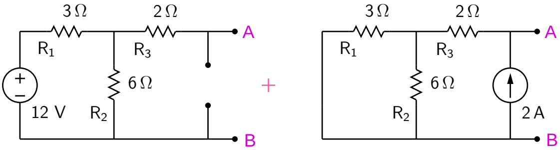
V_{oc}:

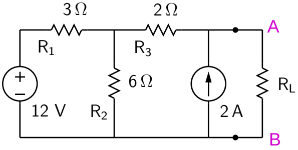


Use superposition to find V_{oc}:

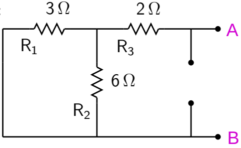


Maximum power transfer: example

Find R_L for which P_L is maximum.



R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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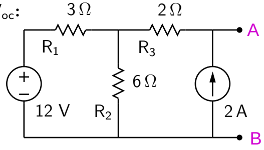
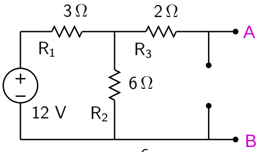
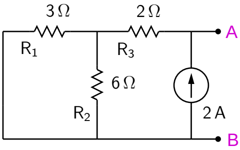
V_{oc}:



Use superposition to find V_{oc}:



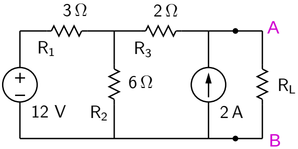
$+$



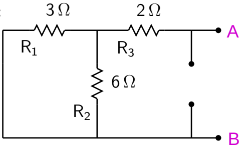
$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\ \text{V}$$

Maximum power transfer: example

Find R_L for which P_L is maximum.



R_{Th}:



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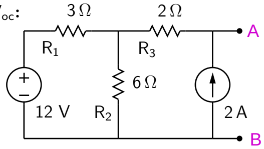
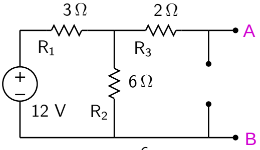
V_{oc}:



Use superposition to find V_{oc}:



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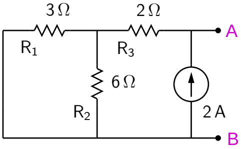
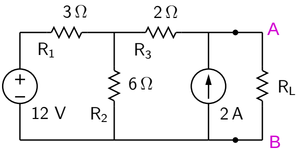


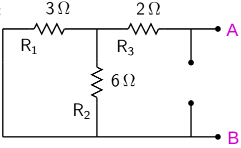
$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\ \text{V}$$

$$V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

Find R_L for which P_L is maximum.



R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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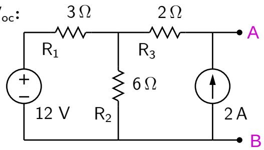
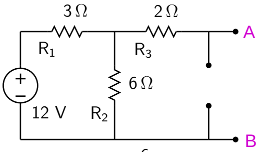
V_{oc}:



Use superposition to find V_{oc}:



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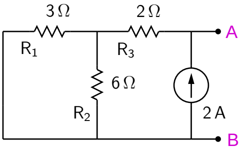
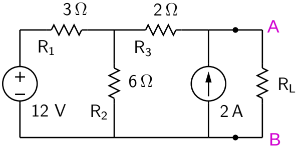


$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\text{ V} \qquad V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

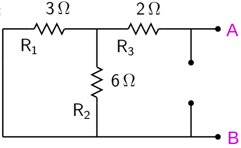
$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16\,\text{V}$$

Maximum power transfer: example

Find R_L for which P_L is maximum.



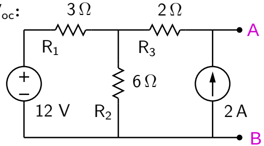
R_{Th}:



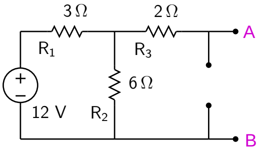
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

V_{oc}:



Use superposition to find V_{oc}:



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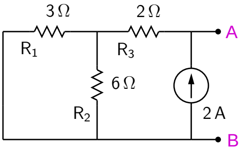
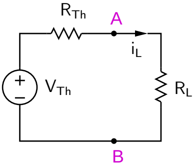


$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\text{ V} \qquad V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16\,\text{V}$$

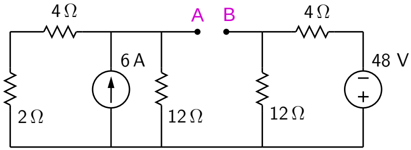


P_L is maximum when $R_L = R_{Th} = 4\,\Omega$

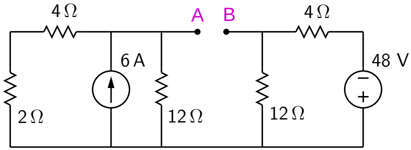
$\Rightarrow i_L = V_{Th}/(2\,R_{Th}) = 2\,\text{A}$

$P_L^{max} = 2^2 \times 4 = 16\,\text{W}$.

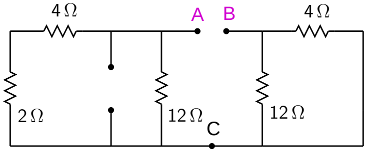
Thevenin's theorem: example



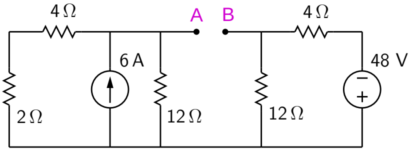
Thevenin's theorem: example



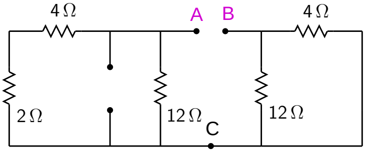
R_{Th}:



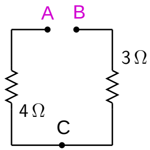
Thevenin's theorem: example

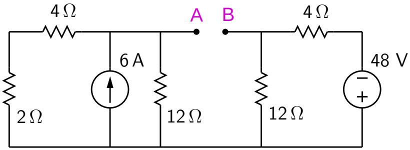


R_{Th}:

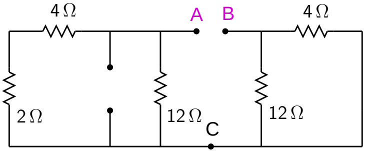


$\equiv$





R_{Th}:



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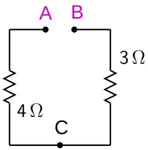
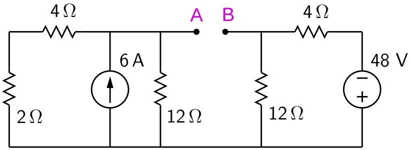
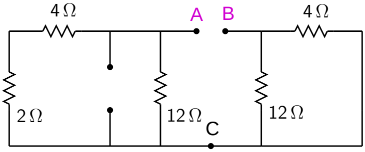


$\Rightarrow$ $R_{Th} = 7\,\Omega$

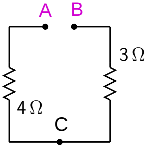
Thevenin's theorem: example



R_{Th}:

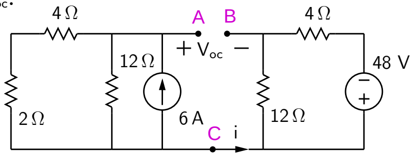


$\equiv$

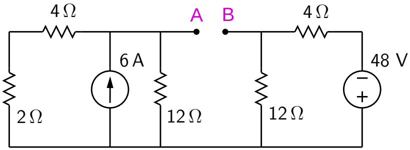


$\Longrightarrow \quad R_{Th} = 7\,\Omega$

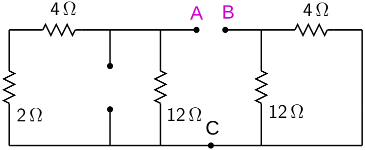
V_{oc}:



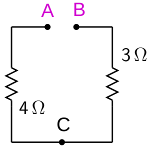
Thevenin's theorem: example



R_{Th}:



$\equiv$



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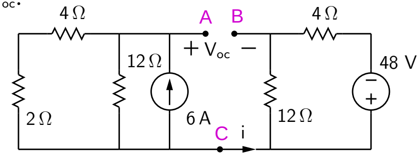
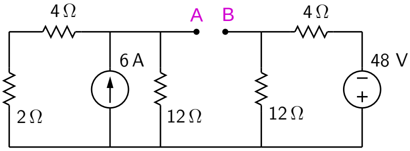


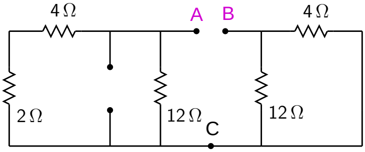
Note: $i = 0$ (since there is no return path).

$$
\begin{aligned}
V_{AB} &= V_A - V_B \\
&= (V_A - V_C) + (V_C - V_B) \\
&= V_{AC} + V_{CB} \\
&= 24\,V + 36\,V = 60\,V
\end{aligned}
$$

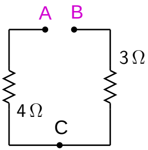
Thevenin's theorem: example



R_{Th}:

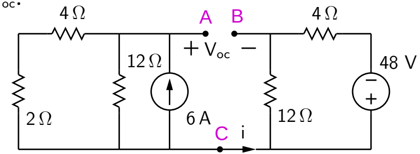


$\equiv$



$\Rightarrow \quad R_{Th} = 7\,\Omega$

V_{oc}:



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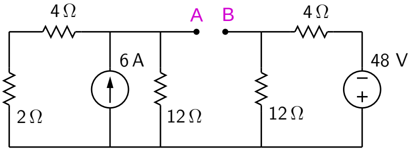
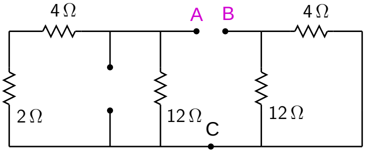
$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= (V_A - V_C) + (V_C - V_B) \\ &= V_{AC} + V_{CB} \\ &= 24\,\text{V} + 36\,\text{V} = 60\,\text{V} \end{aligned}$$

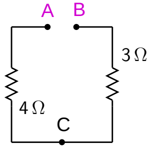
$$\boxed{\begin{aligned} V_{Th} &= 60\,\text{V} \\ R_{Th} &= 7\,\Omega \end{aligned}}$$



R_{Th}:

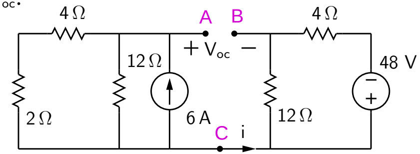


$\equiv$



$\Rightarrow \quad R_{Th} = 7\,\Omega$

V_{oc}:



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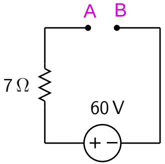
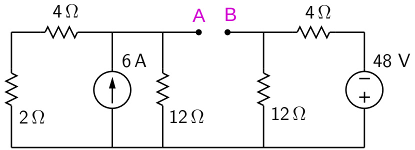


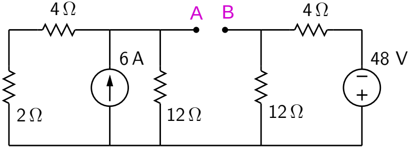
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



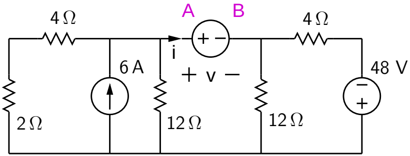
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



Connect a voltage source between A and B.

Plot i versus v.

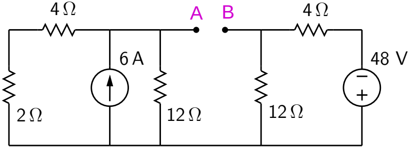


V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.

Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



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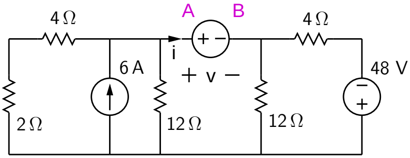
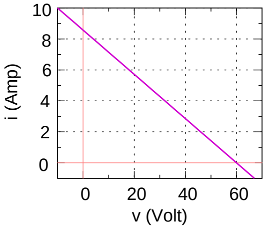


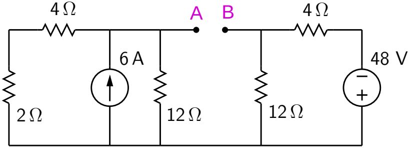
V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.



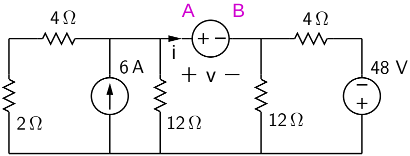
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



Connect a voltage source between A and B.

Plot i versus v.



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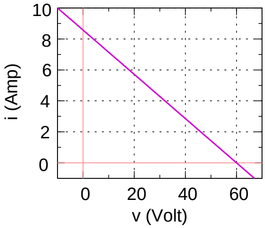
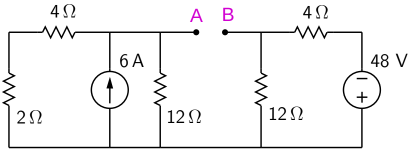


$V_{oc} = 60$ V, $I_{sc} = 8.57$ A

$R_{Th} = V_{sc}/I_{sc} = 7\ \Omega$

Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



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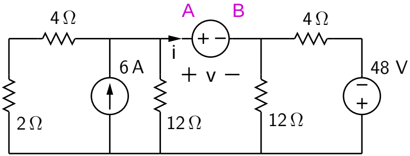

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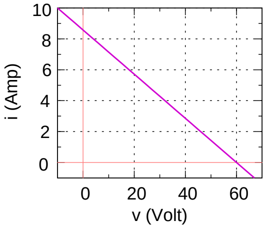
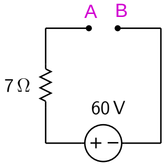


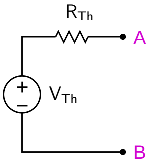
$V_{oc} = 60$ V, $I_{sc} = 8.57$ A

$R_{Th} = V_{sc}/I_{sc} = 7\ \Omega$

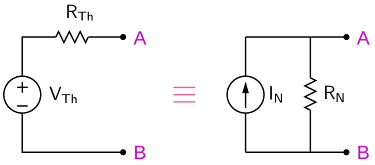
$V_{Th} = 60\,\text{V}$
$R_{Th} = 7\,\Omega$



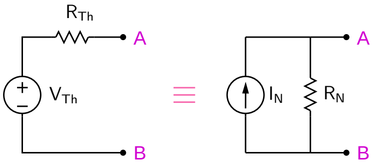
Norton equivalent circuit



Norton equivalent circuit

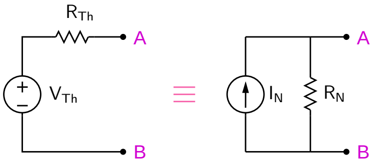


Norton equivalent circuit



∗ Consider the open circuit case.

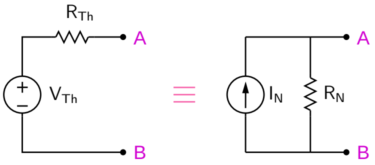
Norton equivalent circuit



* Consider the open circuit case.

  Thevenin circuit: $V_{AB} = V_{Th}$.

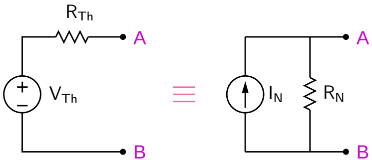
Norton equivalent circuit



* Consider the open circuit case.

  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.

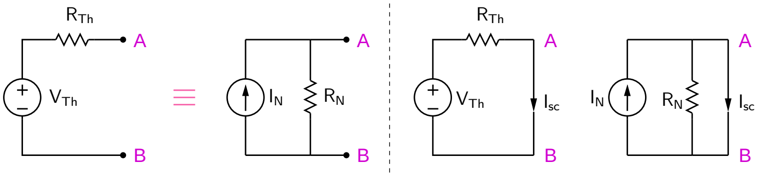
Norton equivalent circuit



* Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.
Norton circuit: $V_{AB} = I_N\, R_N$.
$\Rightarrow V_{Th} = I_N\, R_N$.

Norton equivalent circuit



* Consider the open circuit case.

  Thevenin circuit: $V_{AB} = V_{Th}$.
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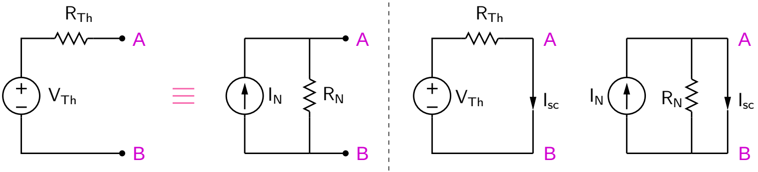
* Consider the short circuit case.

Norton equivalent circuit



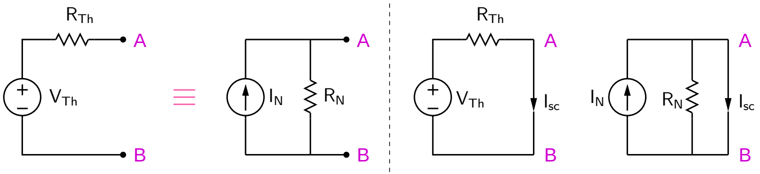
* Consider the open circuit case.

  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.

* Consider the short circuit case.

  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.

Norton equivalent circuit



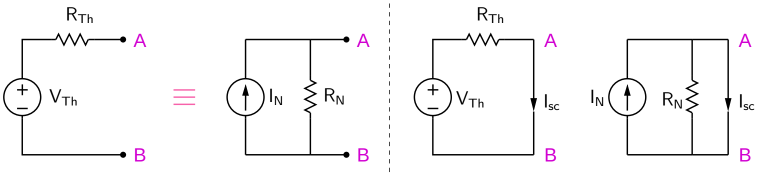
* Consider the open circuit case.

  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N \, R_N$.
  $\Rightarrow V_{Th} = I_N \, R_N$.

* Consider the short circuit case.

  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.

Norton equivalent circuit



* Consider the open circuit case.

  Thevenin circuit: $V_{AB} = V_{Th}$.
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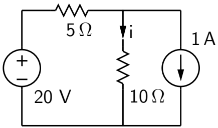
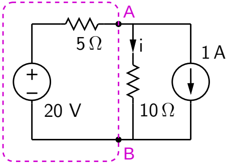
* Consider the short circuit case.

  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow R_{Th} = R_N$.

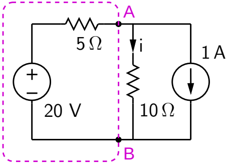
Example



Example



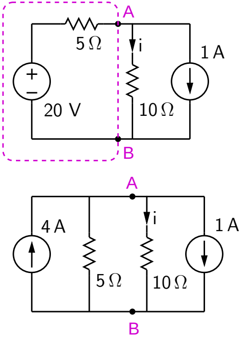
Example



$$R_N = 5\,\Omega$$

$$I_N = \frac{20\,\text{V}}{5\,\Omega} = 4\,\text{A}$$

Example



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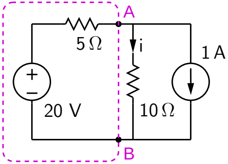

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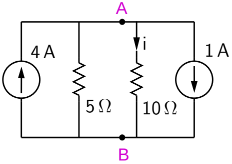
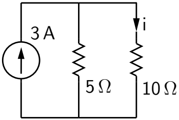
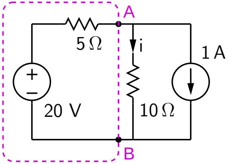




Example



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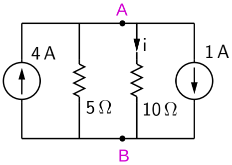
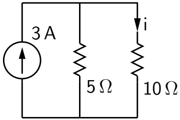
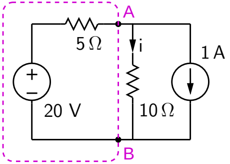




$$i = 3\,\text{A} \times \frac{5}{5+10}$$

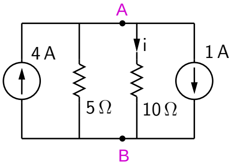
$$= 1\,\text{A}$$

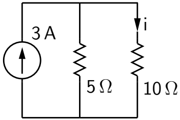
Example



$R_N = 5\,\Omega$

$I_N = \dfrac{20\,V}{5\,\Omega} = 4\,A$





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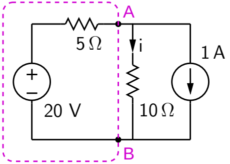
Home work:

* Find i by superposition and compare.

Example



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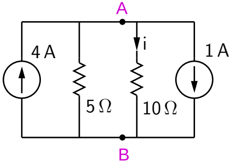

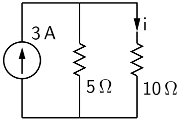

$$i = 3\,\text{A} \times \frac{5}{5+10}$$

$$= 1\,\text{A}$$

Home work:

* Find i by superposition and compare.
* Compute the power absorbed by each element, and verify that $\sum P_i = 0$.