

EE101: Op Amp circuits (Part 3)



M. B. Patil

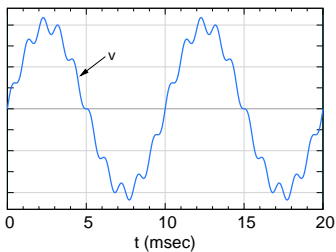
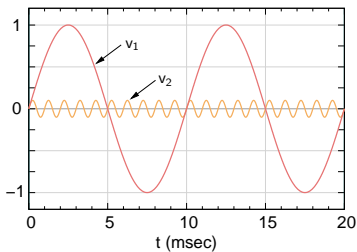
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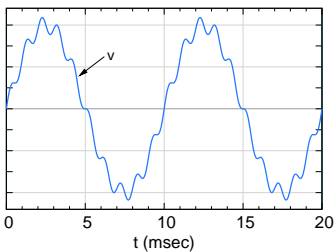
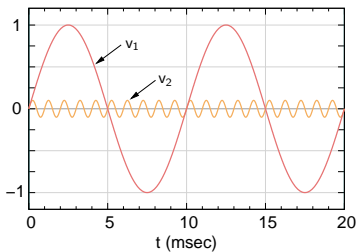
Introduction to filters

Consider $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$.



Introduction to filters

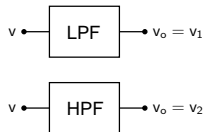
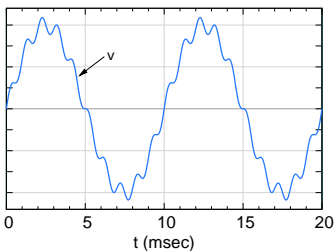
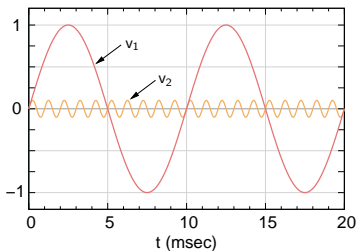
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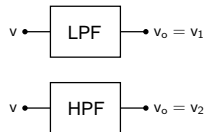
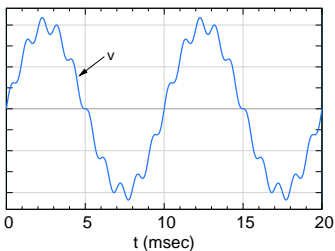
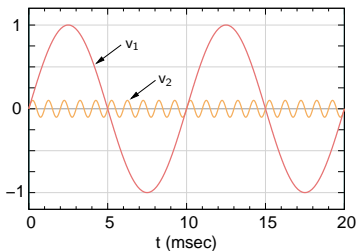


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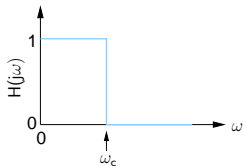
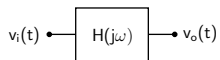


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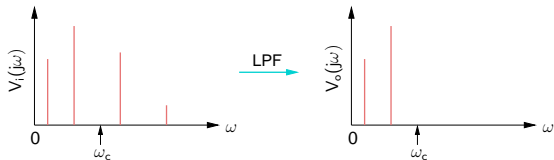
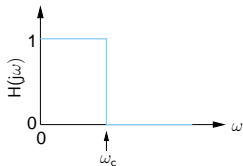
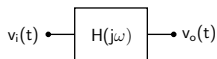
There are some other types of filters, as we will see.

Ideal low-pass filter



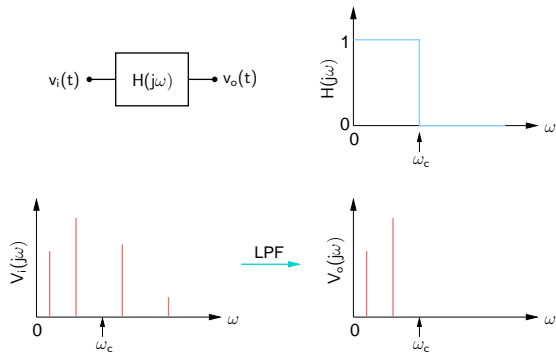
$$V_o(j\omega) = H(j\omega) V_i(j\omega) .$$

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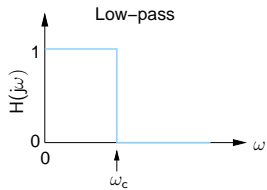
$$V_o(j\omega) = H(j\omega) V_i(j\omega).$$

All components with $\omega < \omega_c$ appear at the output without attenuation.

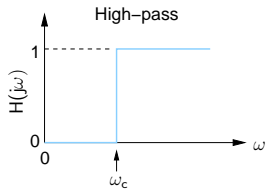
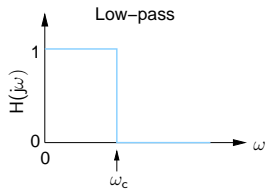
All components with $\omega > \omega_c$ get eliminated.

(Note that the ideal low-pass filter has $\angle H(j\omega) = 1$, i.e., $H(j\omega) = 1 + j0$.)

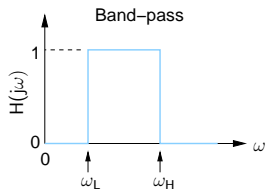
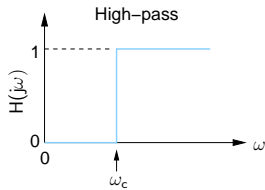
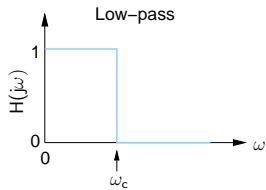
Ideal filters



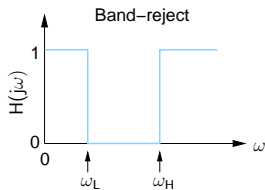
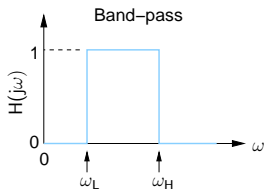
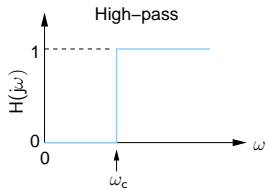
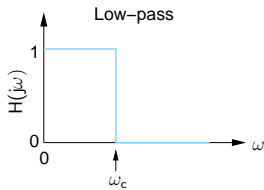
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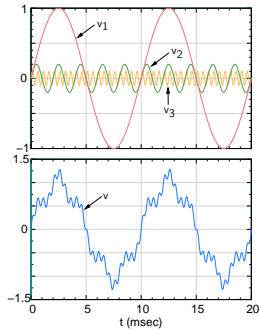
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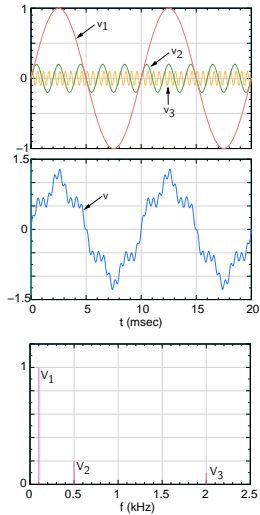
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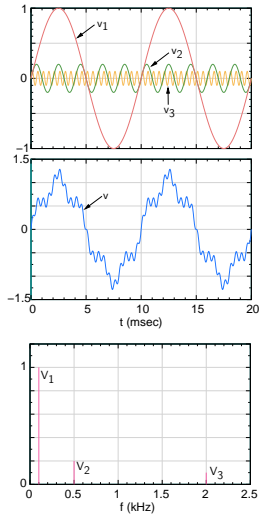
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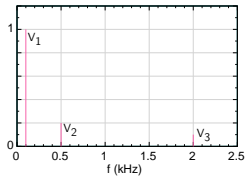
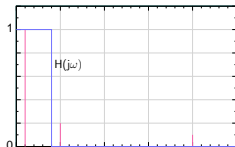
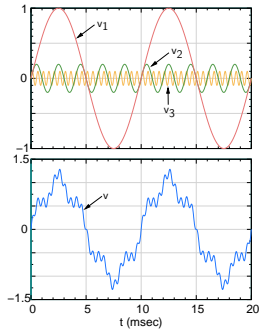


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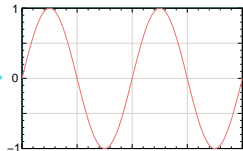
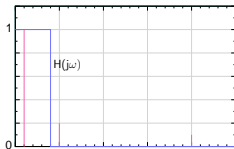
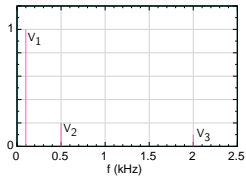
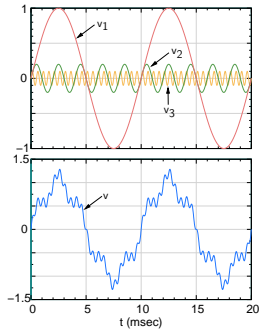
Let us see the effect of
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Ideal filters



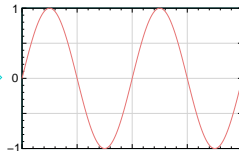
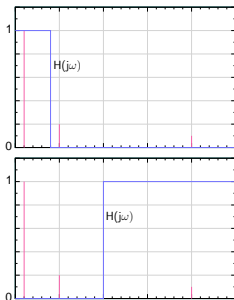
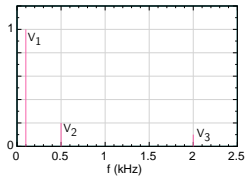
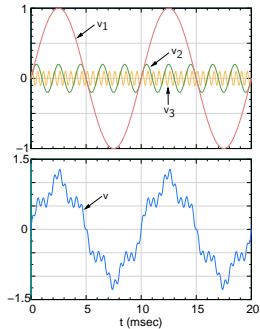
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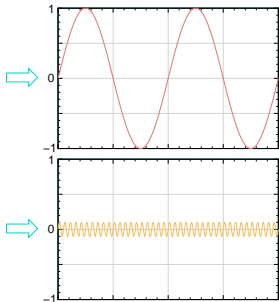
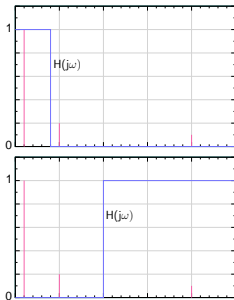
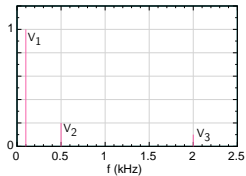
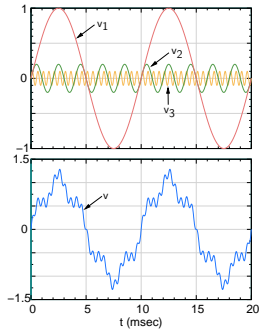
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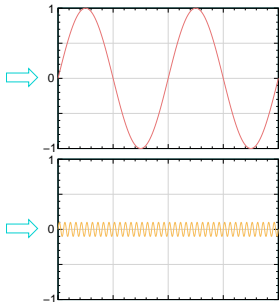
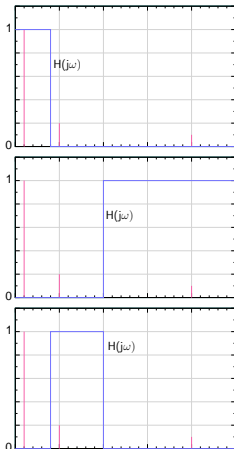
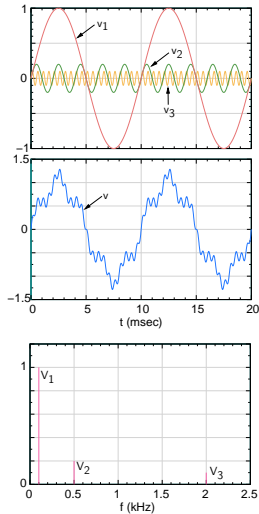
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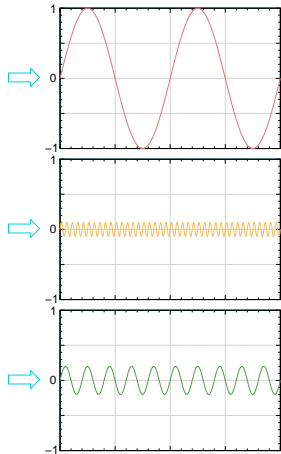
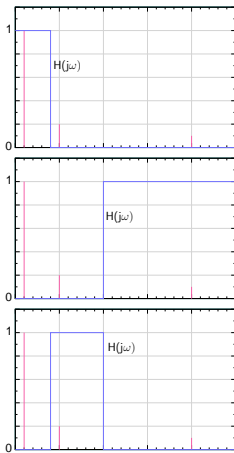
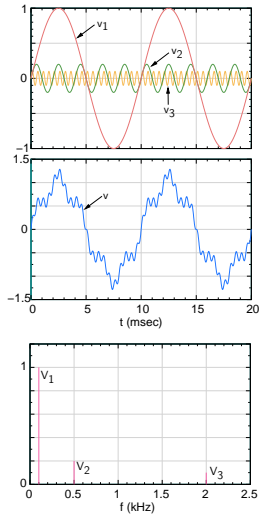
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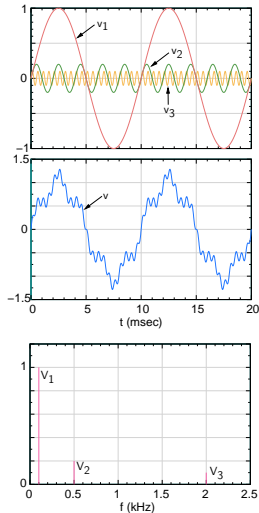
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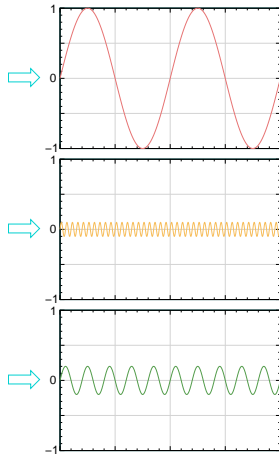
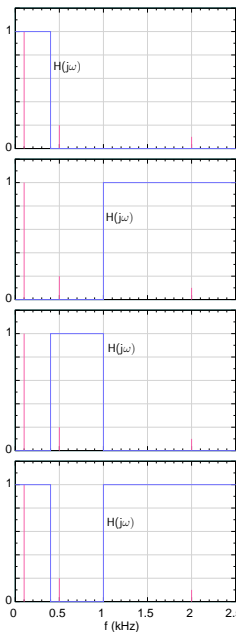


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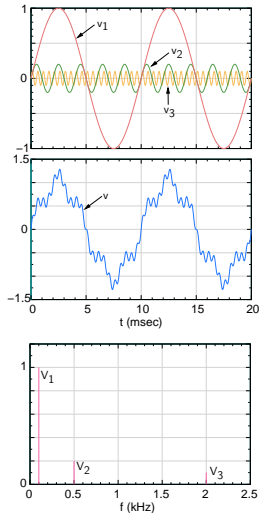
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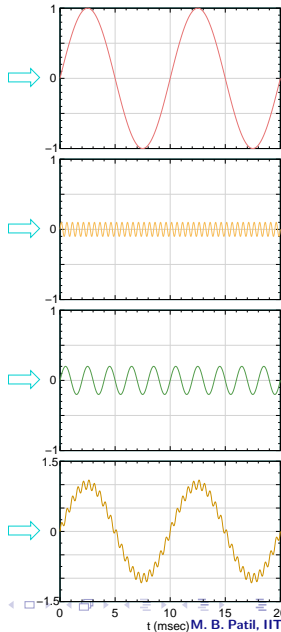
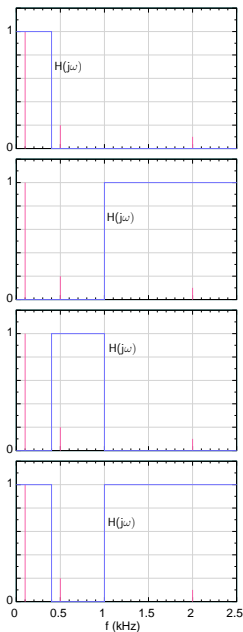
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Ideal filters



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- * In practical filter circuits, the ideal filter response is approximated with a suitable $H(j\omega)$ that can be obtained with circuit elements. For example,

$$H(s) = \frac{1}{a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

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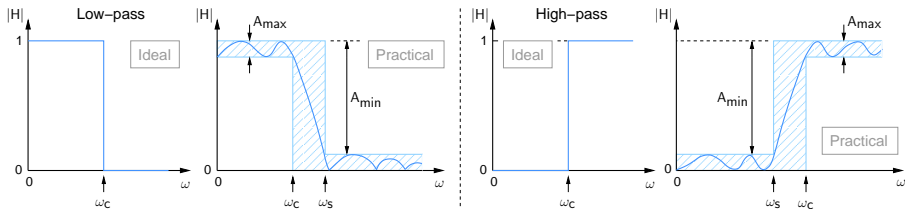
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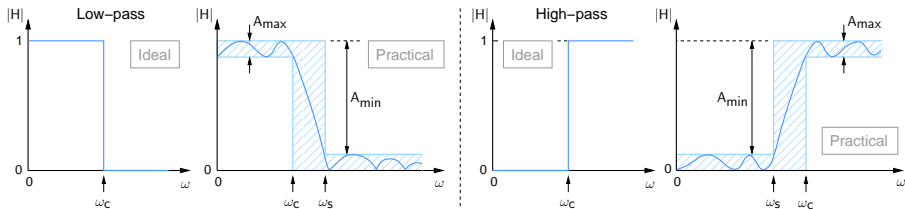
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- * Coefficients for these filters listed in filter handbooks. Also, programs for filter design are available on the internet.

Practical filters

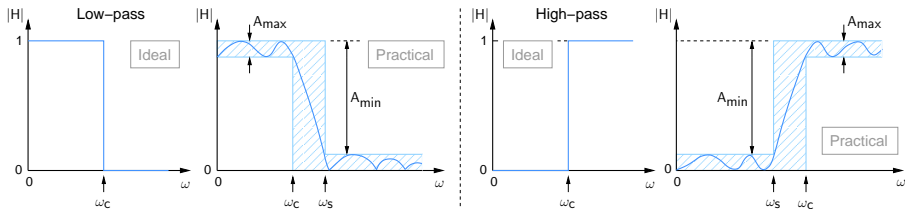


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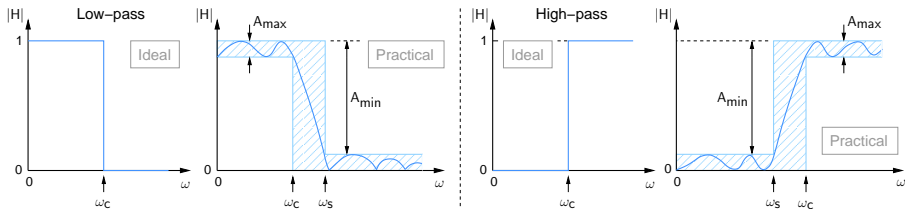
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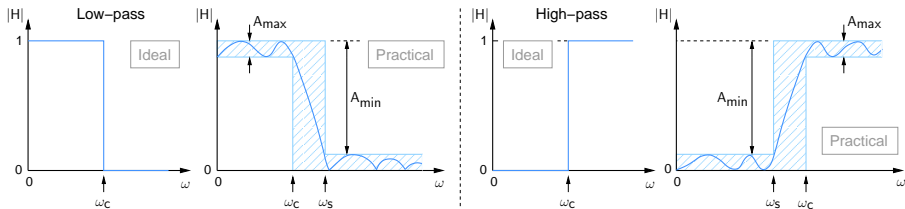
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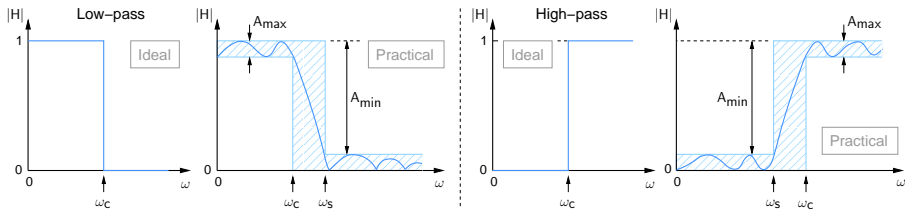
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- * ω_s/ω_c (for a low-pass filter): selectivity factor, a measure of the sharpness of the filter.
- * $\omega_c < \omega < \omega_s$: transition band.

Practical filters

For a low-pass filter, $H(s) = \frac{1}{\sum_{i=0}^n a_i (s/\omega_c)^i}$.

Coefficients (a_i) for various types of filters are tabulated in handbooks. We now look at $|H(j\omega)|$ for two commonly used filters.

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$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 C_n^2(\omega/\omega_c)}} \quad \text{where}$$

$$C_n(x) = \cos [n \cos^{-1}(x)] \quad \text{for } x \leq 1,$$

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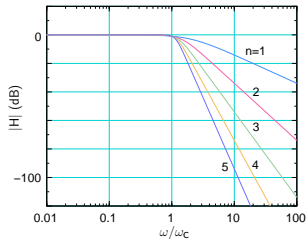
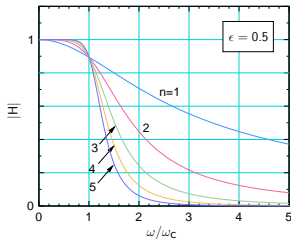
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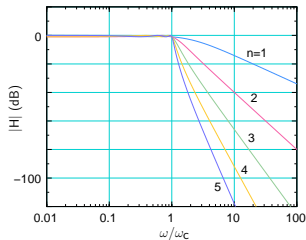
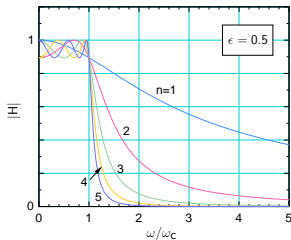
$H(s)$ for a high-pass filter can be obtained from $H(s)$ of the corresponding low-pass filter by $(s/\omega_c) \rightarrow (\omega_c/s)$.

Practical filters (low-pass)

Butterworth filters:

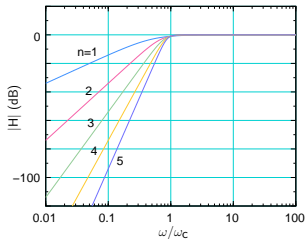
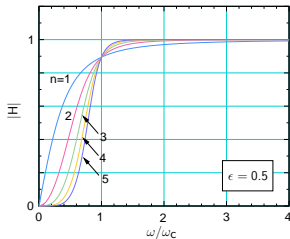


Chebyshev filters:

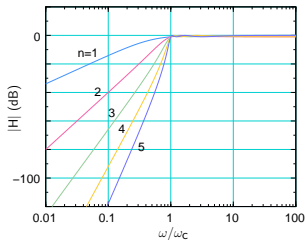
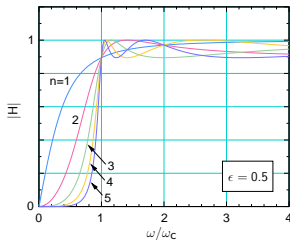


Practical filters (high-pass)

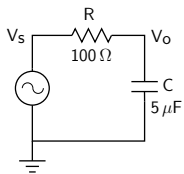
Butterworth filters:



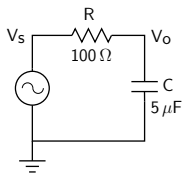
Chebyshev filters:



Passive filter example



Passive filter example

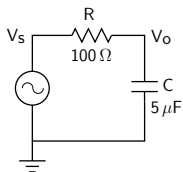


$$H(s) = \frac{(1/sC)}{R + (1/sC)} = \frac{1}{1 + (s/\omega_0)},$$

with $\omega_0 = 1/RC$.

(Low-pass filter)

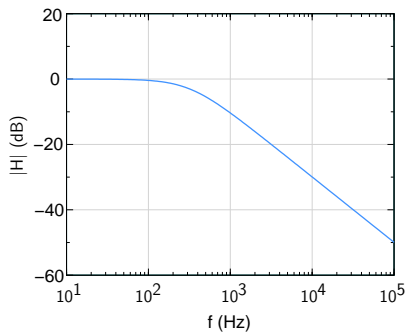
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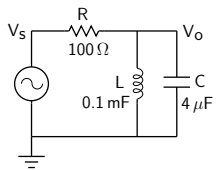
$$\text{with } \omega_0 = 1/RC.$$

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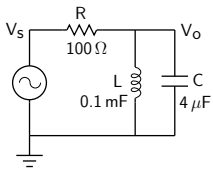


(SEQUEL file: ee101_rc_ac_2.sqproj)

Passive filter example



Passive filter example

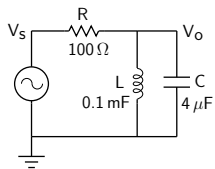


$$H(s) = \frac{(sL) \parallel (1/sC)}{R + (sL) \parallel (1/sC)} = \frac{s(L/R)}{1 + s(L/R) + s^2 LC}$$

with $\omega_0 = 1/\sqrt{LC}$.

(Band-pass filter)

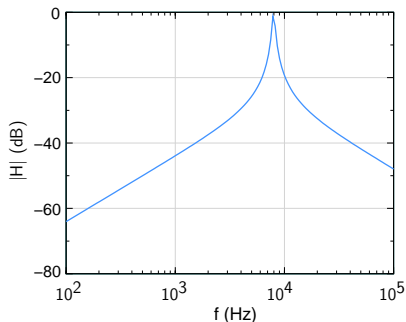
Passive filter example



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$$\text{with } \omega_0 = 1/\sqrt{LC}.$$

(Band-pass filter)



(SEQUEL file: ee101_lc_1.sqproj)

Op Amp filters (“Active” filters)

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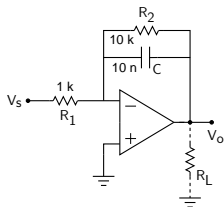
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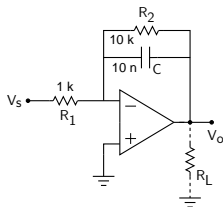
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Op Amp filters: example



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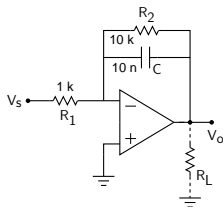


Op Amp filters are designed for Op Amp operation in the linear region
→ Our analysis of the inverting amplifier applies, and we get,

$$\mathbf{V_o} = -\frac{R_2 \parallel (1/sC)}{R_1} \mathbf{V_s} \quad (\mathbf{V_s} \text{ and } \mathbf{V_o} \text{ are phasors})$$

$$H(s) = -\frac{R_2}{R_1} \frac{1}{1 + sR_2C}$$

Op Amp filters: example



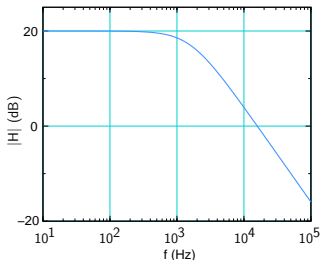
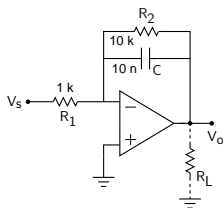
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This is a low-pass filter, with $\omega_0 = 1/R_2C$.

Op Amp filters: example



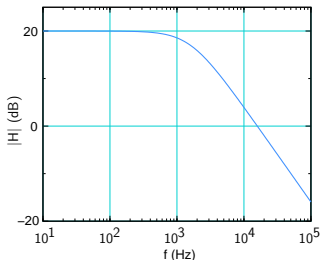
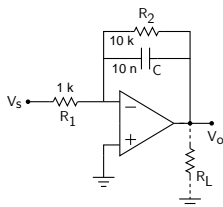
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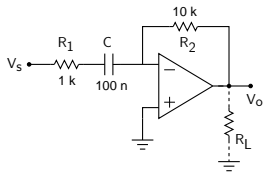
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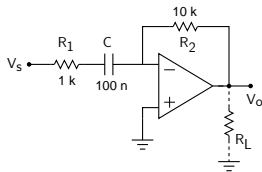
This is a low-pass filter, with $\omega_0 = 1/R_2C$.

(SEQUEL file: ee101_op_filter_1.sqproj)

Op Amp filters: example

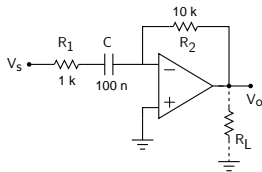


Op Amp filters: example



$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}.$$

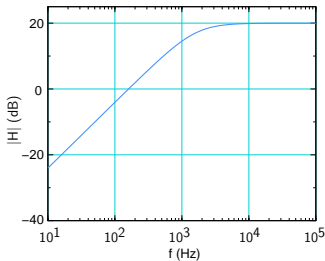
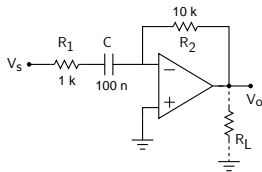
Op Amp filters: example



$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}.$$

This is a high-pass filter, with $\omega_0 = 1/R_1C$.

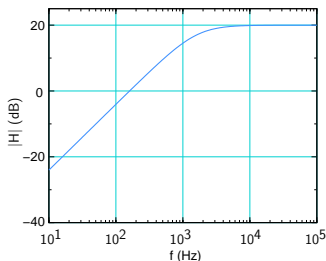
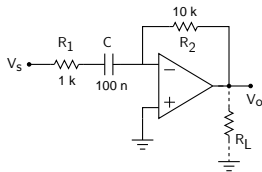
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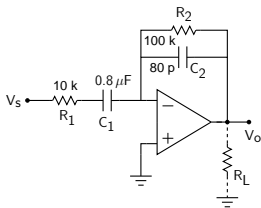


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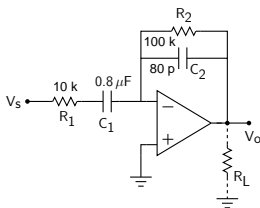
This is a high-pass filter, with $\omega_0 = 1/R_1C$.

(SEQUEL file: ee101_op_filter_2.sqproj)

Op Amp filters: example

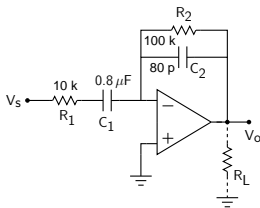


Op Amp filters: example



$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1} \frac{sR_1 C_1}{(1 + sR_1 C_1)(1 + sR_2 C_2)}.$$

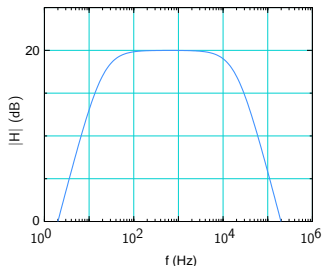
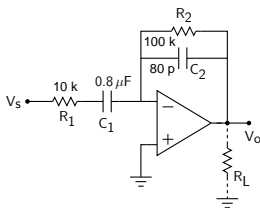
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This is a band-pass filter, with $\omega_L = 1/R_1 C_1$ and $\omega_H = 1/R_2 C_2$.

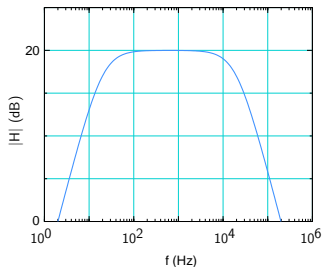
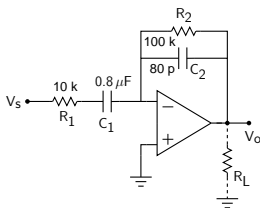
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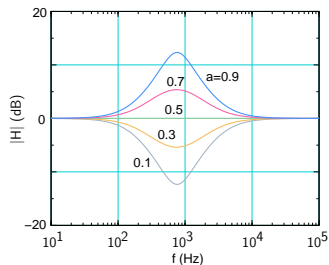
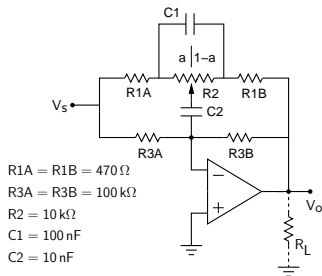


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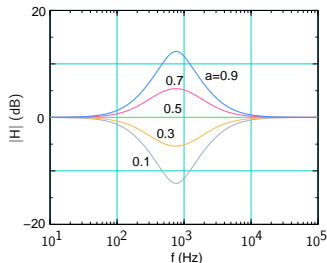
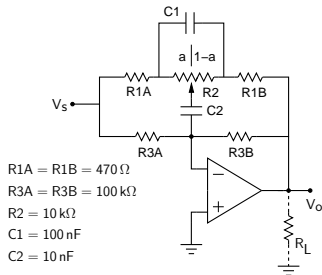
(SEQUEL file: ee101_op_filter_3.sqproj)

Graphic equalizer



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

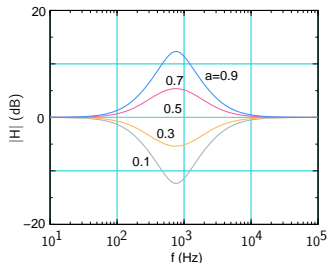
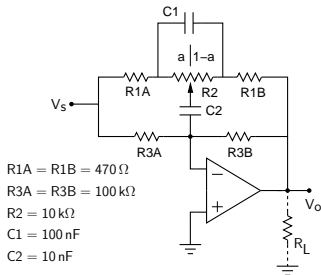
Graphic equalizer



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

- * Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.

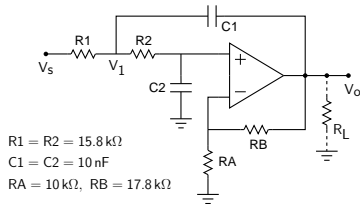
Graphic equalizer



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- * Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.
- * The circuit shown above represents one of the equalizer sections.
(SEQUEL file: ee101_op_filter_4.sqproj)

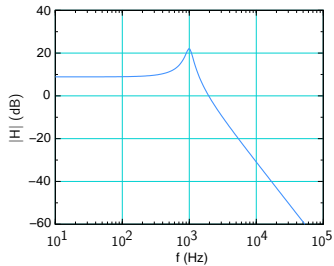
Sallen-Key filter example (2nd order, low-pass)



$$R_1 = R_2 = 15.8 \text{ k}\Omega$$

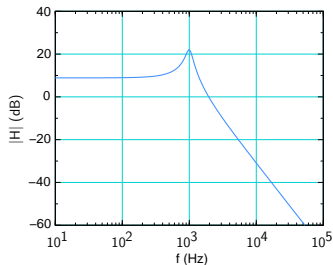
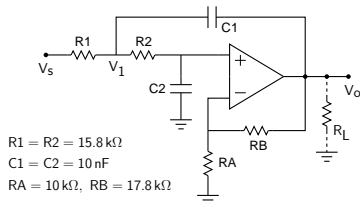
$$C_1 = C_2 = 10 \text{ nF}$$

$$R_A = 10 \text{ k}\Omega, R_B = 17.8 \text{ k}\Omega$$



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

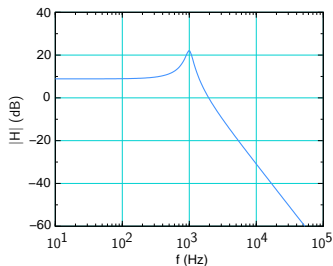
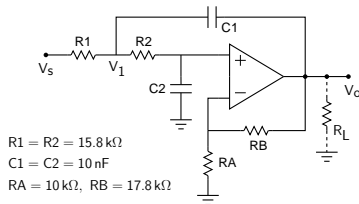
Sallen-Key filter example (2nd order, low-pass)



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o / K.$$

Sallen-Key filter example (2nd order, low-pass)

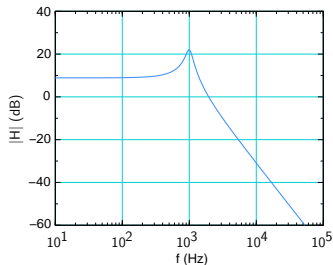
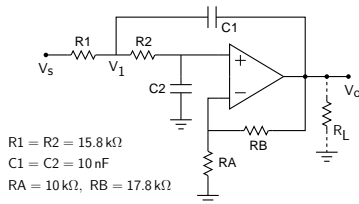


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o / K.$$

$$\text{Also, } V_+ = \frac{(1/sC_2)}{R_2 + (1/sC_2)} V_1 = \frac{1}{1 + sR_2C_2} V_1.$$

Sallen-Key filter example (2nd order, low-pass)



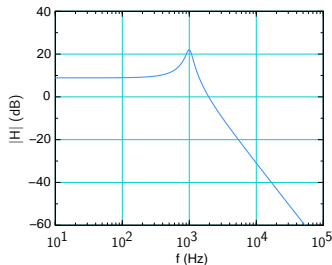
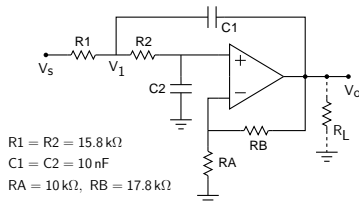
(Ref.: S. Franco, "Design with Op Amps and analog ICs")

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$$\text{Also, } V_+ = \frac{(1/sC_2)}{R_2 + (1/sC_2)} V_1 = \frac{1}{1 + sR_2C_2} V_1.$$

$$\text{KCL at } V_1 \rightarrow \frac{1}{R_1}(V_s - V_1) + sC_1(V_o - V_1) + \frac{1}{R_2}(V_+ - V_1) = 0.$$

Sallen-Key filter example (2nd order, low-pass)



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

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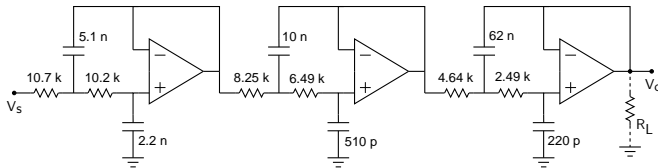
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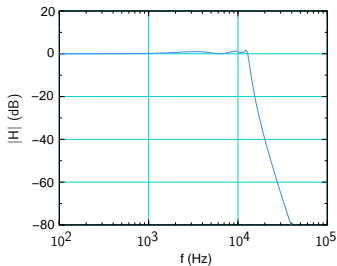
$$\text{Combining the above equations, } H(s) = \frac{K}{1 + s[(R_1 + R_2)C_2 + (1 - K)R_1C_1] + s^2R_1C_1R_2C_2}.$$

(SEQUEL file: ee101_op_filter.5.sqproj)

Sixth-order Chebyshev low-pass filter (cascade design)

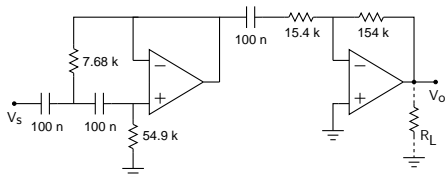


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

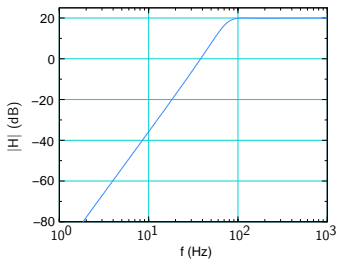


SEQUEL file: ee101_op_filter_6.sqproj

Third-order Chebyshev high-pass filter

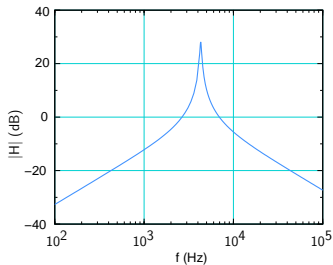
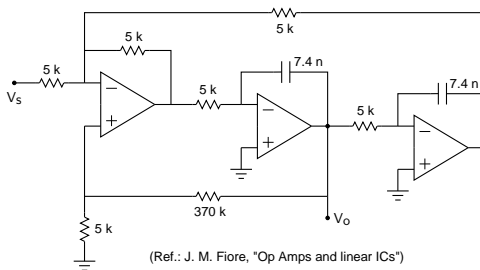


(Ref.: S. Franco, "Design with Op Amps and analog ICs")



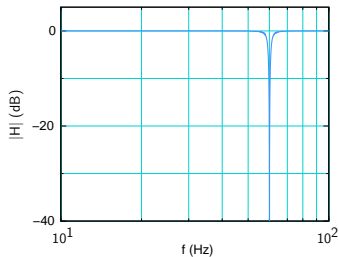
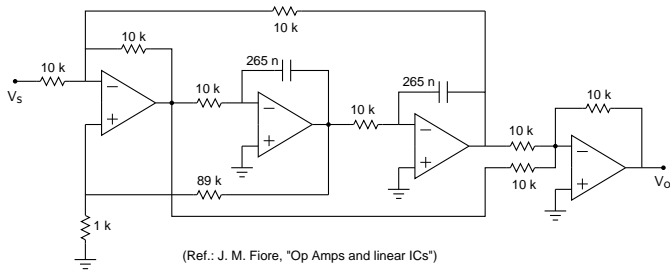
SEQUEL file: ee101_op_filter_7.sqproj

Band-pass filter example



SEQUEL file: ee101_op_filter_8.sqproj

Notch filter example



SEQUEL file: ee101_op_filter_9.sqproj