EE101: Op Amp circuits (Part 3)

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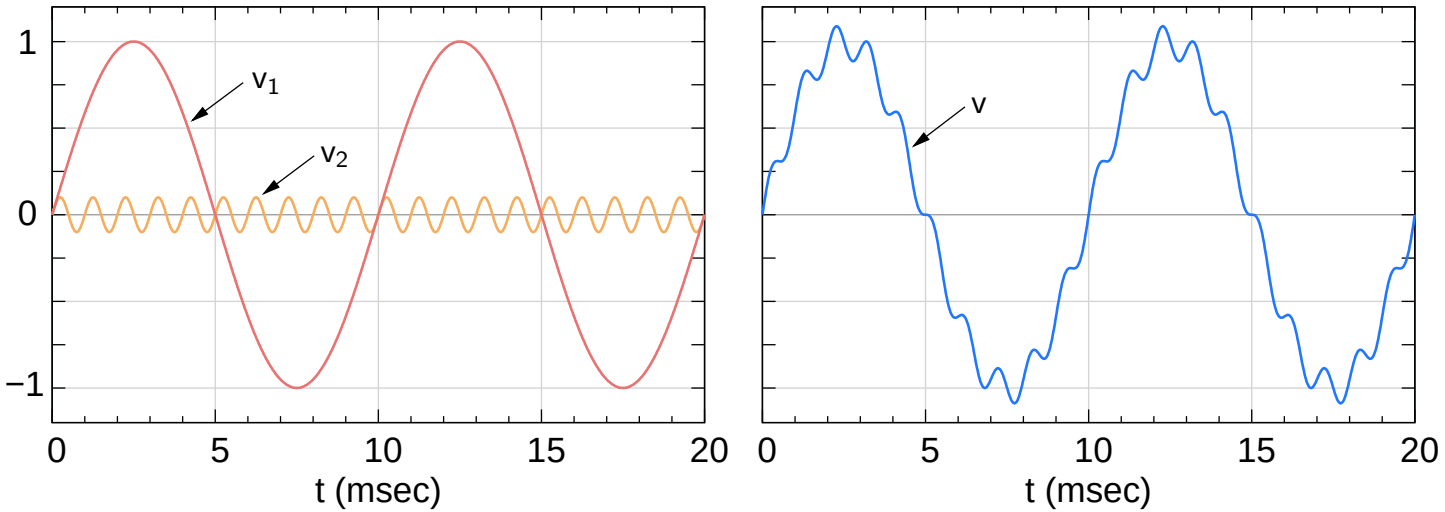
Introduction to filters

Consider $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$.



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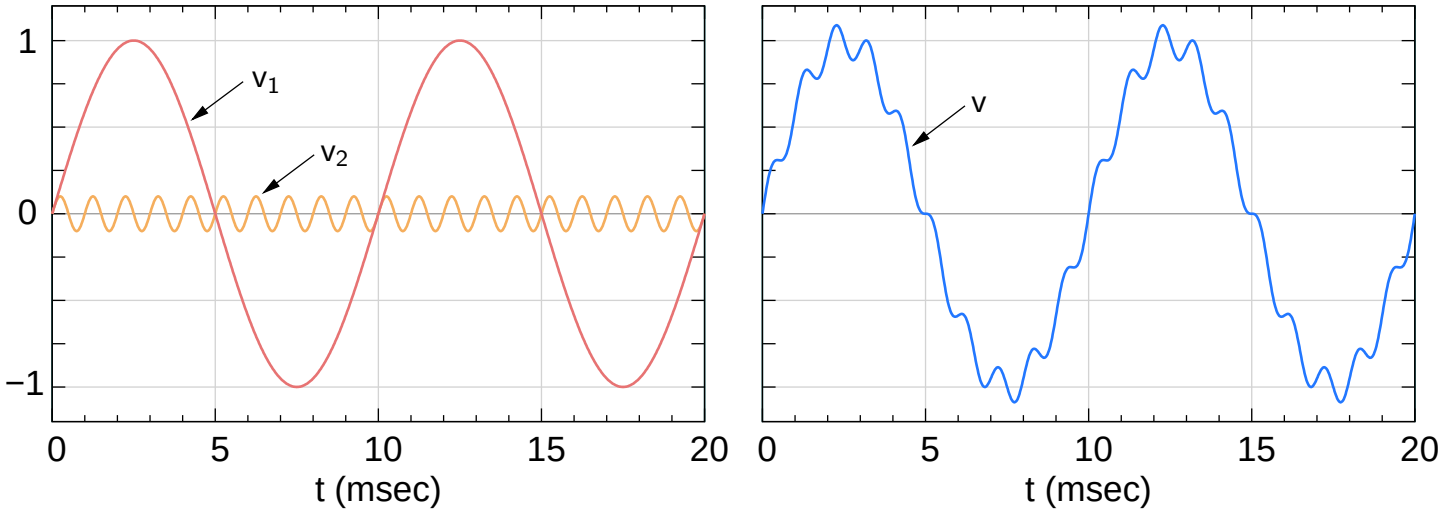
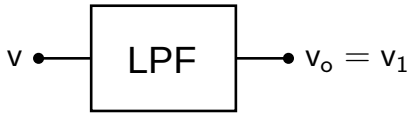
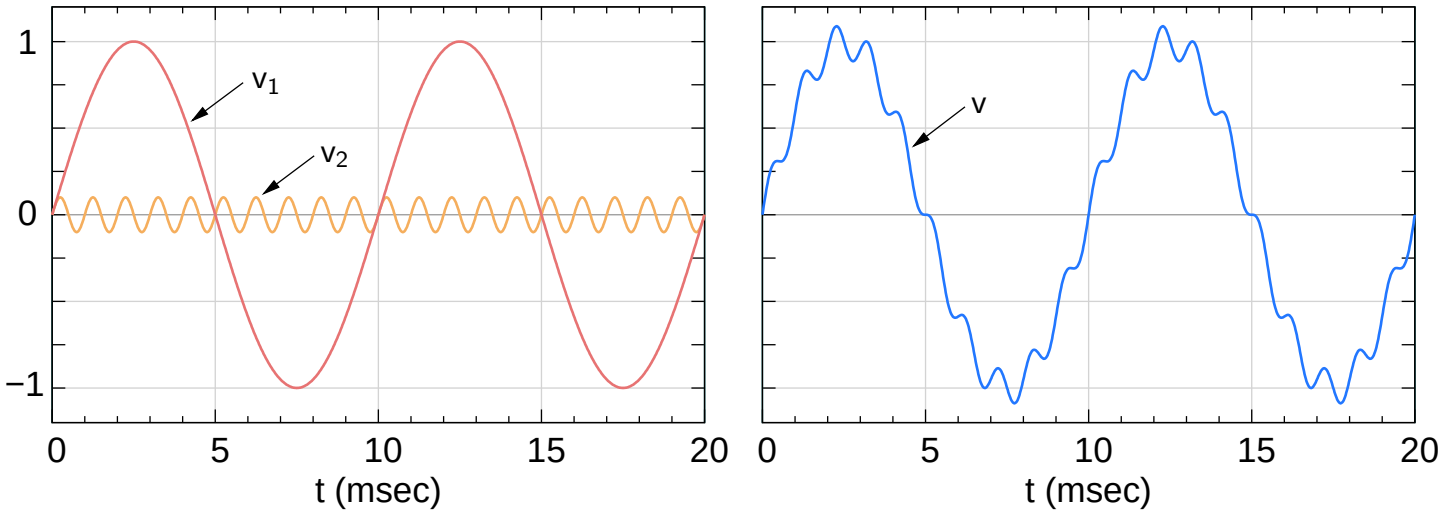
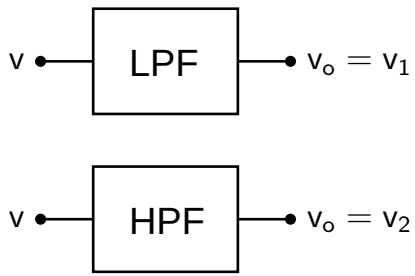




A low-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the low-frequency component $v_1(t)$ and remove the high-frequency component $v_2(t)$.

Consider $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$.

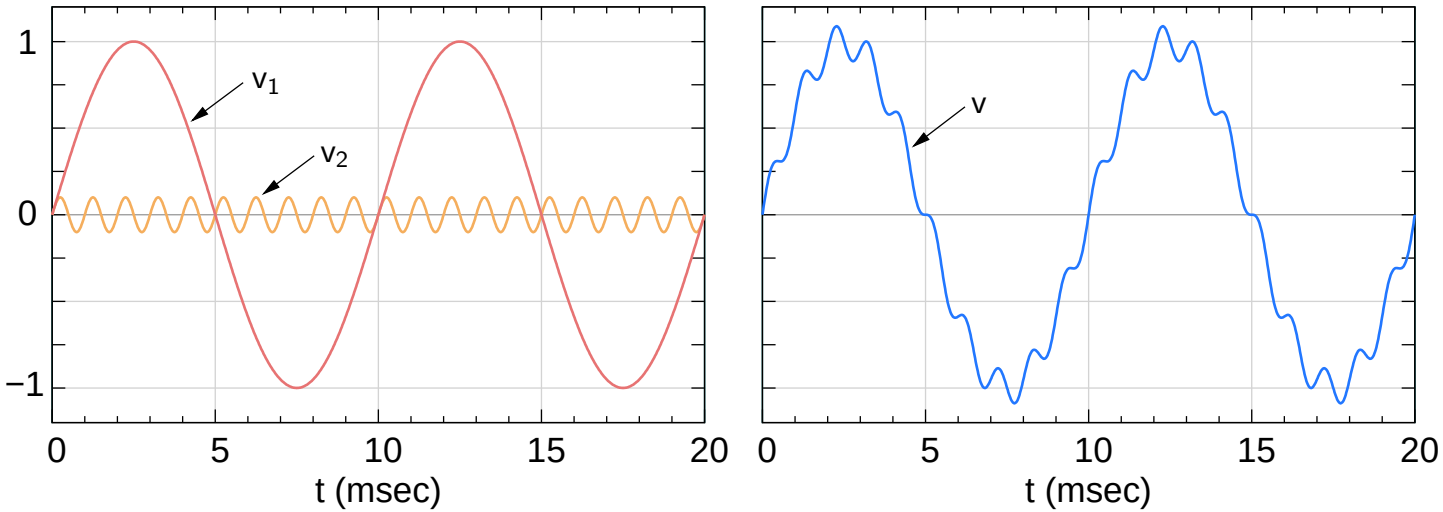


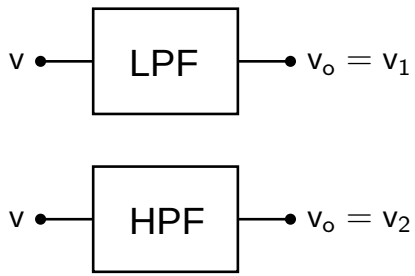


A low-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the low-frequency component $v_1(t)$ and remove the high-frequency component $v_2(t)$.

A high-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the high-frequency component $v_2(t)$ and remove the low-frequency component $v_1(t)$.

Consider $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$.



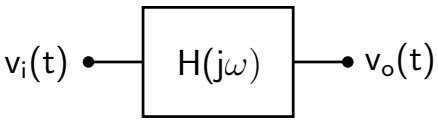


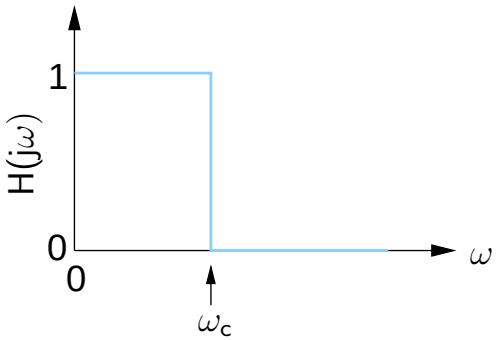
A low-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the low-frequency component $v_1(t)$ and remove the high-frequency component $v_2(t)$.

A high-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the high-frequency component $v_2(t)$ and remove the low-frequency component $v_1(t)$.

There are some other types of filters, as we will see.

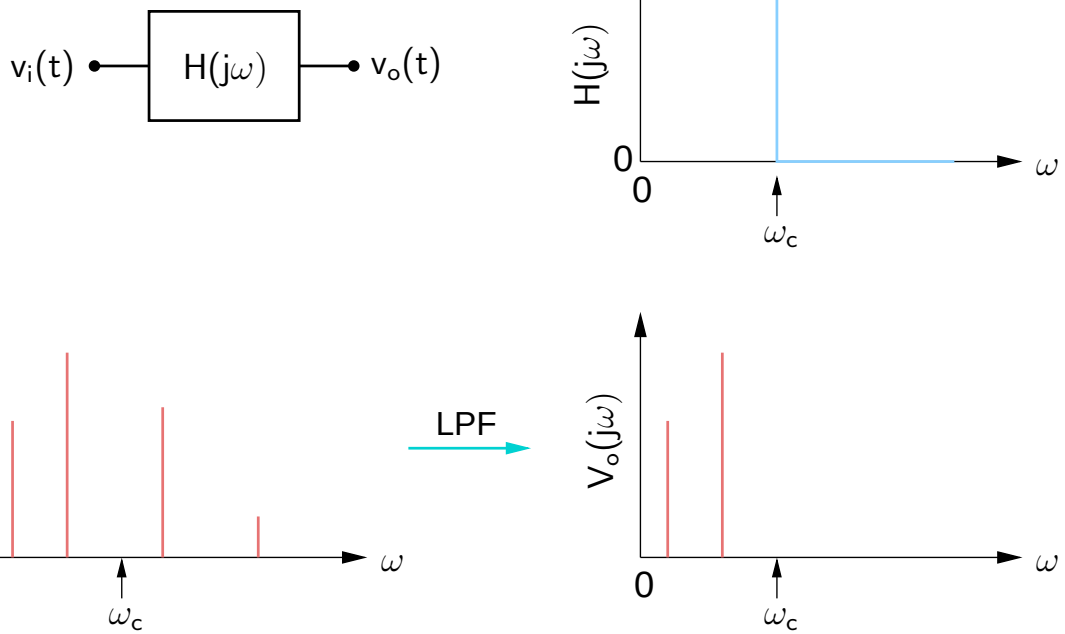
Ideal low-pass filter





$$V_o(j\omega) = H(j\omega)\, V_i(j\omega)\,.$$

Ideal low-pass filter



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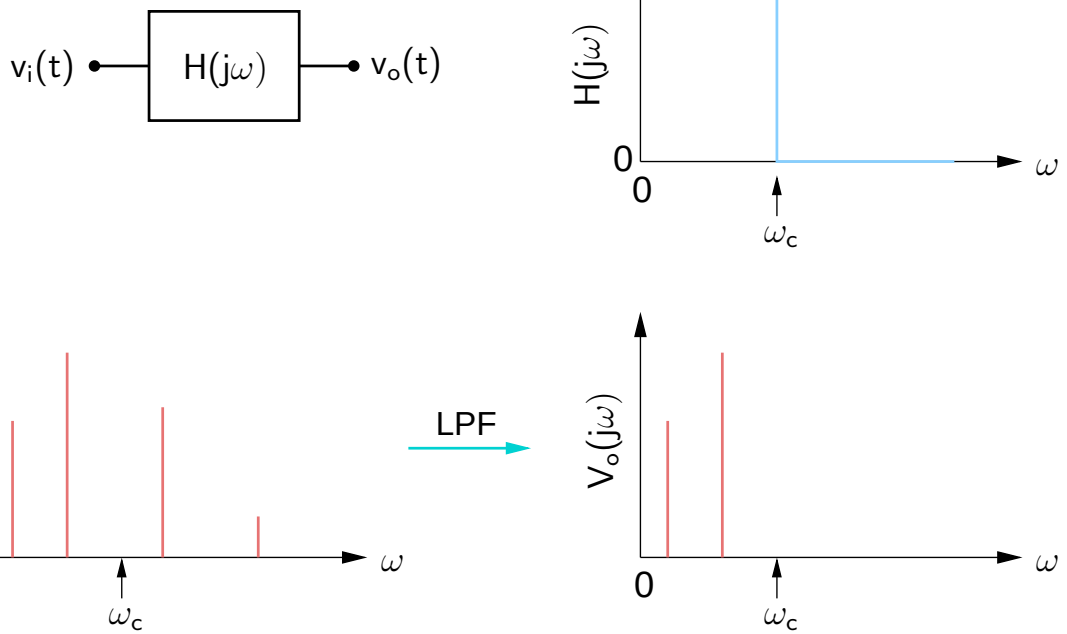
Ideal low-pass filter



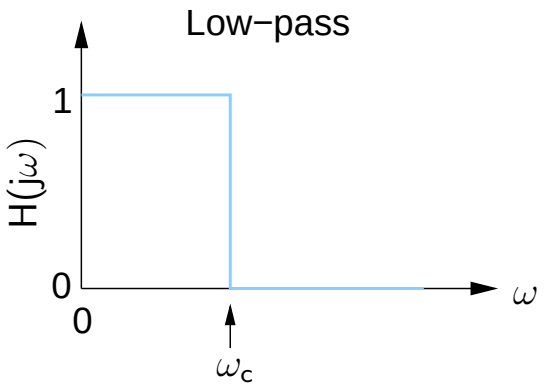
$V_o(j\omega) = H(j\omega)\, V_i(j\omega)$.

All components with $\omega < \omega_c$ appear at the output without attenuation.

All components with $\omega > \omega_c$ get eliminated.

(Note that the ideal low-pass filter has $\angle H(j\omega) = 1$, i.e., $H(j\omega) = 1 + j0$.)

Ideal filters



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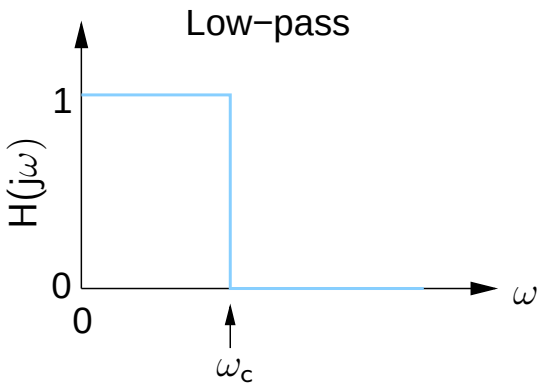

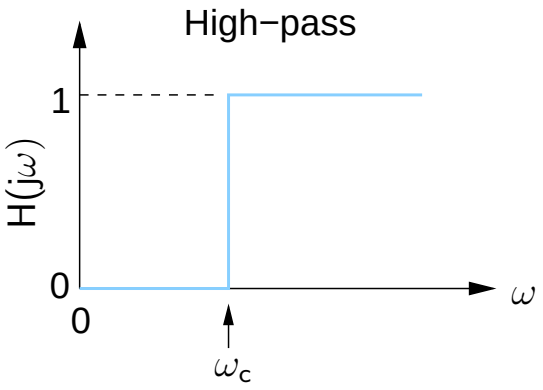

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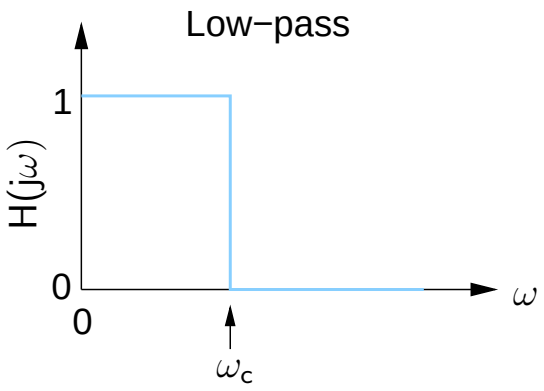

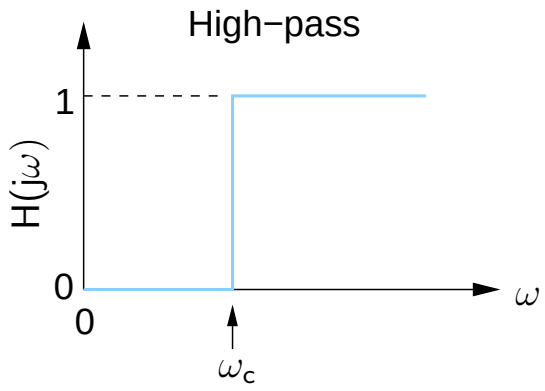

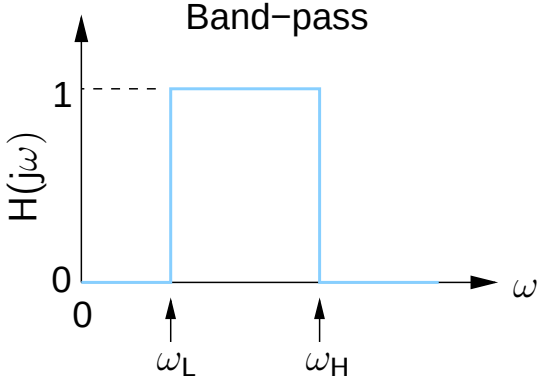

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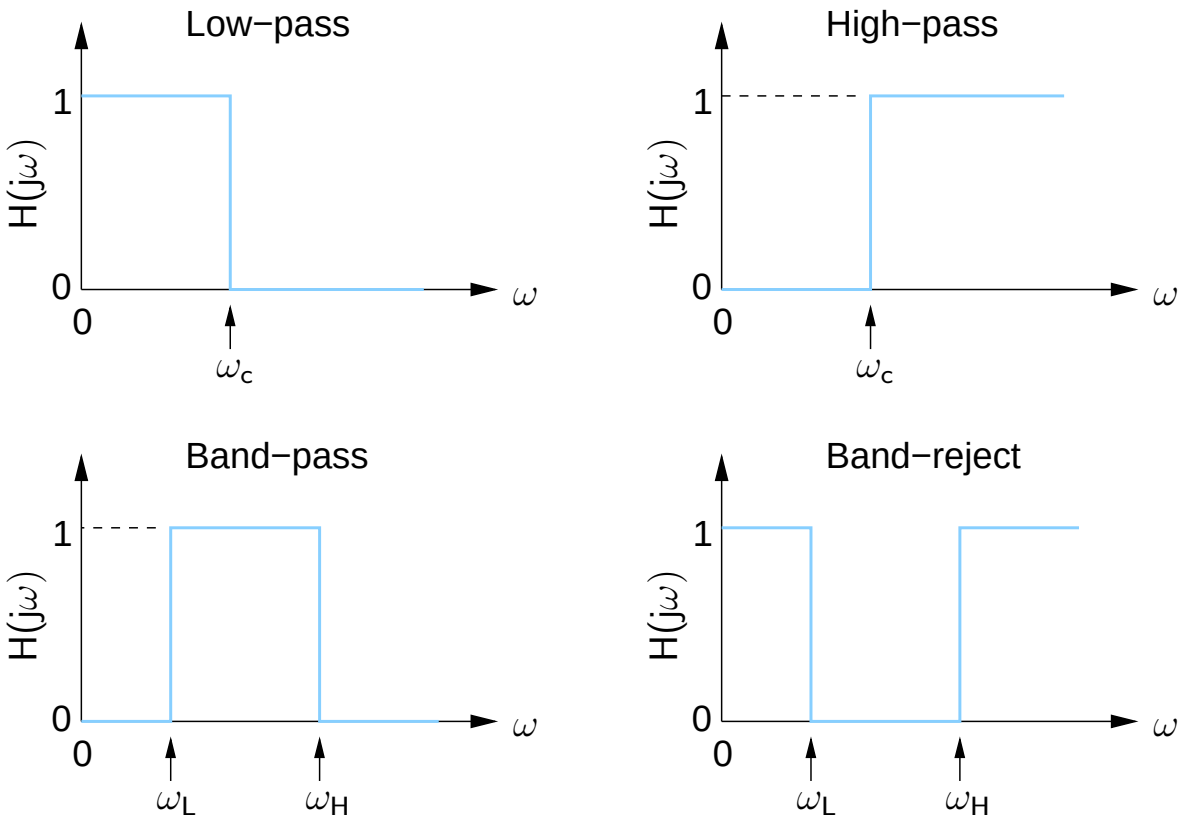

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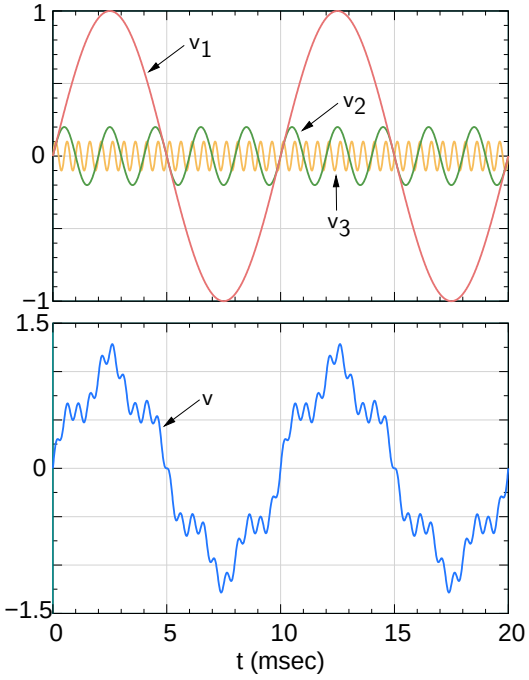

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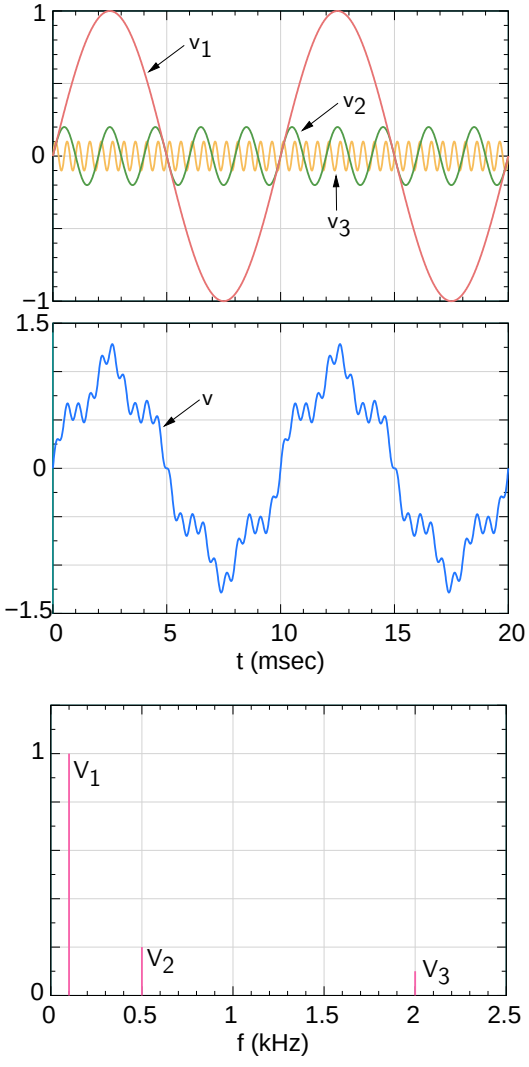

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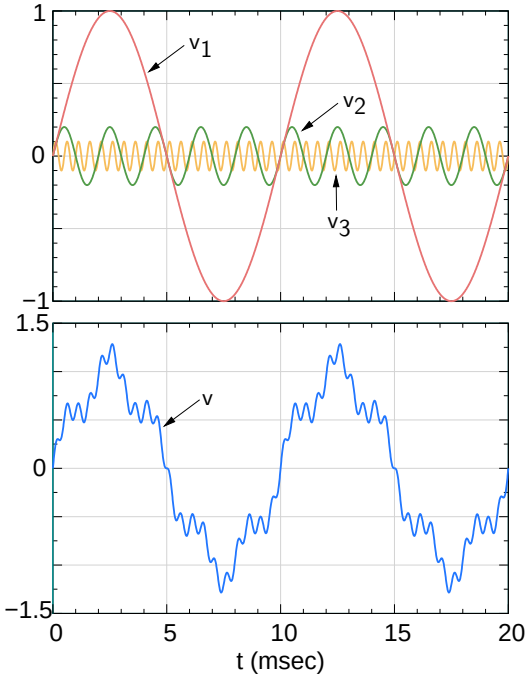

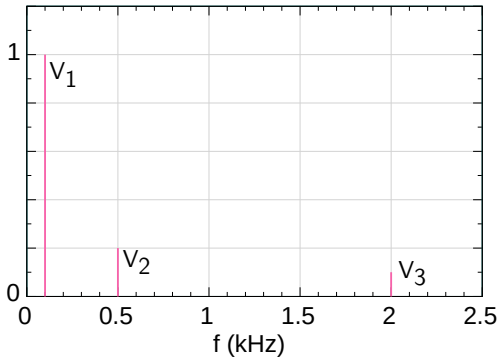

Let us see the effect of
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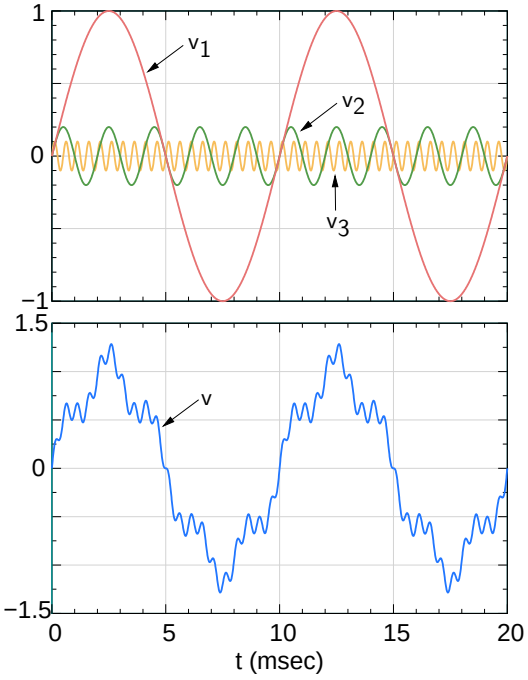

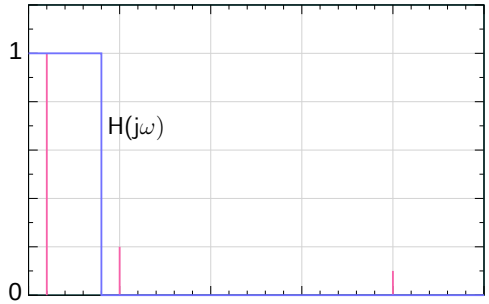

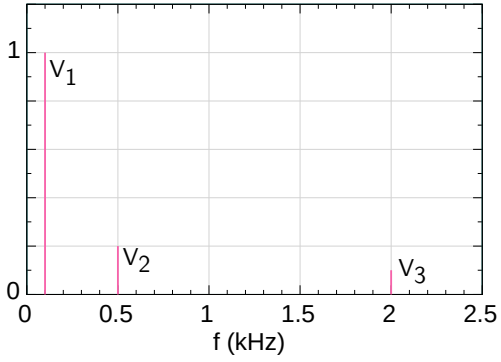

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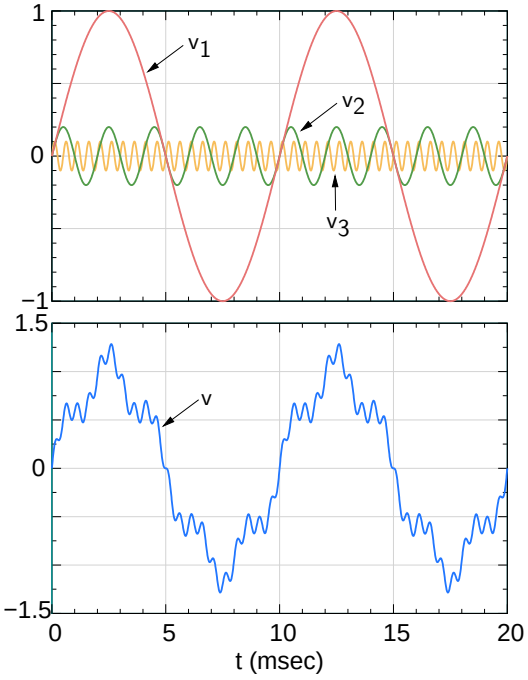

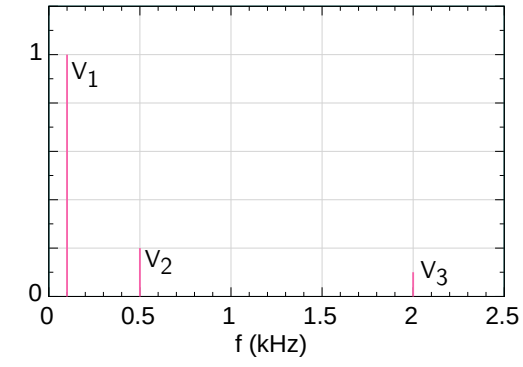

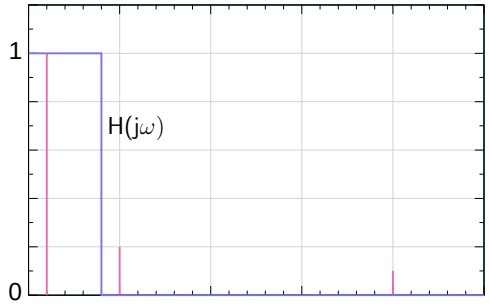

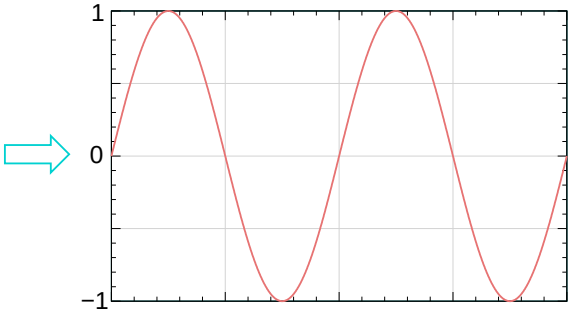

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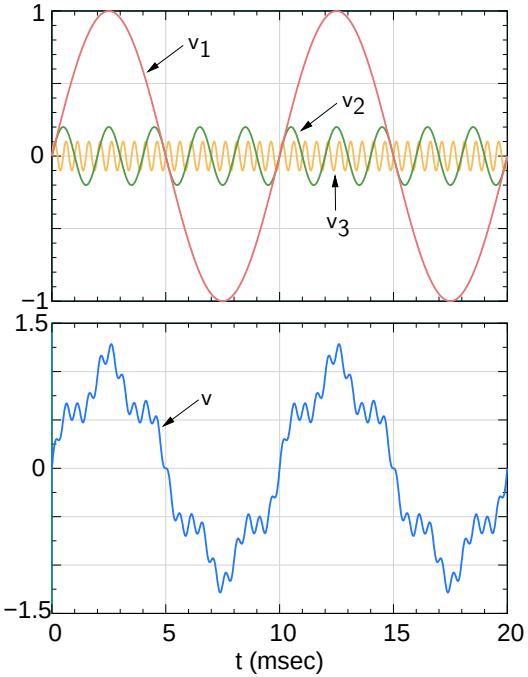

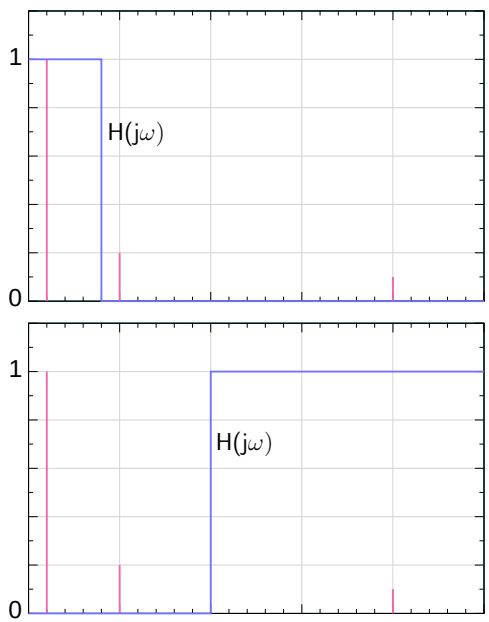

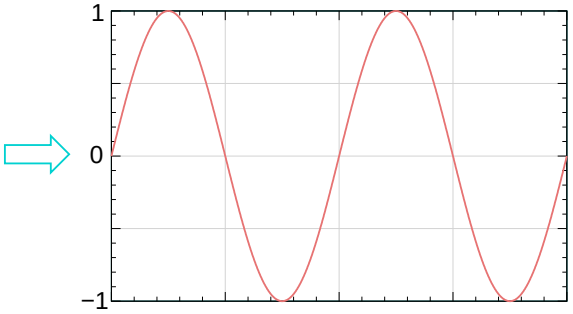

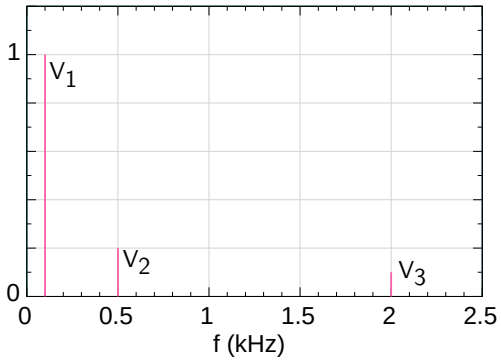

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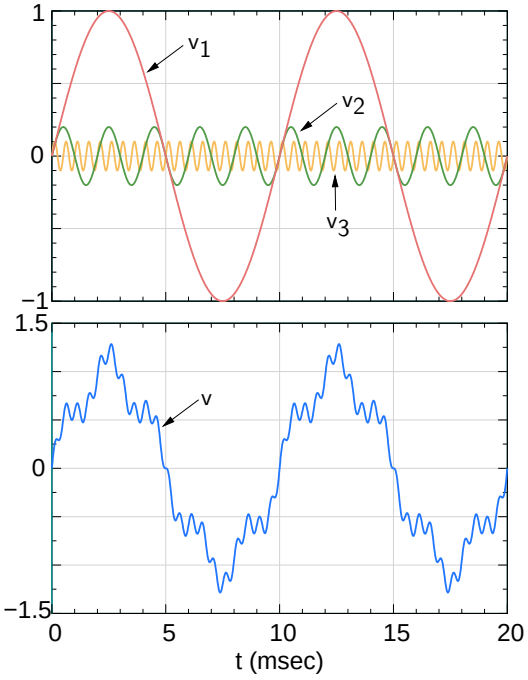

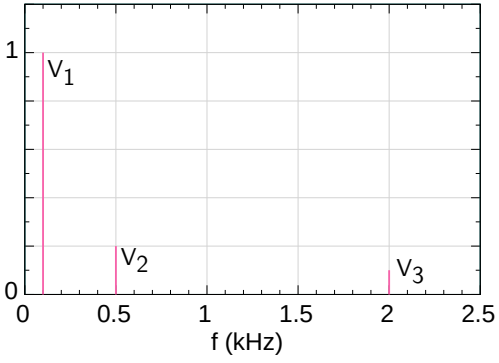

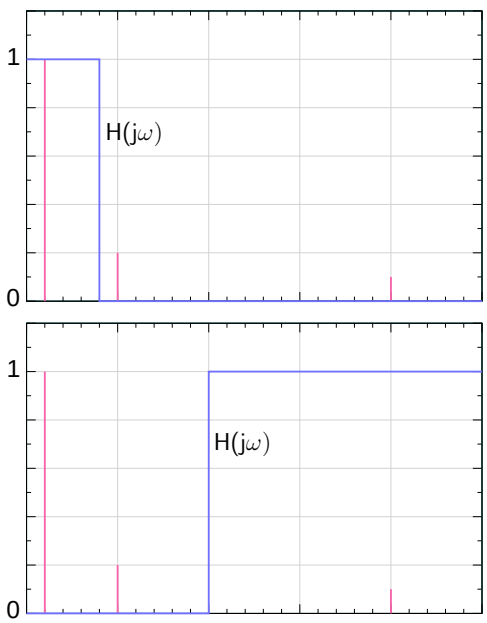

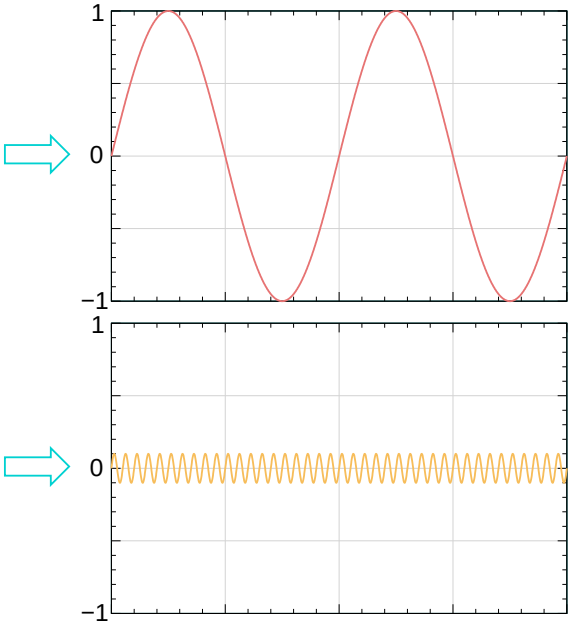

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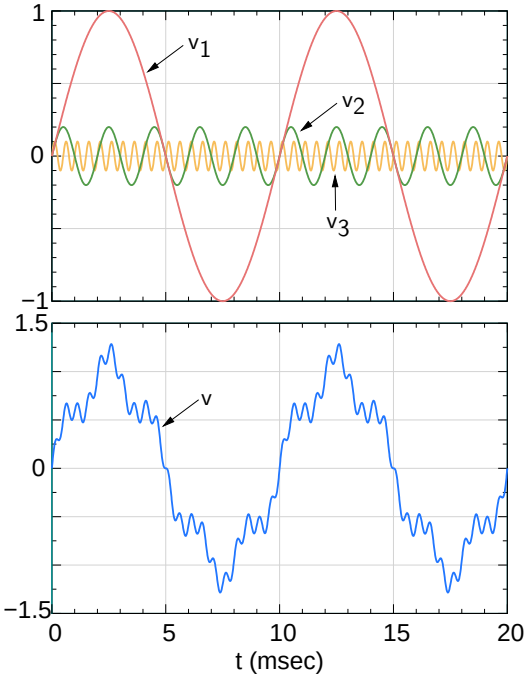

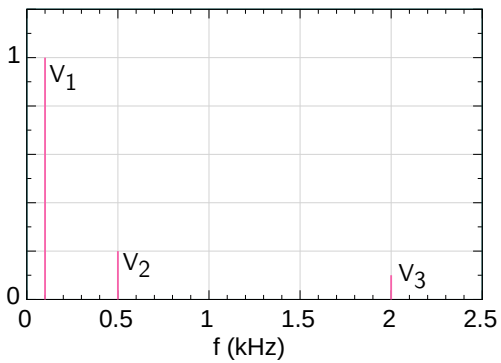

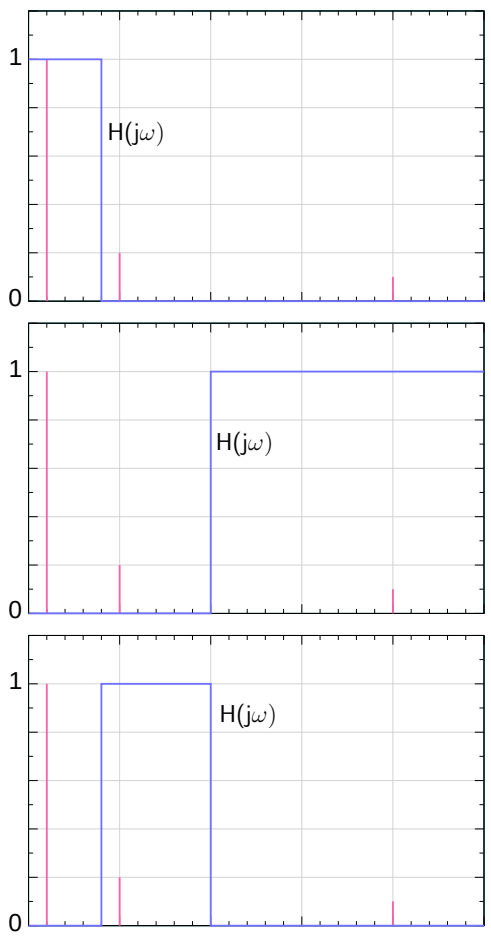

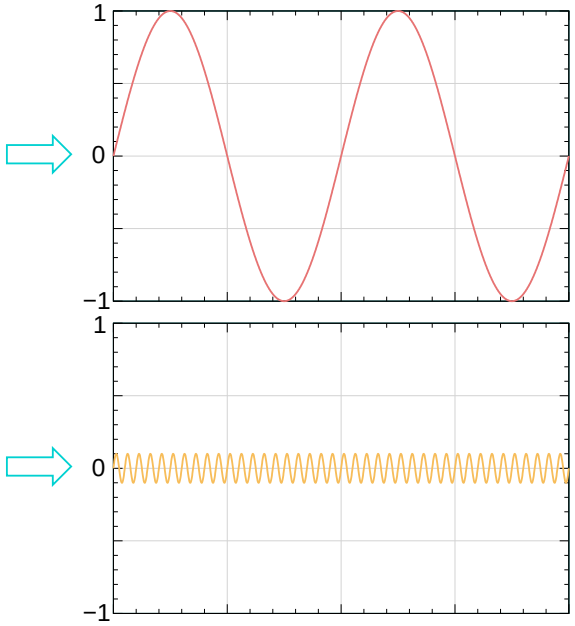

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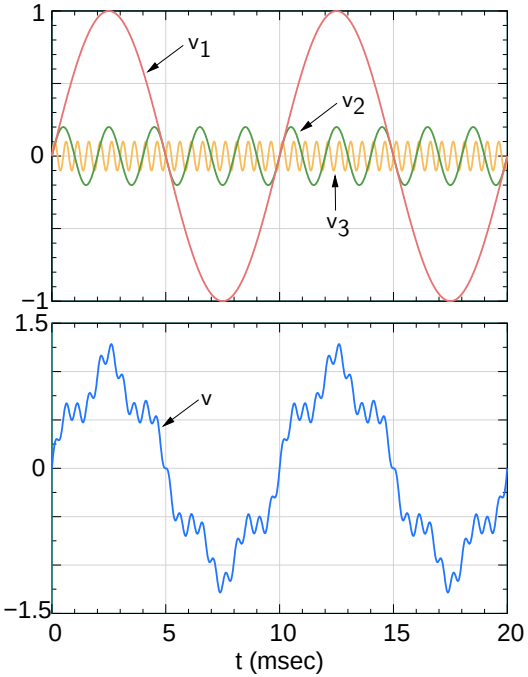

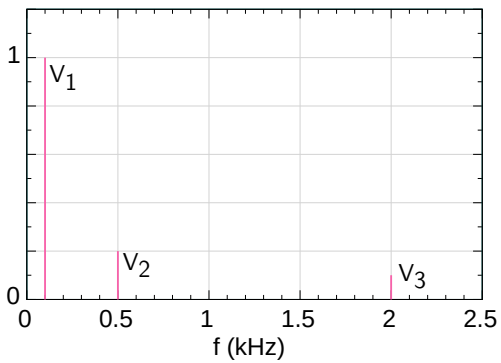

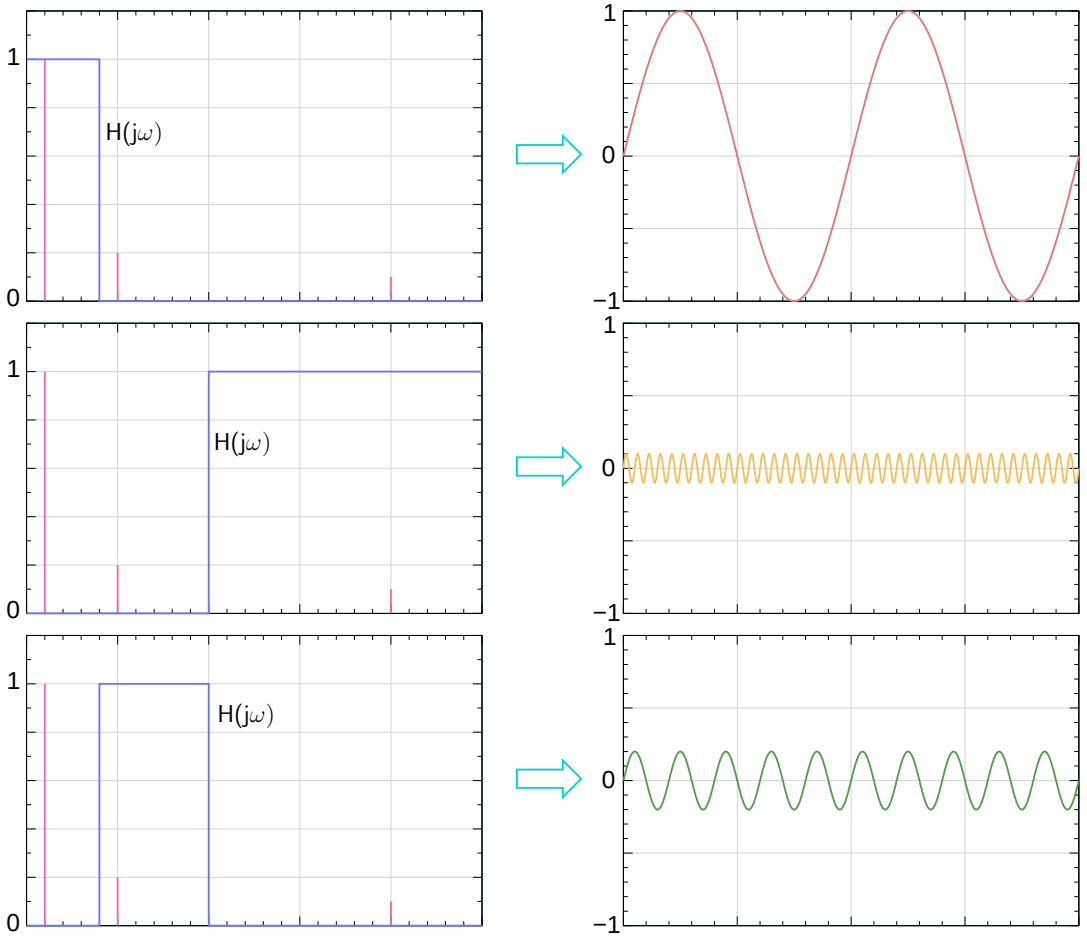

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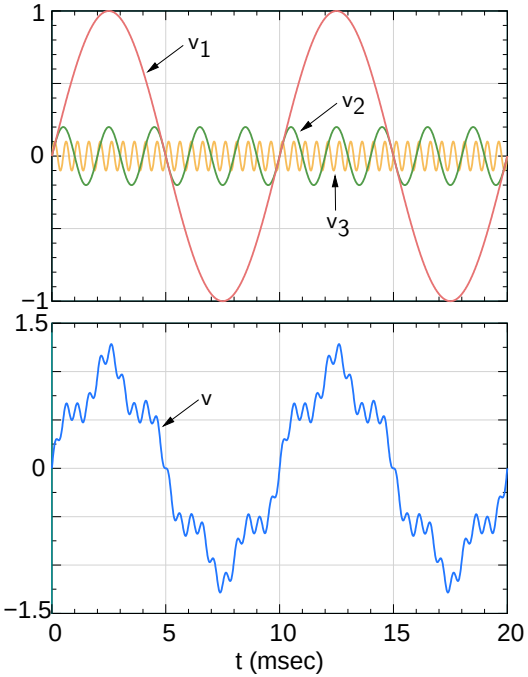

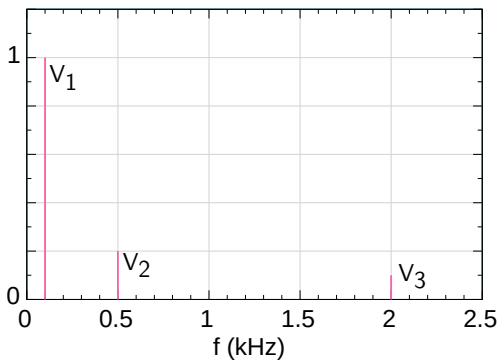

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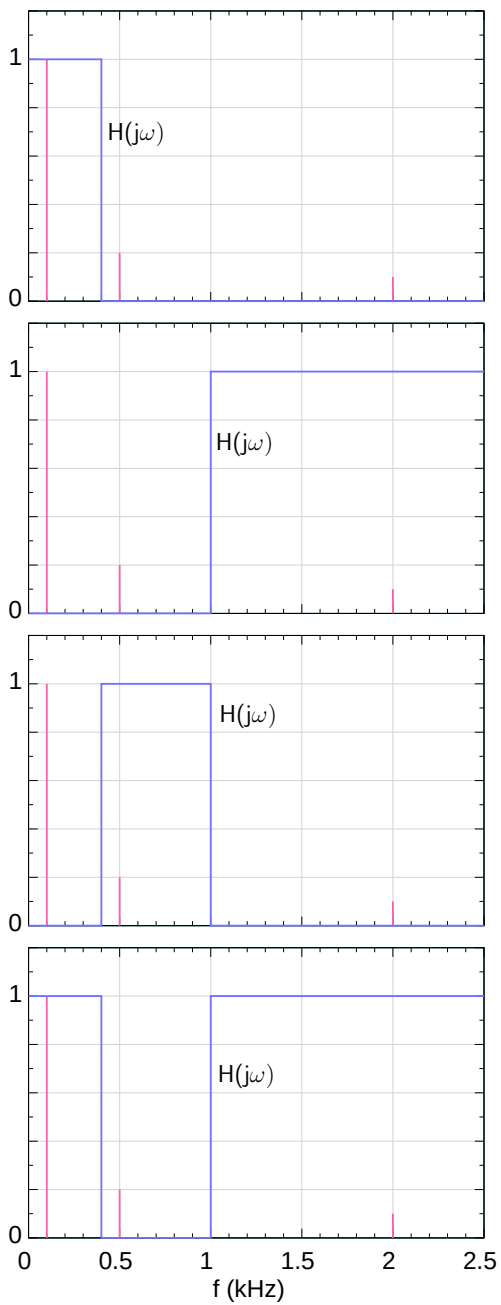

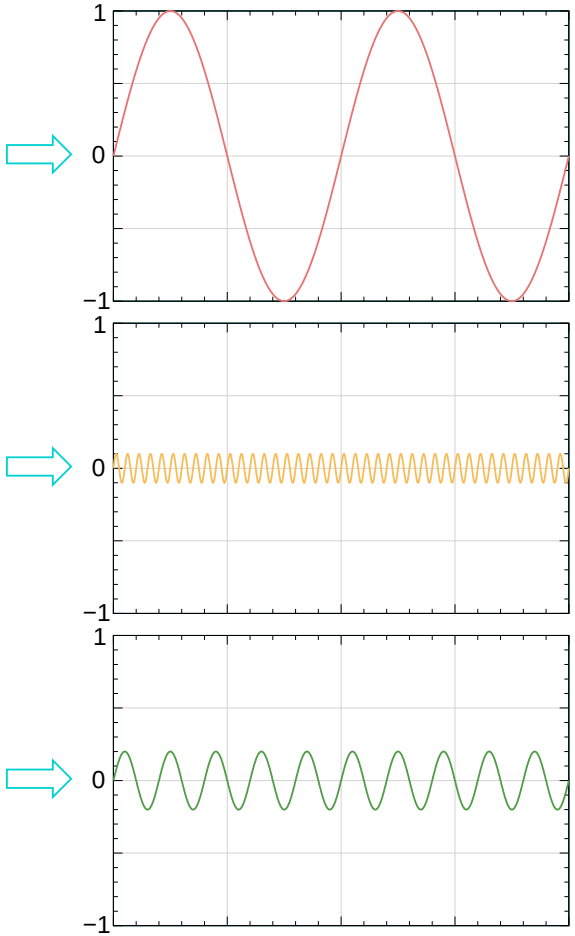

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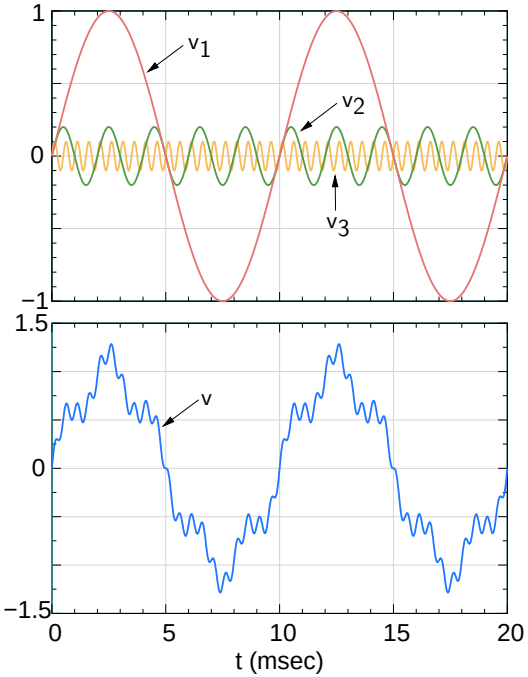

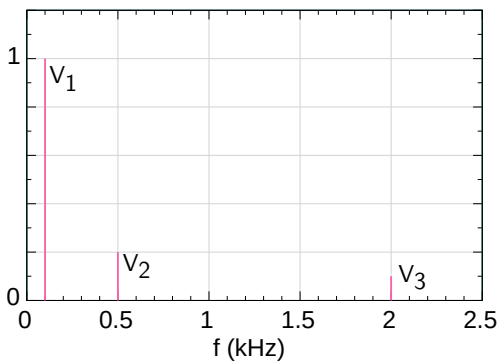

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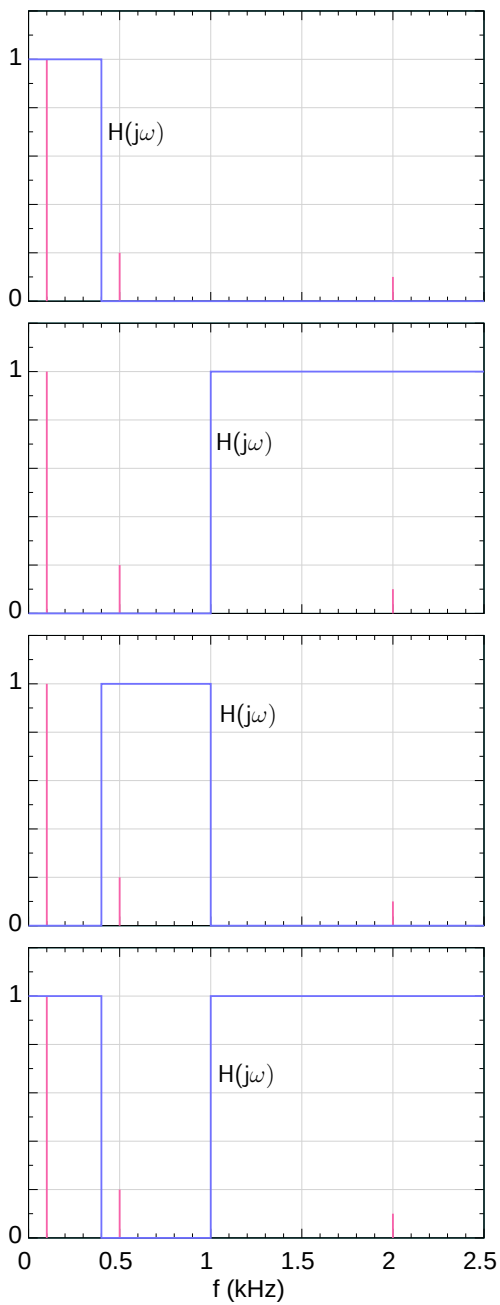

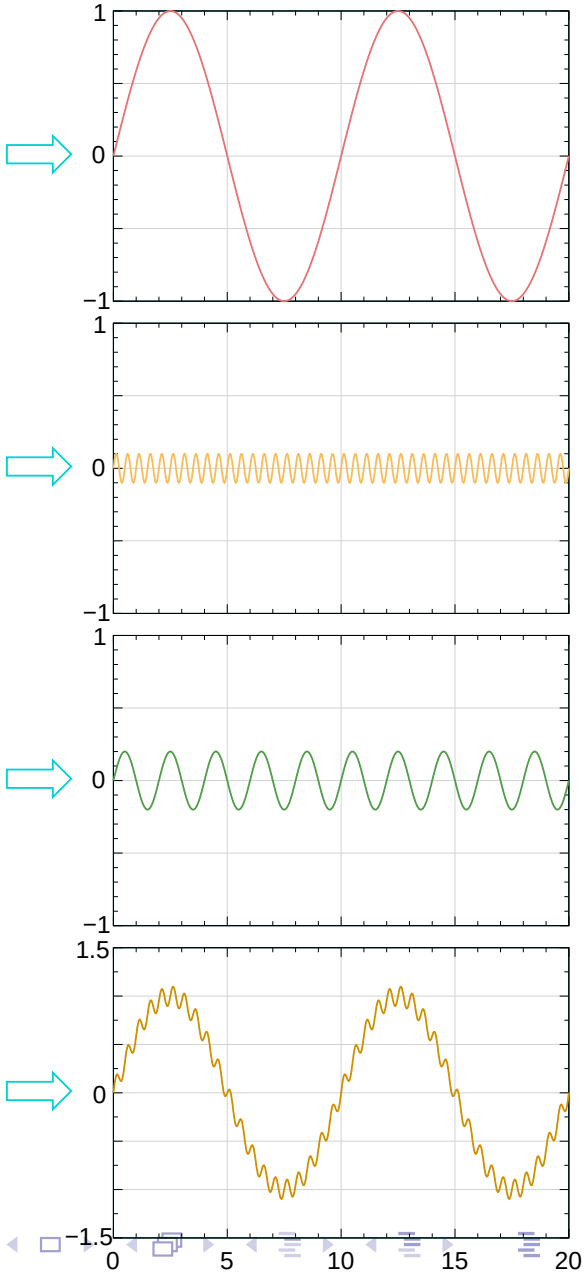

Practical filter circuits

* In practical filter circuits, the ideal filter response is approximated with a suitable $H(j\omega)$ that can be obtained with circuit elements. For example,

$$H(s) = \frac{1}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

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* Coefficients for these filters listed in filter handbooks. Also, programs for filter design are available on the internet.

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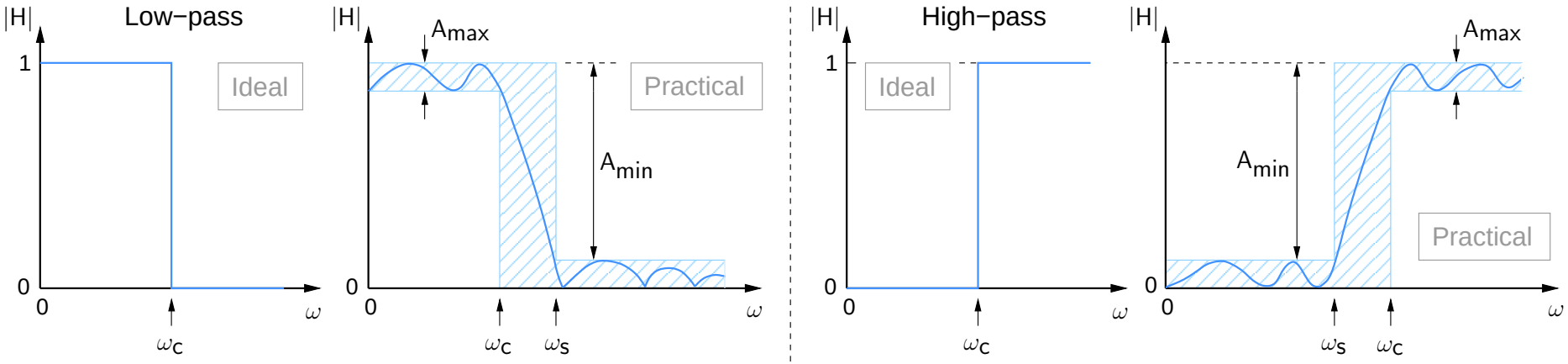

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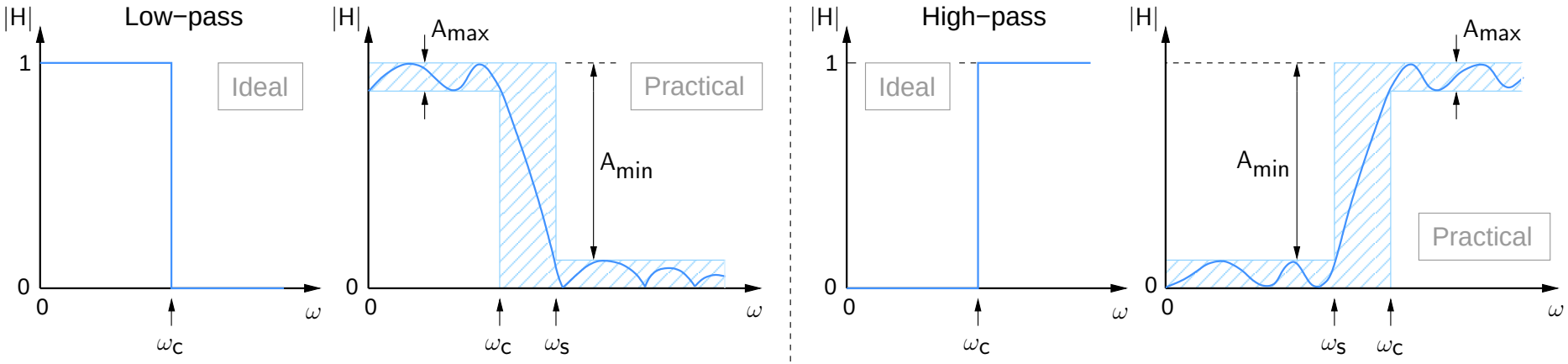

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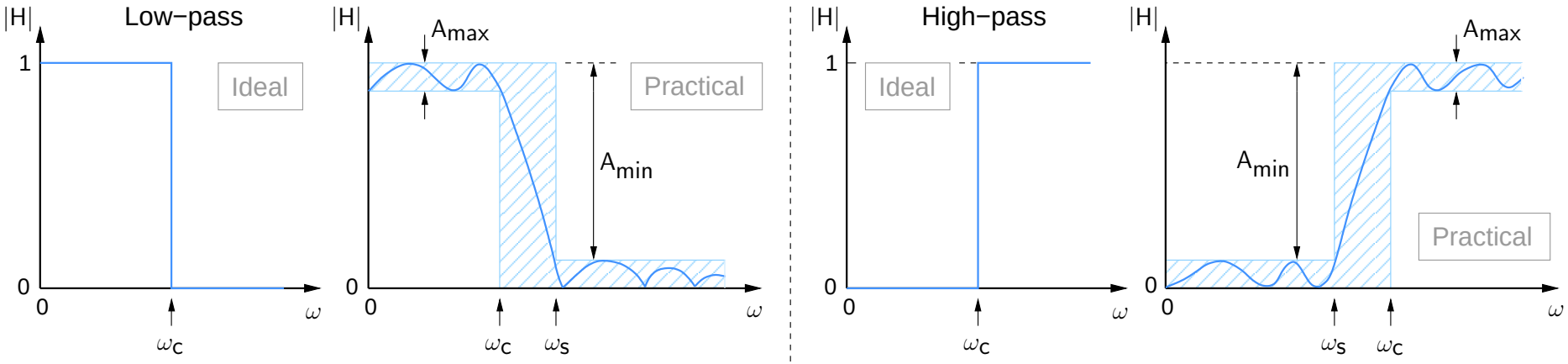

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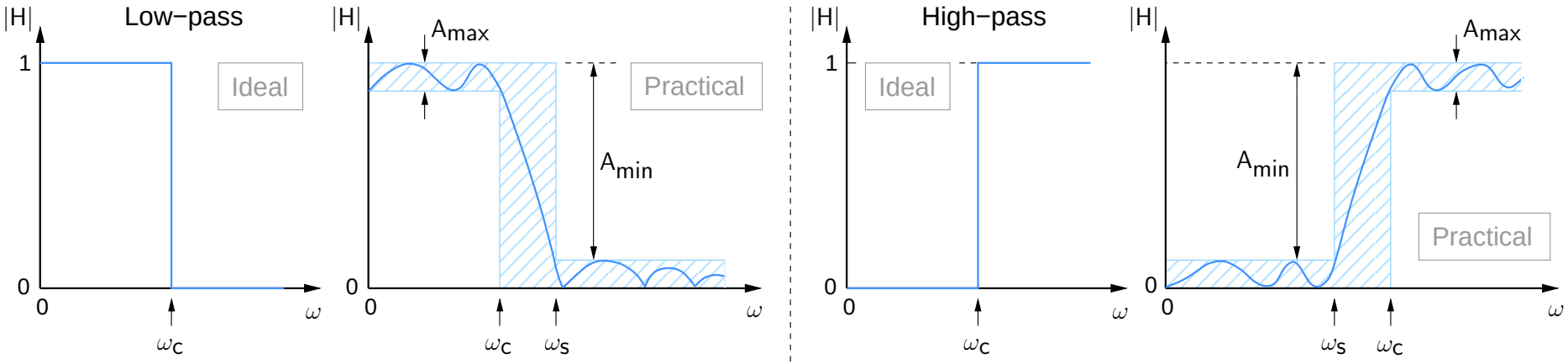

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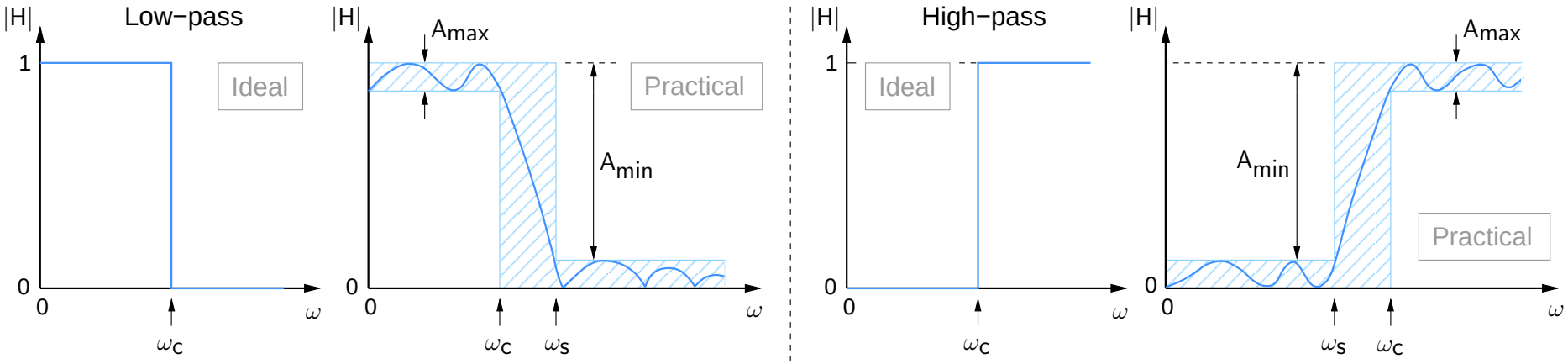

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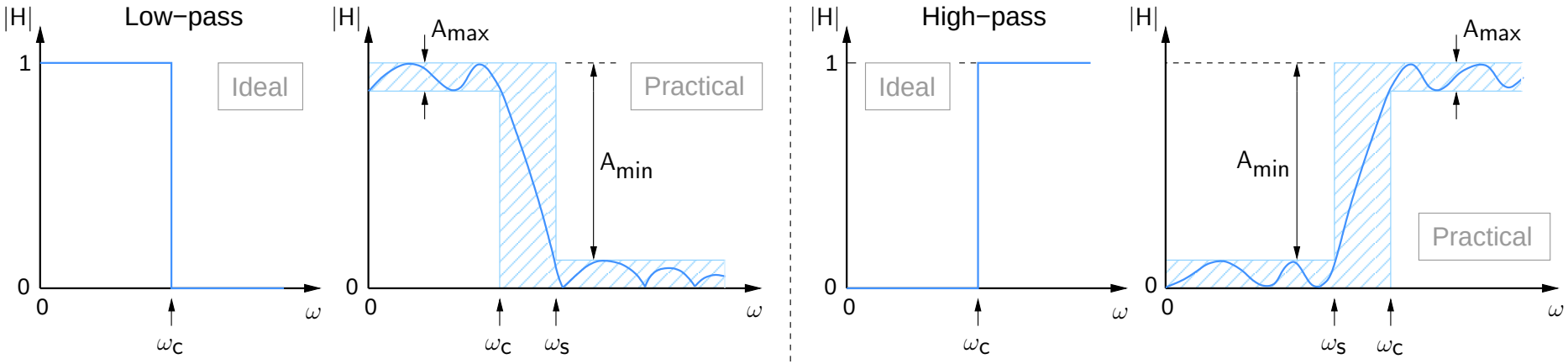

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* $\omega_c < \omega < \omega_s$: transition band.

Practical filters

For a low-pass filter, $H(s) = \dfrac{1}{\sum_{i=0}^{n} a_i (s/\omega_c)^i}$.

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$$C_n(x) = \cos\left[n\cos^{-1}(x)\right] \quad \text{for } x \leq 1,$$

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$H(s)$ for a high-pass filter can be obtained from $H(s)$ of the corresponding low-pass filter by $(s/\omega_c) \longrightarrow (\omega_c/s)$.

Practical filters (low-pass)

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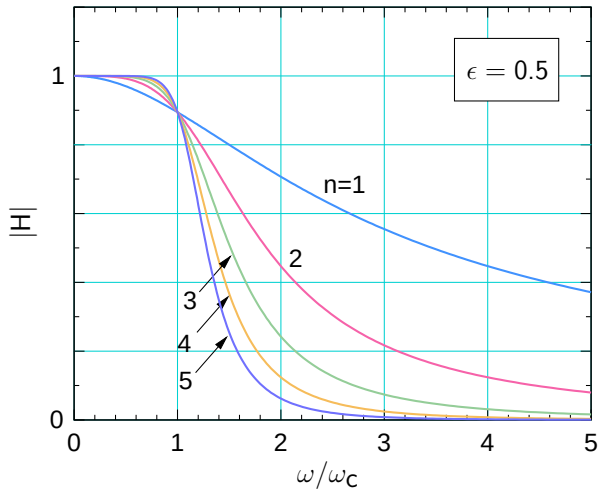
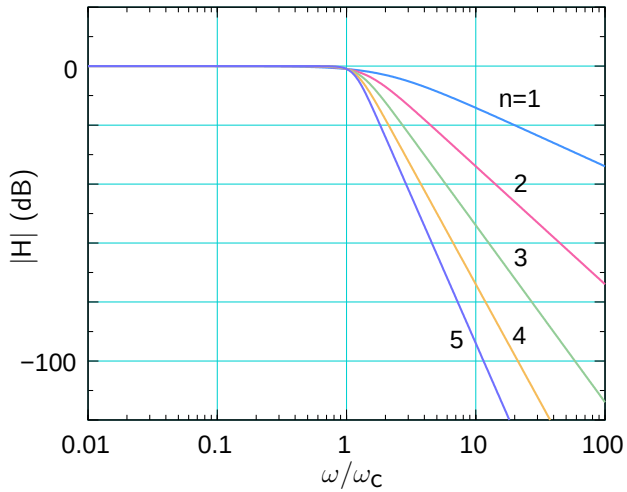
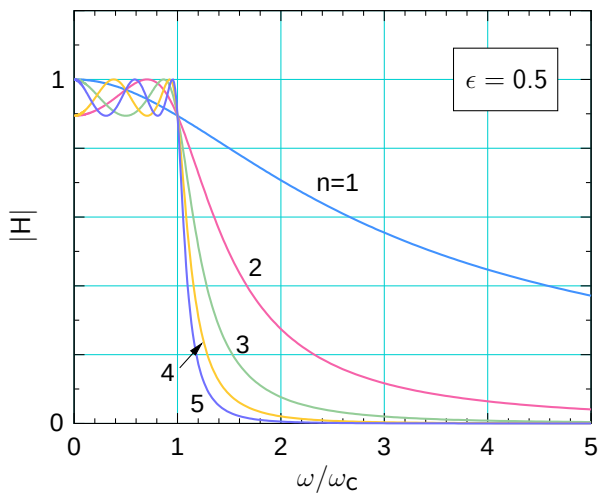
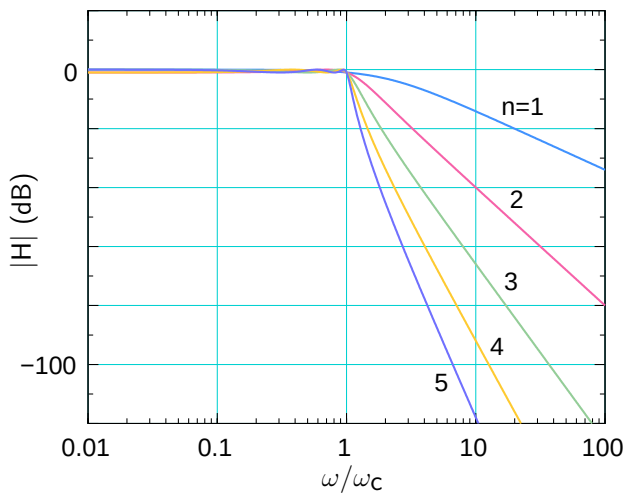




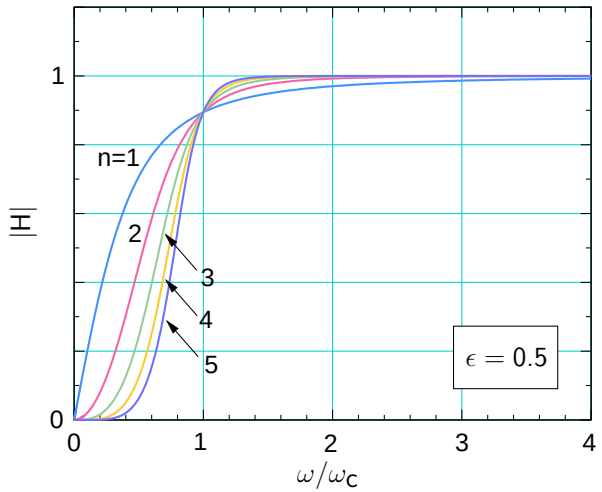
Chebyshev filters:

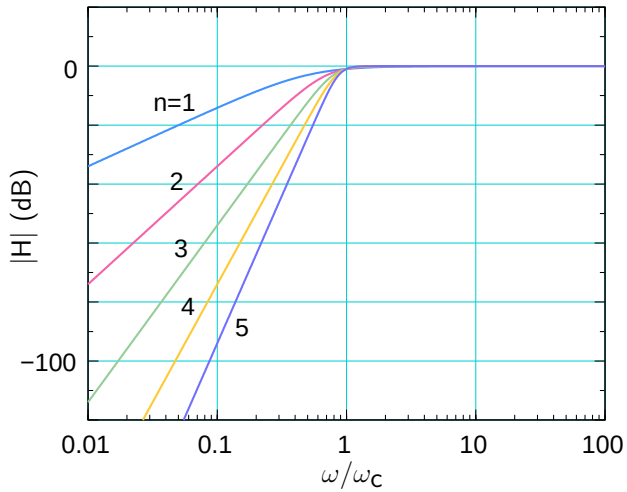




Practical filters (high-pass)

Butterworth filters:





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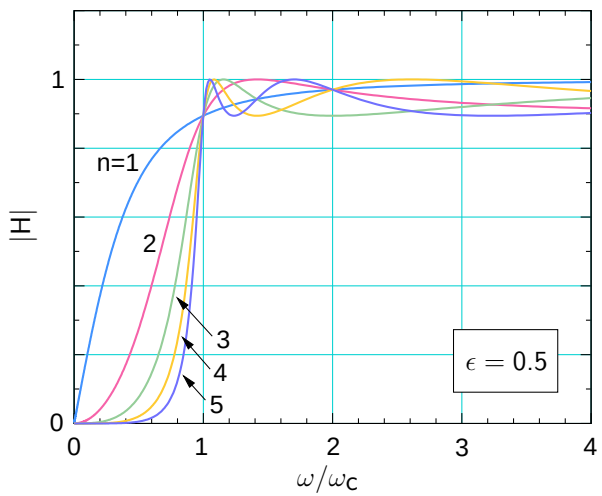
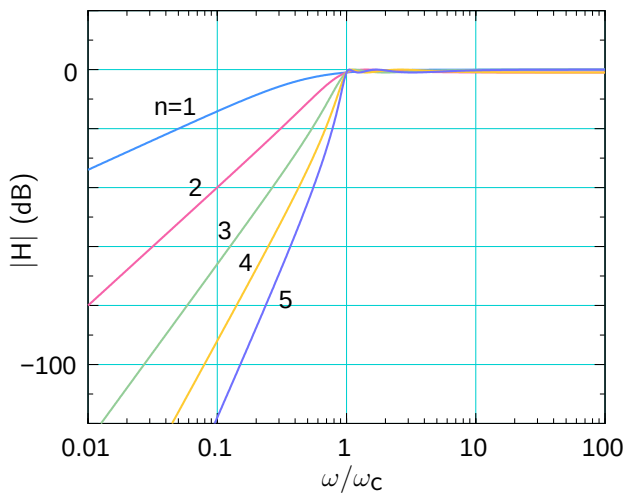
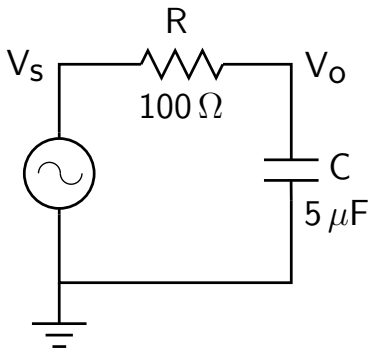




Passive filter example



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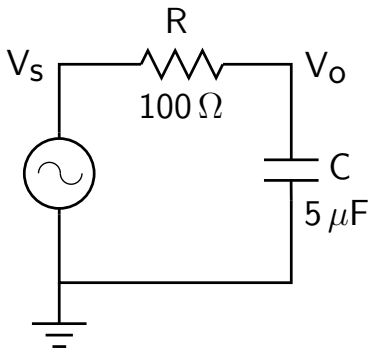

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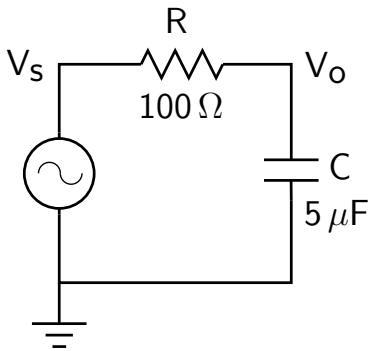

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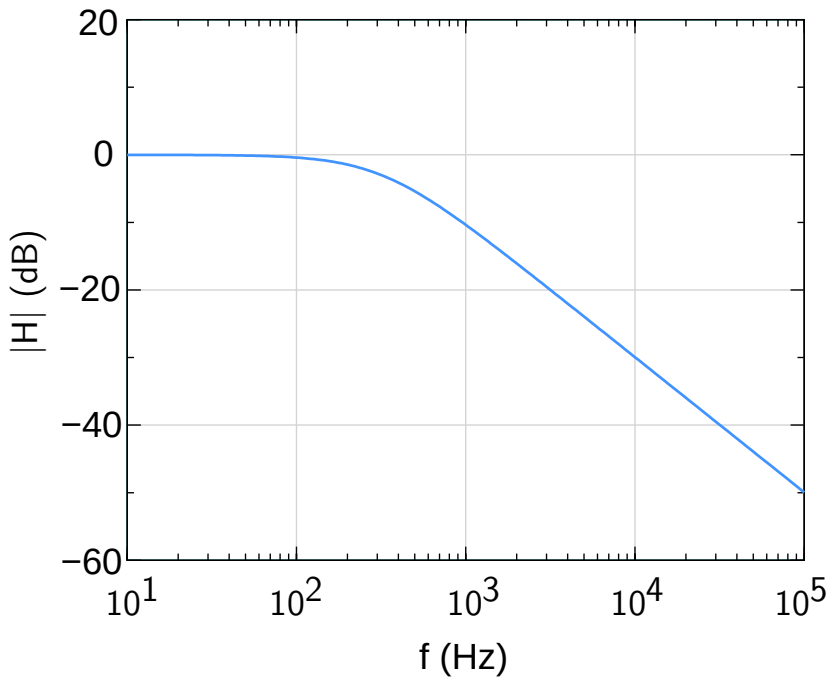

(SEQUEL file: ee101_rc_ac_2.sqproj)

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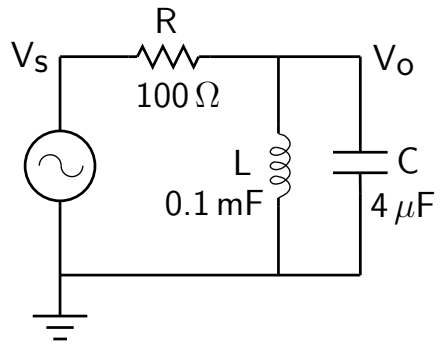

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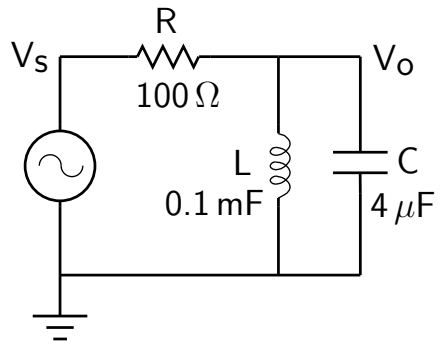

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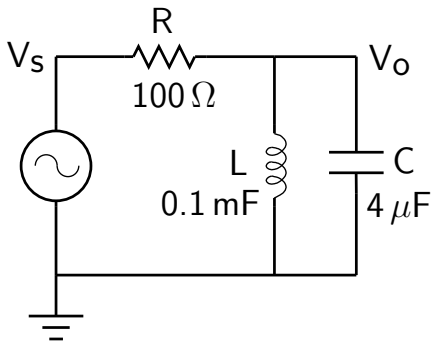

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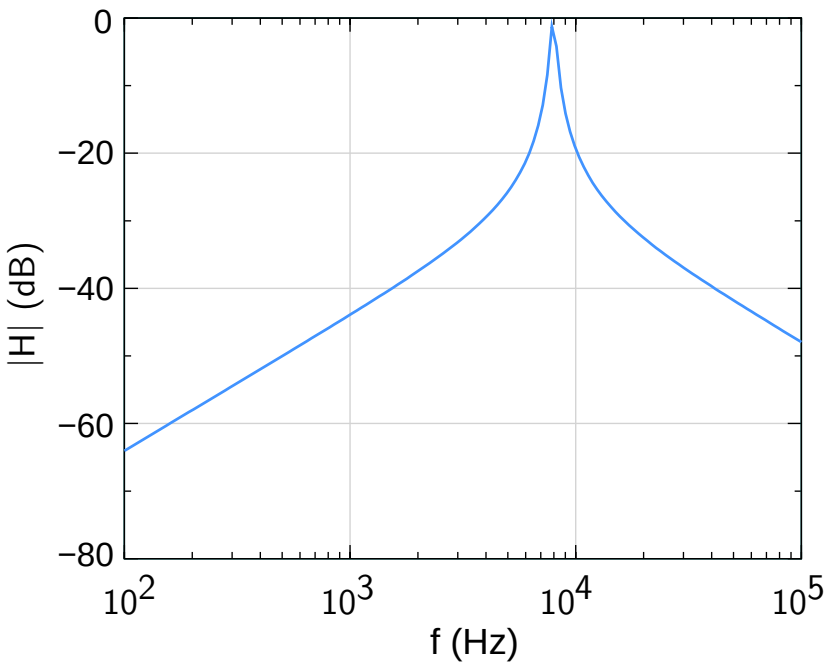

(SEQUEL file: ee101_lc_1.sqproj)

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* Also, if the power requirement is high, Op Amp filters cannot be used $\longrightarrow$ passive filters.

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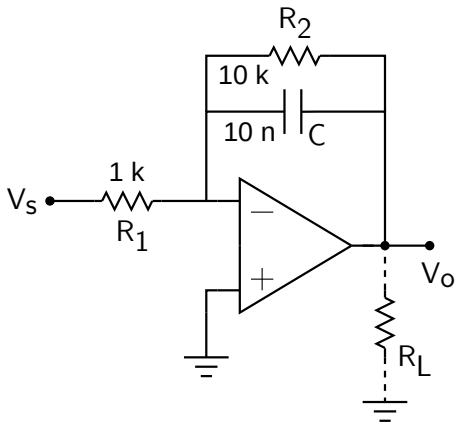

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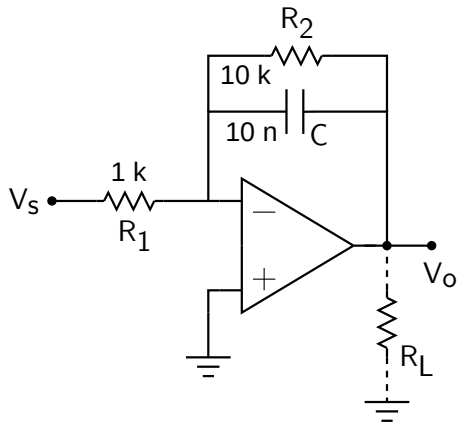

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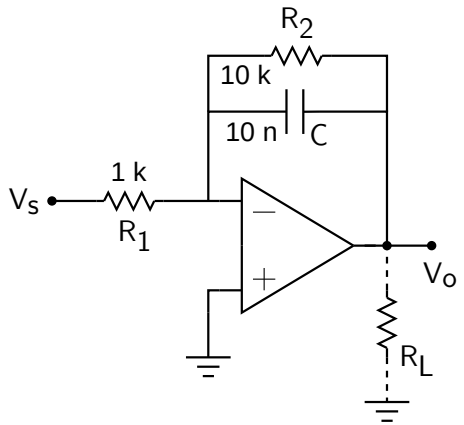

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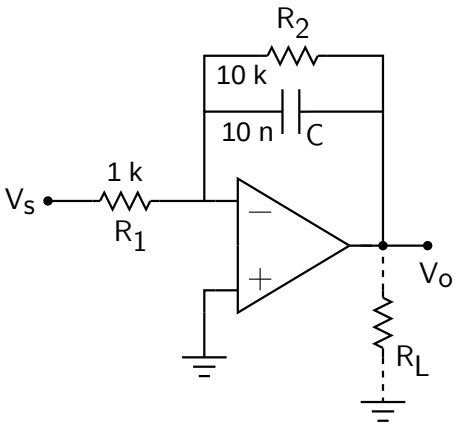

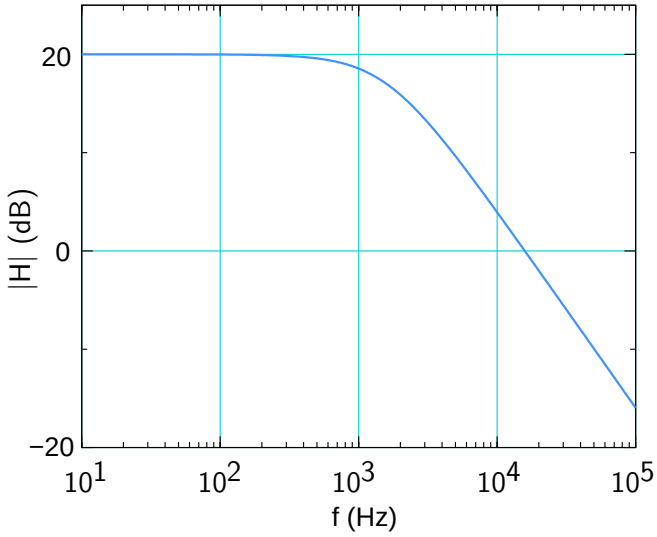

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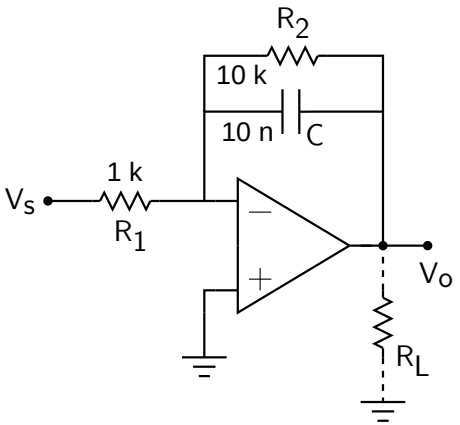

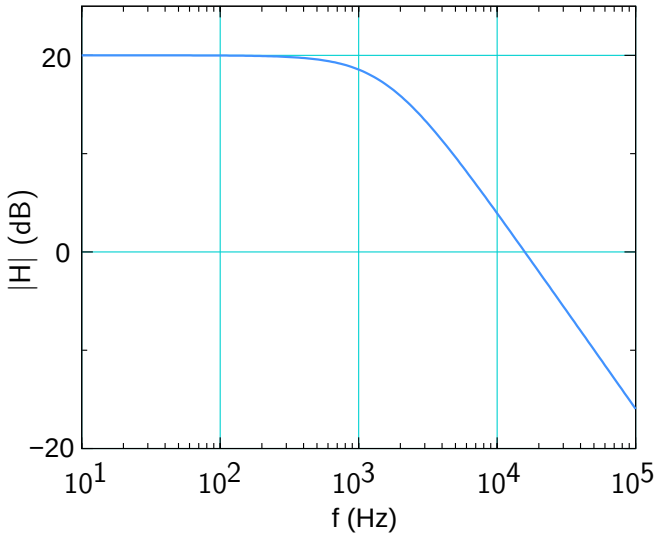

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(SEQUEL file: `ee101_op_filter_1.sqproj`)

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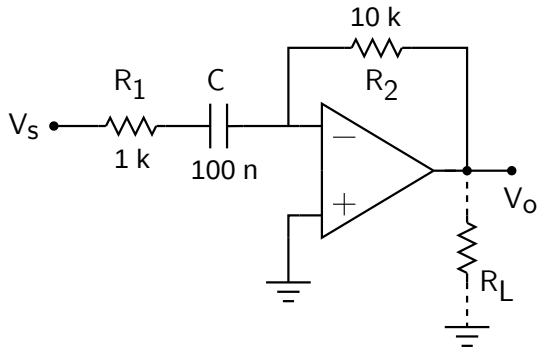

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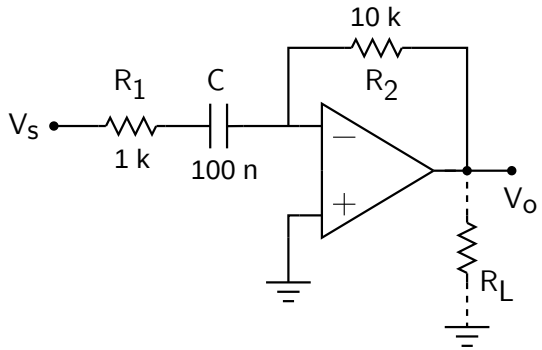
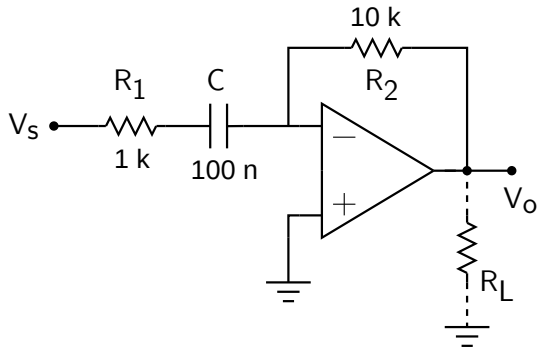


$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}\,.$$

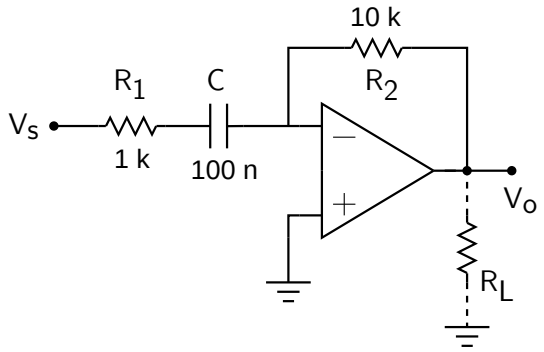
Op Amp filters: example

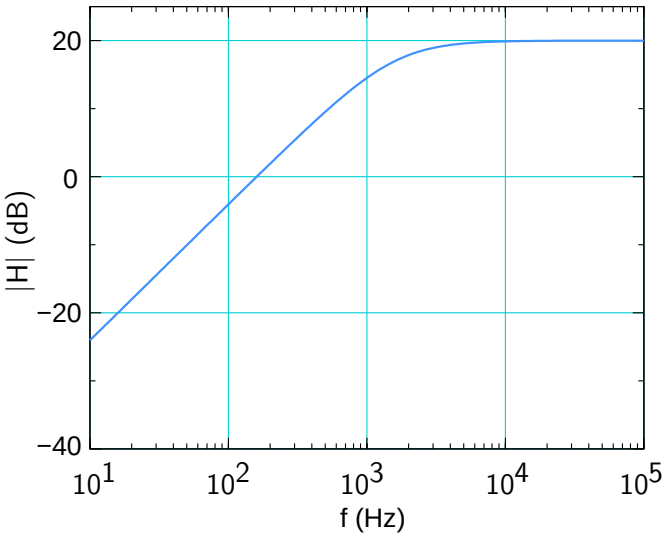


$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}\,.$$

This is a high-pass filter, with $\omega_0 = 1/R_1C$.

Op Amp filters: example

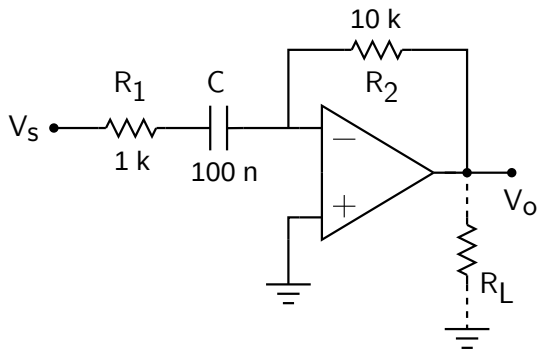


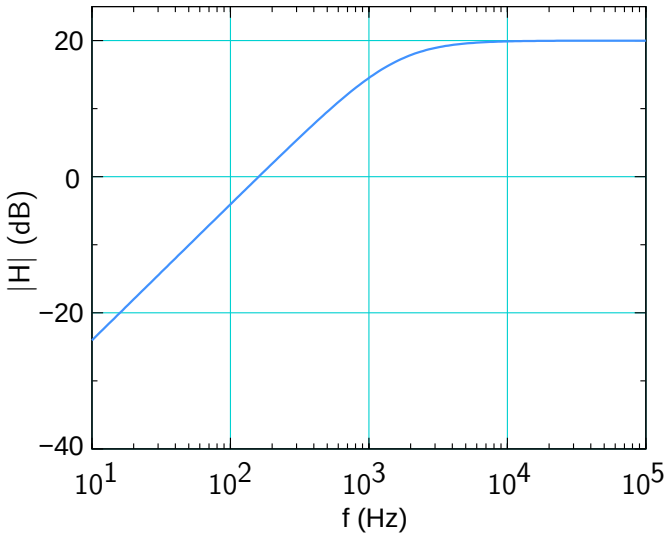


$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}\,.$$

This is a high-pass filter, with $\omega_0 = 1/R_1C$.

Op Amp filters: example



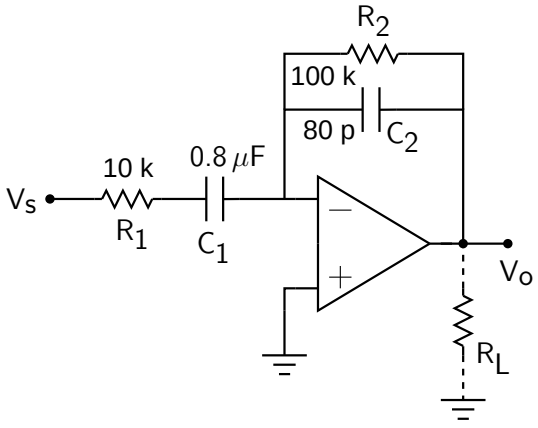


$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}\,.$$

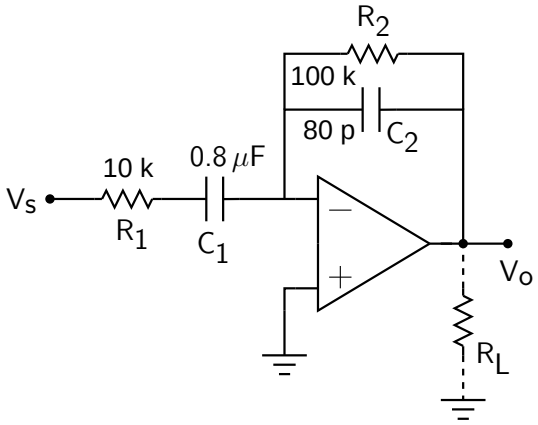
This is a high-pass filter, with $\omega_0 = 1/R_1C$.

(SEQUEL file: `ee101_op_filter_2.sqproj`)

Op Amp filters: example

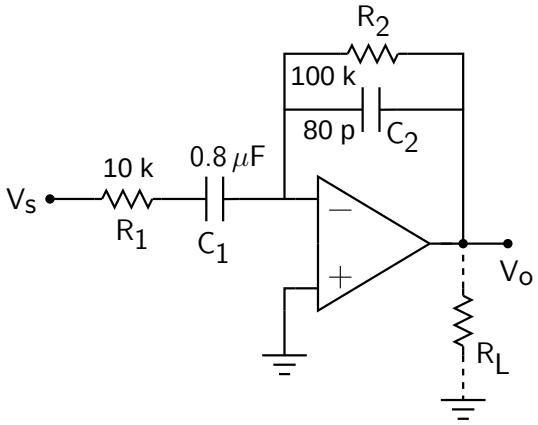


Op Amp filters: example



$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1}\,\frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)}\,.$$

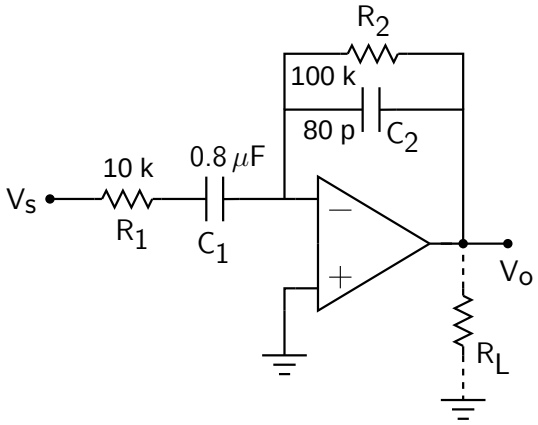
Op Amp filters: example

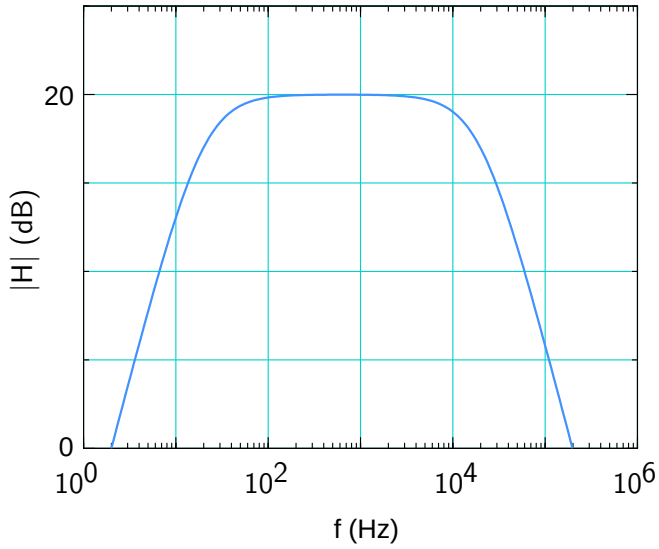


$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1}\,\frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)}\,.$$

This is a band-pass filter, with $\omega_L = 1/R_1C_1$ and $\omega_H = 1/R_2C_2$.

Op Amp filters: example

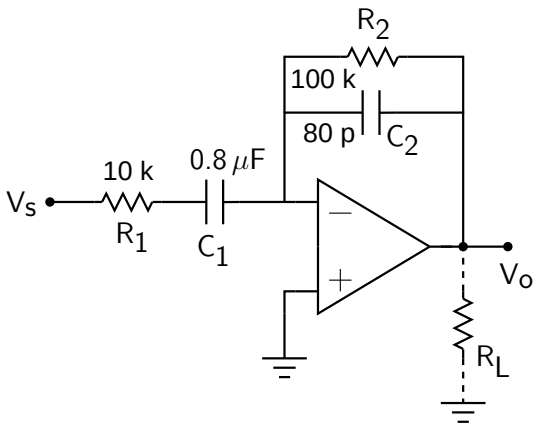


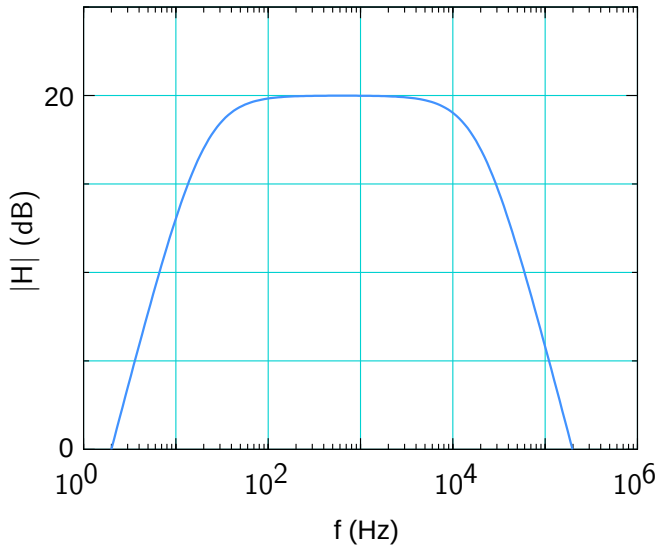


$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1}\,\frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)}\,.$$

This is a band-pass filter, with $\omega_L = 1/R_1C_1$ and $\omega_H = 1/R_2C_2$.

Op Amp filters: example



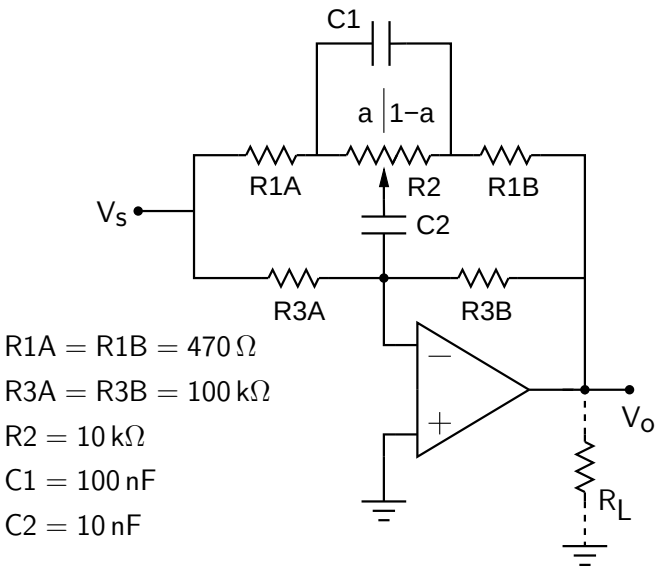


$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1}\,\frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)}\,.$$

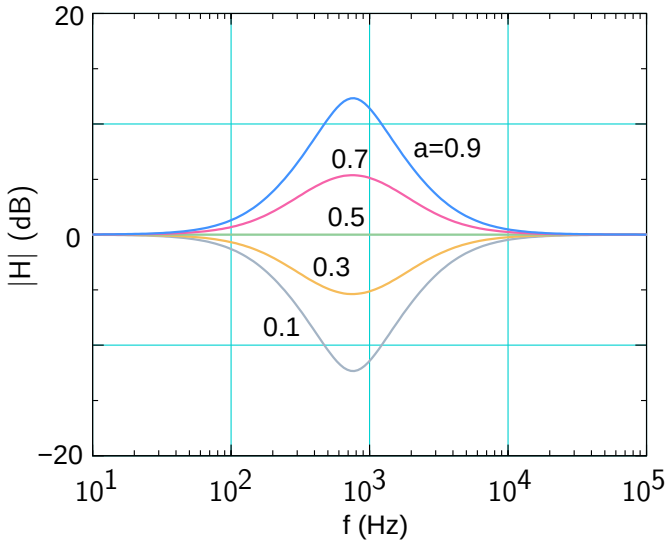
This is a band-pass filter, with $\omega_L = 1/R_1C_1$ and $\omega_H = 1/R_2C_2$.

(SEQUEL file: `ee101_op_filter_3.sqproj`)

Graphic equalizer

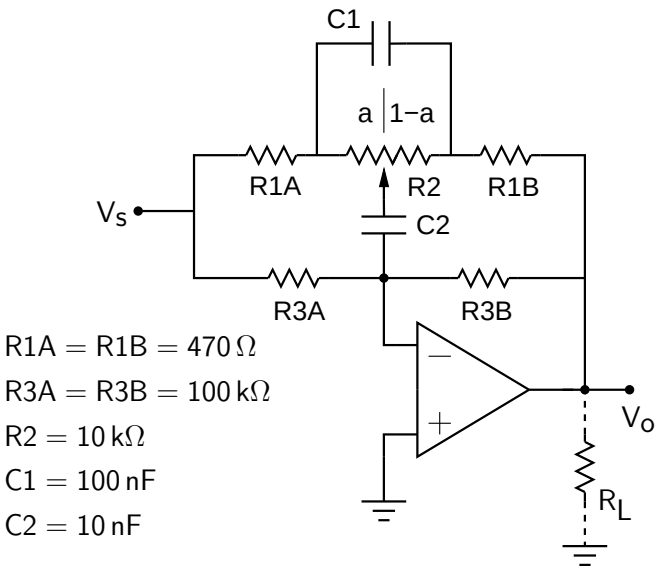


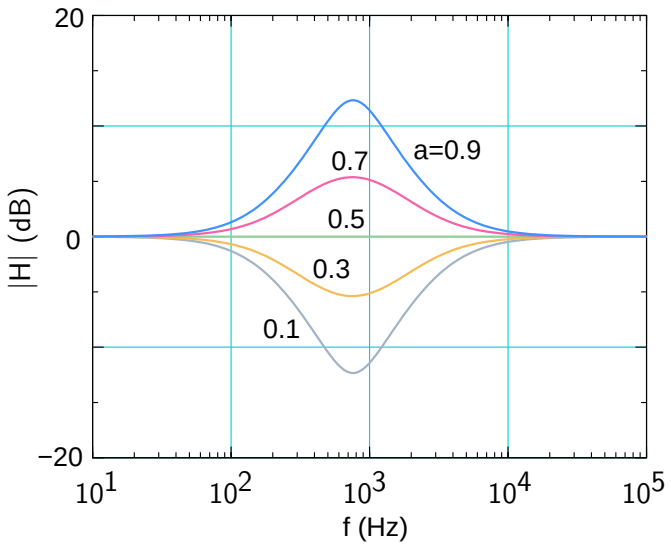
$R1A = R1B = 470\,\Omega$
$R3A = R3B = 100\,k\Omega$
$R2 = 10\,k\Omega$
$C1 = 100\,nF$
$C2 = 10\,nF$



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

Graphic equalizer

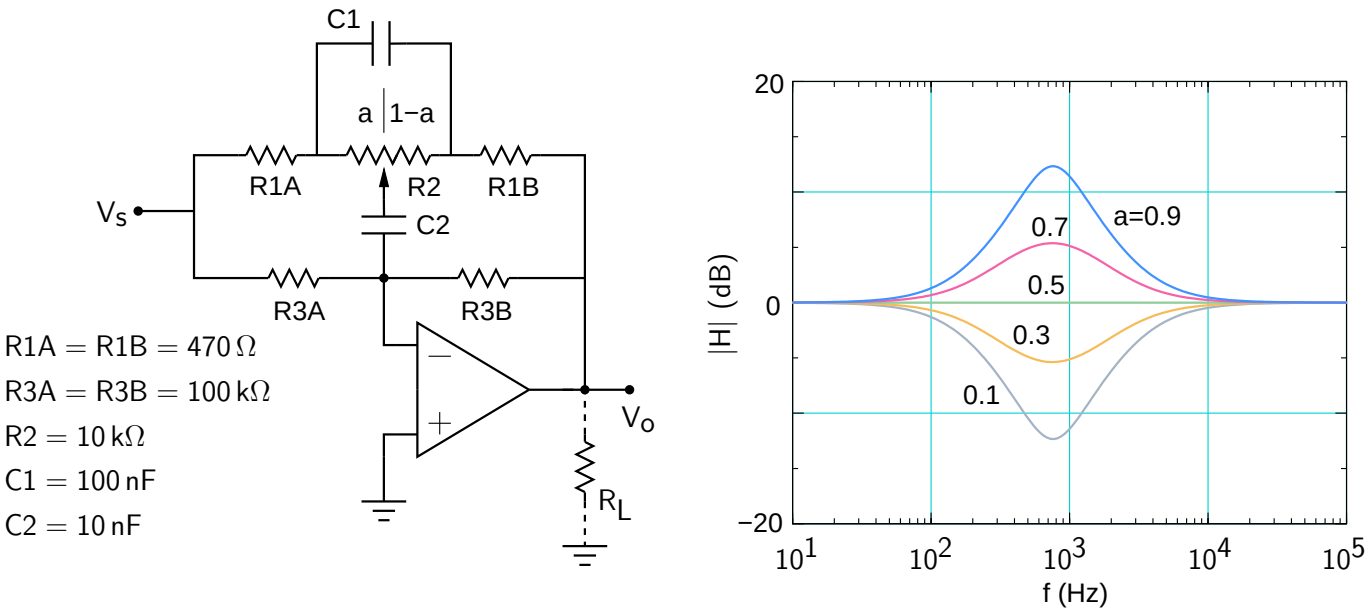




(Ref.: S. Franco, "Design with Op Amps and analog ICs")

* Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.

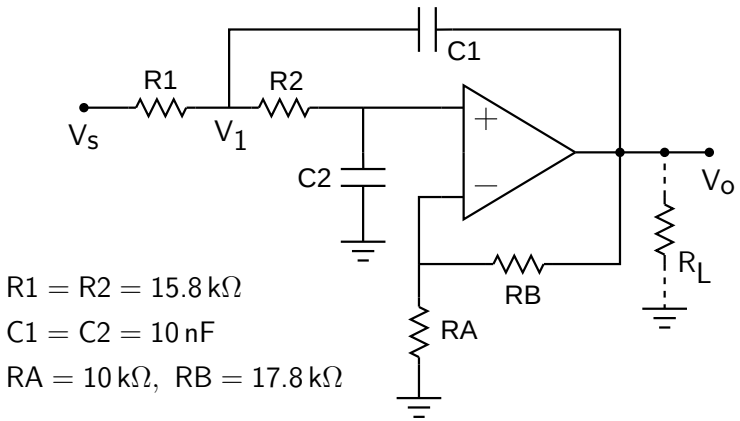
Graphic equalizer



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

* Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.
* The circuit shown above represents one of the equalizer sections.
  (SEQUEL file: `ee101_op_filter_4.sqproj`)

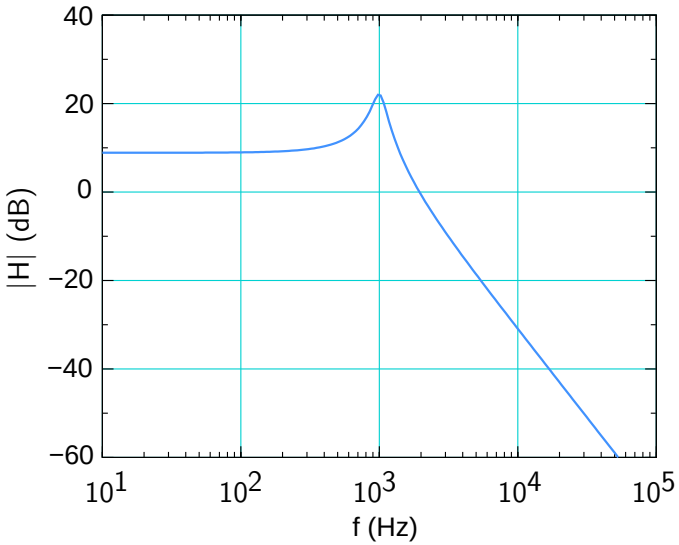
Sallen-Key filter example (2^{nd} order, low-pass)



$R1 = R2 = 15.8\,k\Omega$

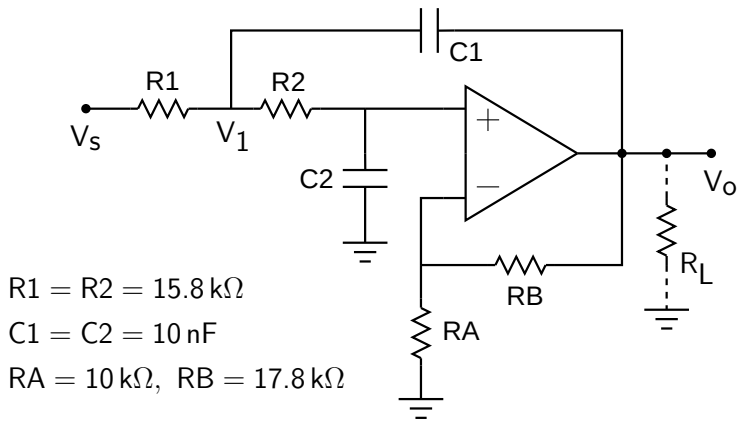
$C1 = C2 = 10\,nF$

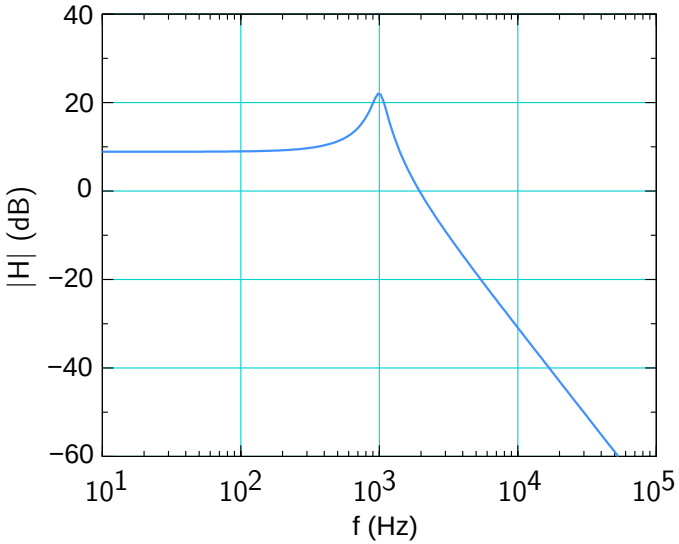
$RA = 10\,k\Omega,\ RB = 17.8\,k\Omega$



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

Sallen-Key filter example (2^{nd} order, low-pass)

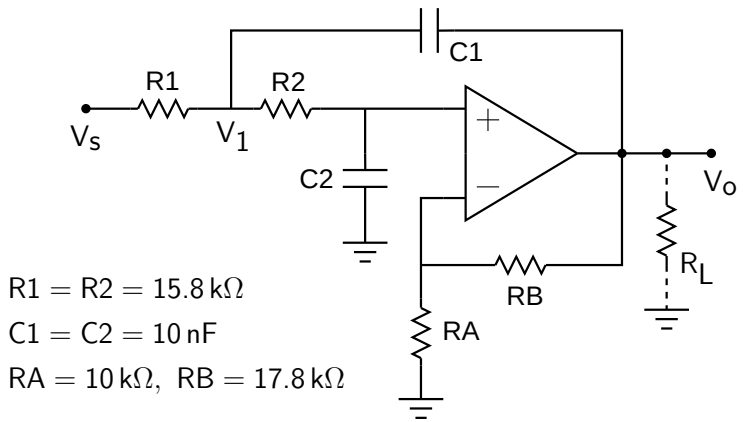


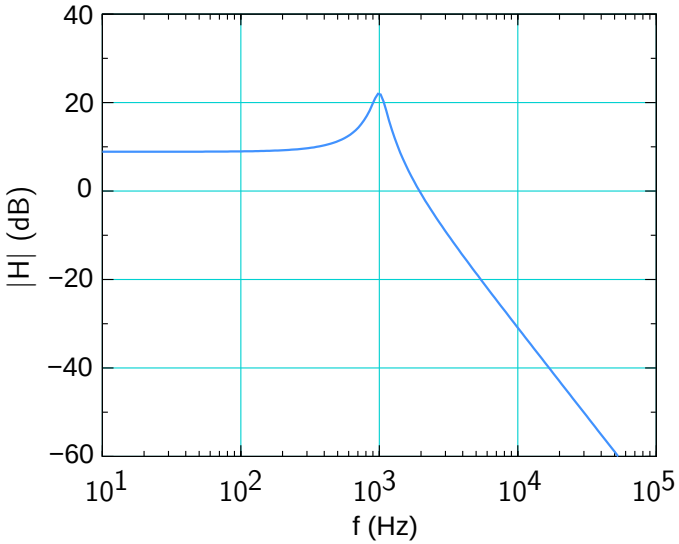


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\,.$$

Sallen-Key filter example (2^{nd} order, low-pass)



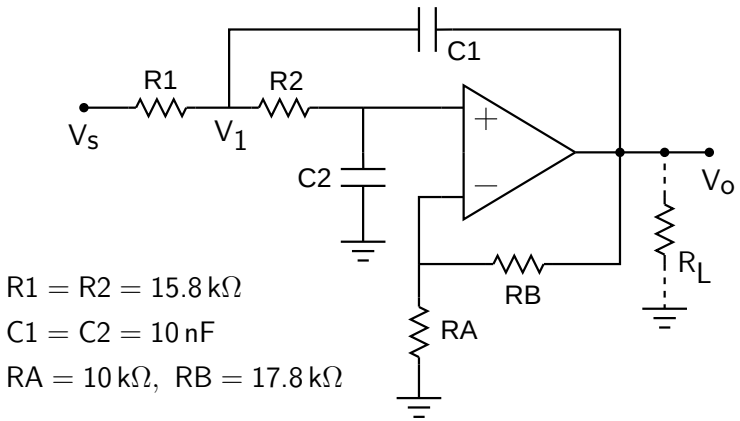


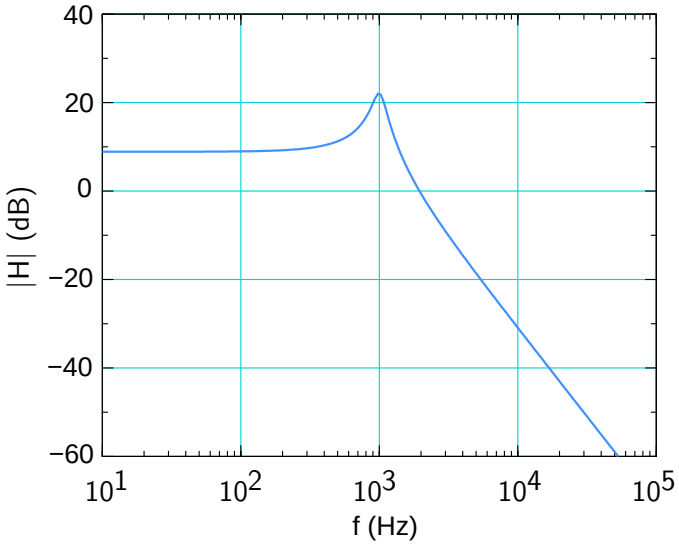
(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\,.$$

Also, $V_+ = \dfrac{(1/sC_2)}{R_2 + (1/sC_2)}\, V_1 = \dfrac{1}{1 + sR_2C_2}\, V_1\,.$

Sallen-Key filter example (2^{nd} order, low-pass)





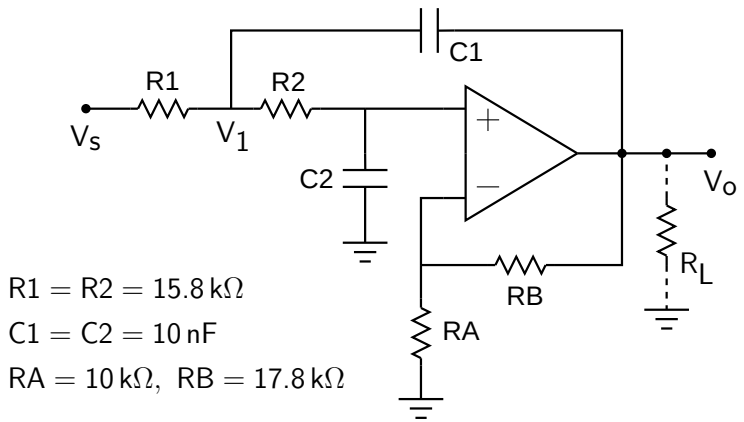
(Ref.: S. Franco, "Design with Op Amps and analog ICs")

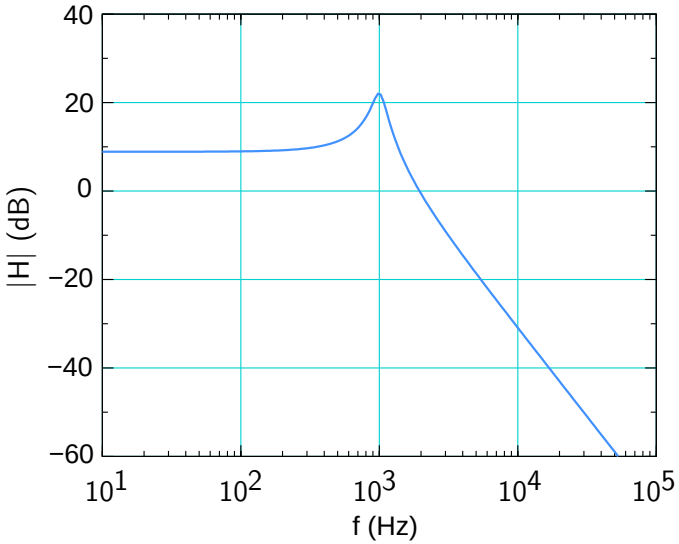
$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\,.$$

Also, $V_+ = \dfrac{(1/sC_2)}{R_2 + (1/sC_2)}\, V_1 = \dfrac{1}{1 + sR_2C_2}\, V_1\,.$

KCL at $V_1 \rightarrow \dfrac{1}{R_1}(V_s - V_1) + sC_1(V_o - V_1) + \dfrac{1}{R_2}(V_+ - V_1) = 0\,.$

Sallen-Key filter example (2^{nd} order, low-pass)





(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\,.$$

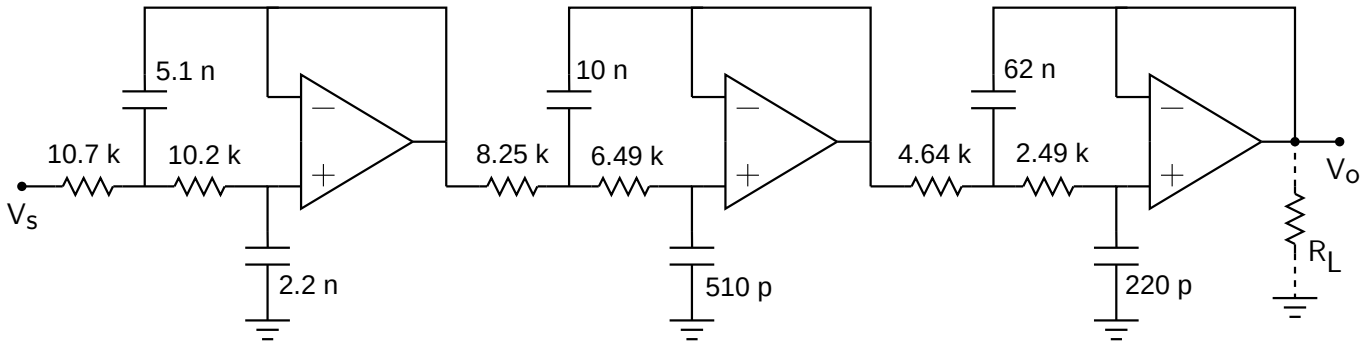
Also, $V_+ = \dfrac{(1/sC_2)}{R_2 + (1/sC_2)}\, V_1 = \dfrac{1}{1 + sR_2C_2}\, V_1\,.$

KCL at $V_1 \rightarrow \dfrac{1}{R_1}(V_s - V_1) + sC_1(V_o - V_1) + \dfrac{1}{R_2}(V_+ - V_1) = 0\,.$

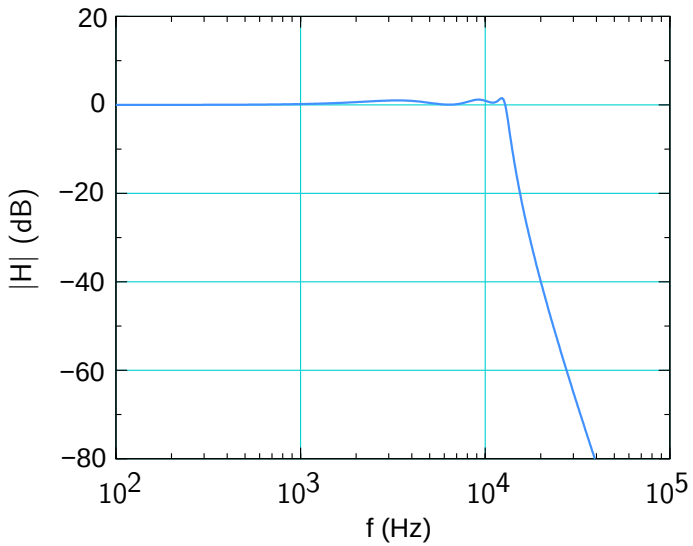
Combining the above equations, $H(s) = \dfrac{K}{1 + s\left[(R_1 + R_2)C_2 + (1 - K)R_1C_1\right] + s^2R_1C_1R_2C_2}\,.$

(SEQUEL file: `ee101_op_filter_5.sqproj`)

Sixth-order Chebyshev low-pass filter (cascade design)

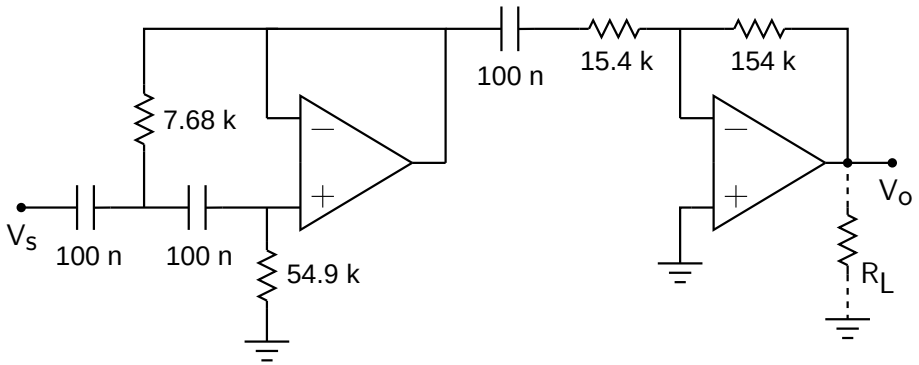


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

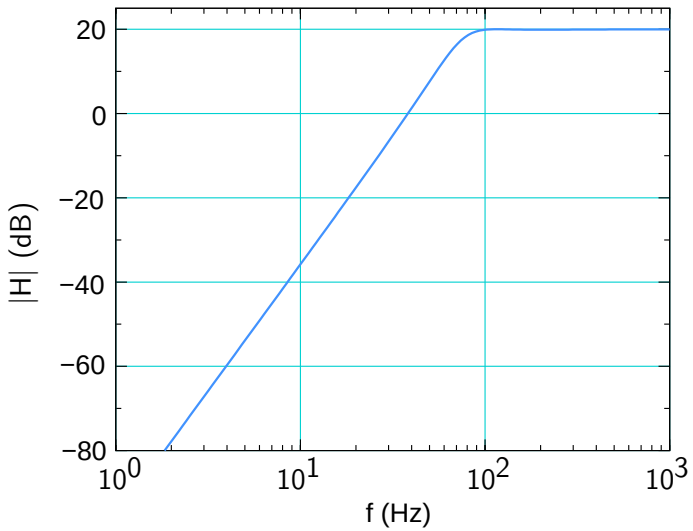


SEQUEL file: ee101_op_filter_6.sqproj

Third-order Chebyshev high-pass filter

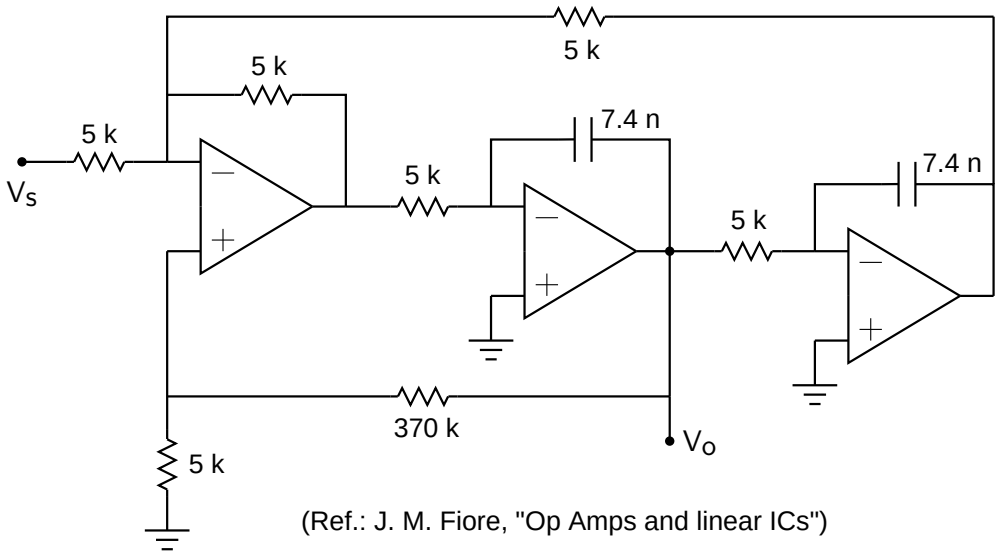


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

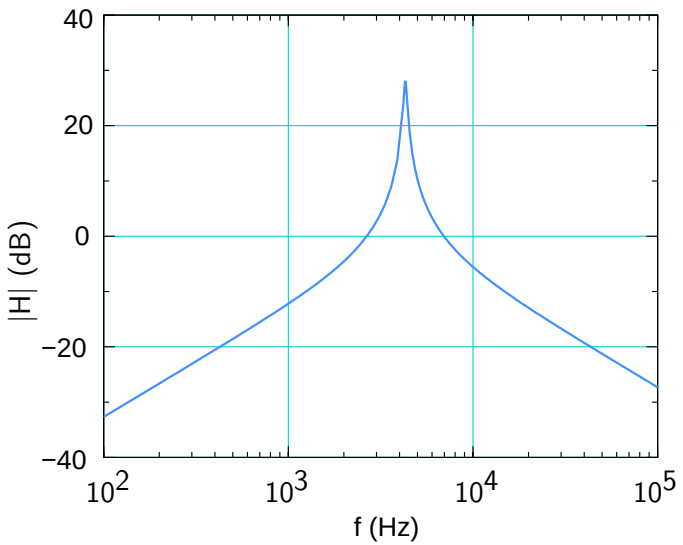


SEQUEL file: ee101_op_filter_7.sqproj

Band-pass filter example

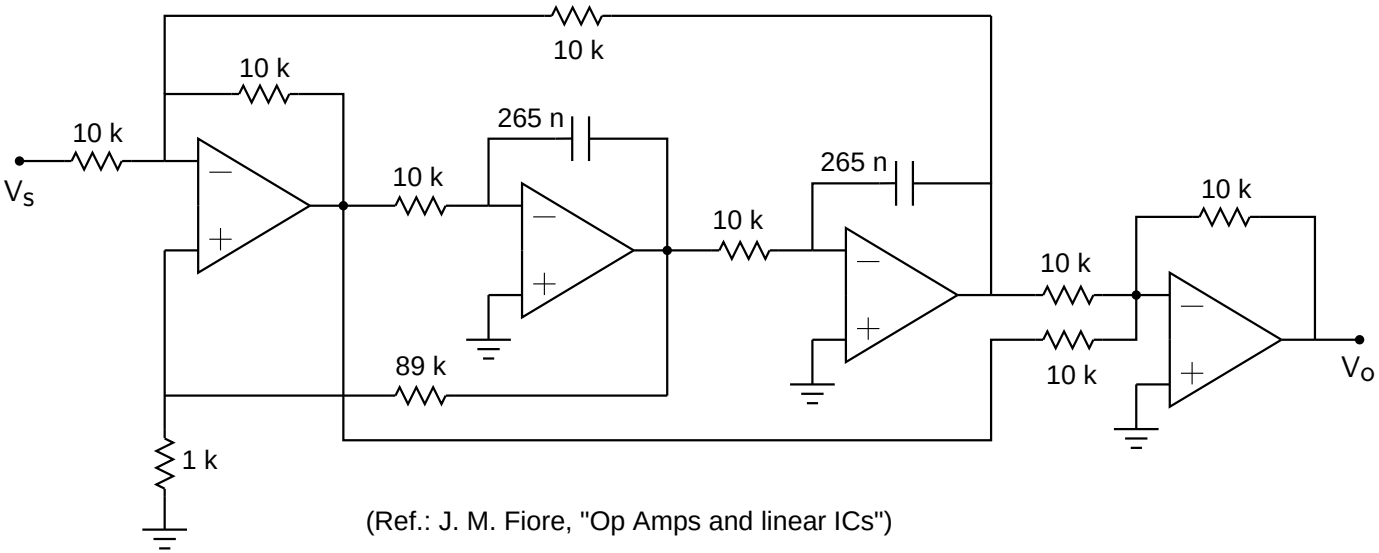


(Ref.: J. M. Fiore, "Op Amps and linear ICs")

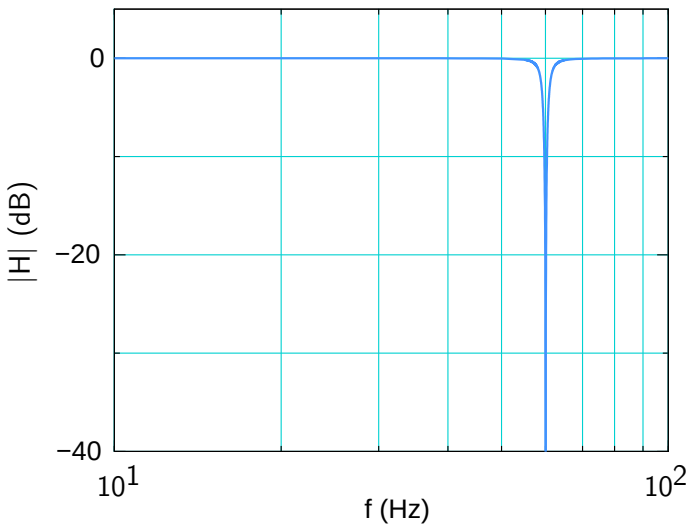


SEQUEL file: ee101_op_filter_8.sqproj

Notch filter example



(Ref.: J. M. Fiore, "Op Amps and linear ICs")



SEQUEL file: ee101_op_filter_9.sqproj