# Power Constrained MPOE Based Prefiltering for Statistical Channel Model

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***Abstract*— In this paper, the power constraint in transmitter based pre-filtering for downlink direct sequence code division multiple access (DS-CDMA) system is proposed. Weights of FIR pre-coding filter are computed by minimizing the conditional Probability of Error (MPOE) and the Mean Square Error (MMSE) for all users subject to power constraint. In both the cases, we optimize the error based on the condition that the bit vector sequence is known at the transmitter. No knowledge of the channel is assumed except its statistical parameters. Our simulation results show that with power constrained MPOE, reduction in power by 4% at low SNR (5 dB) and by 17% at high SNR (25 dB) can be achieved for equal or lesser probability of error as compared to unconstrained pre-filtering algorithm. Moreover, MPOE based power constrained algorithm gives better result as compared to power constrained MMSE algorithm.**

## I. Introduction

In a conventional mobile communication system, the bulk of processing is carried out at the receiver end to overcome multipath effects, fading or distortion of received signal. On the other hand, by using pre-filtering scheme at the transmitter, only a simple matched filter can be used in the receiver [1, 2]. In case of slow varying channel, Time Domain Duplex (TDD) scheme allows channel measurements in the uplink which can be used for preprocessing in downlink due to channel reciprocity. However, the required transmitted power by the base station increases with these pre-processing techniques [3, 4]. In CDMA, the transmitted power for each user from the base station must be reduced to limit interference, while it should be enough to maintain the required $E_b/N_0$ (signal to noise ratio) [5, 6]. CDMA base station deployment in developing countries suffers from power cuts. During power cuts, the base stations run on battery power and thus power control mechanism at the base station becomes important.

Downlink power constrained algorithm which maximizes Signal to Interference and Noise Ratio (SINR) of the receiver subject to constraint that total transmitted power is constant was proposed in [7]. In [8] power was minimized such that $E_b/N_0$ is maintained above a particular value, while total power is below a fixed thresold. Power minimization based on SINR maximization by taking background noise into consideration was analyzed in [9]. An adaptive power constrained algorithm with adaptive step parameter based on intercell interference was considered in [10]. In [11] optimal signature sequences and power constrained strategies have been considered so that the users meet their QoS (Quality of Service) requirement. All the above papers assume the channel model is known fully at the transmitter and does not change for a long period. Moreover the optimization criterion is MMSE.

In this paper, we propose a power constraint based pre-filtering scheme for statistical channel model. By statistical channel model, we assume no knowledge of the channel except its statistical parameters, namely, mean and variance. In [14] statistical channel model based pre-filtering is addressed, but it does not include any power constraint. We develop power constrained algorithms for both, MPOE and MMSE case. The filter weights are computed based on the condition that the transmitted bit vector is known at the base station. The receiver is a conventional matched filter detector. We also assume that the base station has enough computational capabilities to carry out the optimization subject to power constraint.

We carry out simulations to demonostrate that for equal or lesser probability of error, the power constrained MPOE allows reduction in power by 4% at low SNR (5 dB) and by 17% at high SNR (25 dB). Also, MPOE based power constrained algorithm gives better results as compared to power constrained MMSE algorithm.

## II. Signal Model and Problem Formulation

We now describe the signal model and then the power constrained based MPOE and MMSE signal detection for a channel whose statistical model is known.

Consider a DS-CDMA system with K number of users as shown in Fig 1. The $k^{th}$ user transmits $b_k(i)$ bit in $i^{th}$ interval and the length of signalling interval for each user is T [12, 13]. User $k$ is assigned a spreading wavefrom $c_k(.)$ which is supported in [0,T] and is normalised to 1. Let $\mathbf{s}_k = [s_{k1}, s_{k2} .....s_{KN}]$ denote the corresponding spreading sequence. Then,

$$c_k(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} s_{kn} rect[t - (n-1)T_c]$$

where, $rect(t)$ is a rectangular waveform with unit amplitude in [0,T] and N is the processing gain of the system [4]. The baseband signal of the $k^{th}$ user in the $i^{th}$ bit interval can now be expressed as

$$x_k(t) = A_k b_k(i) c_k(t - iT), iT \leq t \leq (i+1)T$$

Let $x(t)$ denote the transmitted signal by the base station in

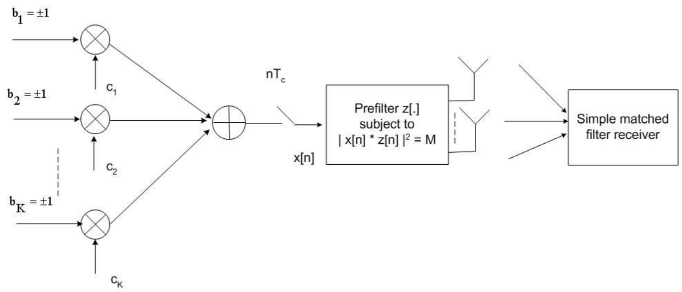


Fig. 1. DS-CDMA channel model for an asynchronous multipath channel with transmitter prefiltering

$i^{th}$ interval,

$$x(t) = \sum_{u=1}^{u=K} A_u b_u(i) c_u(t - iT_{bit}), iT \leq t \leq (i+1)T$$

Assume that $x(t)$ is sampled ($x[n]$) at the chip period ($T_c$) to process the signal in the prefilter.

$$x[n] = x[nT_c] = \frac{1}{\sqrt{N}} \sum_{u=1}^{u=K} A_u b_u(i) \tilde{s}_u[n]$$

where $\tilde{s}_u[n]$ = ...,$s_{u0}$, $s_{u1}$,...,$s_{uN-1}$, $s_{u0}$,... We assume that the prefilter for a particular bit period $i$ is an FIR filter ($z[.][i]$) of length $L_z$. The idea is to compute these filter coefficients using MPOE and MMSE criteria such that power is constrained. The channel is modeled as an FIR filter of length $L_h$, and the channel for $u^{th}$ user at $i^{th}$ bit interval is denoted as $h_u[.][i]$. The noise is zero mean additive white Gaussian (AWGN). The received signal is

$$r_u[n] = h_u[n][.] * x[n] * z[n][.] + \eta_u[n]$$

where $*$ denotes convolution operation.

$$\mathbf{r}_u[n] = [r[iN], ..., r[iN + N - 1]]^T$$

At the receiver, simple matched filter is employed to reduce the complexity.

$$y_u[i] = \mathbf{s}_u^T \mathbf{r}_u[i] = \sum_{k=0}^{N-1} s_{uk} r[iN + k]$$

$$\begin{aligned} y_u[i] = \Bigg[ & \sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} h_u[m][j_1] \sum_{l=0}^{L_z^u - 1} z[l][j_2]. \\ & . \sum_{u=1}^{u=K} A_u b_u[j_2] \tilde{s}_u[iN + k - m - l] \Bigg] \\ & + \sum_{k=0}^{N-1} s_{uk} \eta_u[iN + k] \end{aligned} \quad (1)$$

where

$$j_1 = \left\lfloor \frac{iN + k - m}{N} \right\rfloor, j_2 = \left\lfloor \frac{iN + k - m - l}{N} \right\rfloor \quad (2)$$

## III. Statistical channel model

In most practical situations, full knowledge of the channel may not be available at the trasmitter. We assume that only the statistical parameters, namely, mean and variance of the channel are known. These parameters can be calculated by making large sample measurements and can be assumed to be non varying for reasonable amount of time.

### A. MPOE based pre-filter

Let us assume

$$E(h_u[m][j]) = \gamma_u[m][j] \quad (3)$$

$$\begin{aligned} E\Big[ & (h_u[m_1] - \gamma_u[m_1][j_1])(h_u^*[m_2][j_2] - \gamma_u^*[m_2][j_2]) \Big] = \\ & C_u[m_1, m_2][j_1, j_2] \end{aligned} \quad (4)$$

$$\begin{aligned} E\Big[ & (h_u[m_1] - \gamma_u[m_1][j_1])(h_u[m_2][j_2] - \gamma_u[m_2][j_2]) \Big] = \\ & \tilde{C}_u[m_1, m_2][j_1, j_2] \end{aligned} \quad (5)$$

Since the transmitted bit is known at the transmitter where all processing is being carried out, we use conditional probability of error rather than probability of error [13, 14]. Then, the conditional mean and variance of the real part of received signal ($y_u^R$) are given as

$$\begin{aligned} \mu_{y_u^R|B[i]}[i] = E(y_u^R|B[i]) = & \sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} \gamma_u[m][j_1] \sum_{l=0}^{L_Z - 1} z[l][j_2]. \\ & . \left( \sum_{u=1}^{K} A_u b_u[j_2] \tilde{s}_u[iN + k - m - l] \right) \end{aligned}$$

where $\gamma_u[m][j_1]$ is given in (3), $j_1$ and $j_2$ are given in (2).

$$\begin{aligned} \sigma^2_{y_u^R|B[i]}[i] = & var \left( \Re \left[ \sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} \gamma_u[m][j_1] \left( \sum_{u=1}^{K} A_u b_u[j_2]. \right.\right.\right. \\ & \left.\left.\left. . \tilde{s}_u[iN + k - m - l] \right) \right] \right) + N\frac{\sigma^2}{2} \end{aligned}$$

For simplicity the variance of the channel noise $\sigma^2$ is assumed to be constant for all users. .

$$P_{E|B[i]}[i] = Q\left( \frac{b_u[i] \mu_{y_u^R|B[i]}[i]}{\sigma\sqrt{N/2}} \right)$$

We want to minimize the joint probability of error for all users, namely,

$$P_{Ej}[i] = 1 - P[y_1^R \in \alpha_1, y_2^R \in \alpha_2, ......, y_K^R \in \alpha_K]$$

where $\alpha_i s$ are decision regions for symbol detection. Since both the signal and noise vector for all users are independent of each other, the joint probability of error becomes

$$P_{Ej}[i] = 1 - P[y_1^R \in \alpha_1] P[y_2^R \in \alpha_2] ......P[y_K^R \in \alpha_K]$$

The decision region for BPSK constellation, $\alpha_u$, for any user $u$, is given by $(0, \infty)$ when $b_u[i] = +1$ and $(-\infty, 0)$ when

$b_u[i] = -1$.

By using the above equations, $P_{Ej}$ can be written in closed form as

$$P_{Ej}[i] = 1 - \prod_{u=1}^{K} Q\left(-\frac{b_u \mu_{y_u^R}}{\sigma\sqrt{N/2}}\right)$$

**Power Constraint MPOE**

Now the optimization criterion is minimizing the probability of error at the receiver subject to the constraint that the power level after prefiltering is minimum. The MPOE optimization problem now becomes

$$Min_{z[.]} P_{Ej}$$

subject to

$$|\sum_{i=0}^{L_z-1}\sum_{k=0}^{N-1} z[l][i]x[iN+k-l]|^2 = M \quad (6)$$

i.e. we impose constraint such that the total transmitted power in a bit interval is constant. In the above equation, M is adaptively chosen and $z[l][i]$ is the $l^{th}$ coefficient of filter at particular bit interval $i$. Since we have equality constraint, we use Lagrange multiplier method to find $z[.]$ which minimizes

$$J(z) = P_{Ej} + \lambda\left(|\sum_{i=0}^{L_z-1}\sum_{k=0}^{N-1} z[l][i]x[iN+k-l]|^2 - M\right) \quad (7)$$

where $\lambda$ is the Lagrange multiplier. This cost function can be minimized for $z[.]$ using gradient search method as follows

$$\begin{pmatrix} z_1[0][i] \\ . \\ . \\ . \\ z_1[L_Z-1][i] \end{pmatrix} = \begin{pmatrix} z[0][i] \\ . \\ . \\ . \\ z[L_Z-1][i] \end{pmatrix} - \mu_z \begin{pmatrix} \frac{\partial J}{\partial z[0][i]} \\ . \\ . \\ . \\ \frac{\partial J}{\partial z[L_Z-1][i]} \end{pmatrix}$$

$$\lambda_1 = \lambda - \mu_\lambda \frac{\partial J}{\partial \lambda} \quad (8)$$

$z[.][i] \leftarrow z_1[.][i]$ and $\lambda \leftarrow \lambda_1$, where $\mu_\lambda$ and $\mu_z$ are step size parameters for gradient search.

*B. MMSE based pre-filter*

Let $y_u^R$ be the decision statistic for the u$^{th}$ user, then the cost function in case of MMSE based algorithm can be written as

$$\begin{aligned}\xi^2_{J|B[i]}[i] =& E\left[||y_u^R[i] - b[i]||^2 |B[i]\right] \\ =& \sum_{u=0}^{K} E\left[(\tilde{y}_u^R)^2 + b_u^2 - 2y_u^R b_u)|B\right] \\ =& \sum_{u=0}^{K}\left[E((\tilde{y}_u^R)^2|B) + \sigma^2\frac{N}{2} + 1 - 2b_u E(\tilde{y}_u^R|B)\right]\end{aligned}$$

where $J$ denotes the joint norm for all the users. Now E($(\tilde{y}_u^R)^2$—B) and E($\tilde{y}_u^R$—B) [14] can be expressed in terms of $\gamma$, $C_u$ and $\tilde{C}_u$ (3)-(5).

**Power Constraint MMSE**

Next, we construct the constrained optimization problem in a similar manner as (7). Thus we get

$$Min_{z[.]}\xi^2_{J|B[i]}$$

subject to

$$|\sum_{i=0}^{L_z-1}\sum_{k=0}^{N-1} z[l][i]x[iN+k-l]|^2 = M \quad (9)$$

Once again, we use gradient descent, as in (8), to compute the filter weights.

## IV. Simulation and Results

Simulations were carried out to calculate the filter coefficients for both MPOE and MMSE. BPSK constellation for bits was assumed with equal probability for bits +1 and -1. The processing gain, $N$, was assumed to be 15 and the number of users was taken to be 4. Channel was assumed to be complex Gaussian (both real and imaginary parts are IID Gaussian) having $\sigma$ value as 0.1655 and mean as 0.5.
Results for channel length, $L_h = 2$ and pre-filter length, $L_z = 3$ for varying SNRs are shown. The use of power constrained MPOE results in the transmitted power reduction by 4% at low SNR (5 dB) and by 17% at high SNR (25 dB). Moreover, we observe from the graph (Fig. 2) that for the power constrained algorithm, the probability of error is equal or less than that of without power constraint. For MMSE, all the above parameters were taken to be the same. From Fig. 2, it is clear that probability of error is more in case of MMSE as compared to MPOE for both the power constrained as well as unconstrained algorithms. Fig. 4 shows that reduction in power with power constrained MMSE as compared to unconstrained MMSE. Table 1 depicts the comparison between different schemes for SNR = 15 dB. It clearly exhibits the superiority of power constrained MPOE. In all graphs, power scale is in watts since we assumed bits with amplitude equal to +1V or -1V. Similar results are observed for the case when the channel is fully known at the transmitter and when SINR is varied in place of SNRs. Due to lack of space, we are unable to put the graphs for the same.

TABLE I
Comparison of different algorithms

| SNR=15 dB | | |
|---|---|---|
| Algorithm | Transmitted power(watts) | Probability of error |
| MMSE without power constraint | 7.76 | 0.095 |
| MMSE with power constraint | 7.62 | 0.091 |
| MPOE without power constraint | 2.556 | 0.067 |
| MPOE with power constraint | 2.316 | 0.062 |

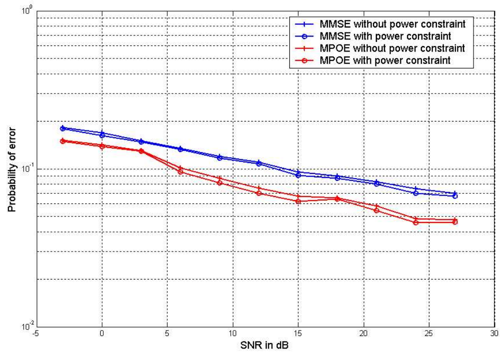


Fig. 2. Probability of error for various SNRs for the statistical channel

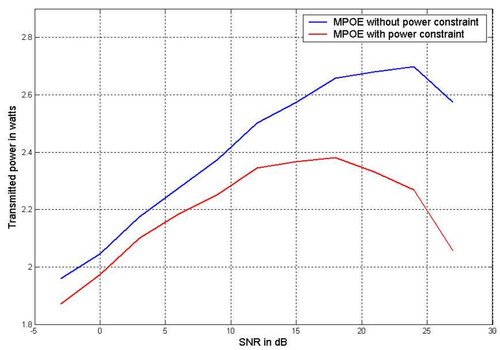


Fig. 3. Trasmitted power of MPOE based prefilter for the statistical channel

## V. CONCLUSION

We have proposed a power constrained scheme in MPOE and MMSE based pre-filtering technique for DS-CDMA system. We do not assume any knowledge about the channel except its statistical parameters. Simulation results show that the power constrained MPOE is superior to both the unconstrained MPOE in terms of power saving and the power constrained MMSE in terms of power saving as well as probability of error.

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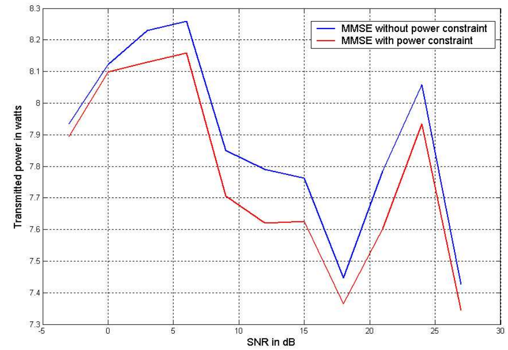


Fig. 4. Trasmitted power of MMSE based prefilter for the statistical channel

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