DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
SEMESTER-END EXAMINATION: AUTUMN SEMESTER 2001

Course Number: EE 603 Course Name: Digital Signal Processing and its applications
Programme: M. Tech./ Dual Degree Microelectronics/ Ph.D. in Elect. Engg. or allied branches.
Date: 23 November 2001   Time: 14:30 – 17:30 (3 hours)
Maximum marks: 80 (Weightage: 40 percent)

Instructions:

1. Each student is permitted to refer only to the following during the examination:
(i) two two-sided A4 sheets of paper with material written on it by him/ her.
(ii) the data on FIR filter design distributed during the lectures.

2. Unless otherwise specified, the sequence x[n] denotes the input to the discrete time system concerned and the sequence y[n] denotes the output of that system.
3. Please begin the solution to each new main question (Q..) on a fresh page of the answer book.

Q1. (16 marks) Find the minimum ORDER (only) of a discrete-time Butterworth bandstop filter to meet the following specifications:

Sampling frequency = 200 kHz; Stopband – 40 kHz to 70 kHz; Transition bands on either side of stopband = 5 kHz; Passband magnitude response must lie between 1 and 0.9; Stopband magnitude response must lie between 0 and 0.1.

Please show all intermediate calculations clearly.

Q2. (20 marks) Figs. 2-1 to 2-4 show four Discrete-Time Fourier Transforms (DTFTs), respectively of the four sequences $h_0[n]$, $h_1[n]$, $h_2[n]$, $h_3[n]$. In each of these DTFTs, the phase response is assumed to be zero for all ω. Assume $\pi > 4\omega_c$.

(a) Obtain $h_0[n]$.
(b) Hence, or otherwise obtain $h_1[n]$. (Hint: convolution in frequency domain)
(c) Hence, or otherwise obtain $h_2[n]$. (Hint: modulation in time domain)
(d) Hence, or otherwise obtain $h_3[n]$.
(e) With $\omega_c = \pi/8$, design a causal linear phase FIR filter of order 4 (impulse response length = 5) to approximate the response of Fig. 2-4 with minimum mean squared error in the frequency response. Write down the difference equation relating the input and output sequences. (Numerical values of coefficients are expected in the description).
(f) Obtain the magnitude of the frequency response of the designed FIR filter at $\omega = 2\omega_c$.

FIG. 2-1

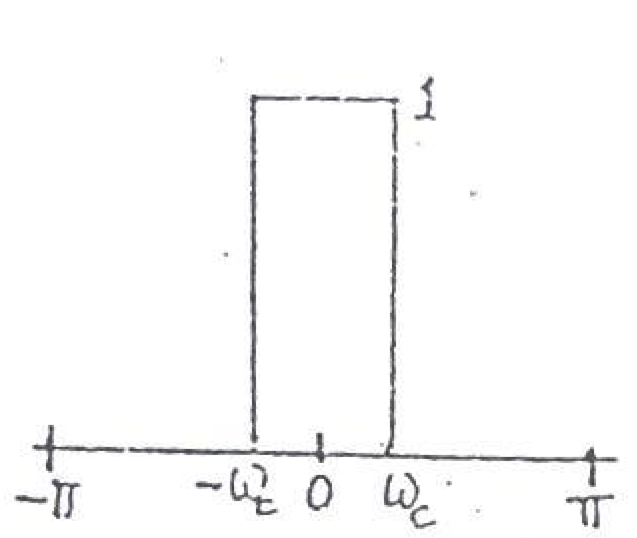


FIG. 2-2

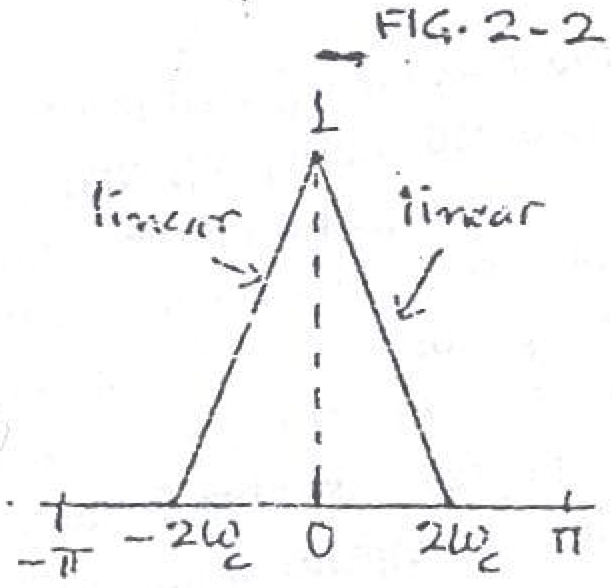


FIG. 2-3

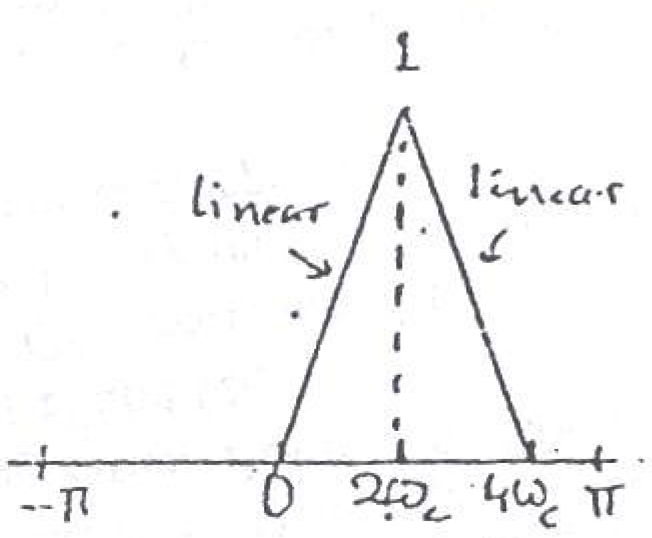


FIG. 2-4

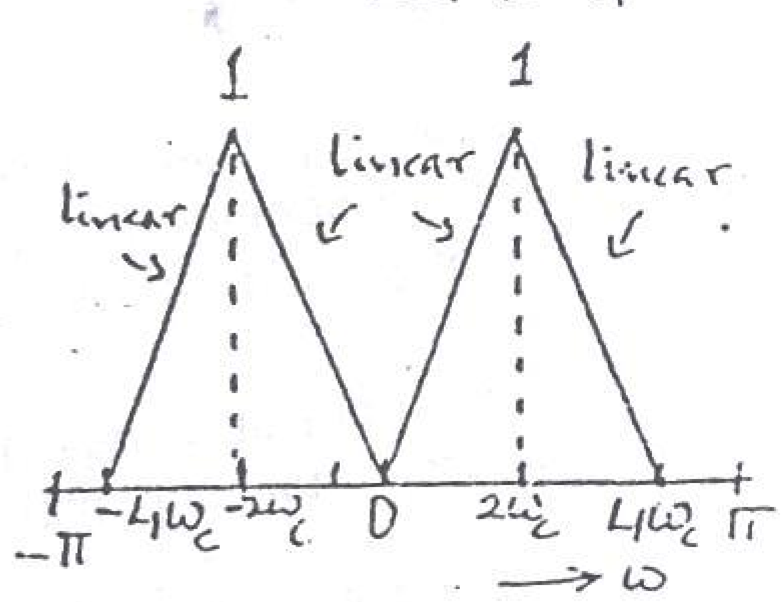
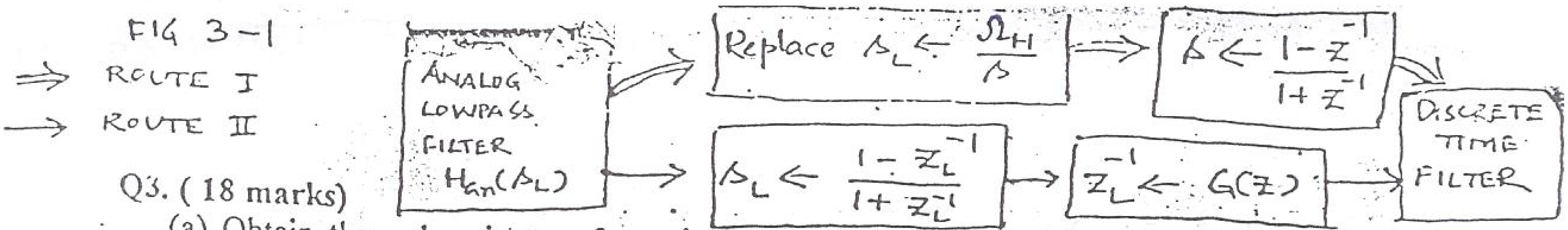


FIG 3-1



Q3. (18 marks)

(a) Obtain the z-domain transformation G(z) in Fig. 3-1, so that the same discrete time filter results on employing either Route I or Route II shown in the figure. Clearly express G(z) in terms of z^{-1} and Ω_H.

(b) Obtain G(z) for $\Omega_H = 200$. Let G(z) correspond to a causal discrete- time system. Does G(z) correspond to a stable system? If so, obtain the magnitude of the frequency response of the system corresponding to G(z). (Hint: what kind of a filter would it be?)

(c) Obtain the lattice parameters K_1, C_0, C_1; for a lattice realization of the system G(z) of part (b) above according to the diagram of Fig. 3-2.

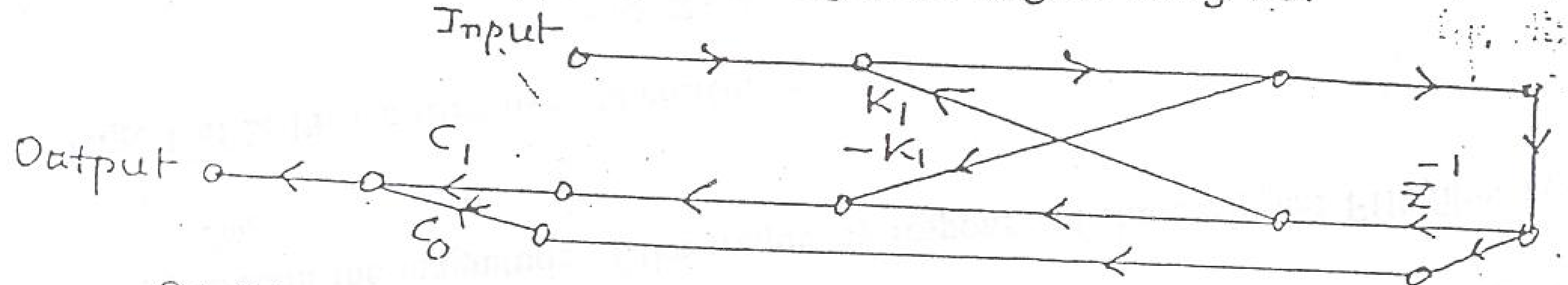


FIG 3-2

Q4. (18 marks) You are provided with the following hardware elements: (i) 3-point DFT blocks; (ii) multiplying elements which multiply by factors of the form $e^{-(jk2\pi/9)}$ for various integers k going from 1 to 9. It is desired to build a hardware module to implement a 9-point Fast Fourier Transform (FFT) efficiently using only these elements, and as few of each of these elements as possible. A part of the hardware module is shown in Fig. 4-1 where the input points to the FFT, x_0 to x_8 are given in decimated order.

(a) Obtain and draw clearly a hardware scheme for implementing the 9-point FFT with the above guidelines, completing the diagram of Fig. 4-1. Note that it is not necessary to keep the output FFT points X_0 to X_8 in order of index – they could appear in any convenient order from top to bottom. The diagram must clearly show the interconnections required and the multiplying elements used as needed. The reasoning must also be shown clearly.

(b) Evaluate the precise number of **nontrivial** (other than +1 or −1) multiplications, and additions, required in each 3-point DFT block for direct implementation of the 3-point DFT.

(c) Hence evaluate the precise number of **nontrivial** multiplications, and additions, required in this 9-point FFT. Compare them with the numbers required for the direct implementation of the 9-point DFT.

Q5. (2+2+4 = 8 marks) Let α be a positive real number. The sequence $g[n] = \alpha^n u[n]$ is applied to a linear, shift-invariant (LSI) discrete time system, and the output sequence produced is denoted by q[n]. Here u[n] denotes the unit step sequence.

(a) Express the unit impulse sequence δ[n] in terms of the sequence g[n].

(b) Hence express the impulse response of this discrete time LSI system in terms of q[n].

(c) For one such discrete time LSI system, it was observed that q[n] = δ[n]. Obtain its unit impulse response, and hence its unit step response (response to a unit step input).

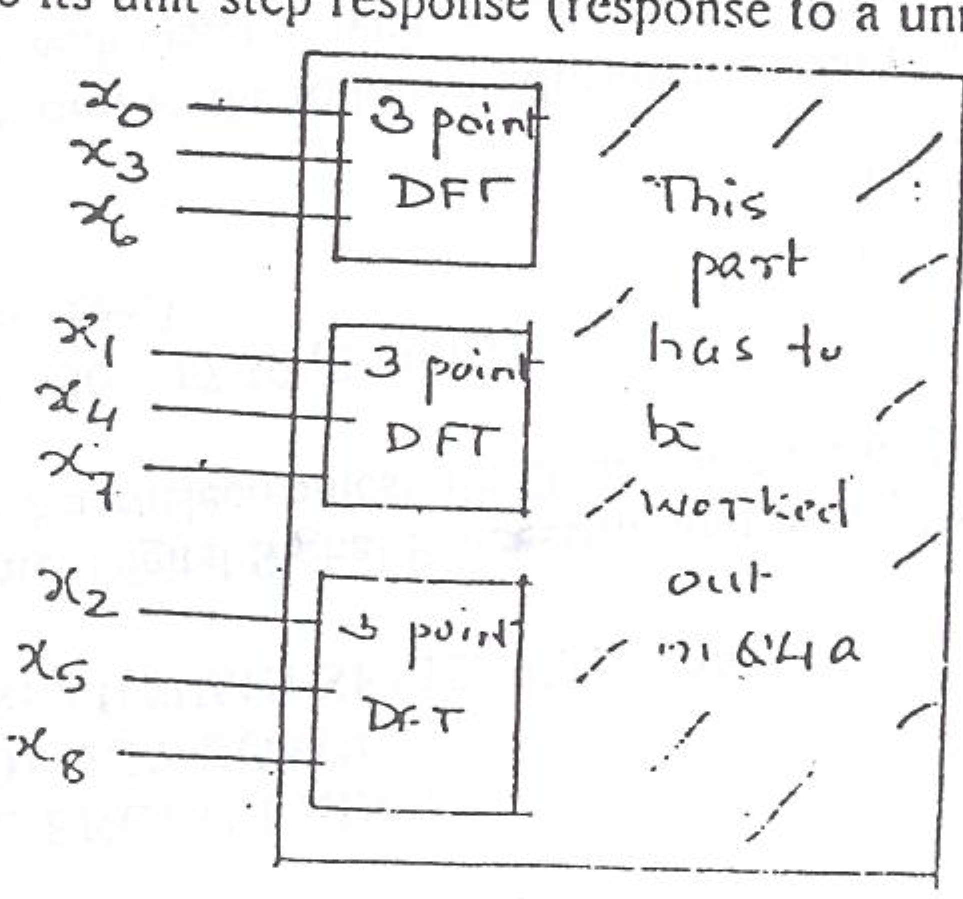


FIG 4-1

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
SEMESTER-END EXAMINATION: AUTUMN SEMESTER 2002

Course Number: EE603
Course Name: Digital Signal Processing and its Applications
Programme: Graduate Level Course in Electrical Engg. or Allied Branches.
Date: 22 November 2002 **Time:** 14:30-17:30
Maximum marks: 80 (Weightage = 40 percent)

Instructions:

1. Each student is permitted to refer only to the following during the examination:
(i) one two-sided A4 sheet of paper with material written on it by him/ her
(ii) all the Tutorial Sheets and associated lecture handouts provided as a set during the course.
2. Unless otherwise specified, the sequence x[n] denotes the input to the discrete time system concerned and the sequence y[n] denotes the output of that system.
3. Please begin the solution to each main question (Q..) on a fresh page of the answer book, even if you happen to attempt a main question in two or more different places.

Q1. (16 marks) Find the minimum ORDER (only) of a discrete-time Butterworth bandstop filter to meet the following specifications:

Sampling frequency = 400 kHz; Stopband = 90 kHz to 150 kHz; Transition bands on either side of stopband = 15 kHz; Passband magnitude response must lie between 1 and 0.9; Stopband magnitude response must lie between 0 and 0.1.

Please show all intermediate calculations clearly; each important intermediate step carries some credit.

Q2. (20 marks) The following blocks are available as ready-made pieces for your use:
(i) two-point Discrete Fourier Transform (DFT) blocks as in Fig. 2-1
(ii) three-point DFT blocks as in Fig. 2-2
(iii) multipliers of the form $\exp(-j2\pi k/6)$; for integer values of $k = 1,2,3,4,5$.

The systems shown in Fig.2-3 and 2-4 correspond to two different 6-point Fast Fourier Transform (FFT algorithms) acting on the 6 input points: $x_0, x_1, x_2, x_3, x_4, x_5$; to produce the output DFT points X_k; k=0..5.

(a) Draw the internal signal flow graph of the 2-point and 3-point DFT blocks in Fig. 2-1 and 2-2. Evaluate the precise number of non-trivial multiplications (other than by 1 and –1), and additions, in each of these blocks.
(b) Complete the block diagram of Fig. 2-3 using only three-point DFT blocks, and, multipliers as described in (iii) above. Draw the completed block diagram showing interconnections clearly.

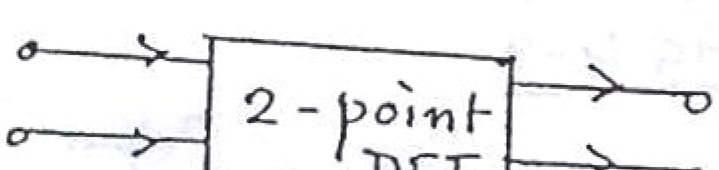


Fig.2-1

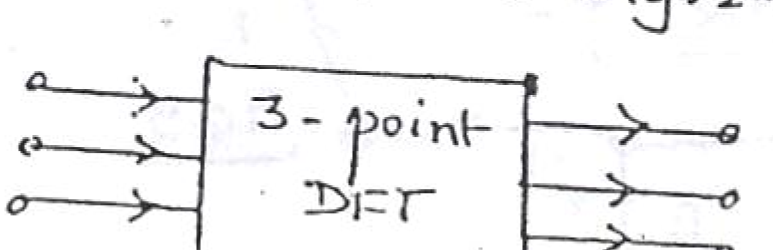


Fig 2-2

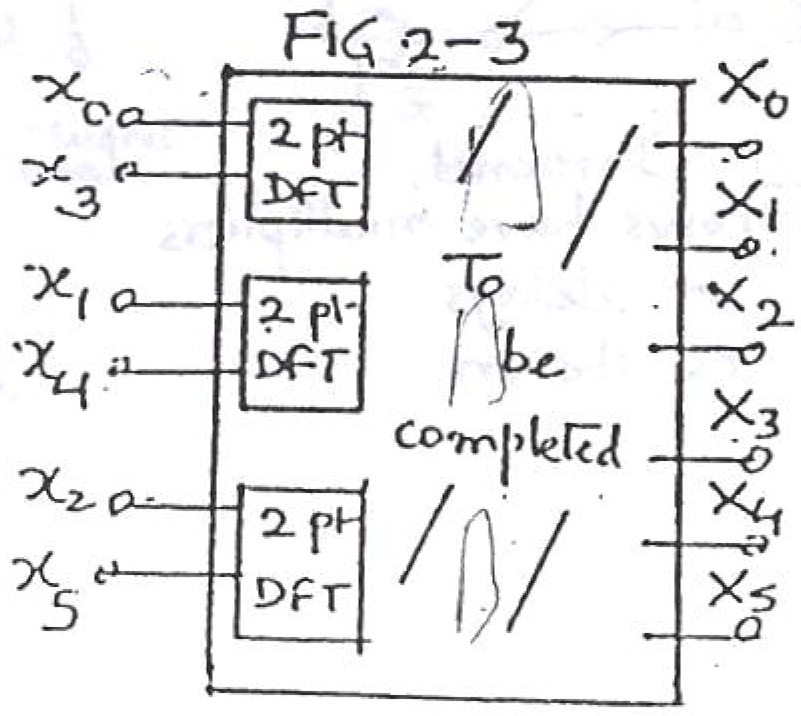
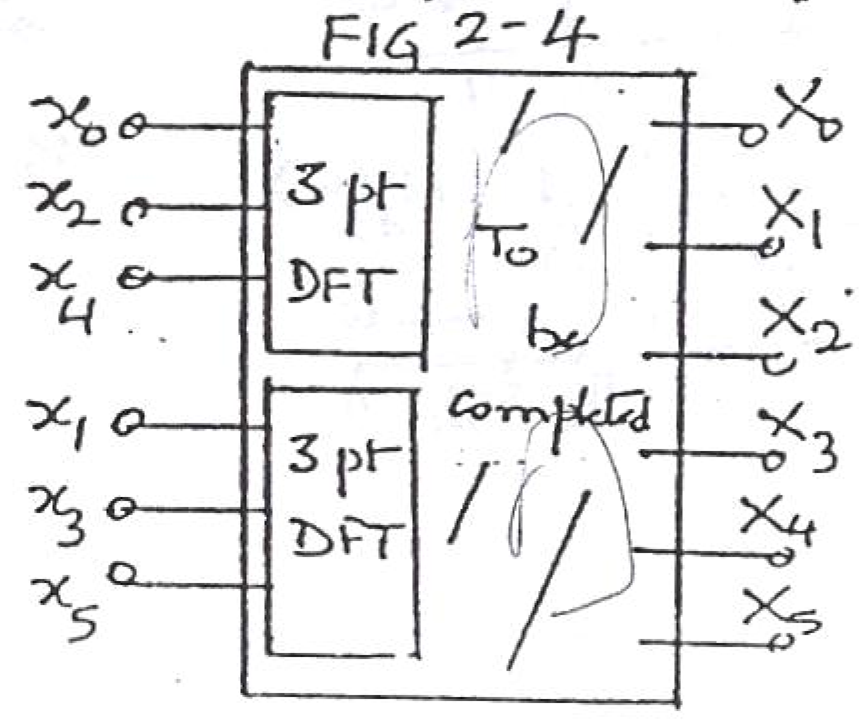
FIG 2-3


FIG 2-4


(c) Complete the block diagram of Fig. 2-4 using only two-point DFT blocks, and, multipliers as described in (iii) above. Draw the completed block diagram showing interconnections clearly.

(d) Calculate the computational complexity of each of the structures of Fig.2-3 and Fig.2-4, in terms of total number of nontrivial multiplications and additions.

Q3. (12 marks)

(a) Obtain the ideal (real) impulse response corresponding to the ideal frequency response $H_{d1}(\omega)$ shown in Fig. 3-1; and hence to $H_{d2}(\omega)$ shown in Fig. 3-2. (Hint: Frequency domain convolution). The phase response of the ideal filters is zero for all frequencies.

(b) Obtain the impulse response of a Finite Impulse Response (FIR) Filter, with impulse response lasting from time index –2 to +2; whose frequency response $H_{FIR}(\omega)$ has a minimum total magnitude squared error from the ideal response $H_{d2}(\omega)$ over the band ω: [$-\pi$ to π]. Please write numerical values for the impulse response samples.

(c) Write down a system description for this FIR filter, after delaying it just sufficiently to make it causal. What effect does this delay have on the frequency response?

Q4. (16 marks)

(a) Write down the transfer function relating the input z-transform X(z) and output z-transform Y(z) in Fig. 4-1 and Fig. 4-2.

(b) Under what conditions will the system of Fig. 4-1 be stable, assuming it is causal? Can its inverse be stable and causal?

(c) Consider the system function $H(z) = (1-2z^{-1})(1-0.5z^{-1}+0.25z^{-2})$. Assume it corresponds to a causal system. Is the system stable? Explain.

(d) Draw a cascaded realization for the system function H(z) of part (c) as a cascade of two structures: one, as in Fig. 4-1 and the other, as in Fig. 4-2. In the cascade, the structure realized according to Fig. 4-2 should correspond to a system, which is stable, and so is its inverse. Give numerical values of the coefficients. Explain.

Q5. (16 marks)

A discrete time, causal, linear shift-invariant system is realized by the structure shown in Fig. 5. The multipliers α and β may be real or complex.

(a) Write down the system function.

(b) Obtain the impulse response of the system.

(c) The impulse response of this system is recorded at time index n=1 and n=2; giving the real values h_1 and h_2 respectively. Obtain α and β in terms of h_1 and h_2.

(d) Hence, or otherwise, obtain the regions in the h_1 - h_2 plane corresponding to each of the cases when the impulse response is:

(i) a sum of two real exponential sequences.

(ii) a sinusoidal sequence with exponentially decaying amplitude.

(iii) a sinusoidal sequence with exponentially growing amplitude.

FIG 3-1

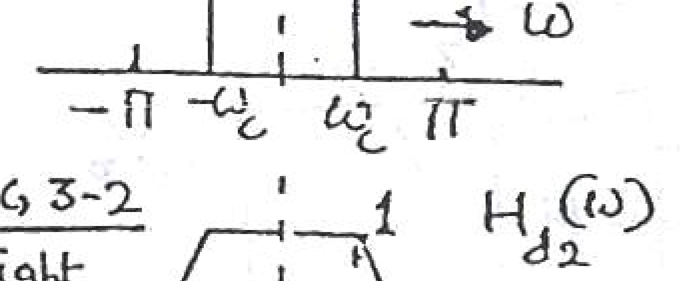
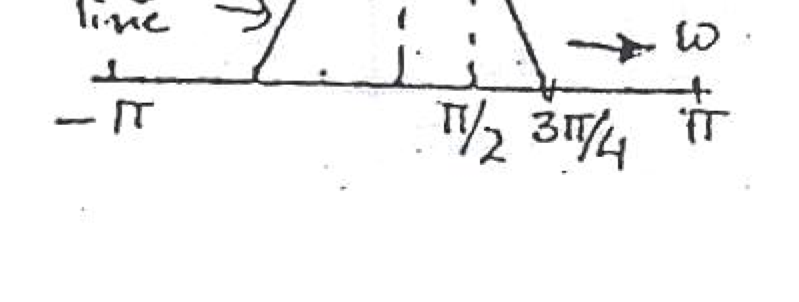


FIG 3-2



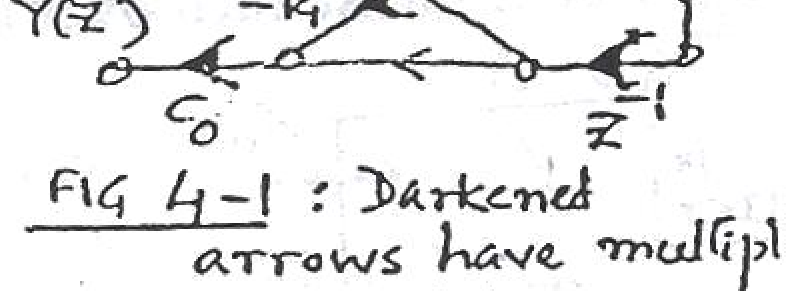


FIG 4-1 : Darkened arrows have multipliers or delays as shown.

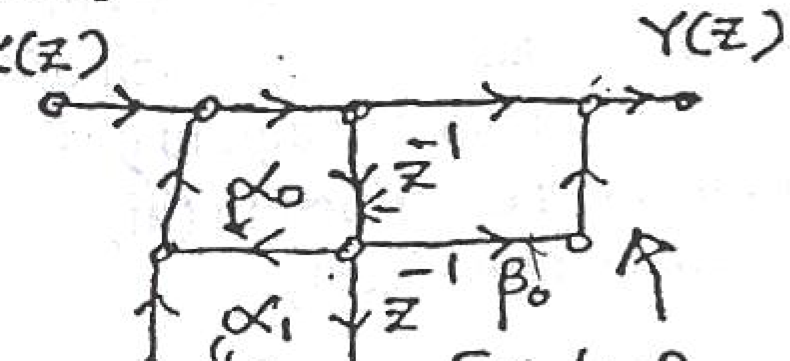


FIG 4-2

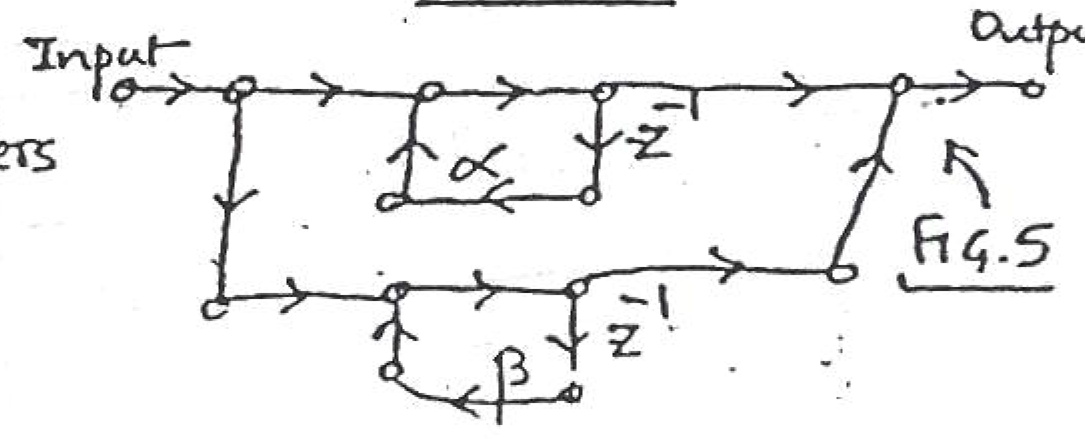


FIG.5

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
MID-SEMESTER EXAMINATION: AUTUMN SEMESTER 2003

Course No: EE 603 **Course Name: Digital Signal Processing and its Applications**
Date: 29 September 2003 (Monday). **Time:** 14:30 – 16:30 hours (2:30 – 4:30 p.m.)
Maximum marks: 50(Weightage = 25 percent)

Instructions:

1. Students may refer to one A4 (two-sided) sheet of paper, with matter written by himself/ herself during the examination.
2. Kindly begin the solution to each main question (Q1, Q2, etc.) on a fresh page of the answer booklet.

Q1. (3+2+6+3+4+3 = 21 marks). Consider the causal, rational, discrete time linear shift-invariant (LSI) system S, described by the system function:

$H(z) = (4 - z^{-1}) / \{(1 - 2z^{-1})(1 - (1/3)z^{-1})\}$.

(a) Show that the system S may be realized by a cascade combination as shown in the Signal Flow Graph (SFG) of Fig. 1-1. Write down a set of values for b_0, a_1 and a_2.
(b) Is the system S stable? Explain.
(c) Show that the system S may be realized as a parallel combination of two rational, causal, first order LSI (sub-)systems: one stable (sub-)system S_s and one unstable (sub-)system S_u. Write down a computable system description for each of these two (sub-)systems. Please note that the outputs of these two (sub-)systems, when added, must result in the output of the system S above. Draw a Signal Flow Graph detailing this parallel combination.
(d) Obtain the impulse responses of the stable and unstable (sub-)systems S_s and S_u.
(e) Obtain the frequency response of the stable (sub-) system S_s, and obtain the output of that (sub-)system to the input $x[n] = \cos(2\pi n/3)$.
(f) Obtain the output of the unstable (sub-)system S_u if the input sequence $x[n] = 3^n$ be applied to it.

Q2. (3+7 = 10 marks). The dynamics of a biological culture in a laboratory are described in the following paragraph:

The mass of the culture is measured every day at a fixed time of the day. The mass is found to be equal to: α times the mass measured the previous day (a consequence of growth over a day), plus the mass introduced externally that day, minus β times the mass measured two days before (a consequence of decay and demise of a portion of the culture)

(a) Express this relation as a difference equation, using y[n] to denote the mass measured on day n, x[n] to denote the externally introduced mass that day.

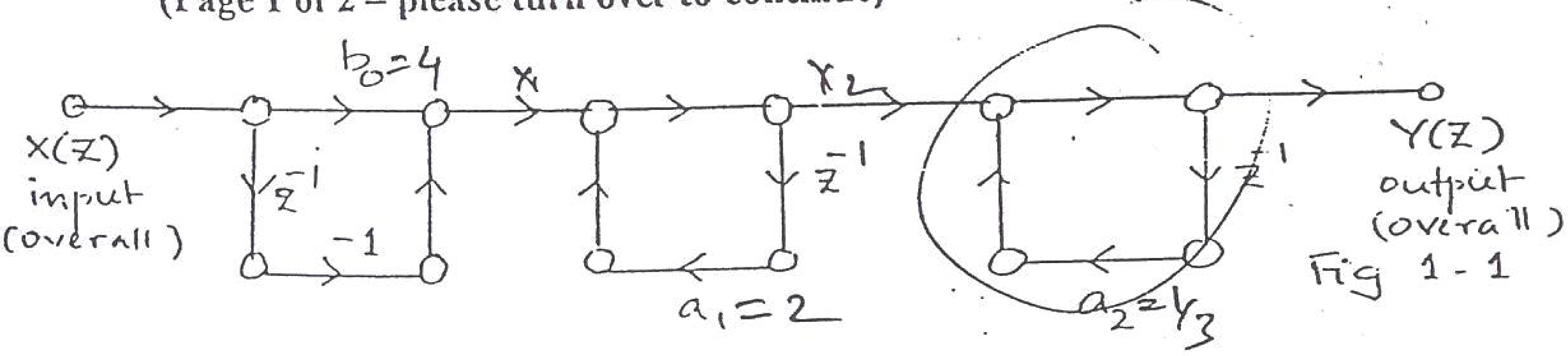


Fig 1 - 1

(b) ~~Let $\beta = 0.49$~~ and $\alpha = 1.4$ for this part. Obtain the mass of the culture from day 0 onwards as a function of the day number n, knowing that the mass measured on day $n = -1$ was 5 units, and that on day $n = -2$ was 4 units. Assume that no external mass is introduced on day 0 or subsequently. Explain briefly, whether the mass is monotonically increasing, monotonically decreasing or neither, as n goes from 0 towards ∞. If neither, comment briefly on how it varies with n.

Q3. (4+3+4 = 11 marks).

A digital differentiator, has the idealized frequency response $H(\omega)$ given by:

$H(\omega) = j\omega$ for $-\pi \leq \omega \leq \pi$.

(a) Obtain the ideal impulse response h[n] of the digital differentiator.
(b) Tabulate the samples of this ideal impulse response for n going from –5 to +5.
(c) Obtain the total energy in the impulse response.

Q4 (3+5 = 8 marks).

(a) Obtain the Discrete-Time Fourier Transform (DTFT) $X(\omega)$ of the sequence: $x[n] = \alpha^n u[n]$. u[n] is the unit step sequence. Assume that α is a real and positive number obeying $0 < \alpha < 1$.

(b) Two sequences $x_1[n]$ and $x_2[n]$ defined, as below, have the respective DTFTs, $X_1(\omega)$ and $X_2(\omega)$:

$x_1[n] = 1$, $n = 0, 1, 2, 3$,and $= 0$ otherwise.

$x_2[n] = 1$, $n = 0, 1, 2$ and $= 0$ otherwise.

Obtain the inverse 3 – point DFT of the 3 – point sequence Y[k], k = 0,1,2 described by: $Y[k] = X_1(2\pi k / 3)\, X_2(2\pi k / 3)$; k = 0,1,2. Explain your reasoning clearly. (Hint: You do not need to carry out any complex number arithmetic at all Simply use the properties of the DFT and inverse DFT).

End of question paper: Best wishes!

4 4 N Jayathri

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT BOMBAY
SEMESTER- END EXAMINATION: AUTUMN SEMESTER 2003

Course No: EE603 (Graduate Level Course)
Course Name: Digital Signal Processing and its applications
Date: 28 November 2003 Time: 14:30 – 17:30 (3 hours)

Maximum marks: 70 = 60 for questions below + 10 for challenging problem attempts (35 percent weightage in total)

Instructions:

1. This question paper has 2 pages including this one.
2. During the examination, you may refer to the printed tutorials/ handouts distributed by the Instructor during the lectures, as also to two additional A4 size sheets of paper with matter written by you, individually, on both sides of each sheet.
3. Kindly begin answering each main question (Q1, Q2 etc.) on a new page of the answer booklet.

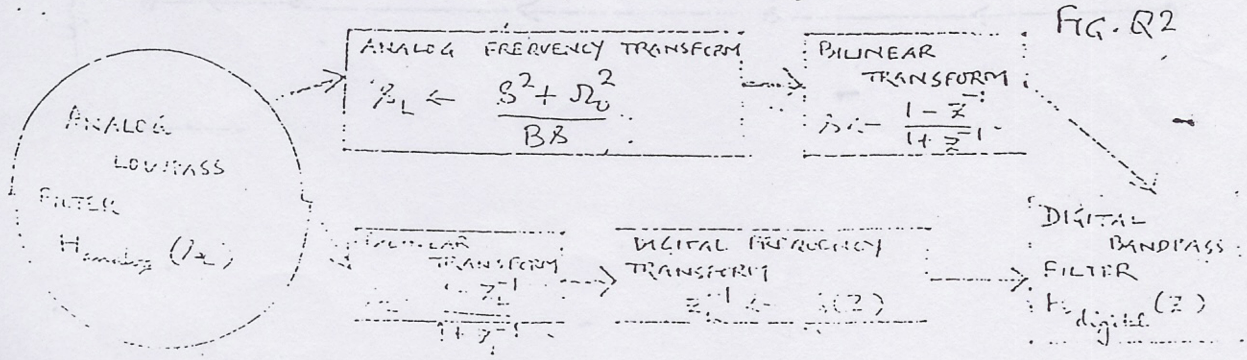
Q0. (10 marks): Challenging problem attempts. Several challenging problems have been announced during the lectures. Some of you have submitted attempts to some of these challenging problems. This question is to recognize these attempts submitted to date, and to award them credit out of 10 marks.

Q1. (12 marks). Draw two neat, labelled signal flow graphs (SFGs), for a 4-point Fast Fourier Transform (FFT) algorithm using Decimation-in-time, and Decimation-in-frequency separately. Work out the precise number of non-trivial multiplications and additions for each of these SFGs. Multiplications by j (square root of –1) are non-trivial. Multiplications by –1 are trivial. Compare with the number of non-trivial (precisely) multiplications and additions for the direct DFT calculation)

Q2. (12 marks). Two different paths are shown, in Fig. Q2, for frequency transformation, from an analog lowpass filter to a digital bandpass filter.

(a) Obtain the transfer function G(z), which will make these two paths equivalent in the transformation. Clearly write G(z) as a ratio of polynomials in z^{-1}, with coefficients in each polynomial clearly calculated.
(b) Can G(z) be the system function of a stable, causal, rational system? If so, what would be the magnitude of its frequency response? Briefly explain.

FIG. Q2



$$y[n] = \alpha\, y[n-1] + C_0\, x[n] - D_0\, x[n-1]$$

where α, C_0 and D_0 are constants. $x[n]$ denotes the input sequence and $y[n]$ the output sequence. When the input sequence $x[n] = \beta^n u[n]$ is applied to the system, the output sequence is $y[n] = A_0\, \alpha^n\, u[n]$, where $u[n]$ denotes the unit step sequence. Here, α is not equal to β.

(a) Obtain C_0 and D_0 in terms of A_0, β and α. Explain your reasoning clearly. $C_0 = A_0$

(b) Obtain the output of the same system when the input sequence is $\alpha^n u[n]$. $D_0 = \beta A_0$

Q4. (12 marks). A Finite Impulse Response (FIR) Filter has a real impulse response $h[n]$, with the non-zero samples of $h[n]$ lying in the range $n = 0,\ldots,4$. The impulse response $h[n]$ exhibits symmetry about $n = 2$; i.e. $h[0] = h[4]$; $h[1] = h[3]$. The frequency response $H(\omega)$ of the FIR filter takes the values:

$H(0) = C_0$, $H(\pi/4) = A_0 \exp(-j2\pi/4)$ and $H(\pi/2) = B_0 \exp(-j2\pi/2)$; where C_0, A_0 and B_0 are real constants.

(a) Obtain the phase response of the FIR filter.

(b) Obtain the frequency response $H(\omega)$ of the FIR filter in terms of only the parameters $h[0]$, $h[1]$ and $h[2]$.

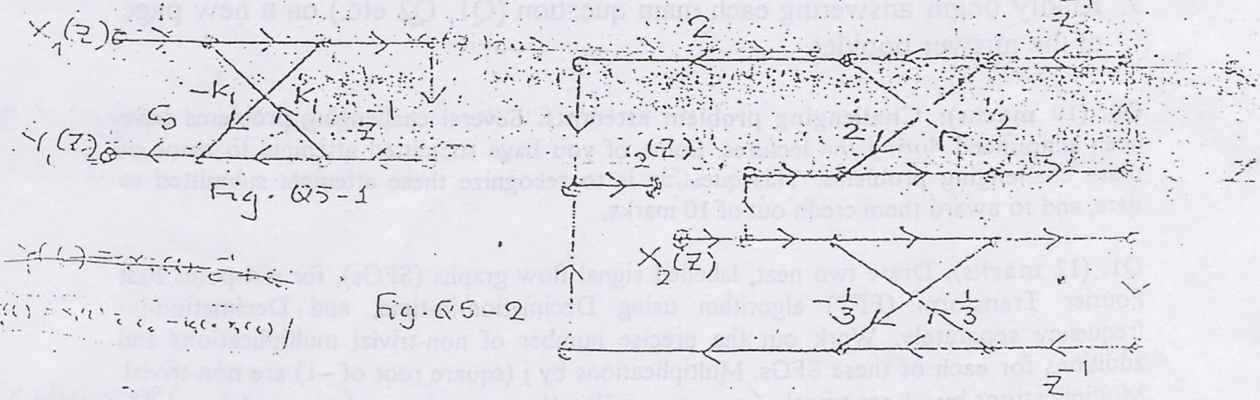
(c) Obtain the impulse response of the filter $h[n]$ in terms of C_0, A_0 and B_0.

Q5. (12 marks).

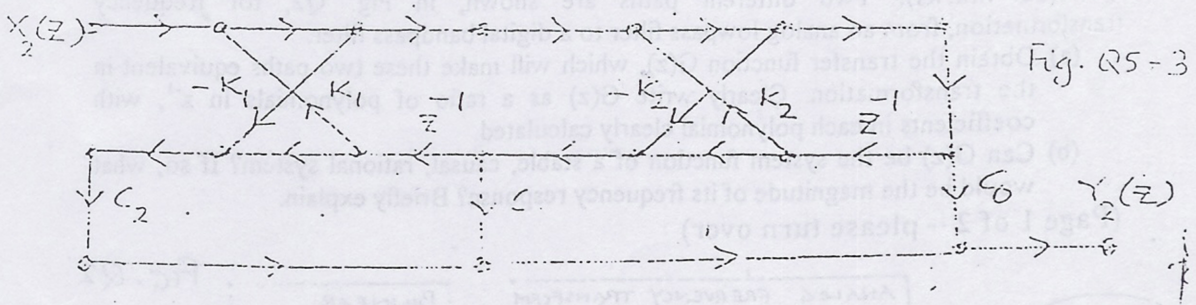
(a) Obtain the transfer function $Y_1(z) / X_1(z)$ in Fig. Q5-1.

(b) Hence, or otherwise, obtain the transfer function $Y_2(z) / X_2(z)$ in Fig. Q5-2.

(c) Realize the transfer function $Y_2(z) / X_2(z)$ of Fig. Q5-2 with the lattice structure of Fig. Q5-3, i.e. obtain the lattice parameters K_1, K_2, C_0, C_1 and C_2 in Fig. Q5-3.



Page 2 of 2: End of question paper- with best wishes.



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DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
MID-SEMESTER EXAMINATION: AUTUMN SEMESTER 2002

Course Number: EE 603 Course Name: Digital Signal Processing and its Applications
Programme: Graduate Level Core/ Elective
Maximum marks: 50 (Weightage: 25 percent)

Instructions:

1. Kindly indicate your reasoning and intermediate steps clearly. Some credit is reserved for intermediate steps even if the problem is not solved completely.
2. Unless otherwise specified, the sequence x[n] denotes the input to the discrete time system concerned and the sequence y[n] denotes the output of that system.

Q1. (12 marks) A linear shift-invariant (LSI) system has the impulse response h[n] described by h[n] = 2, 1, 3, -1 at n = -1, 0, 1, 2 respectively and zero otherwise.

(a) Is the system causal? Explain briefly.

(b) Is the system stable? Explain briefly. $\sum_n |h[n]| < \infty$

(c) Let the input to the system be x[n] = nu[n]. Obtain the output sequence, and describe it clearly.

(d) Let the input to the system be a periodic sequence x[n] with period 3, described by x[0] = 4, x[1] = -5, x[2] = 3. Obtain the output sequence, and describe it clearly.

Q2. (15 marks). The pole-zero plot of a rational system with system function H(z) is shown in Fig. 2. H(1) = 1.

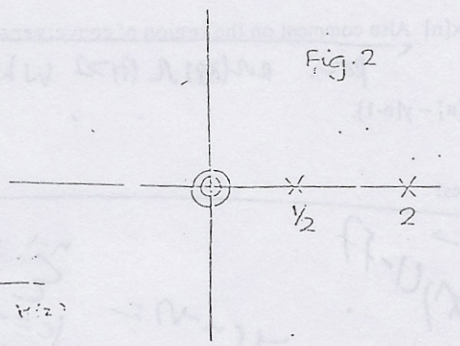
(a) Obtain the system function.

(b) Find the region of convergence (ROC) of H(z) and the impulse response, of the corresponding stable system. To this stable system is applied the input x[n] = sin(0.3πn) for all n. Obtain the output sequence.

(c) Find the region of convergence (ROC) of H(z) and the impulse response, of the corresponding causal system. To this causal system is applied the input $x[n] = 6^n$ for all n. Obtain the output sequence.

Where to be done with H(1)=1 ??

Fig. 2



Page 1 of 2
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84,
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245

text 4.3

Q3. (10 marks).

The input to a causal LSI system is: $x[n] = u[-n-1] + 0.5^n u[n]$. The system output has the z – transform

$$Y(z) = (-0.5z^{-1}) / \{ (1 - 0.5z^{-1})(1 + z^{-1}) \}$$

(a) Determine the system function $H(z)$ and obtain its Region of Convergence (ROC).

(b) Determine $y[n]$, the output sequence, while specifying the ROC of $Y(z)$

Q4 (13 marks).

(a)

Obtain the Discrete Time Fourier Transform (DTFT), $X(e^{j\omega})$ of the sequence $x[n]$ described by

$x[n] = 1, n = 0, 1;$
$= 0$ for all other n.

(b) Hence or otherwise obtain the 6-point Discrete Fourier Transform (DFT) $X_1(k)$; $k=0, 1, 2, 3, 4, 5$; of the following 6-point sequence $x_1[n]$, explaining your reasoning clearly.

$x_1[n] = 1$ for $n = 0, 5$
$= 0$ for $n = 1, 2, 3, 4$

(c) Obtain the inverse DFT of the 6-point DFT $X_2(k)$ described by:

$X_2(k) = X_1(k)\, X(e^{j2\pi k/6})$; $k = 0, 1, 2, 3, 4, 5.$

Explain your reasoning clearly.

(d) Obtain the z- transform $Y(z)$ of the sequence $y[n]$ described by

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

in terms of the z-transform $X(z)$ of $x[n]$. Also comment on the region of convergence of $Y(z)$.

ROC: ROC(X) ∩ |z|>1

Hint: Use convolution, or look at $y[n] - y[n-1]$.

End of question paper. Best wishes!

(3) ROC X(z): $0.5 < |z| < 1$

$\frac{Y(z)}{X(z)} = \frac{-0.5z^{-1}}{(1-0.5z^{-1})(1+z^{-1})} \Big/ \ldots$

$= \frac{1-z^{-1}}{1+z^{-1}}$

$= \frac{1}{1+z^{-1}} - \frac{z^{-1}}{1+z^{-1}}$

$= \frac{1}{1-(-1)z^{-1}} - \ldots$

$h[n] = (-1)^n - (-1)^{n-1}$

$|z| > |-1|$

$y[n-1] = \sum_{k=-\infty}^{n-1} x(k)$

$= \sum_{k=-\infty}^{n} x(k) - x(n)$

$= y(n) - x(n)$

$y(n) = y(n-1) + x(n)$

$Y(z) = z^{-1}Y(z) + X(z)$

$Y(z) = \frac{X(z)}{1-z^{-1}}$

$\frac{Y(z)}{X(z)} = \frac{1}{1-z^{-1}}$, $|z|>1$

6

3

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT BOMBAY
SEMESTER- END EXAMINATION: AUTUMN SEMESTER 2004

Course No: EE603 (Graduate Level Course)
Course Name: Digital Signal Processing and its applications
Date: 22 November 2004 (Monday) Time: 14:30 – 17:30 (3 hours)
Maximum marks: 70 (35 percent weightage)

Instructions:

1. This question paper has 2 pages including this one.
2. During the examination, you may refer to the printed tutorials/ handouts distributed by the Instructor during the lectures, (and anything that you may have written on them yourself), as also to two additional A4 size sheets of paper with matter written by you, individually, on both sides of each sheet.
3. Unless otherwise specified, n refers to the discrete time index, ω to the normalized angular frequency and z to the complex z-transform variable.
4. Please begin the answer to each main question Q1., Q2., etc. on a fresh page of the answer booklet.

Q1. (20 marks)

(a) The ideal frequency response of a discrete time half-band differentiator is described by:

$$H(\omega) = \begin{cases} j\omega; & -\pi/2 \le \omega \le \pi/2; \\ 0; & \pi/2 < \omega < \pi; \\ 0; & -\pi < \omega < -\pi/2. \end{cases}$$

Sketch this ideal frequency response, in magnitude, from $-\pi$ to $+\pi$, and in phase separately, in the interval $-\pi/2$ to $+\pi/2$. (Note that the phase response for $\pi/2 < |\omega| < \pi$ is undefined and irrelevant). Obtain the corresponding ideal impulse response.

(b) Obtain the (non-causal) Finite Impulse Response (FIR) Filter, with an impulse response lasting from n = -3 to n = +3, whose frequency response would approximate the ideal frequency response of Q1(a) above, with the minimum possible mean squared error.

(c) Show that the system function H(z), of this FIR filter, may be expressed as: $H(z) = (z - z^{-1})\, H_0(z)$; where $H_0(z)$ is the system function of a symmetric FIR filter with a real impulse response lasting from n = -2 to n = +2. Obtain $H_0(z)$, and the associated impulse response.

(d) Evaluate the magnitude and phase of the frequency response, of the FIR filter of Q1(b) above, at the normalized angular frequencies $\omega = 0$ and $\omega = \pi$.

Q2. (20 marks)

Systematically derive an inequality, which would specify the minimum order of an inverse Chebyshev lowpass filter, with specifications as follows: Passband edge = ω_p, Stopband Edge = ω_s, both on the normalized angular frequency axis; and $0 < \omega_p < \omega_s < \pi$.

Passband tolerance = δ_1 meaning that the passband magnitude response must be between 1 and 1 - δ_1. Stopband tolerance = δ_2, meaning that the stopband magnitude response must be between 0 and δ_2. The passband must be monotonic, and stopband, equiripple Evaluate the minimum order required, when $\omega_p = 0.5\pi$, $\omega_s = 0.6\pi$, $\delta_1 = 0.1$, $\delta_2 = 0.1$.

Q3. (15 marks)

The system function H(z) of a causal, rational system is given by:

$$H(z) = 1 / \{ (1 - 0.5 z^{-1})^2 (1 - 0.3 z^{-1})^2 \}$$

(a). Decompose H(z) into a sum of two causal, rational system functions: $H_1(z)$ and $H_2(z)$, each of which has a double pole.

(b). Hence, realize H(z) as a parallel combination of two lattice stages of the form of Fig. Q3-1, and obtain the parameters, separately, of each of the two stages in the parallel combination.

Q4. (15 marks)

(a). Obtain the Inverse 3-point Discrete Fourier Transform (3-point Inverse DFT) of the 3-point sequence $\{X[k], k = 0, 1, 2\}$, described by:

$$X[k] = 1 / \{ 1 - 0.5 \exp(-j\omega) \}; \omega = (2\pi k / 3) \text{ for } k = 0, 1, 2.$$

Explain your reasoning clearly. (Hint: Time domain aliasing of underlying sequence)

(b). Draw a neat labeled Signal Flow Graph, for a 3-point Discrete Fourier Transform (DFT) of the 3 input points y[0], y[1], y[2], resulting in the DFT points Y[0], Y[1], Y[2].

(c). Now we use two of these 3-point blocks, to compute a 6-point DFT of the six points $\{y_1[n], n = 0, 1, 2, 3, 4, 5\}$ as follows, with some additional computation. To one of the 3-point DFT blocks, give the inputs $\{y_1[n], n = 0, 2, 4\}$ and let it produce the outputs $\{Y_{11}[k], k = 0, 1, 2\}$. To the other, give the inputs $\{y_1[n], n = 1, 3, 5\}$ and let it produce the outputs $\{Y_{12}[k], k = 0, 1, 2\}$. Draw the complete Signal Flow Graph which would take these points $\{Y_{11}[k], Y_{12}[k], k = 0, 1, 2\}$ as input nodes, and produce as outputs, the 6-point DFT $\{Y_1[k], k = 0, 1, 2, 3, 4, 5\}$ of the original 6-point sequence $\{y_1[n], n = 0, 1, 2, 3, 4, 5\}$. The non-trivial multipliers in this signal flow graph, wherever they occur, may only take the values: $W_6^{-1} = \exp(-j\, 2\pi / 6)$ or $W_6^{-2} = \exp(-j\, 2\pi\, 2 / 6)$.

Page 2 of 2: End of question paper- with best wishes. Please make sure that you have written your roll number on the regular main answer booklet and each supplementary sheet.

Please check the notice board outside Instructor's office for announcements regarding evaluated exam answer scripts. All assignments – group and individual must be turned in latest by 27 November 2004 (Saturday) to earn some credit. Possible penalty for submission of assignments beyond 20 Nov 2004, since the due date was 17 Nov 2004.

EE 603 DIGITAL SIGNAL PROCESSING AND ITS APPLICATIONS
SEMESTER-END EXAMINATION : AUTUMN 2004

FIGURES PAGE

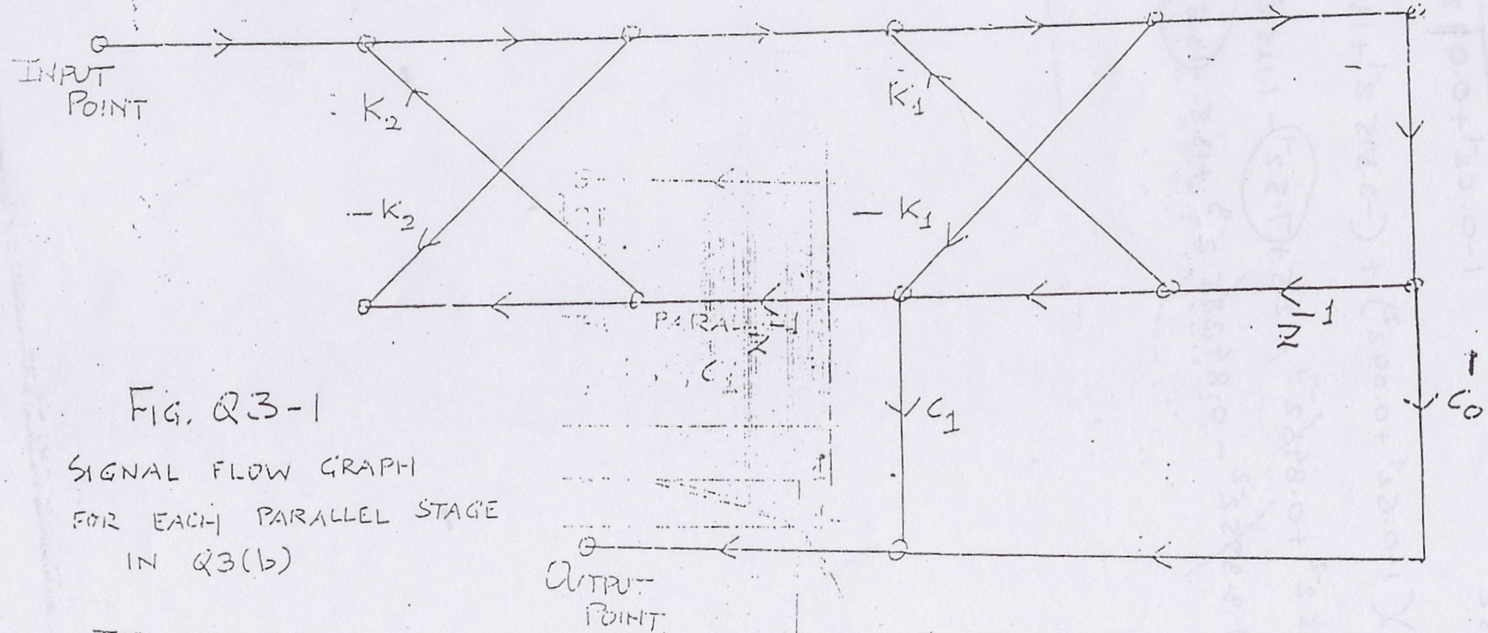
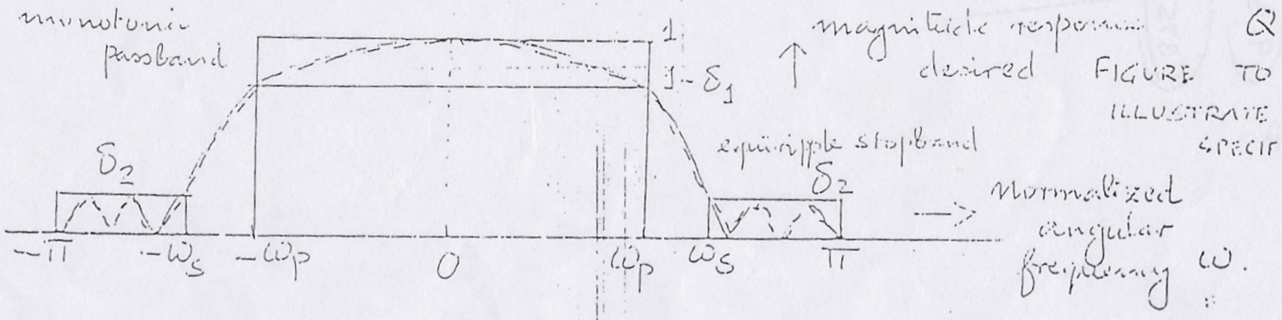


FIG. Q3-1

SIGNAL FLOW GRAPH FOR EACH PARALLEL STAGE IN Q3(b)

THE PARAMETERS FOR ONE SUCH STAGE IN THE PARALLEL COMBINATION REFER TO THE VALUES OF K_1, K_2, C_0, C_1 FOR THAT STAGE



Q2 FIGURE TO ILLUSTRATE SPECIFICATIONS

(8)

(8)

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY
MID-SEMESTER EXAMINATION: AUTUMN SEMESTER 2004

Course No: EE 603 Course Name: Digital Signal Processing and its Applications
Date: 14 September 2004 (Tuesday). Time: 14:30 – 16:30 hours (2:30 – 4:30 p.m.)
Maximum marks: 50(Weightage = 25 percent)

Instructions:

1. This is a closed notes, closed book examination – students may not refer to/ use any literature or data during the examination. Non-programmable calculators may, however, be used.
2. Kindly begin the solution to each main question (Q1, Q2, etc.) on a fresh page of the answer booklet.
3. In any discrete system description, unless otherwise specified: y[n] refers to the output sequence and x[n], to the input sequence.

Q1. (12 marks). A discrete time linear shift invariant system is described by:

$$y[n] = x[n] + x[n-1] + x[n-2]$$

(a) Obtain its unit impulse response.
(b) Obtain its frequency response.
(c) Obtain the output of the system to the input sequence $x[n] = \cos(0.5\pi n)$. Write it clearly as one sampled sinusoid showing the change of amplitude and phase.
(d) Obtain the normalized angular frequency ω_0 for which the output would be the zero sequence, when the input is $\cos(\omega_0 n)$.

$Y(\omega) = \frac{1}{2}\sin\omega_0 n$

$\omega_0 = \pi$ $Y(\omega_0) = 0$

Q2. (20 marks).

(a) Prove that a periodic input to a discrete time shift-invariant system, with period N, results in a periodic output with the same period. (Note: the system may not be linear – the proof must not use linearity).
(b) A discrete time linear shift-invariant (LSI) system S1 has its unit impulse response described by $h[n] = \delta[n] - 4\,\delta[n-1] + 3\,\delta[n-2]$, where $\delta[n]$ is the unit impulse sequence. The input to this system is the periodic input x[n], with period 4, and $x[0] = 5$, $x[1] = 6$, $x[2] = -8$, $x[3] = 10$. Obtain the output sequence.
(c) Define the sequence $p_N(n)$ to be $\Sigma_k\, \delta[n-kN]$; where N is a positive integer, and Σ_k denotes sum over all integers k. Sketch the sequence $p_5(n)$ in the interval –6 to +8.
(d) Prove that any periodic sequence g[n] with period N, may be expressed as a linear combination of $p_N(n-j)$; $j = 0, 1, 2, \ldots N-1$. Illustrate this clearly for the sequence x[n] of Q2(b) above.

(Page 1 of 2 – please turn over to continue to Page 2)

(e) Obtain the output of the system S1 of Q2(b), to the input $p_3(n)$, as defined as per Q2(c). With x[n] defined as in Q2(b), show that the output of the same system S1, to the input $x[n] + p_3(n)$, is periodic and find the period of this output. Note that you do not actually need to evaluate the output to $x[n] + p_3(n)$.

Q3. (6 marks). Obtain the Inverse Discrete Time Fourier Transform (Inverse DTFT) of the (periodic) function $H(\omega)$ of (normalized angular frequency) ω defined by Fig. 3-1 below, in one period of the variable ω: Evaluate the inverse DTFT explicitly for n: $-2, -1, 0, 1, 2$.

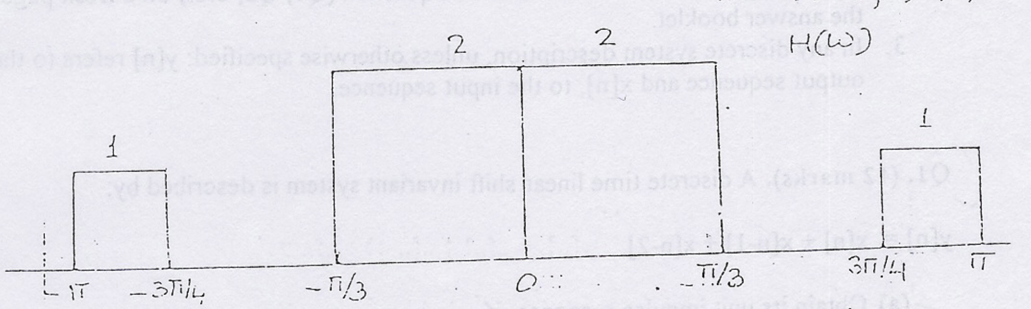


Fig 3-1

$H(\omega)$ is real and even, as a function of ω.

Q4. (12 marks). The system function of a linear shift invariant system S2 is given by: $H_2(z) = \exp(z^{-1})$, with a Region of Convergence $|z| > 0$.

(a) Is the system causal? Explain.
(b) Obtain the unit impulse response $h_2[n]$ of the system.
(c) Obtain the absolute sum of the unit impulse response and hence show that the system is stable. $h(n) = e^{\sum |h(n)|} =$
(d) Let the input to the system S2 be the sequence $h_2[n]$ itself. Obtain the output sequence. Show that this output sequence would also be obtained if the same input sequence $h_2[n]$, were given to another memoryless system S3 described by: $y[n] = q[n]\,x[n]$, where q[n] is a (fixed) sequence, $q[n] = 0$ for all $n < 0$. Find this sequence q[n] and obtain its z-transform.

$q(n) = 2^{n}$

End of question paper: Best wishes!

$$H(z) = e^{z^{-1}}$$

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$= \frac{1}{0!} + \frac{\cancel{e}\,z^{-1}}{1!} + \frac{z^{-2}}{2!} \cdots$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!}\left[z^{-2k}\right]$$

$$\therefore h(n) = \sum_{k=0}^{\infty} \frac{1}{k!}\,\delta(n-k) \Rightarrow \text{stable}$$

$$X(z) = e^{\cancel{z^{-1}}} \quad \therefore Y(z) = H(z)\cdot X(z) = e^{z^{-2}}$$

$$= 1 + \frac{z^{-2}}{1!} + $$

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EXTRA

N Gayathri ✓

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT BOMBAY
SEMESTER- END EXAMINATION: AUTUMN SEMESTER 2003

Course No: EE603 (Graduate Level Course)
Course Name: Digital Signal Processing and its applications
Date: 28 November 2003 Time: 14:30 – 17:30 (3 hours)

Maximum marks: 70 = 60 for questions below + 10 for challenging problem attempts (35 percent weightage in total)

Instructions:

1. This question paper has 2 pages including this one.
2. During the examination, you may refer to the printed tutorials/ handouts distributed by the Instructor during the lectures, as also to two additional A4 size sheets of paper with matter written by you, individually, on both sides of each sheet.
3. Kindly begin answering each main question (Q1, Q2 etc.) on a new page of the answer booklet.

Q0. (10 marks): Challenging problem attempts. Several challenging problems have been announced during the lectures. Some of you have submitted attempts to some of these challenging problems. This question is to recognize these attempts submitted to date, and to award them credit out of 10 marks.

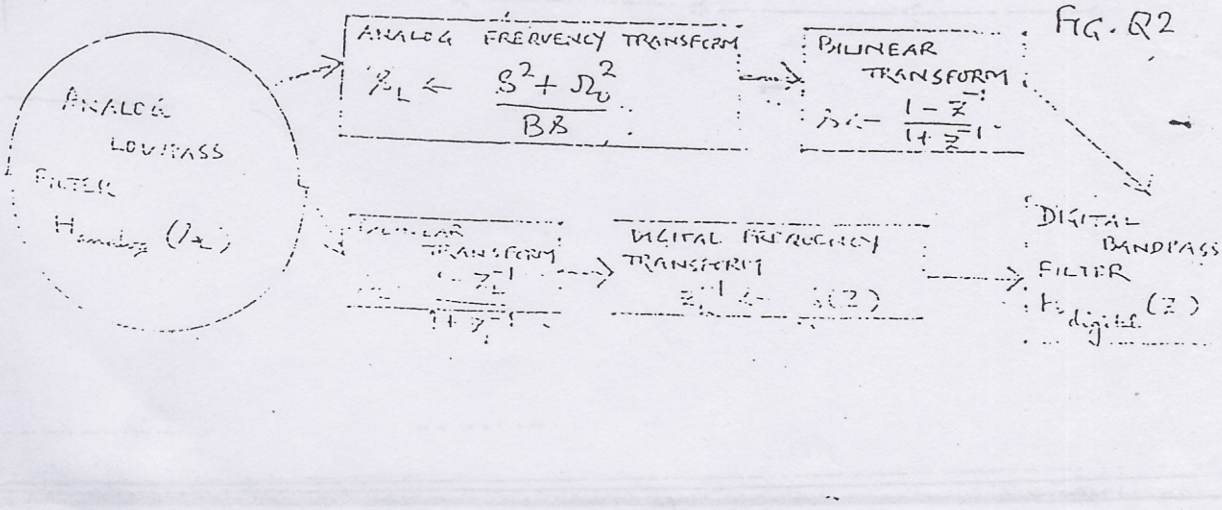
Q1. (12 marks). Draw two neat, labelled signal flow graphs (SFGs), for a 4-point Fast Fourier Transform (FFT) algorithm using Decimation-in-time, and Decimation-in-frequency separately. Work out the precise number of non-trivial multiplications and additions for each of these SFGs. Multiplications by j (square root of –1) are non-trivial. Multiplications by –1 are trivial. Compare with the number of non-trivial (precisely) multiplications and additions for the direct DFT calculation)

Q2. (12 marks). Two different paths are shown, in Fig. Q2, for frequency transformation, from an analog lowpass filter to a digital bandpass filter.

(a) Obtain the transfer function G(z), which will make these two paths equivalent in the transformation. Clearly write G(z) as a ratio of polynomials in z^{-1}, with coefficients in each polynomial clearly calculated.
(b) Can G(z) be the system function of a stable, causal, rational system? If so, what would be the magnitude of its frequency response? Briefly explain.

(Page 1 of 2 -- please turn over)

FIG. Q2



$$y[n] = \alpha\, y[n-1] + C_0\, x[n] - D_0\, x[n-1]$$

where α, C_0 and D_0 are constants, $x[n]$ denotes the input sequence and $y[n]$ the output sequence. When the input sequence $x[n] = \beta^n u[n]$ is applied to the system, the output sequence is $y[n] = A_0\, \alpha^n\, u[n]$, where $u[n]$ denotes the unit step sequence. Here, α is not equal to β.

(a) Obtain C_0 and D_0 in terms of A_0, β and α. Explain your reasoning clearly. $C_0 = A_0$, $D_0 = \beta A_0$

(b) Obtain the output of the same system when the input sequence is $\alpha^n u[n]$.

Q4. (12 marks). A Finite Impulse Response (FIR) Filter has a real impulse response $h[n]$, with the non-zero samples of $h[n]$ lying in the range $n = 0,\ldots,4$. The impulse response $h[n]$ exhibits symmetry about $n = 2$; i.e. $h[0] = h[4]$; $h[1] = h[3]$. The frequency response $H(\omega)$ of the FIR filter takes the values:
$H(0) = C_0$, $H(\pi/4) = A_0 \exp(-j2\pi/4)$ and $H(\pi/2) = B_0 \exp(-j2\pi/2)$; where C_0, A_0 and B_0 are real constants.

(a) Obtain the phase response of the FIR filter.

(b) Obtain the frequency response $H(\omega)$ of the FIR filter in terms of only the parameters $h[0]$, $h[1]$ and $h[2]$.

(c) Obtain the impulse response of the filter $h[n]$ in terms of C_0, A_0 and B_0.

Q5. (12 marks).

(a) Obtain the transfer function $Y_1(z) / X_1(z)$ in Fig. Q5-1.

(b) Hence, or otherwise, obtain the transfer function $Y_2(z) / X_2(z)$ in Fig. Q5-2.

(c) Realize the transfer function $Y_2(z) / X_2(z)$ of Fig. Q5-2 with the lattice structure of Fig. Q5-3, i.e. obtain the lattice parameters K_1, K_2, C_0, C_1 and C_2 in Fig. Q5-3.

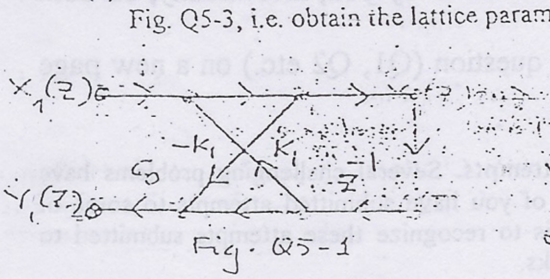


Fig Q5-1

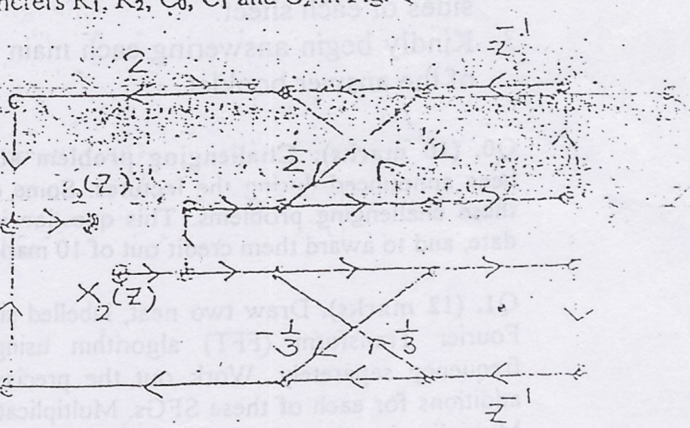


Fig Q5-2

Page 2 of 2: End of question paper- with best wishes.

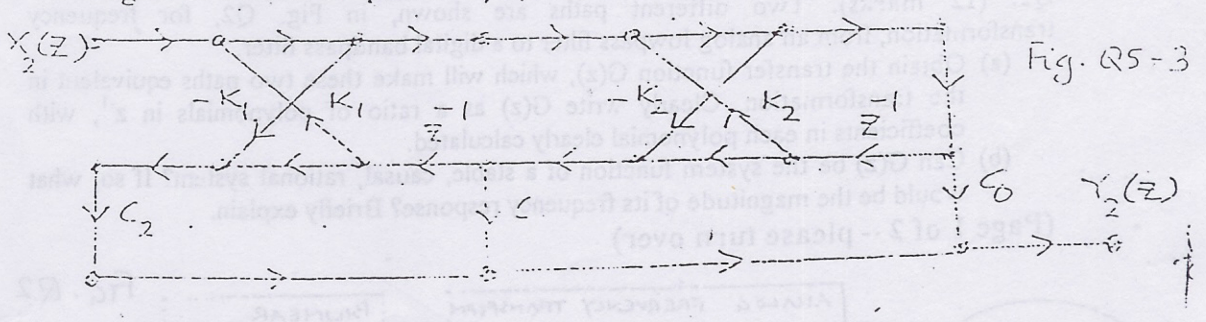


Fig Q5-3

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DEPARTMENT OF ELECTRICAL ENGINEERING, IIT BOMBAY

AUTUMN SEMESTER 2006-07 : MID-SEMESTER EXAMINATION.

COURSE NO : EE 603 COURSE NAME: Digital Signal Processing and its Applications

DATE : 11 September 2006 TIME : 09:30 – 11:30 a.m.

MAXIMUM MARKS = 50 (WEIGHTAGE = 25 percent)

INSTRUCTIONS :

1. This is a closed-book, closed-notes examination.
2. Scientific, non-programmable calculators may be used.
3. Please begin answering each MAIN question: Q1, 2 etc. on a fresh page of the answer booklet.

Q1 - A linear, shift-invariant, causal system is described by the difference equation: (15 marks)

$$y[n] = \frac{3}{4} y[n-1] - \frac{1}{8} y[n-2] + x[n]$$ $A(z-\frac{1}{4}) + B(z-\frac{1}{2}) = 2$ $B(-\frac{1}{4}) = \frac{1}{4}$

with $x[n]$ = input sequence, $y[n]$ = output sequence. $B=-1$

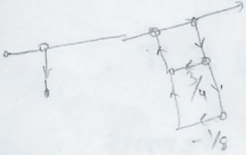
(a) Obtain its system function, specifying the region of convergence clearly. $A(\frac{1}{4}) = \frac{1}{2}$

(b) Obtain its impulse response. $a^n u(n)$ $\sum_{n=0}^{\infty} a^n z^{-n}$

(c) Is the system stable? Explain.

$\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$ (d) Draw a realization of this system in Direct Form I. $\frac{1}{1-az^{-1}}$

(e) Show that this system may be realized with two linear shift-invariant systems, both causal, placed in cascade; both described by a difference equation of the form: $y[n] = \alpha y[n-1] + x[n]$, with α constant. Obtain α for the two systems in cascade.



Q2 - (a) For a system which is homogeneous and shift-invariant, show that: If the input $x[n]$ obeys $x[n] = -x[n+N]$ AND $x[n] = x[n+2N]$ for all integer n, for some positive integer N; then so does the output $y[n]$. (10 marks)

$a^n e^{-j\omega n}$ $ae^{-j\omega n}$ $(ae^{-j\omega})^n$ $a^n x[n]$

(b) The input $x[n]$ to a linear shift-invariant system with impulse response $h[n] = \underset{\uparrow 0}{2} \; -1 \; 0 \; 3$, is periodic with period 8, and its 8 samples for $n=0$ to $n=7$ are described by:

$n \to$	0	1	2	3	4	5	6	7
$x[n] \to$	3	−9	6	12	−3	9	−6	−12

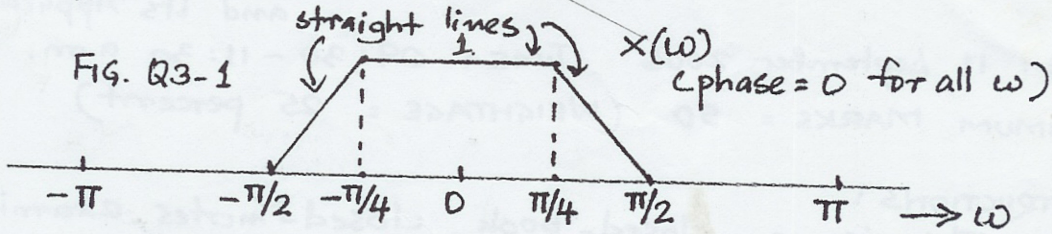
Obtain the output, $y[n]$. [PAGE 1/2 : please turn over to continue ...]

$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

Q3 - The Discrete Time Fourier Transform (DTFT) $X(\omega)$ of the sequence $x[n]$ is shown in Fig. Q3-1.

15 marks

FIG. Q3-1



(a) Obtain and sketch the DTFT of the sequence $(-1)^n x[n]$.

(b) Consider the sequence
$$s[n] \begin{cases} = x[n], & \text{for all even } n \\ = 0, & \text{for all odd } n \end{cases}$$

Express $s[n]$ in terms of $x[n]$ and $(-1)^n x[n]$.

(c) Hence obtain the DTFT of $s[n]$.

(d) Evaluate the sums: $\sum_{n=-\infty}^{+\infty} |x[n]|^2$ and $\sum_{n=-\infty}^{+\infty} |s[n]|^2$

Q4 - We investigate, in this question, the importance of poles on the unit circle.

The difference equation: applicable only for $n \geq 0$:

10 marks

$$y[n] = (2\cos\theta)\, y[n-1] - y[n-2] + x[n]$$

generates a sequence $y[n]$ with a 'driving' sequence $x[n]$ for $n \geq 0$, and initial conditions: $y[-1] = A$, $y[-2] = B$. ; and it is known $-\pi \leq \theta \leq \pi$.

(a) With $x[n] = 0$ for all $n \geq 0$, obtain $y[n]$ in terms of A and B. Assume that A and B are real. Explain why this is called a digital oscillator.

(b) What would be the nature of the 'output' $y[n]$ with 'input' $x[n] = \cos\theta n$, $n \geq 0$? Hence comment on the stability of the causal, linear, shift-invariant system described by the difference equation of this question. (then applicable for all n)

[PAGE 2/2 - End of question paper. With best wishes!]

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DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

SEMESTER-END EXAMINATION: AUTUMN SEMESTER 2005

Course Number: EE 603 **Level:** Graduate Elective
Course Name: Digital Signal Processing and its Applications
Date: 23 November 2005 **Time:** 14:30 to 17:30 hours (3 hours)
Maximum marks = 70 (35 percent weightage)

Instructions:

1. This question paper has three (3) pages. All the figures are on Page 3.
2. During this examination, the candidate may refer to: (i) upto two A4 sheets (4 sides) of paper, with matter written by the candidate himself/ herself. (ii) the tables and data on FIR filter design provided as a part of the course material.
3. Please begin the answer to each main question (Q1, Q2 etc.) on a fresh page of the answer booklet.
4. Please show the important intermediate steps clearly in problem solutions.

Q1. (6 + 6 + 5 = 17 marks)

(a) The ideal frequency response $H(e^{j\omega})$ desired from a discrete time system, with a real impulse response h[n], is shown in magnitude and phase, in Fig. Q1-1 and Fig. Q1-2 respectively for the normalized angular frequency range: $\omega \in]0, \pi[$. It is described by: $H(e^{j\omega}) = j$, $0 < \omega \leq (\pi/3)$, $H(e^{j\omega}) = 2j$, $(2\pi/3) \leq \omega \leq (3\pi/4)$, and $H(e^{j\omega}) = 0$, for the rest of the region of ω between 0 and π. Obtain h[n].

(b) Use the rectangular window, to truncate the ideal impulse response of Q1-(a), between $n = -3$ and $n = +3$. Obtain, numerically these 7 impulse response samples. Let these constitute the impulse response of a Finite Impulse Response (FIR) filter. Now obtain the impulse response of another FIR filter, with impulse response extending from $n = 0$ to $n = 6$, whose frequency response is identical to this one, except for an additional linear phase term. Realize this filter using at most 6 delays, at most 4 multipliers and a minimum possible number of adder/ subtractor units. Each adder/ subtractor unit (Fig. Q1-3) can either add or subtract two inputs as required – depending on a third binary control input (1 for subtract, 0 for add). Indicate whether add or subtract is required, for each such unit used.

(c) Obtain the sequence $h_1[n]$, such that $h[n] = h_1[n] - h_1[n-2]$, and $h_1[n]$ is a sequence, whose non-zero samples are confined to the range $n = 0$ to $n = 4$, symmetric about $n = 2$.

Q2. (6 + 4 + 7 = 17 marks).

(a) An analog filter $H_{analog}(s_L)$ may be converted to an equivalent discrete time filter by two equivalent routes as shown in Fig. Q2-1. Obtain the rational function G(z) of the variable z, which will ensure that these two routes result in the same discrete time filter.

(b) Regard G(z) as the system function of a causal, discrete time, linear, shift-invariant system. Does it correspond to a stable system? Explain. If it does, obtain the magnitude of its frequency response as a function of the normalized angular frequency ω and comment on what kind of filter it constitutes.

(c) Obtain the system function relating X(z) to Y(z) in Fig. Q2-2. Show that G(z) of Q2-(b) may be realized by a Signal Flow Graph of the form of Fig. Q2-2, by an appropriate choice of C_0, K_1 and K_2. Find these values of C_0, K_1 and K_2.

Q3. (8 + 5 + 5 = 18 marks).

(a) Let two sequences a[n] and b[n] have the respective z-transforms: $A(z) = (1 - 0.5z^{-1})^{-1}$ and $B(z) = (1 - 0.25z^{-1})^{-1}$ where the Regions of Convergence of both A(z) and B(z) include the unit circle. Obtain a closed form expression for the Inverse 3-point Discrete Fourier Transform x[n], n = 0, 1, 2; of the 3 points X[k], k = 0, 1, 2; described by $X[k] = A(\exp((j2\pi k)/3))\, B(\exp((j2\pi k)/3))$, k = 0, 1, 2. Hence, or otherwise show that the 3-point sequence x[n], n = 0, 1, 2 is a linear combination of the two 3-point sequences a[n], n = 0, 1, 2; and b[n]; n = 0, 1, 2.

(b) Draw the signal flow graph of a 3-point Discrete Fourier Transform (DFT) of the three points y[0], y[1], y[2] resulting in the three points Y[0], Y[1], Y[2]. Calculate the precise number of *non-trivial* multiplications and additions in this 3-point DFT.

(c) Now let us treat the 3-point DFT of Q3-(b) as a block. Show clearly, by completing with interconnections, multiplies, and labels; the block diagram of Fig. Q3-1, how 6 such blocks, along with some external non-trivial multiplications can be used to realize a 9-point Fast Fourier Transform {X[k], k = 0, ..., 8} of the 9 points {x[n], n = 0, 1, 2, ..., 8}.

Q4. (3 + 4 + 4 + 7 = 18 marks). Consider the causal, discrete time, linear, shift-invariant (LSI) system realized in the signal flow graph of Fig. Q4-1. The multipliers α and β are assumed to be either real, or, complex conjugates.

(a) Obtain the system function of this LSI system.

(b) Obtain the impulse response h[n] of this LSI system – writing the expression for h[n] in such a way that it is applicable to both cases: $\alpha \neq \beta$; $\alpha = \beta$. (Hint: use convolution.)

(c) The impulse response of this system is recorded at time index n = 1 and n = 2, giving the real values h_1 and h_2. Write down the expressions relating h_1 and h_2, to α and β.

(d) Hence, or otherwise calculate the expressions for the values of α and β, in terms of the values of h_1 and h_2 observed. Identify the condition(s) on h_1 and h_2 for distinguishing the two cases: $\alpha \neq \beta$, $\alpha = \beta$. For the case $\alpha \neq \beta$, identify the condition(s) for distinguishing between real, and complex conjugate, α and β.

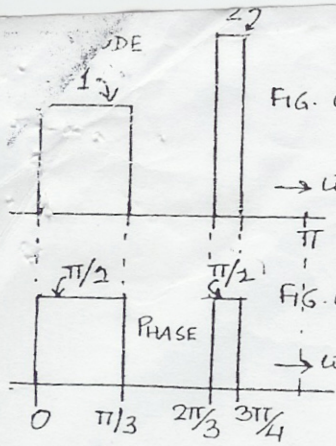

FIG. Q1-1

FIG. Q1-2

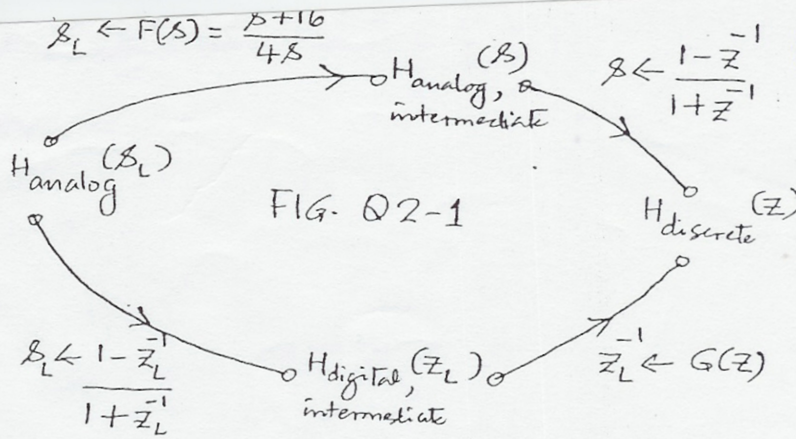

FIG. Q2-1

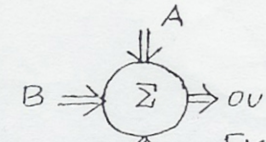

FIG. Q1-3

CONTROL INPUT

1 ⇒ A − B = OUT

0 ⇒ A + B = OUT

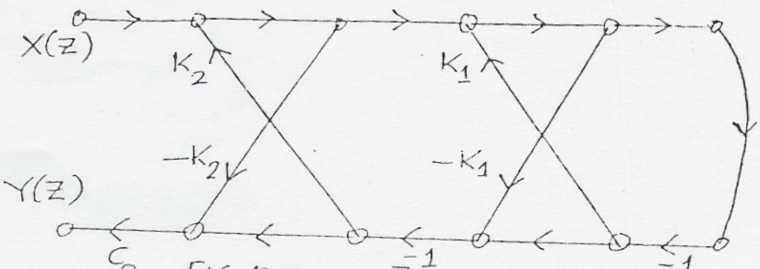

FIG. Q2-2

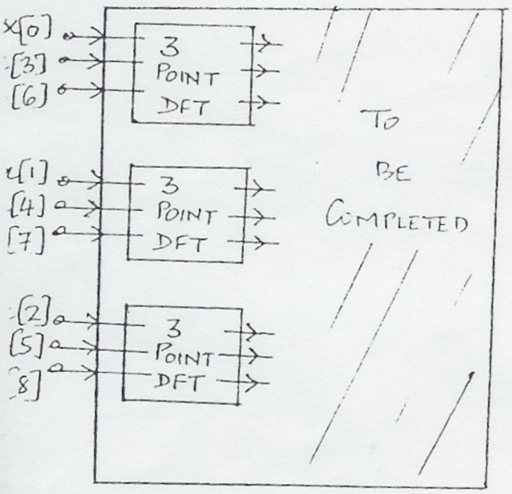

FIG. Q3-1

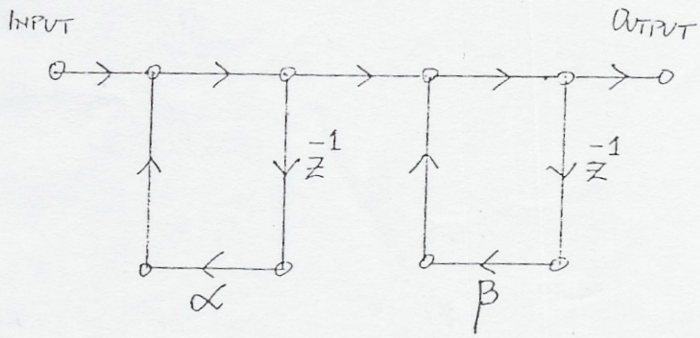

FIG. Q4-1

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FIGURES

SEMESTER - END EXAMINATION

AUTUMN SEMESTER 2005

EE603 DIGITAL SIGNAL PROCESSING AND ITS APPLICATIONS

(18)

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

SEMESTER-END EXAMINATION: AUTUMN SEMESTER 2006-07

Course Number: EE 603 **Level:** Graduate Elective
Course Name: Digital Signal Processing and its Applications
Date: 20 November 2006 (Monday) **Time:** 09:30 to 12:30 hours (3 hours)
Maximum marks = 70 (35 percent weightage)

Instructions:

1. This question paper has three (3) pages. All the figures are on Page 3.
2. During this examination, the candidate may refer to: (i) upto one A4 sheet (2 sides) of paper, with matter written by the candidate himself/ herself. (ii) the course handouts, design tables and tutorial sheets provided to all students.
3. Please begin the answer to each main question (Q1, Q2, etc.) on a fresh page of the answer booklet.
4. Please show the important intermediate steps clearly in problem solutions.

Q1. (20 marks)

(a) It is desired to design a digital lowpass filter, with the following specifications on the normalized angular frequency axis: Passband edge = $\pi/4$; stopband edge = $\pi/3$; passband magnitude response to lie between 1 and 0.9; stopband magnitude response to lie between 0 and 0.1. (*Please note*: magnitude, not magnitude squared !)
Obtain the minimum **order** (only) of: (i) a digital Butterworth lowpass filter (ii) a digital Chebyschev lowpass filter, which will meet these specifications.

(b) An analog lowpass filter $H_{an}(s_L)$ has a magnitude response as drawn in Fig. Q1-1. The passband edge = 1 angular frequency unit, stopband edge = 1.3 angular frequency units, passband magnitude response lies between 1 and 0.8; stopband magnitude response lies between 0 and 0.2. Obtain, and sketch, the magnitude response of an analog filter $H_{final}(s)$ with the most stringent possible specifications, resulting from the following sequence of transformations: The variable s_L is replaced by the function $(1 / s_1)$ to obtain an intermediate analog filter $H_{int}(s_1)$. Then, the variable s_1 in $H_{int}(s_1)$ is replaced by the function $((s^2 + 16) / (2s))$ to obtain the analog filter $H_{final}(s)$. What kind of filter is $H_{final}(s)$? In the sketch, show the numerical values of the edges and tolerances of the passband and stopband clearly.

Q2. (20 marks)

(a) Obtain the ideal impulse response h[n] of a digital Hilbert transformer: i.e. a discrete time filter, with the frequency response described by:
$H(\omega) = -j$ for $-\pi < \omega < 0$; $H(\omega) = +j$ for $0 < \omega < +\pi$. The magnitude and phase of $H(\omega)$ are sketched in Fig. Q2-1 for clarity.

(Page 1 of 3 – please turn over to Page 2 to continue …)

(b) A non-causal Finite Impulse Response (FIR) Filter has impulse response $h_{FIR}[n]$ with the following properties: $h_{FIR}[n]$ is nonzero only in the range $-3 \leq n \leq +3$, and $h_{FIR}[n] = h[n]$ for $-3 \leq n \leq +3$, where h[n] is the ideal impulse response obtained in Q2 – (a). Show that the frequency response of this FIR filter may be expressed as $H_{FIR}(\omega) = \sin(\omega)\, P(\omega)$, where $P(\omega)$ is a polynomial in $\cos(\omega)$. Obtain the polynomial $P(\omega)$.

(c) Obtain the precise number of non-trivial multiplications, and two-input additions, required to evaluate a 5 – point Discrete Fourier Transform (DFT) directly (i.e. using the expression for a DFT). Multiplications other than, by 0, + 1 and – 1, are considered non-trivial.

(d) Complete and redraw, with proper labels, the figure Fig. Q2-2, showing clearly the additional interconnections and additional nontrivial multipliers – so that X[k], k = 0 to 9, the 10 – point DFT of the 10 points x[n], n = 0 to 9 is obtained at the outputs of the 2-point DFT blocks shown. It is not necessary that the outputs X[k] occur in order from top to bottom. Evaluate and hence compare (i) the precise overall number of non-trivial multiplications and two-input additions required in this completed Fig. Q2-2 (ii) the precise number of non-trivial multiplications and two-input additions required if the 10-point DFT were evaluated directly.

Q3. (20 marks).

(a) Obtain the (causal) system function Y(z) / X(z) in the signal flow graph of Fig. Q3-1, where K and C are real constants. What is the condition on the parameter K in Fig. Q3-1, for this system function to be stable? Evaluate the magnitude of this system function on the unit circle in the z-plane, $z = \exp(j\omega)$, assuming that this system function is stable.

(b) Show that the system function:
$G(z) = (A_1 + A_2 z^{-1} + A_3 z^{-2}) / ((1 - 0.5\, z^{-1})(1 - 0.25\, z^{-1}))$
may be realized by a structure of the form of Fig. Q3-2, under some condition(s) on the real constants A_1, A_2, A_3. Obtain the condition(s) on A_1, A_2, A_3 for this to be possible. Evaluate the parameters K_1, K_2, C_1, C_2 in terms of A_1, A_2, A_3 ; given that the required condition(s) are satisfied.

(c) Obtain and sketch the locus of valid pairs (A_2, A_3) in Q3-(b) with $A_1 = 1$, in the two-dimensional plane of A_2-A_3, which would satisfy the required condition(s).

Q4. (10 marks).

Write a short note explaining the applications of digital signal processing in any **two** of the following themes; in about 100 words each.

(a) 3 – D sound localization using HRTFs
(b) Global Positioning Systems (GPS)
(c) Speech Recognition – based on Mel Frequency Scale and Cepstrum
(d) Methods for Pitch Detection – autocorrelation and AMDF approaches
(e) Heart Rate Variability Analysis

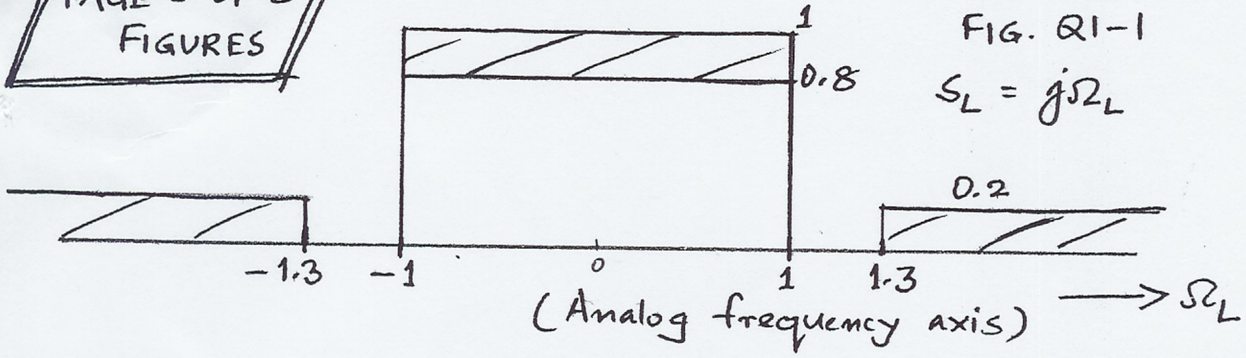

FIG. Q1-1

$s_L = j\Omega_L$

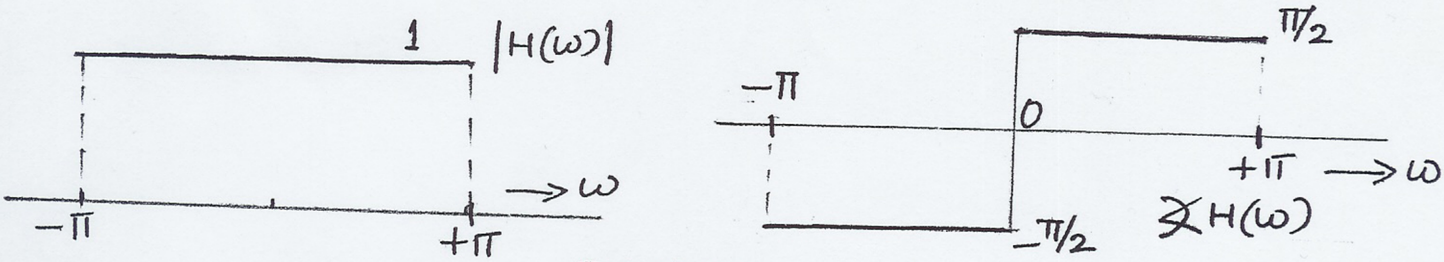

FIG. Q2-1

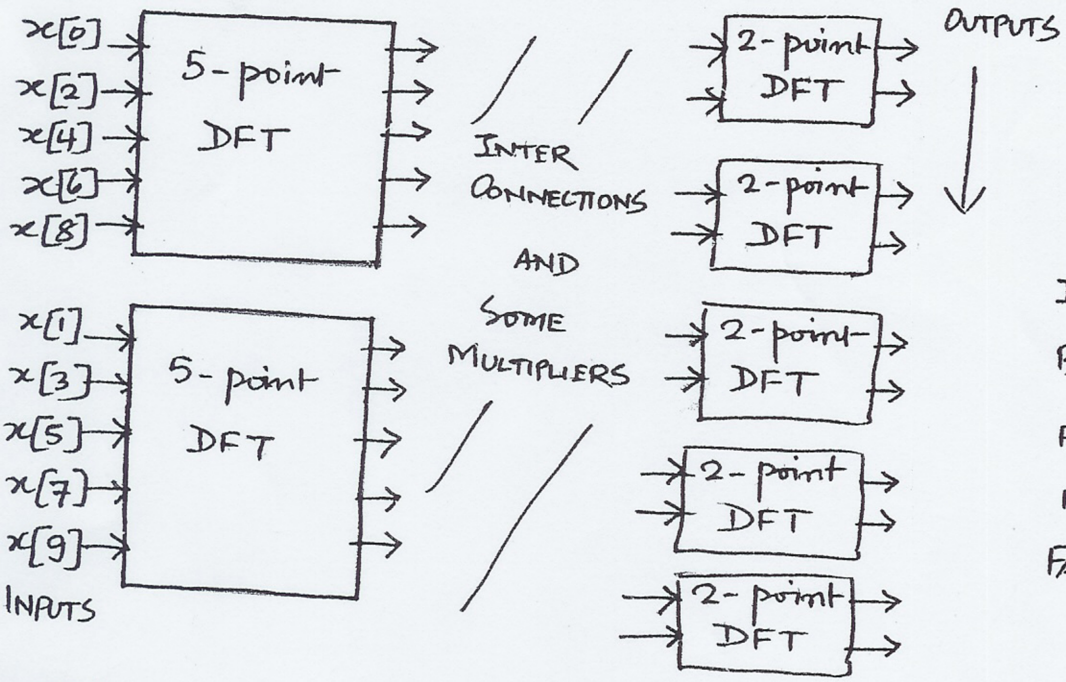

FIG. Q2-2

INCOMPLETE BLOCK DIAGRAM FOR 10-POINT FAST FOURIER TRANSFORM

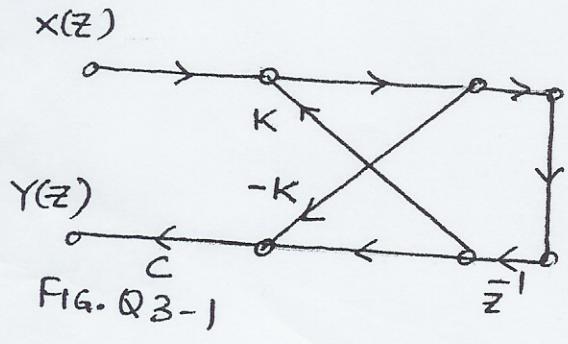

FIG. Q3-1

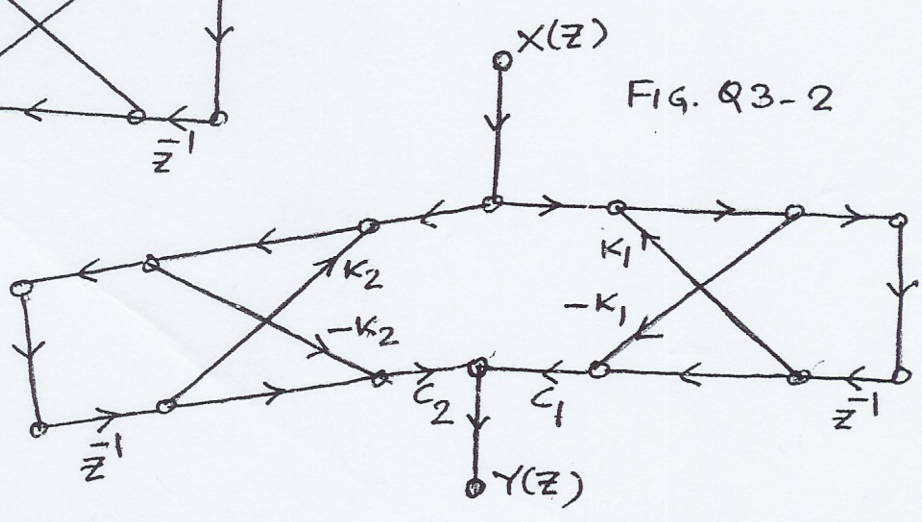

FIG. Q3-2