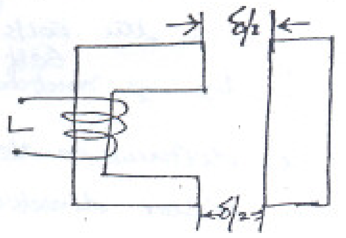
Inductive Sensors.



L = Self inductance $= \dfrac{N^2 \mu A}{\delta}$

N = number of turns,
A = area of cross-section, m^2
μ = permeability of the material, H/m
δ = total air gap, $\frac{\delta}{2} + \frac{\delta}{2} = \delta$, m.

Comparison between R, C, L type of sensors :-

i) R - Strain Gauges - Source impedance is low, gets affected by stray capacitances, they are delicate, practically cannot be used at extremely high frequencies, but theoretically - bandwidth = ∞. Value of resistance involved is low (order of hundreds of Ω)

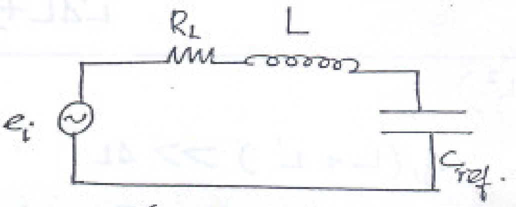
ii) C - Capacitive transducers - Source impedance is very high, these are susceptible to stray capacitances, these are delicate. The value of capacitances involved are low. The frequencies at which these transducers are used is very high (order of MHz).

iii) L - Inductive transducers - The source impedance is higher than strain gauges but lower than capacitive transducers. These are very robust and sturdy. There are no stray capacitances involved. The value of inductances involved is high. Frequency range between 5 kHz - 100 kHz.

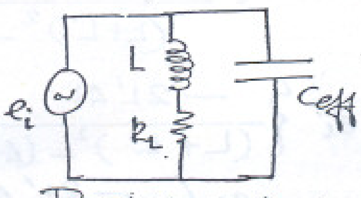
Q. How to measure 'δ', which corresponds to ~~frequency~~ displacement
The method for measuring 'δ' in an inductive transducer is more or less similar to the way of measuring 'C' in capacitance sensors.

1. Variable 'f' method → Series Resonance Method. → Parallel Resonance Method.

R_L = Leakage inductance.

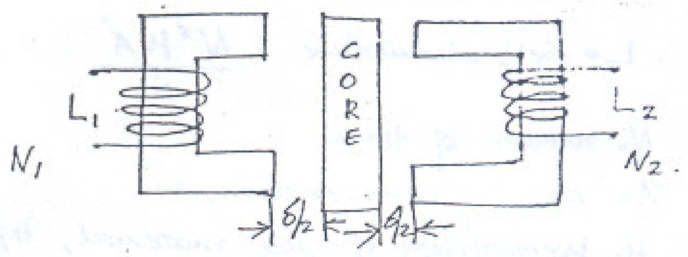


Series method.



Parallel method.

How to measure differential inductance



$N_1, N_2 \rightarrow$ no. of turns on both the coils.

$L_1, L_2 \rightarrow$ ~~mutual~~ self inductances,

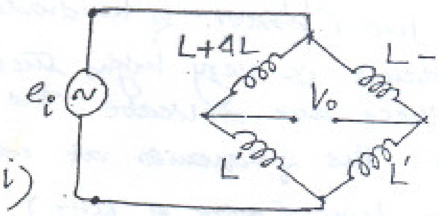
x = distance moved from the core in either direction.

$$L_1 - L_2 = \Delta L = N^2\mu A\left[\frac{1}{\delta - x} - \frac{1}{\delta + x}\right]$$

$$\rightarrow \propto \frac{x}{\delta^2 - x^2}$$

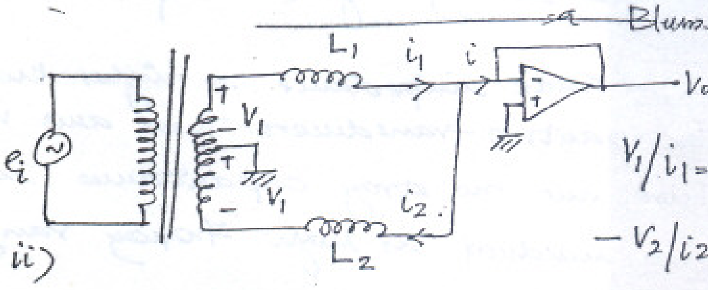
The inductance can be measured using a wheatstone's bridge, in which all the arms are made of inductances (AC bridge).

i)



The two 'L'' will be replaced by the two secondary windings of a transformer. The circuit then 'becomes' or 'gets transformed' into a Blumlein's Bridge.

ii)



$$V_1/i_1 = j\omega L_1, \quad \therefore i_1 = \frac{V_1}{j\omega L_1}$$

$$-V_2/i_2 = j\omega L_2, \quad \therefore i_2 = -\frac{V_1}{j\omega L_2}$$

$$\therefore i_1 + i_2 = i = \frac{V_1}{j\omega L_1} - \frac{V_1}{j\omega L_2} = \frac{V_1}{j\omega}\left[\frac{L_2 - L_1}{L_2 \times L_1}\right]$$

in i) $$V_0 = e_i\left\{\frac{L - \Delta L}{L' + L - \Delta L} - \frac{L + \Delta L}{L' + L + \Delta L}\right\}$$

$$= e_i\left\{\frac{(LL' + L^2 + L\Delta L - L'\Delta L - L\Delta L - \Delta L^2) - (LL' + L^2 - L\Delta L + L'\Delta L + L\Delta L - \Delta L^2)}{(L + L')^2 - (\Delta L^2)}\right\}$$

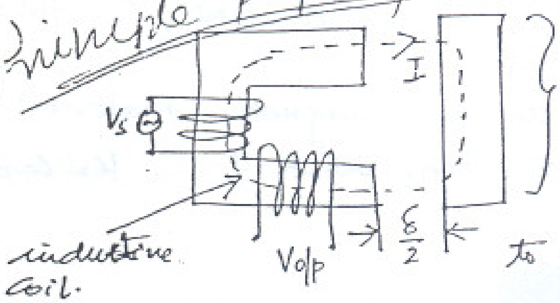
$$= e_i\left\{\frac{-2L'\Delta L}{(L + L')^2 - (\Delta L)^2}\right\} \quad \text{if } (L + L') >> \Delta L$$

then $$e_0 \approx e_i\left\{\frac{-2L'\Delta L}{(L + L')^2}\right\} = e_i\left\{-\frac{\Delta L}{2L}\right\}, \quad \text{if } L' = L$$

The methods involved in measuring L or ΔL is same as compared to the measurement of 'c', but the frequencies involved are much smaller.

$f \approx$ 5 KHz - 100 KHz.

Simple property



Length of core = l.

Total inductance = Sum of inductance due to the magnet + inductance due to air-gap and Vo/p $\propto$ total inductance.

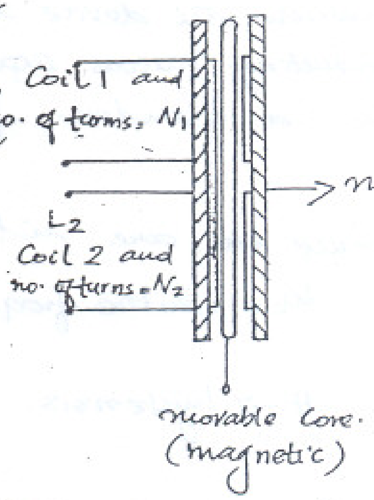
$V_{o/p}$ will depend on the coupling of flux with magnetic path of length 'l' and 'δ' the air gap.

This property is mainly used by LDI's and LVDT's.

LDI – Linear difference inductance / reluctance.

LVDT – Linear variable differential transducer.

LDI ckt



Strictly f-dependent

The movable core is made up of a magnetic material. The whole set up is air tight.

→ made of a material with high permeability ($\approx$ 3000 - 4000). This is to ensure that no stray electromagnetic effects come into picture and affect the inner core.

{ one coil at the top and one at the bottom. The core is inserted through a slit

$Q = \frac{L\omega}{R}$ = quality factor of the coil.

L = inductance of the coil, H . ω = frequency of operation, rad/s.

R = resistance of the coil winding, Ω.

There can be various losses in the coil (because, the coil has a resistance,

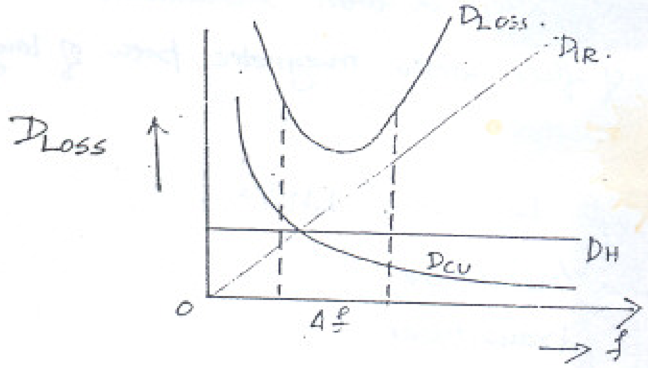
1) Loss due to hysteresis → since magnetic materials are involved. The loss due to hysteresis is a constant for a particular frequency, say D_H.

2) Copper coils are normally used to make the winding of an induc[tor] hence there is a resistance involved and this introduces copper loss. At higher frequencies the resistance or reactance increases and current reduces ($L\omega R = X_L \cdot R$).

Say the dissipation $= D_{CU} \propto \frac{1}{f} = \frac{C}{f}$, C = Constant; f = frequency.

3) Iron losses are involved due to the magnetic material being used in the core. Dissipation, $D_{IR} \propto f = K \cdot f$. K = Constant.

So total loss $= D_H + D_{CU} + D_{IR} = D_{LOSS}$.



— indicates the overall loss.
— indicates the Copper loss.
— indicates the iron loss.
— indicates the hysteresis loss

'Δf' is the optimal frequency range in which the device is operated and this bandwidth is arrived after conducting various experiments, a[nd] plotting the overall loss Vs frequency. So a random frequency of operation cannot be chosen.

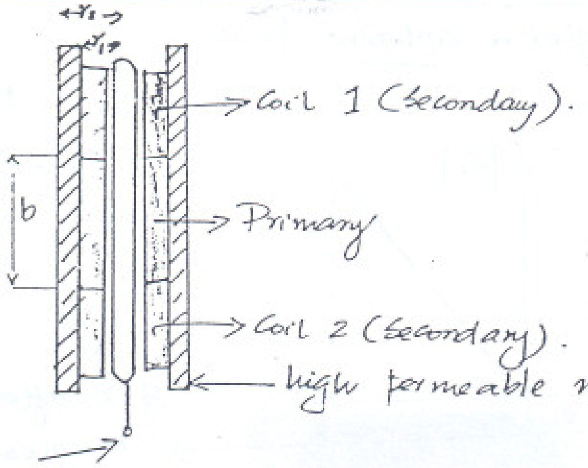
These losses are taken into account since we are interested - high values of Q. ω_c = Centre frequency

$$Q = \frac{\omega_c}{R} \propto \frac{1}{\left(\frac{C}{f} + Kf + H\right)}$$, $H \rightarrow$ hysteresis.

So the biggest disadvantage is that Q is highly dependant o[n] frequency.

So the best solution to the above problem is by going for LVDTs wh[ich] have a better design

Linear Variable Differential Transformer.



r_2 and r_1 are the rad indicated and the reference is the centre of the core

e_1 = Voltage of coil 1 (secondary)

e_2 = Voltage of coil 2 (secondary)

When the core is located at the exact midpoint the voltages 'e_1' & 'e_2' are equal. Hence $e_1 - e_2 = 0$.

When the core moves upwards $e_1 > e_2$, and $e_1 - e_2 = +ve$.

When the core moves downwards $e_1 < e_2$, and $e_1 - e_2 = -ve$.

$$e_0 = e_1 - e_2 \propto f \cdot I_p \cdot x\left[-\frac{x^2}{2b^2}\right] \text{ or } e_0 = K \cdot f \cdot I_p \cdot x\left[1 - \frac{x^2}{2b^2}\right] \quad \text{---(1)}$$

$K f I f$

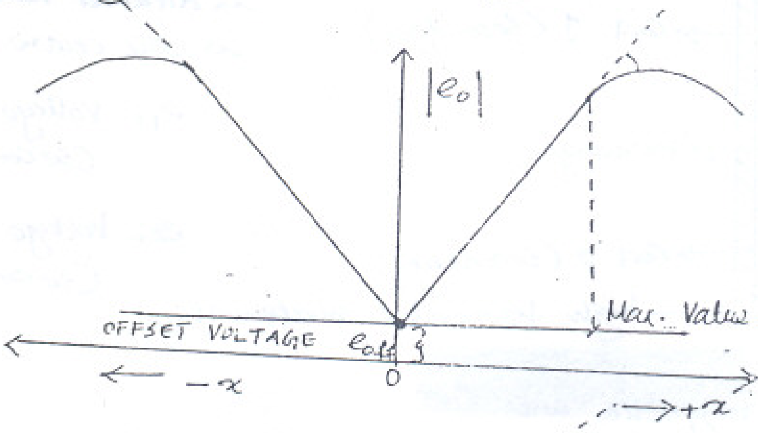
b = length of the primary coil, m and x = distance by which the core m to either side, m.

K = Constant, that will be a combination of permeability of free space (μ_0), number of turns (N_1, N_2), length of the coils and the radii r_1, r_2.

Derivation of (1)

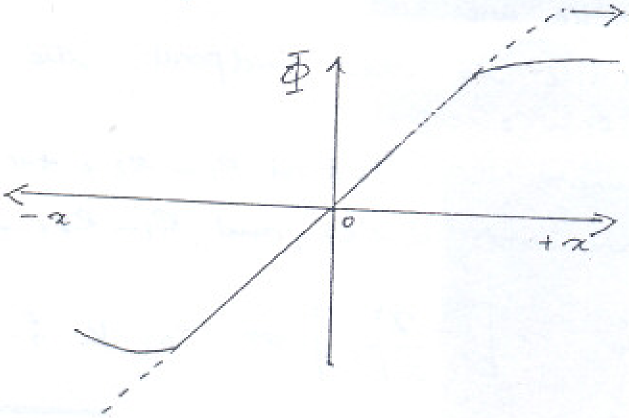
$[\ldots, N_2, l, \ldots]$

In ① the term $\frac{x^2}{2b^2}$ introduces a non-linearity and this starts becoming significant after a certain point



Max. Value of x after which the graph becomes non-linear

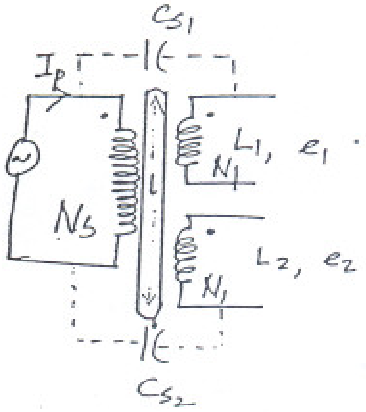
Magnitude response.
— EVEN function.



Phase response.
--- ODD function

+ve $x \rightarrow 0° \rightarrow$ change
−ve $x \rightarrow 180° \rightarrow$ change.

In the magnitude response a small offset voltage is observed even when the displacement of the core = 0. This is because of the stray capacitances indicated below. These capacitances come into picture because we operate the core at a frequency and not DC. (for DC, C_{S1}, C_{S2} would have been open circuited). C_{S1} and C_{S2} = $\frac{1}{j\omega C_{S1}}$, $\frac{1}{j\omega C_{S2}}$.

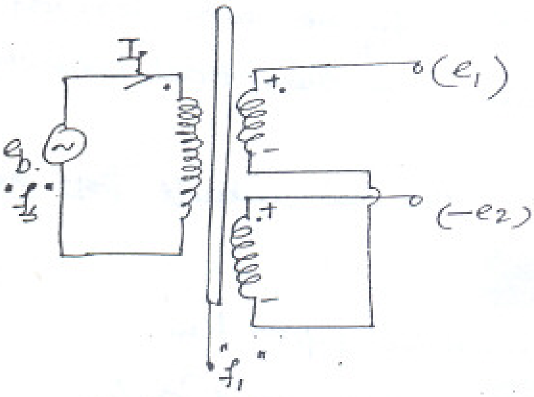


Typical freq. of operation = 5KHz – to 100KHz

'l' = length of the core, usually the length is made equal to '3b'.

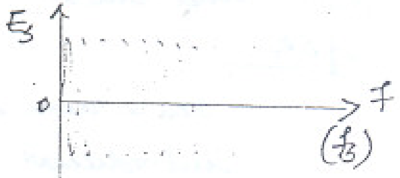
$\boxed{L = 3b}$

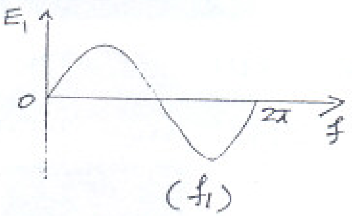
Phase gives an excellent indication of the direction of movement. We are interested in $|e_1| - |e_2| = e_0$. So we need '$-e_2$' for further operations, hence the connection is made like this

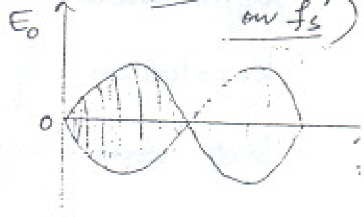


"f_s" = frequency of the input signal to the primary.

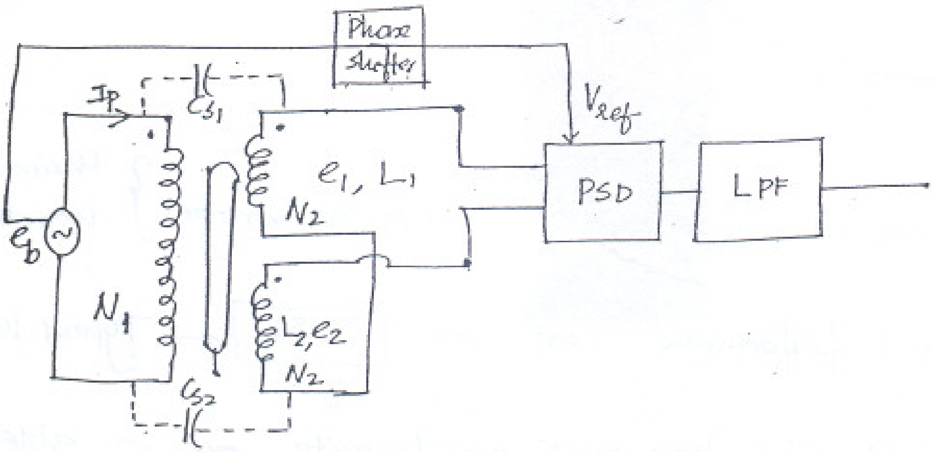
"f_1" = the frequency to which the core is subjected to vibrate ('f_m' modulated on f_s')

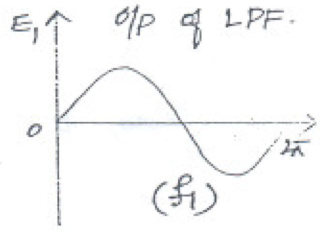






Using a PSD to measure the phase shift :-





Output of the low pass filter contains the frequency information about the vibration of the core.

The PSD needs a reference voltage (and phase) and it is derived from the source supplying the voltage to the primary coil.

But in practice the reference voltage is passed through a phase shifter where the phase of V_{ref} is slightly changed.

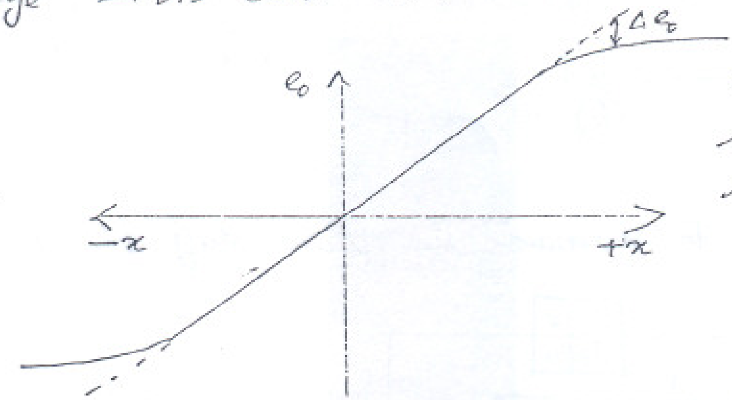
In reality the capacitances C_{s1} and C_{s2} introduce a phase shift to e_1 and that needs to compensated. So a phase shifter is introduced. The amount of phase shift depends on the value of the capacitances.

Now $f_1 = \frac{f_s}{10}$ gives good result. This figure was arrived after practical observation and does not have any theoretical backing.

Nowadays, most of the LVDTs come with an inbuilt PSD and the performances are enhanced thenceforth.

Resolution of an LVDT $\rightarrow$ can measure upto μm.

But some large LVDTs can measure mm's and cm's.



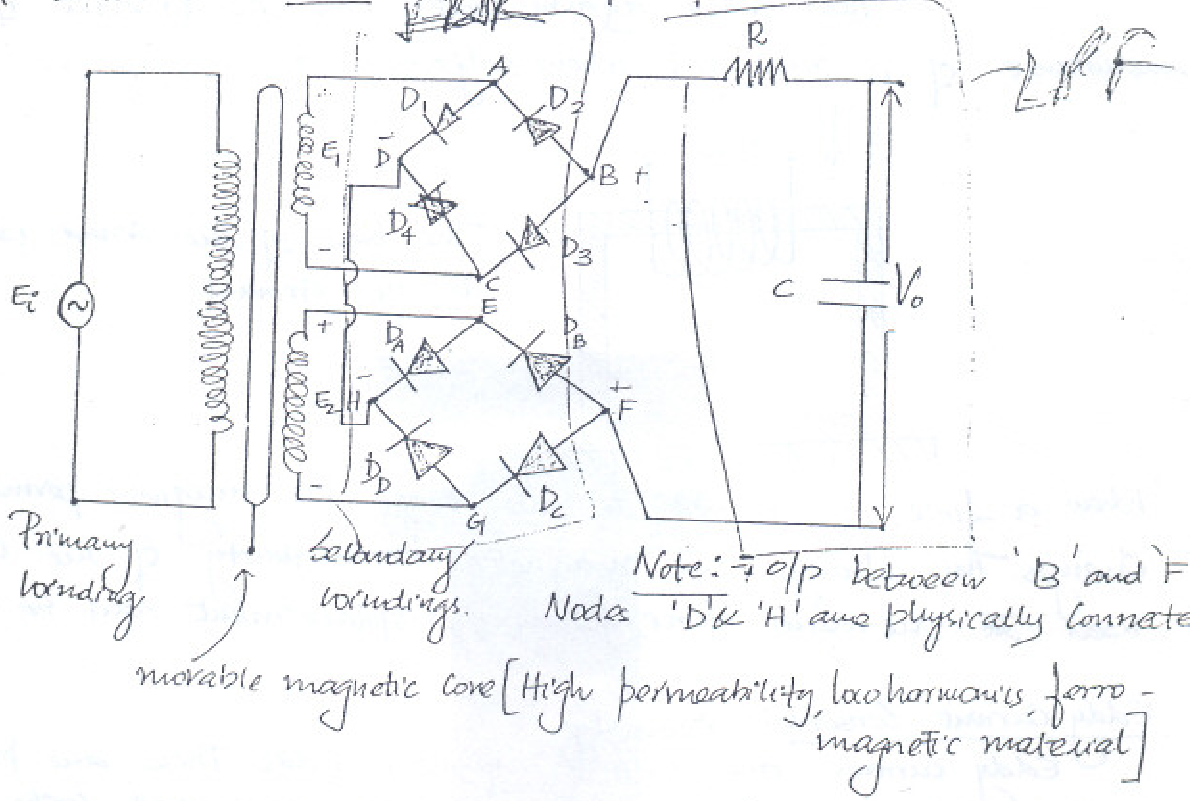
Δe_0 = non linearity introduced because the term $(x^2/2b^2)$ in ①

now, $b = x_{max} \cdot \frac{1}{\sqrt{2e}}$, e = acceptable error. } Maximum tolerable non-linearity.

So for good performance we use $\boxed{l = 3b}$ ← Typical values.

This factor decides (b) how much non-linearity one is able to tolerate in a measurement, and the specification of that quantity decides the range of 'x' or the value of 'x' upto which the operation is acceptable.

Another method to obtain the differential Voltage and phase, a method which does not involve a phase sensitive detector.



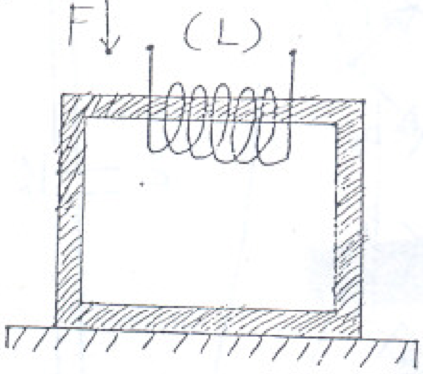
During the positive half cycle diodes 'D_1' and 'D_3' are forward biased and hence they conduct. Point 'D' will be held at negative and point 'B' will be positive. Diodes 'D_2' and 'D_4' are reverse biased. Also, diodes 'D_A' & 'D_C' conduct and points 'H' is held at negative and 'F' is at positive. So the output across 'C' i.e; Vo will be the difference of E_1 and E_2. Diodes 'D_B' and 'D_D' will be reverse biased.

During the negative half cycle diodes 'D_1' and 'D_3' are reverse biased. 'D_2' and 'D_4' conduct again 'D' is held at negative and 'B' will be positive. Diodes 'D_A' and 'D_C' will be reverse biased. 'D_B' and 'D_D' will be forward biased and hence conducting. 'H' will be negative and 'F' will be positive. The potential 'Vo' will be equal to the difference in potentials, E_1 and E_2.

The excitation Voltage is an ac. The rectifier network is followed by a low pass network where the signal gets demodulated. Vo can be used to interpret the movements of the core.

Magnetostrictive Sensors :-

This sensor again works on the principle of change in inductance of a material when subjected to a force or a displacement.

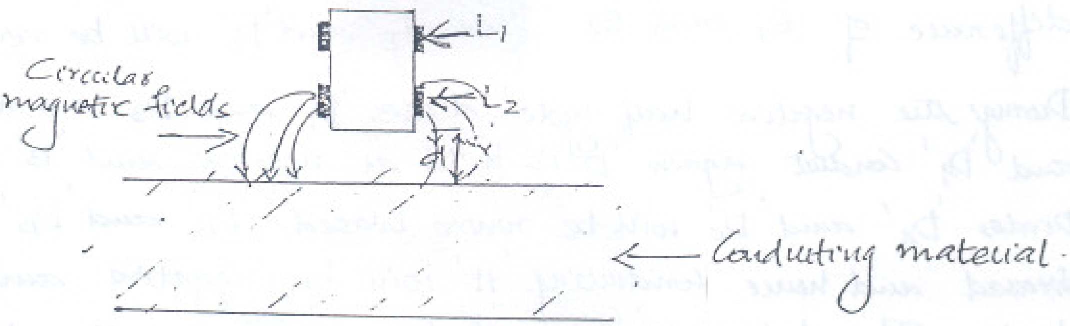


The base of the sensor is fixed firmly to the ground.

When a force is applied to the core, the magnetic permeability (μ) changes. This change in magnetic permeability of the core can be used to measure force, pressure, displacement and so on.

Eddy current Sensors :-

Eddy currents are caused by circular magnetic fields. These are produced when a material having a magnetic/Electric field associated with it is brought near a conducting plate. The amount of current introduced will be inversely proportional to the distance between them. Eddy probes make use of this principle to detect irregularities/corrosion in metallic pipes/tubes.

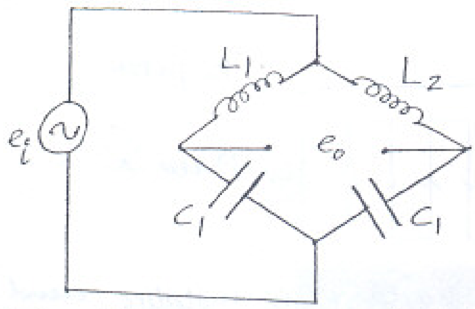


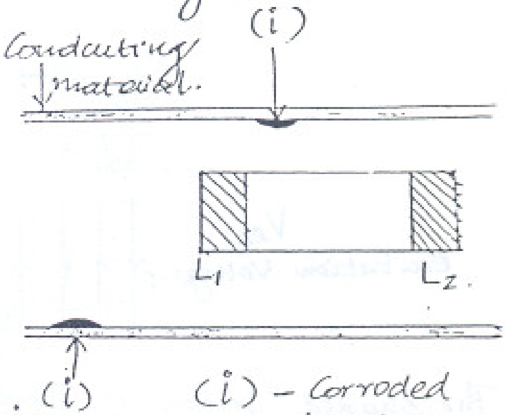
L_1 – reference coil and L_2 – operating coil.

Due to the distance 'd', Eddy currents are formed and a potential will be introduced in the reference coil L_1 due to the field in L_2 the operating coil.

Frequency of operation :- 5 KHz to 2-5 MHz.

The main applications of Eddy probes are in checking the boiler tube irregularities and measuring displacements.

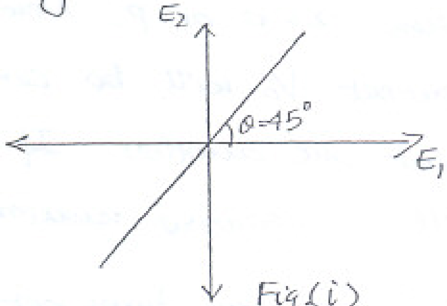




$E_1 \rightarrow$ potential of L_1 and $E_2 \rightarrow$ potential of L_2;

(i) - Corroded territory.

When the Eddy probe is inserted and moved along the interior of the pipe, Eddy currents are introduced. As long as the Corroded parts are not encountered the potentials E_1 and E_2 are same and the Lissajous pattern is a straight line with slope = 1. Fig(i)



Fig(i)

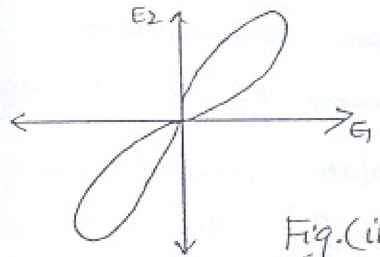


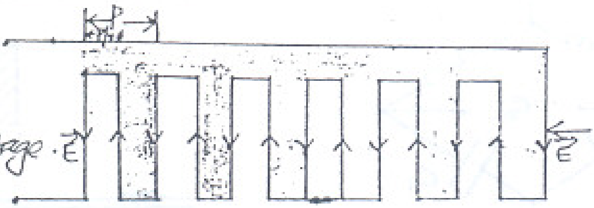
Fig.(ii)

When a Corroded territory is encountered, the potentials E_1 and E_2 differ from one another because the conductivity has changed. So this introduces a phase change and change in frequency too. The Lissajous pattern observed will be no longer like Fig(i) but will be something like Fig(ii) or something different. The shape of the pattern observed determines the position of the irregularity / Corroded part along its length.

Disadv: But the position along the circumference ~~along~~ of the pipe cannot be determined because L_1 and L_2 are similar, hence cannot be used to differentiate the regions in any sense.

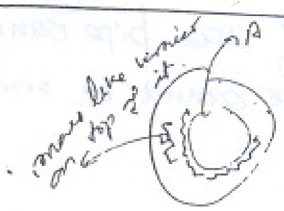
Inductosyn :- [a patented product and widely used to measure displacement]

'x' = small displacement.



The shaded portion of plate [pencil shaded] is metallic and is etched on a plastic base. These are readily available. This is also available in circular shapes besides planar. P= pitch of the teeth. There runs a similar plate, identical to plate A just above the top of plate A. The alignment of the two plates correspond to one another during the null position. If the o/p of plate B is assumed to be V_0, we can use V_0 to measure displacement. $V_0 = V_{ex}$ when $x = 0$ or P. Now, if the plate is moved by '$x = \frac{P}{4}$', then the output V_0 will be zero since the fields of 'A plate' and 'B plate' cancel out one another. If the plate moves by '$x = \frac{P}{2}$', then the output will be negative maximum ∵ fields in both the plates are additive, but the sum turns out to be negative. Now when the movement is from '$x = \frac{P}{2}$' to '$x = P$' the events occur in the reverse order.

$$\boxed{V_0 = K \cdot V_{ex} \cdot \cos\left\{2\pi \cdot \frac{x}{P}\right\}}, \quad \begin{cases} x = 0, & V_0 = V_{ex} \\ x = \frac{P}{4}, & V_0 = 0 \\ x = \frac{P}{2}, & V_0 = -V_{ex} \\ x = \frac{3P}{4}, & V_0 = 0 \\ x = P, & V_0 = V_{ex} \end{cases}$$



K= Constant obtained after calibration.

Inductosyn needs to be calibrated before using it.

Resolution: 10 μm, $V_0 \simeq$ in the range of millivolts. The frequency and operation: 100 KHz - 2 MHz.

Now, if the value of displacement, x', is more than 'P' then the output repeats itself. So how to measure $x > P$?

if $x' = n \cdot P + x$ = total distance moved.

n = no. of whole pitches moved.

x = [Total displacement — no. of whole pitch displacements] = x' -

How to get 'n'?

Observe the no. of polarity changes in the o/p equation,

$$\boxed{V_0 = K \cdot V_{ex} \cdot \cos\left[2\pi \cdot \frac{x}{P}\right]}$$

Now, o/p voltage for $x' = np + x$ will correspond to $x' = x$ itself. So if 'n' can be found out precisely then x' can be found out bec 'P' is already known.

Now $\left[2\pi \cdot \frac{x}{P}\right]$ is assumed to be 'θ'.

How to measure 'θ'? 'phase sensitive detector'.

From the output voltage V_0, we extract two voltages V_1 and V_2

to get ϕ

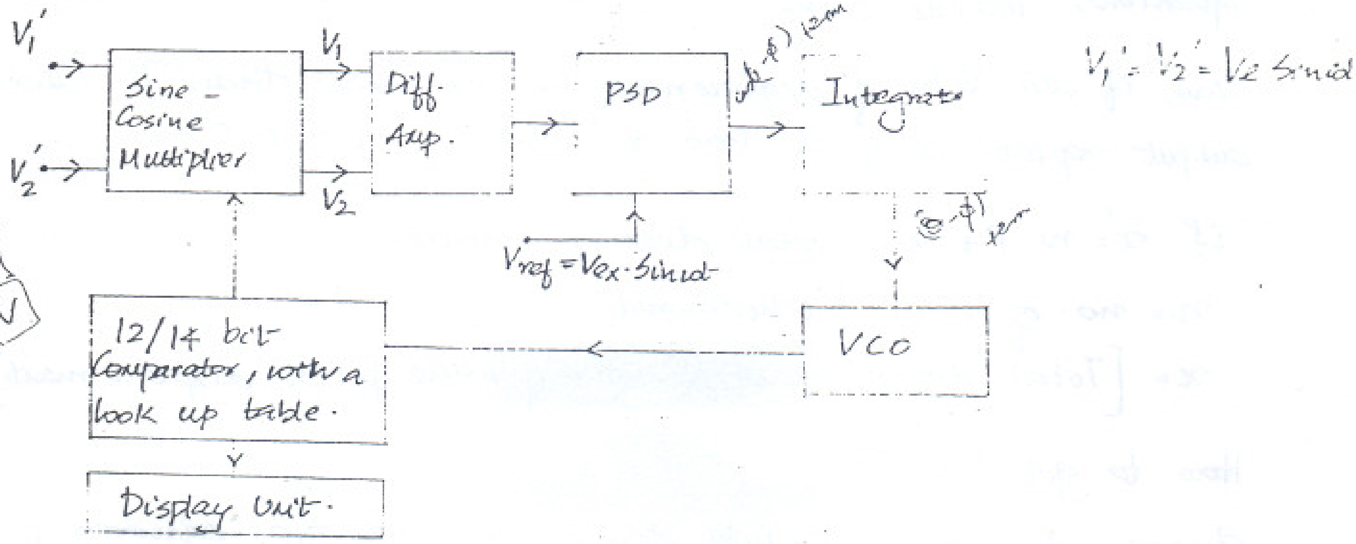
$V_1 = V_{ex} \cdot \sin\omega t \cdot \cos\theta$

$V_2 = V_{ex} \cdot \overset{\sin\theta}{\cancel{\cos\theta}} \cdot \sin\omega t$.

{V_2 is V_1's cosine component shi by 90°}.

"$V_{ex} \sin\omega t$" ← because the applied excitation is a high frequency signal. This when combined with '$\cos\theta$' produces a modulated out V_1 and V_2 are multiplied together with the aid of a 'sine-cosin multiplier.

Circuit Diagram to Condition the signal output of an INDUCTOSYN.



The 14-bit Comparator with a look up table that will contain the values of 'Cos θ' for one whole cycle. So ($V_e \sin\omega t$) can be multiplied with Cos θ and 'Cos θ' can be shifted by 90° to obtain 'Sin θ' which can be again multiplied with '$V_e \sin\omega t$' to obtain two voltages V_1 and V_2. The values or magnitudes of V_1, V_2 are small, so we need an amplifier at the next stage to detect their difference and then amplify. Now the output of the differential amplifier is fed to the PSD (reference voltage = $V_e \sin\omega t$) and the output is fed to an integrator.

o/p of PSD = DC term + AC term.

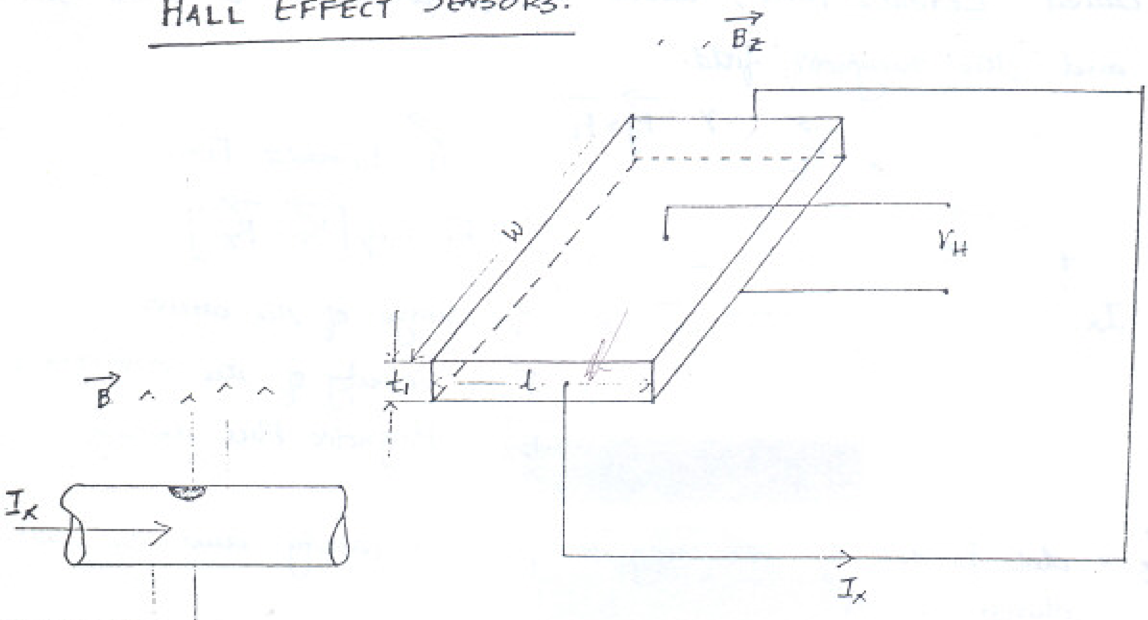
DC term contains a $\cos(\theta - \Phi)$ term.

The output of the integrator will be a sine function and that is fed to a VCO. Integrator is used to produce a term '$\theta - \Phi$'.

The o/p of the VCO = oscillations of a particular frequency corresponding to the input Voltage.

The VCO will stop when $\theta = \Phi$. When $\Phi = \theta$, the o/p of the lookup table and 14-bit Comparator can be observed on the display unit. This Value of 'θ' Corresponds to $2\pi \cdot \frac{x}{P}$, 'P' being fixed, 'x' can be easily calculated. $\boxed{x = \frac{\theta \cdot P}{2\pi}}$

HALL EFFECT SENSORS.



Principle :-

Whenever a current carrying conductor is inserted or exposed to a transverse magnetic field, a potential is developed across the terminals / surface of the conductor inserted. The potential depends on the strength of magnetic field, amount of current carried in the conductor and the nature of the conductor (dimensions) → charge distribution (electrons distb.), semiconductor, ...

B_z = Magnetic flux density, Tesla or weber/metre^2.

V_H = Hall potential, V.

w = width of the conductor, m

L = Length of the conductor, m.

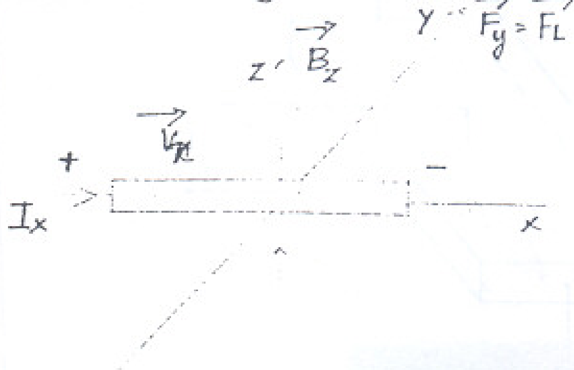
t_1 = thickness of the conductor, m.

I_x = current flowing in the conductor, A.

The Hall effect was initially observed by Lord Kelvin in 1840's but it was actually studied indepth by E H Hall in 1879. This was discovered when Hall was working on the "characterization of metals".

The Hall potential is formed due to the accumulation of charges on a surface of the conductor in a magnetic field. This causes a potential gradient, hence V_H is obtained.

Now, a currant carrying conductor in a transverse magnetic field experien a force called Lorentz Force, which is perpendicular to both the flow of current, and the magnetic field.



$\overrightarrow{F_L}$ = Lorentz Force

$\overrightarrow{F_L} = q\left[\overrightarrow{V_x} \cdot \overrightarrow{B_z}\right]$

q = charge of the carrier

$\overrightarrow{V_x}$ = Velocity of the carriers, m/s

$\overrightarrow{B_z}$ = Magnetic Flux density

$\overrightarrow{V_x} \cdot \overrightarrow{B_z}$ = dot product of the magnetic field intensity and the velocity of charge carriers.

Since the charge gets accumulated on the surface a force ... and $\overrightarrow{F_C}$ = force due to charges = $q \cdot \overrightarrow{E_H}$.

$\overrightarrow{E_H}$ = Electric field, V/m

$\overrightarrow{E_H} = V_H / m$.

Under steady state conditions $\overrightarrow{F_c} = \overrightarrow{F_L}$. Both the forces are equal

$\therefore\ q \cdot \overrightarrow{E_H} = q \cdot \left[\overrightarrow{V_x} \cdot \overrightarrow{B_z}\right]$

or, $\overrightarrow{E_H} = \overrightarrow{V_x} \cdot \overrightarrow{B_z}$ ——(1)

If, J_x = current density of the material, A/m²

$J_x = n \cdot q \cdot \overrightarrow{V_x}$ ——(2), where n = density of charges = number of charge in unit volume of the material

'n' will be high for conductors, and low for semiconductors.

$J_x = \dfrac{I_x}{w \cdot t}$ ——(3), substituting (3) in (2) we get,

$\dfrac{I_x}{w \cdot t} = n \cdot q \cdot \overrightarrow{V_x}$ $\therefore\ \overrightarrow{V_x} = \dfrac{I_x}{nqwt}$ ——(4)

Substituting, (4) in (1)

$$\vec{E_H} = \vec{V_x}\cdot\vec{B_Z} = \frac{\vec{B_Z}\cdot I_x}{nq\,wt} \quad \text{—(5)}$$

$$\text{Now, } \vec{E_H} = \frac{\vec{V_H}}{w} = \frac{\text{Potential difference}}{\text{Length}}$$

[Hence length is measured across the width]

$$\therefore \frac{\vec{V_H}}{w} = \frac{\vec{B_Z}\cdot I_x}{nq\,wt} \qquad \therefore \boxed{V_H = \frac{B_Z\cdot I_x}{nqt}} = \text{Hall potential.}$$

now $\frac{1}{nq} = R_H$ = Hall constant for a particular material.

R_H depends on the number of charge carriers in that material and the relationship is inversely proportional. 'R_H': units = $\frac{\text{Volt}\cdot\text{metre}}{\text{Ampere}\cdot\text{Tesla}}$.

$(R_H) \longrightarrow$

$Cu = 5.3\times10^{-11}$ Vm/AT.
$Ag = 5\times10^{-7}$ Vm/AT

} R_H for metals is low as 'n' is Large

$Si = 1\times10^{-2}$ Vm/AT
$Ge = 2\times10^{-3}$ Vm/AT.

} R_H for semiconductors is high as 'n' is small.

The problems encountered when semiconductors are used :-

1. The charge carriers are temperature dependant. So the Hall constant will not be constant even though the material remains the same.
2. I_x has to be made a constant current source [As the value of the current in the semiconductor should not change] if V_H has to be measured precisely and accurately.

Hall effect sensors are used to measure :-

1. Magnetic Flux
2. DC or AC values of current.
3. Flow
4. Displacement
5. Power
6. movement and direction of movement of magnets.

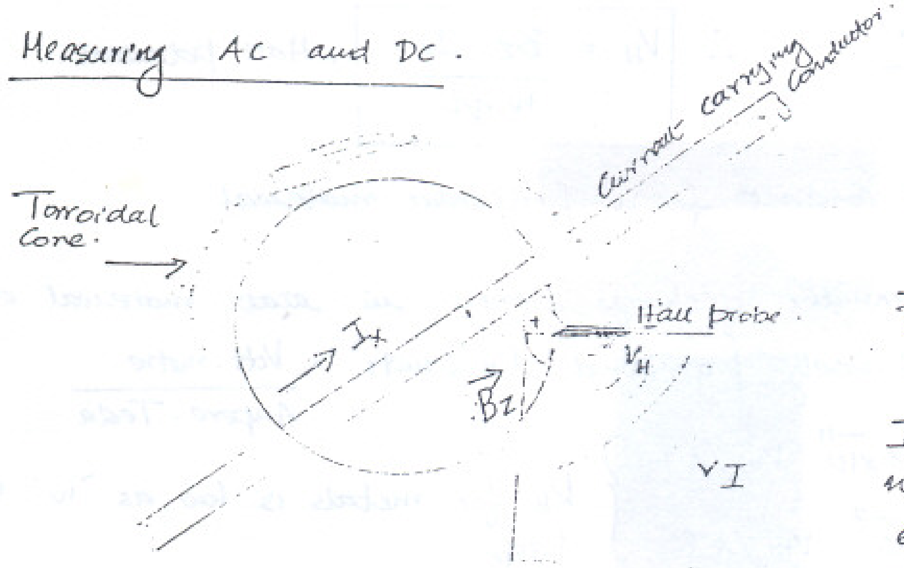
Manufacturers of Hall effect sensors :-

1. Honeywell
2. Microswitch
3. Allegro
4. Microsensors

By the Hall effect sensor arrangement previously illustrated, we can measure the magnetic field intensity if we can measure V_H accurately.

$$V_H = \frac{R_H \cdot I_x \cdot B_Z}{t} \qquad \therefore \boxed{B_Z = \frac{t \cdot V_H}{R_H \cdot I_x}}$$

Measuring AC and DC.



Magnetic Flux units:-
1×10^4 Gauss $= 1$ Tesla.

$\vec{B_Z}$ = induced magnetic flux, T.

I = Constant current that needs to flow in the Hall effect sensor.

The set-up uses a torroidal core which has a discontinuity at one end through which a Hall-effect sensor is introduced. The current carrying conductor is inserted inside the torroidal core which generates a magnetic field. Now, this magnetic field will be directly proportional to the Hall effect voltage developed.

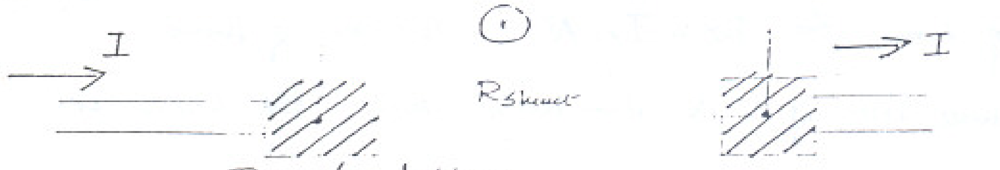
$\therefore I_x \propto B_Z \propto V_H$.

$\therefore$ By measuring V_H, we can measure, I_x. This involves least interference.

Very large currents can be measured using this technique.

How to measure currents of the order of 10^4 to 10^5 Amperes.

For this, we use a shunt.

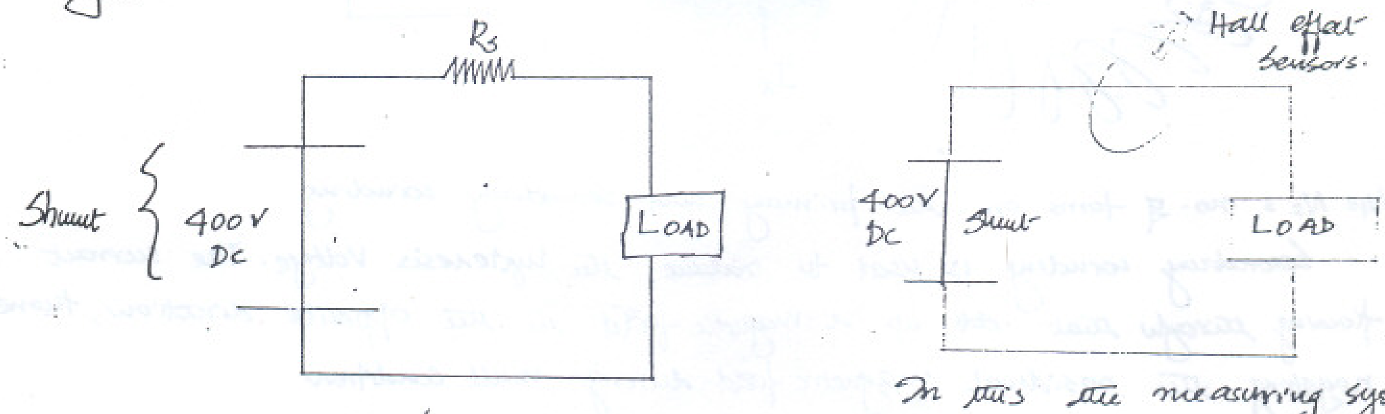


The large current which needs to be measured is made to flow through a shunt resistance which is of the order of m Ω. Now this potential drop across R_{sh} is measured to find out the value of current. The typical values of potential obtained are 75 mV.

For extremely large currents we mount the shunt vertically, so that the heat dissipated will be towards the surroundings, this is more effective when the shunt is mounted vertically.

When high currents are involved, the heat dissipated will also be very high, hence the ambient temperature increases. Now, the shunt resistance are made of Constantan / Manganin and the holding frames are made of Copper or Brass. When high temperatures are involved, this introduces thermocouple effects because a thermocouple can be formed between the copper - constantan [40 μV/°C] and this can create erroneous reading. To prevent this we need to go for better heat dissipation technique or galvanic isolation methods.

Galvanically Isolated



When the currents are large, N=1, i.e., a single turn torroid is sufficient.

In this the measuring system is galvanically isolated. No physical or electrical connection exists.

But how to measure currents of very small values?

For this purpose we can use a wounded torroid core. When a wounded torroid is used the field (magnetic) induced will be directly proportional to the number of turns. So $B_x = I_x . N$. N = no. of turns.

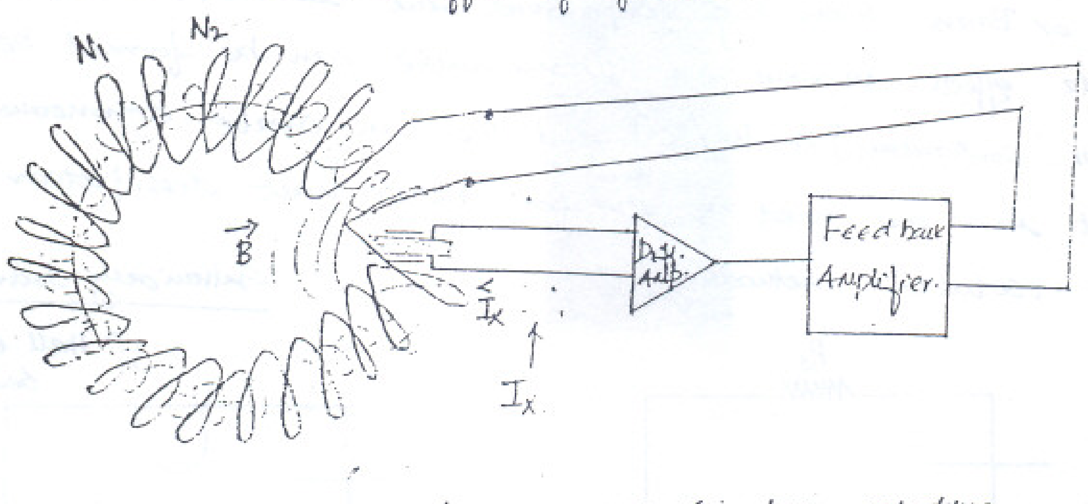
I_x can be small, but if N is very large then B_x increases.

Now, the Hall voltage can be used to measure AC too.

The frequencies that can be handled by Hall effect sensors are from 0Hz (DC) to 100 KHz. The frequency range depends on the magnetic characteristics of the material.

Since magnetic characteristics come into picture, non-linearity creeps in and the major problem encountered is hysteresis. So the design of the sensor is very critical, considerations about the air-gap, no. of turns, type of material become very important.

How to eliminate the effect of hysteresis?



$N_1 = N_2$ = no. of turns on the primary and secondary winding.

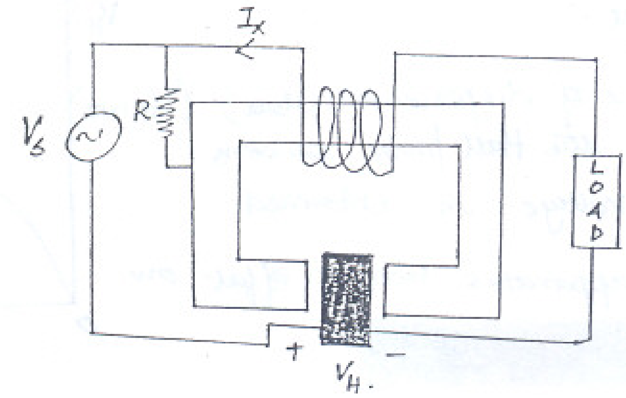
Secondary winding is used to reduce the hysteresis voltage. The current flowing through that sets up a magnetic field in the opposite direction, thereby negating the residual magnetic field during null condition.

The whole effect of this arrangement is valid only during the null condition. We need to ensure that $\vec{B_x} = -\vec{B_{fb}}$ or $I_x . N_1 = -I_f . N_2$

At null

Effect of nonlinearity & temperature is reduced

Hall effect sensors can be used to measure power :-

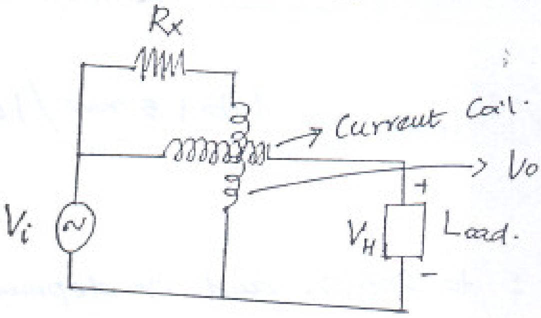


Power = $VI \cos \Phi$

$P = V_H \, I_x \cos \Phi$.

The load can be a wattmet

This principle is used in dynamometer wattmeter:



$V_H \propto V_i \cdot I \cdot \cos \Phi$

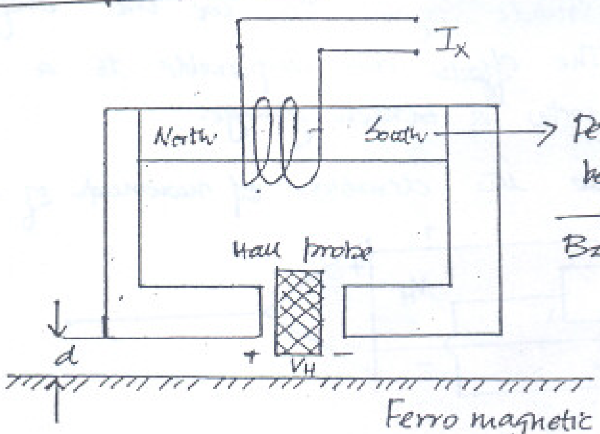
Proper calibration is reqd to set-up the calibration constant such that the

Hall sensor can form a part of the Dynamometer wattmeter.

Hall probe characteristics :- 100 MHz, Current sensitivity = 70 A/μs.

Response time of a Hall probe → 0.5 to 1 μs (rise time).

Measurement of displacement

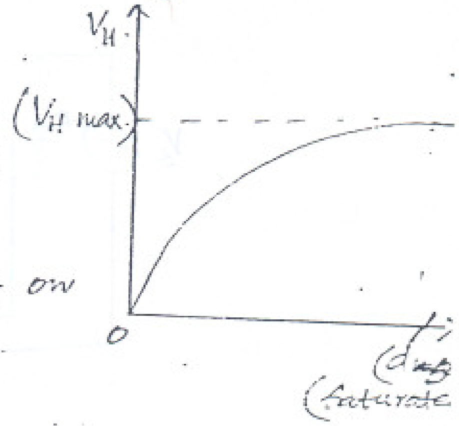


→ Permanent magnet or an electromagnet can be used.

$\vec{B_z}$ = Magnetic field associated with the magnet.

Ferro magnetic Material.

When the whole set-up is made to touch the ferromagnetic material, all the field generated by the magnet will interfere with the ferromagnetic material ... Hall probe = 0 hence $V_H = 0$ indicating zero displacement. Now when the apparatus is further moved away further away from the ferromagnetic material the field associated to the Hall probe increases. Therefore, V_H increases indicating an increase in the value of displacement.



Now, after a particular value of displacement (d_{sat}) the field associated with the Hall probe remains a constant and maximum voltage is obtained.

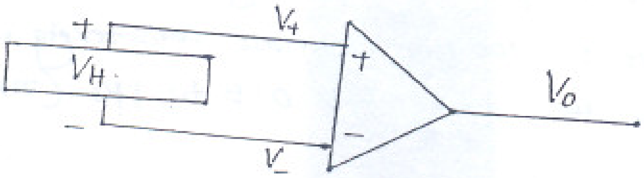
Further movement of the apparatus has no effect on the Hall Voltage, it remains a constant.

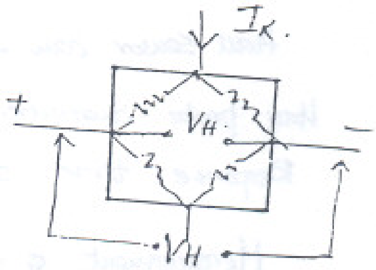
This technique can be used to measure displacement and vibration as well. The range of displacement = a few mm. Vibration can be measured upto 50 KHz.

Range of V_H = 1 to 1.5 mV / 0.1×10^{-3} Tesla, or 1 to 1.5 mV / 1 Gauss.

Values of $I_x \approx$ 10 to 40 mA.

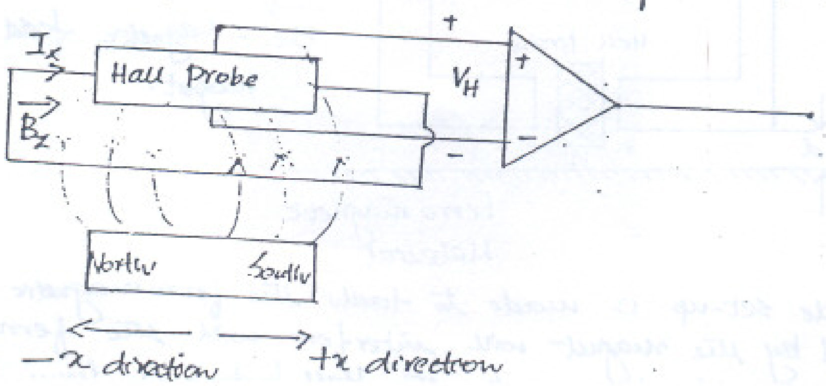
Impedance of the Hall probe $\approx$ 5 to 200 Ω, and it depends on the material.



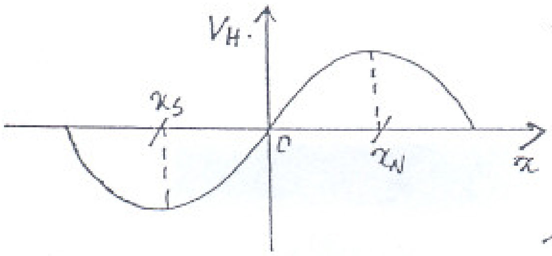


A Hall probe experiences common mode signals. So, we use differential amplifiers to extract the V_H values. The effects are comparable to a strain guage. An analogy can be drawn out with a strain guage.

Hall probes are used to sense the direction of movements of magnets :-



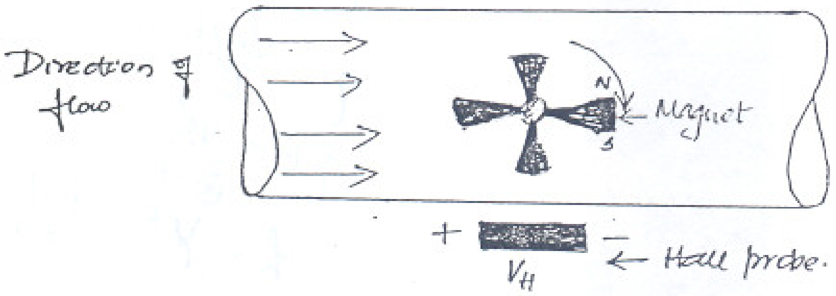
Initially when the magnet is at the null or zero position, the flux associated with both poles are equal and cancel out. Now when the magnet is moved towards the right the flux associated will increase, but will reduce to zero after some time. The same happens when the magnet is moved to the left, but since it is the other pole that is getting i the output will be opposite in nature.

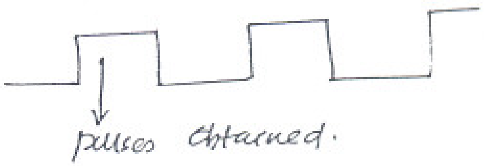


x_N = distance when the v of V_H is dominated by the north pole.

x_S = distance where the value of V_H is dominated the south pole.

Hall effect transducers to measure flow :-





The Hall effect principle can be used to measure the flow of liquids. A small turbine is inserted inside the pipe. A magnet is attached to a vane of the turbine. A Hall probe is placed outside to sense the magne of the passing magnet. When a magnet passes the probe, a pulse is generated at the output.

(No. of pulses obtained in one second) × 60 = rpm of the turbine (rotor), i an indication of the velocity of flow.

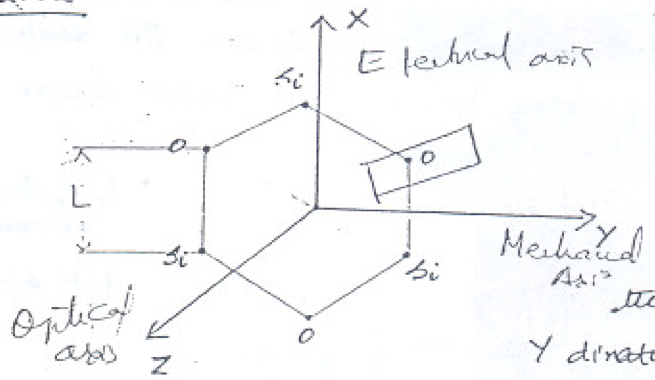
Frequency of pulses $\propto$ Velocity of flow.

Piezoelectric Sensors.

These sensors belong to the class of active transducers. When a - is applied, these sensors produce an output voltage which depe on the amount of force applied. The potential is developed as a resu of the charge being developed on the surface of the materials.

Naturally obtained piezoelectric materials :- Quartz, Rochelle salts, Tourmaline.

Quartz — SiO_2.



X – Electrical Axis

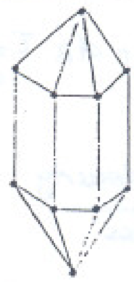
Y – Mechanical Axis

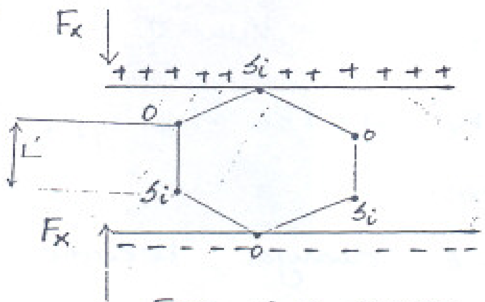
Z – Optical Axis.

Charge gets accumulated only the force is applied in either X Y direction. $V_{o/p} = 0$, if Force is apl in the Z – direction.

The potential is developed because there is a change in the ~~nucleus~~ molecular structure or crystal formation.

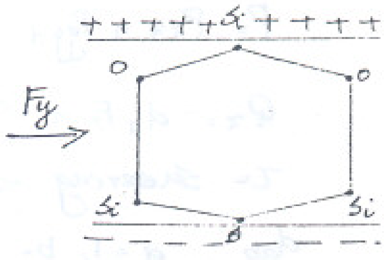
Crystal Structure





Force in X direction

$\underline{\underline{L' < L}}$



Force in Y direction

When a force is applied the structure gets deformed and the molecular distance or the intermolecular length gets shortened. As a result, cha accumulates on the surface, which yields a potential.

$Q = dF.$

Q = charge, Columb.
F = Force, Newton.

$d = Q/F = \dfrac{Q/A}{F/A}$

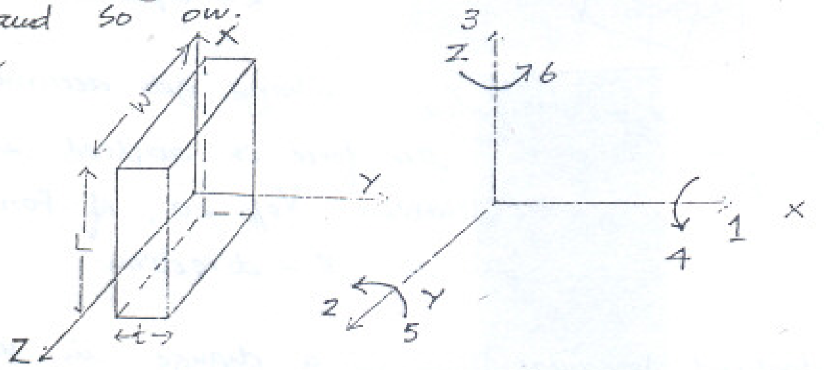
$d = \dfrac{\text{charge density}}{\cancel{\text{Force/Area}}\ \text{Unit stress}} = \dfrac{Q}{F}$

$\boxed{Q \propto F}$

When a force is applied in the X-direction, the charge developed will not bear any relation to the change in length or breadth of the crystal. Infact, the potential will be independant of the length and breadth. But, if we apply a potential in the Y-direction, the charge developed will depend on L_x and L_y [x and y lengths].

No charge will be developed if force is applied on the Z-axis. The virtue of development of a charge when force is applied in a certain direction is the property of the material.

The force can be a combination of two forces. In such case it is called shearing force. They can be of thickness shear or face shear, and so on.



1, 2, 3 → Linear expansion modes.

4, 5, 6 → Shear modes.

$P = P_{xx} + P_{yy} + P_{zz}$. Total value of P in shear mode.

$Q_x = d_{11} F_x + d_{12} F_y + d_{13} F_z + d_{14} \tau_{yz} + d_{15} \tau_{zx} + d_{16} \tau_{xy}$.

τ = shearing force.

d_{ab}, $a = 1$, $b = 1 - 6$ changes, because $\frac{\text{charge density}}{\text{Unit stress}}$ has a different value when different axis are considered.

$Q = d \times F$, But if we define $P = \frac{Q}{A}$ = total charge density

$P = \frac{dF}{A}$.

$P_x = d_{11} \sigma_{xx} + d_{12} \sigma_{yy} + d_{13} \sigma_{zz} + d_{14} \tau_{yz} + d_{15} \tau_{zx} + d_{16} \tau_{xy}$.

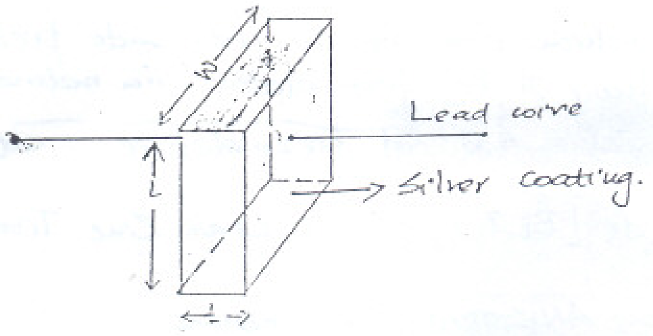
$\sigma_{xx} = \frac{F_x}{A}$, $\sigma_{yy} = \frac{F_y}{A}$, $\sigma_{zz} = \frac{F_z}{A}$ = Corresponding stresses.

Interesting point :-

Now, Quartz is an insulator. What ever may be the charge developed on its surface, how do we extract the potential?

For this, we use the technique called <u>Sputtering</u> :-

A coating of Silver (Ag) is deposited on the surface of the crystal and then the lead wires are brought out.



This produces a capacitance effect and

$$C_x = \epsilon_r \epsilon_0 \frac{Lb}{t}$$

$$V_x = \frac{Q_x}{C_x} = \frac{d_{11} F_x}{C_x}$$

$$V_x = \frac{d_{11} F_x}{\epsilon_0 \epsilon_r \frac{Lb}{t}} = \frac{d_{11} \cdot t \cdot \sigma_x}{\epsilon_r \epsilon_0}, \quad \sigma_x = \text{Stress.}$$

$$\therefore V_x = g_{11} \cdot t \cdot \sigma_x, \quad g_{11} = \frac{d_{11}}{\epsilon_r \epsilon_0}.$$

$$\therefore g_{11} = \frac{V_x / t}{\sigma_x} = \frac{V_x / t}{F_{xx} / \text{Area}} = \frac{\text{Electric Field}}{\text{Unit Stress}} = \frac{\text{Volt/metre}}{\text{Newton/metre}}$$

another parameter can be defined,

$$h = \frac{\text{Electrical Field}}{\text{Unit strain}}, \quad g = \frac{V/t}{\varepsilon_x \cdot E} \Rightarrow gE = \frac{V/t}{\varepsilon_x}$$

now $\frac{V/t}{\varepsilon_x} = h$ $\quad \therefore h = gE.$

E = Young's modulus of elasticity for that material.

$$\boxed{\frac{\text{Stress}}{\text{Strain}} = E = \frac{\sigma}{\epsilon}}$$

Electrostriction :- It is the property of a material in which an applied voltage can generate a force in a certain direction. The material changes its shape.

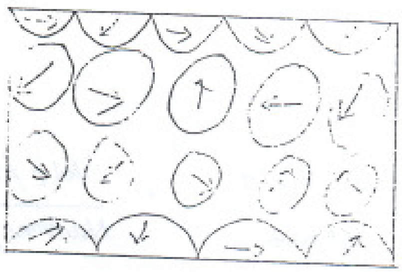
This property is used in quartz oscillators.

There are some materials which can be converted into piezo electric materials. These materials have to be ferro electric in nature. Such materials are called ~~artificial~~ artificial piezo electric materials.

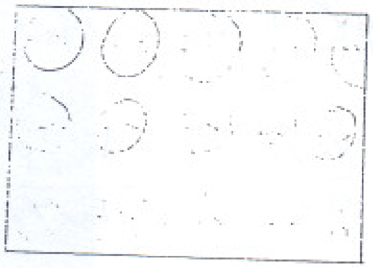
eg :- Bismuth titanium oxide [$BiTiO_2$] and Lead Zinc Titanate.

Initially these materials are 'ANISOTROPIC' in nature.

ANISOTROPIC → The direction of electric field of the individual domain are not aligned in one single direction. The directions are random in nature.



ISOTROPIC or POLARIZED.



Anisotropic materials can be made isotropic, by heating them above the Curie temperature and then applying an magn electric field. Now, when the material is cooled, it can behave as a piezo electric material. now develops a charge / potential when a force is applied on it.

These are tailor-made materials.

Curie temperature : the temperature at which a ferromagnetic material loses its magnetic properties. This temperature is used to produce artificial magnets.

Material	Operating Temperature.
1. Quartz	550°C
2. Tourmaline	1000°C
[illegible]	125°C

Now we know that $h = \dfrac{\text{Electric field}}{\text{Unit } \cancel{\text{stress}} \text{ strain}} = \dfrac{V_x/t}{\Delta t/t} = \dfrac{V_x}{\Delta t} = \dfrac{V}{\Delta t}$

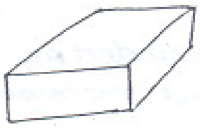
$\boxed{V_x = V}$

$d = \dfrac{Q}{F} = \dfrac{CV}{F}$

$\therefore h \cdot d = \dfrac{CV^2}{F\Delta t}$ $\quad \therefore \dfrac{1}{2} h \cdot d = \boxed{\dfrac{1}{2} \cdot \dfrac{CV^2}{F\Delta t} = K = \dfrac{1}{2} h \cdot d}$

$K = \dfrac{\text{Electrical Energy}}{\text{Mechanical Energy}}$ is a measure of the electrical energy output per unit mechanical energy input.

Numerical :-



$F = 1N$, $d = 110 \times 10^{-12}$ C/N, $L = b = 1$ cm and $t = 1$ mm. find the voltage output. $\epsilon_0 = 8.83 \times 10^{-12}$ F/m, $\epsilon_r = 1200$.

$$C = \frac{A\epsilon_0\epsilon_r}{d} = \frac{lb \cdot \epsilon_0\epsilon_r}{t} = \frac{(1\times10^{-2})^2 \cdot 1200 \times 8.83\times10^{-12}}{(1\times10^{-3})} = \underline{\underline{1.06\,nF}}$$

$d = \dfrac{Q}{F}$ $\quad \therefore Q = dF = 110 \times 10^{-12}$ columbs.

$$V = \frac{Q}{C} = \frac{110\times10^{-12}}{1.06\times10^{-9}} = 0.104\,V \approx \underline{\underline{103.8\,mV}}$$

Strain = ? $\quad E = 8.83 \times 10^{9}\ N/m^2$ = Young's modulus for the piezoelectric material.

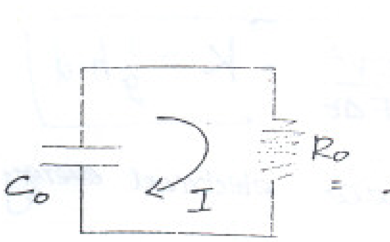
$F/A = \text{stress} = 1N/(1\times10^{-2})^2 = 1\times10^{4}\ N/m^2.$

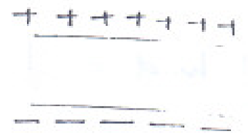
Now, $\text{Strain} = \dfrac{\text{Stress}}{E} = \dfrac{1\times10^{4}}{8.83\times10^{9}} = 1.13\ \mu\text{ strain} = \dfrac{\Delta t}{t}.$

$$\therefore \Delta t = 1.13\times10^{-6} \times 1\times10^{-3} = 1.13\times10^{-9} = 11.3\times10^{-10} = \underline{\underline{11.3\ \text{Å}}}$$

Piezo electric materials cannot be used to measure d.c. or static forces; why?

Modelling of a piezo electric material.

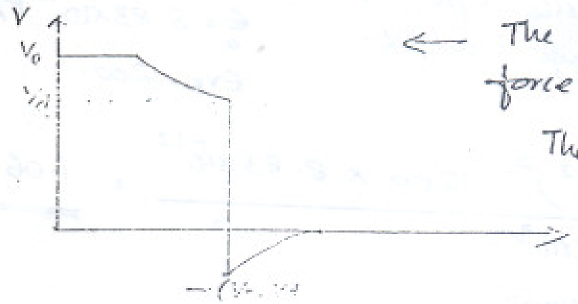




= internal impedance.

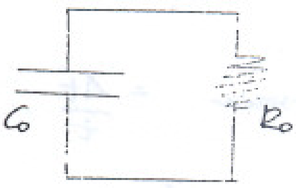
Point of removal of force

As soon as the force is applied and if the force is held a constant charge starts leaking through its internal impedance.

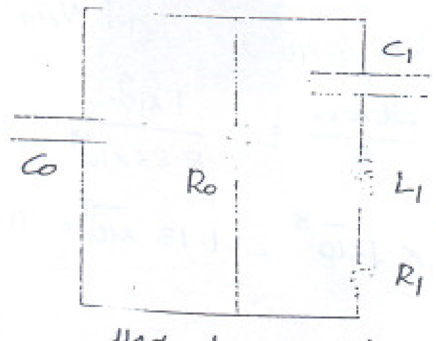


← The output of a piezoelectric crystal when force is applied and removed.

The charge starts leaking and the output droops to Vd. Now the force being removed can be considered to be an application of force in the opposite direction. So the output jumps to $-(V_0 - V_d)$ instead of settling at zero, now it decays from $-(V_0 - V_d)$ slowly to zero.



Low frequency model



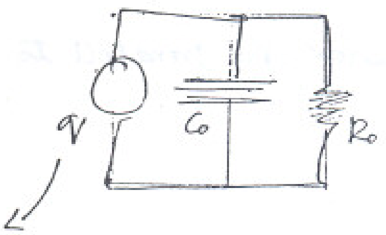
High freq. model.

L_1, R_1, C_1 are the components to determine the natural frequency

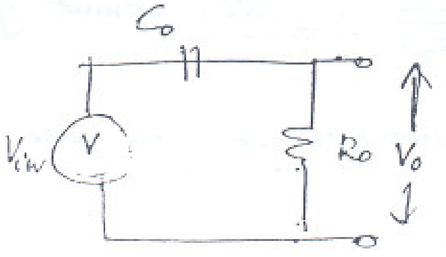
L_1 = Electrical equivalent of internal mechanical damping.

R_1 = electrical equivalent of Mechanical mass.

C_1 = Piezo constant [eqvt. to K]



charge developed



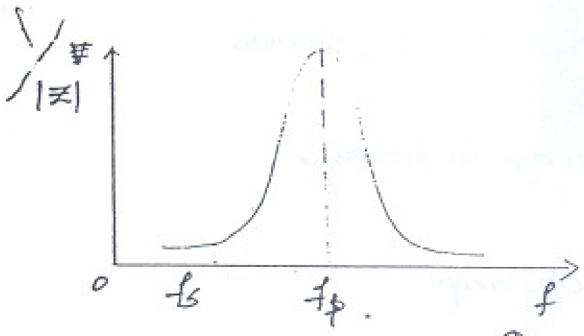
equivalent model.

$$V_0 = V_{in} \cdot \frac{R_0}{R_0 + \frac{1}{sC_0}} \Rightarrow \frac{V_0}{V_{in}} = \frac{sC_0R_0}{1+sC_0R_0} = \frac{j\omega\tau}{1+j\omega\tau} \qquad \tau = R_0C_0.$$

$$\therefore \left|\frac{V_0}{V_{in}}\right| = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$$, so this behaves as a high pass circuit

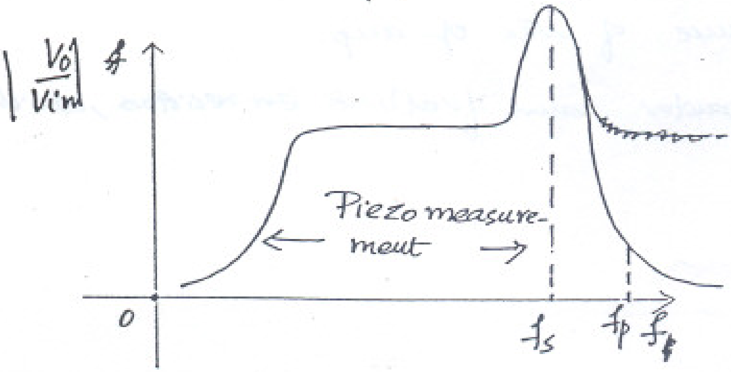
When higher frequencies are assumed, the series combination of $R_1, L_1, \& C$ can be in one resonance, and the whole set up can be in anot resonance.

$$f_s = \frac{1}{2\pi\sqrt{L_1C_1}} \quad \text{and} \quad f_P = \frac{1}{2\pi\sqrt{L\frac{C_1C_0}{C_1+C_0}}}.$$



f_P = natural frequency of the crys

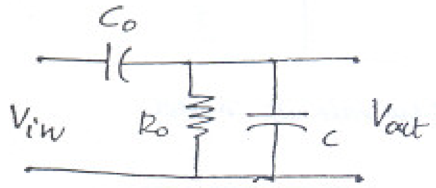
f_s = resonant frequency of R_1, L_1, C_1 at the resonant frequency the l & impedance is very low.



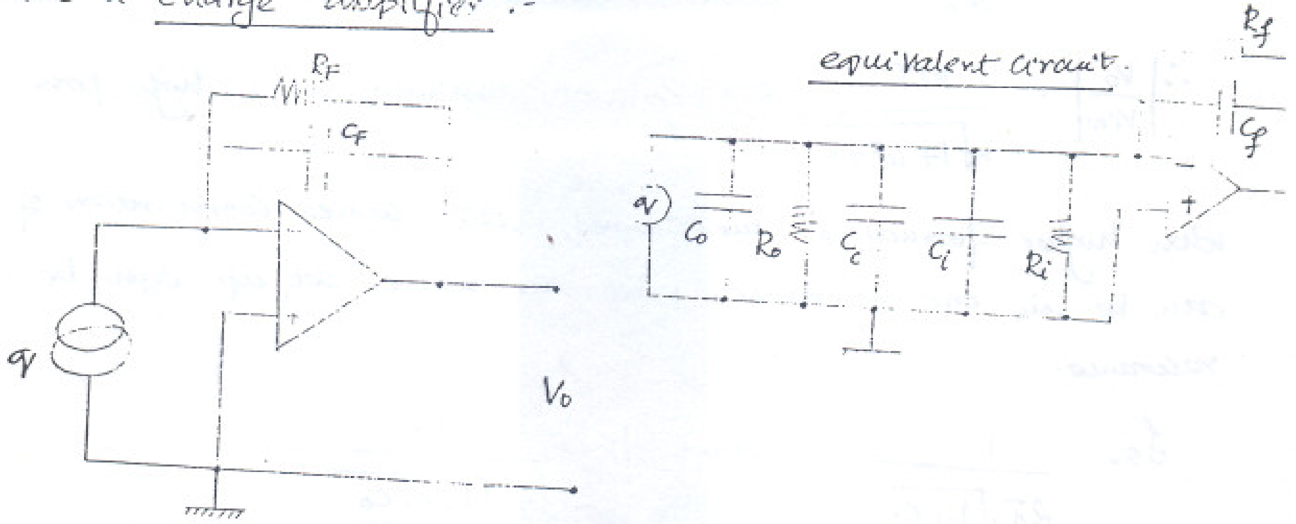
'f_s' → highest voltage output resonance.

$f_s > 10$ times the low freq. band widths then only u use this for faithful m

$$\frac{V_o}{V_{in}} = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$$, this cannot go upto DC (0Hz), to take in. & 0Hz measurements we can use a capacitor in paralell to R_o.



To improve the performance and to get higher sensitivities we use a charge amplifier :-



q= charge developed on the piezo crystal = $d.F$, Columbs.

C_o= Capacitance of the piezo crystal.

R_o= o/p resistance of the crystal, Leakage resistance.

C_c= Cable capacitance.

C_i= ~~i/p~~ input capacitance of the op-amp.

R_i= internal input capacitance of the op-amp.

C_f and R_f are feedback capacitor and feedback ~~ex~~ resistors, which are under our control.

The operational amplifier used in a charge amplifier should have certain characteristics:-

1. <u>Very high input impedance:-</u>

The o/p impedance of a piezo electric crystal is very high so to transfer charge from the crystal to the amplifier, the i/p impedance of the amplifier has to match with that of the o/p impedance of the crystal. Typically $R_i = 10^{13}\,\Omega$.

2. <u>Very low bias currents</u> :- Now the o/p impedance of the crystal is very high, so bias currents can produce a significant drop if the values are large. So I_b^+ and I_b^- should be very small. We do not put much stress on the offset currents.

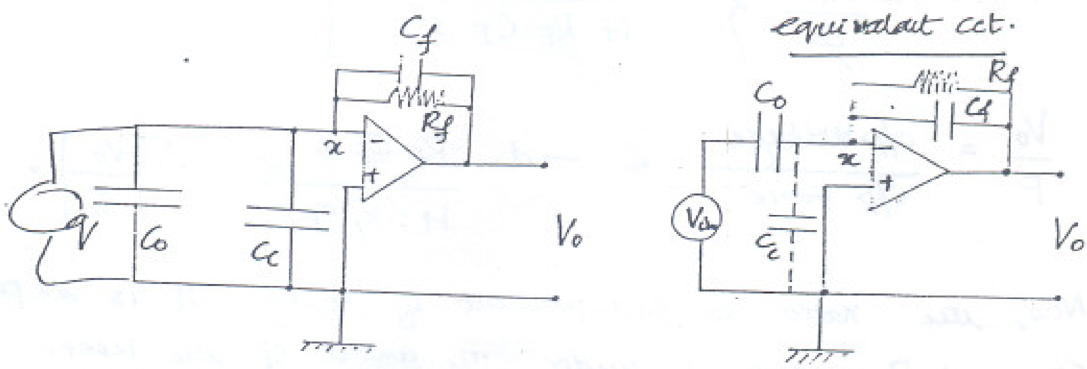
3. <u>Very high open loop gain</u> :-

If the open loop gain is not very high then the offset voltage problems come into picture. V_{os} can appear at the -ve terminal of the op-amp and (+) terminal is no longer at virtual ground. The charge stored in 'q', cannot be transferred to C_f without leakage.

4. <u>Very high slew rate.</u>

For these purposes we use FET <u>op-amps.</u>

Assuming ideal conditions, i.e., C_i and R_i can be neglected as they can be assumed to provide ∞ impedance and resistances R_o can also be ignored. $R_o \to \infty$. Just taking C_c into account.



Now, (+) = Virtual ground. So what ever current flows through C_0 should also flow through C_f, if the principle of virtual short is obeyed. Point 'x' is at ground potential, so

$$V_0 = \frac{q}{C_f}, \quad \because \text{ the whole charge is available at that point.}$$

$$q = V_0 \cdot C_f.$$

The output voltage is independant of the value of capacitance of crystal or the cable capacitance. It is just dependant on the charge developed on the crystal.

Now, if there is a bias current in the op-amp, it will hamper the value of capacitance, as it can charge the capacitor. To elim. we put a feedback resistor $R_F \parallel$ to C_F. But in doing so, note that we are deteriorating the high frequency characteristics of the piezoelectr sensor.

$$V_{in} = \frac{q}{C_0}, \quad V_0 = \frac{-Z_f}{Z_i} \cdot V_{in} = -\left\{ \frac{\left(R_f \cdot \frac{1}{sC_F}\right) \Big/ \left(R_F + \frac{1}{sC_F}\right)}{\frac{1}{sC_0}} \right\} V_{in}$$

$$V_0 = -V_{in} \left\{ \frac{sC_0 \cdot R_F C_F \cdot s}{1 + R_F C_F s} \right\}$$

$$\boxed{C V_{in} = q = dF}$$

$$\boxed{C = C_0}$$

$$\text{now } V_{in} = \frac{dF}{C} = \frac{dF}{C_0}$$

$$\therefore V_0 = -\frac{dF}{\cancel{C_0}} \left\{ \frac{s\cancel{C_0} \cdot R_F C_F \cdot s}{1 + R_F C_F s} \right\}$$

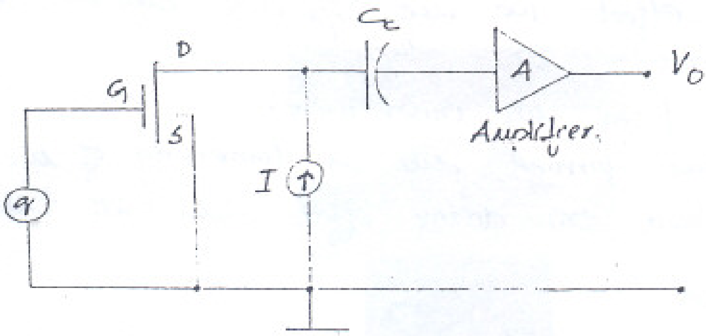
$$\frac{V_0}{F} = \frac{\text{o/p voltage}}{\text{i/p Force}} = -d \cdot \frac{R_F C_F s}{1 + s R_F C_F}. \quad \therefore \left|\frac{V_0}{F}\right| = \frac{\omega \tau_F}{\sqrt{1 + \omega^2 \tau_F^2}}.$$

Now, the ratio is independant of C_0, C_c. It is dependant only on C_F and R_F, which is under the control of the user.

We get a very promising performance when the operating frequencies are to high

If the openloop gain of the op-amp is not very high, we encounter the problem of offset voltage in the output (V_{os}). During this phenomenon, virtual earth is violated and some charges can leak, thereby not ensuring the total charge getting transferred to the feedback capacitor.

* Piezo electric effect was discovered in 1880, but using piezo electric methods to measure parameters was first done in 1950.



Early use of MOSFET and the circuit was patented.

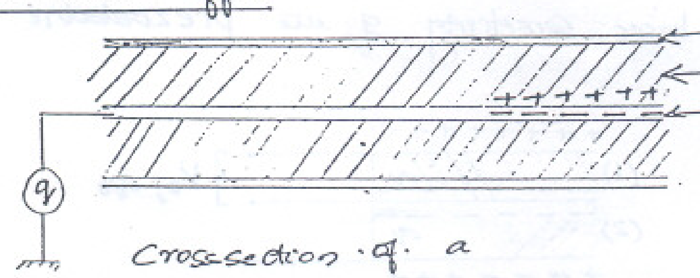
Since the charge has to get transferred to the amplifier, we make use of a MOSFET, which has a very high input impedance comparable to that of a piezo crystal.

I → Constant current source which biases the MOSFET.

C_c → Coupling capacitor. This is used because a piezo electric transducer operates well at higher frequencies. So AC comes into picture, hence a coupling between the input (source) and the amplifier required.

This was one of the earliest circuits to measure acceleration using piezo electric crystals.

Tribo electric effects :-



Cross-section of a COAXIAL CABLE

In a coaxial cable, the inner sheath (wire), Insulator, and the outer sheath are wound or drawn over one another tightly. But due to flexing the tight bonding ~~between~~ the inner wire and the insulator can become weak. This leads to formation of air gaps due to loose bonding. Now this can show piezoelectric effects, since the inner wire is used to carry charge, the output gets affected by stray ~~eff~~ components / parameters like triboelectric effects.

To remove this effect we use special cables which have gap powder in it to ensure two things:-

1. No charge leaks from the inner wire.
2. No air gaps are formed due to loosening of the insulator + wire further ensuring that triboelectric effect does not take place.

<u>Specifications of 'd'.</u>

d_{mnk}. m, n, K = ~~0~~, 1, 2, 3 (can be). They are used to indicate the axis X, Y, Z.

'Z' axis cannot be electrically polarized. So, K' can be neglected if 'no electrical polarization' is ensured for the Z-axis.

d_{mn}.

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

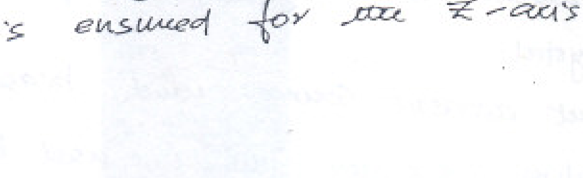
Shear Constant matrix.
Force Constant

d_{11}, d_{22}, d_{33} are different ← Force Constants.

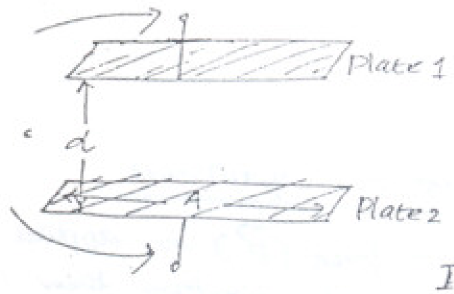
$$\left.\begin{aligned} d_{12} &= d_{21} \\ d_{13} &= d_{31} \\ d_{23} &= d_{32} \end{aligned}\right\}$$ Since the material used is the same.
→ Shear Constants.

To further improve the basic sensitivity of the piezoelectric sensor we use bimorph technique.

Bimorph Technique :-



Capacitive Transducers.



$C = \frac{A\epsilon_0}{d}$ · Capacitance, F

A= Area of the plate, m^2

ϵ_0= Permittivity of free space, F/m = 8.85×10^{-12} F/

d= distance between the two plates, m.

If the medium was not air, but some other [...]erial then $C = \frac{A\epsilon_0\epsilon_r}{d}$, ϵ_r= relative permittivity. ϵ_r for air= 1.

· also called absolute permittivity.

$$\log C = \log\left\{\frac{A\epsilon_0\epsilon_r}{d}\right\}$$

$$\log C = \log A + \log \epsilon_0 + \log \epsilon_r - \log d$$

Differentiate this to get ①.

$$\therefore \frac{\Delta C}{C} = \frac{\Delta A}{A} + \frac{\Delta\epsilon_r}{\epsilon_r} - \frac{\Delta d}{d} \quad \text{——①}$$

[I]n accelerometers we use change in displacement to produce a change in [c]apacitance.

<u>Numerical</u> :- Calculate C, if Area= 1cm x 1cm, ϵ_r= 8.85×10^{-12} F/m, d= 1mm and the dielectric is free space.

$$C = \frac{A\epsilon_0}{d} = \frac{(10\times10^{-3})^2 \times 8.85\times10^{-12}}{1\times10^{-3}} = \frac{(0.01)^2}{0.001} \times 8.85 \times 10^{-12} = 0.885\times10^{-12}\,F$$

$$\simeq 0.9\,pF$$

'd' is always assumed to be the distance between the plates in vertical direction.

We see from the above example that the capacitance involved is of very small magnitudes.

Now if d= 1.1 mm, i.e. the distance changes by 0.1mm or 100 μm.

$$C = \frac{(10\times10^{-3})^2 \times 8.85\times10^{-12}}{(1.1\times10^{-3})} = 8.045\times10^{-13}\,F = 0.8045\times10^{-12}\,F$$

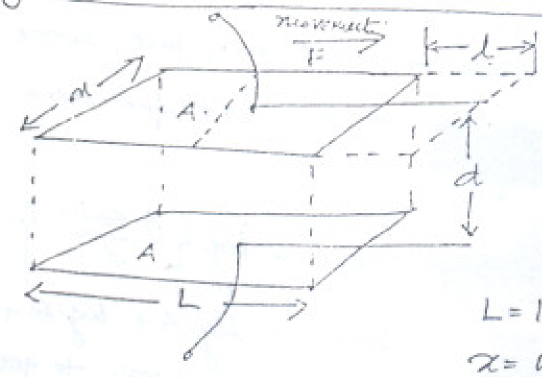
∴ change in capacitance for a change in 100 μm= 8.04×10^{-14} F= 80 femto

The change is extremely small.

Capacitive transducers employ the following principles to produce change in capacitance :-

- change in distance between plates
- change in area (common area)
- change in relative permittivity.

Employing change in area to produce a change in capacitance.



If a force ($\vec{F}$) is applied in the horizontal direction then one of the plates moves by a distance 'l', thereby producing a change in capacitance because the area has changed.

L = length of the plate, m

x = width of the plate, m

Area = Lx, m^2 before the force was applied.

$$C = \frac{A\epsilon_0}{d} = \frac{Lx\epsilon_0}{d}.$$

Now when the plate moves laterally by a distance 'l', the effective length now becomes 'L − l', metres. Width 'x' remains a constant.

$$\therefore \quad C' = \frac{(L-l)x\epsilon_0}{d} = \frac{Lx\epsilon_0}{d} - \frac{lx\epsilon_0}{d} = C - \frac{lx\epsilon_0}{d}$$

$$\therefore \quad C - C' = \frac{lx\epsilon_0}{d}, \quad \boxed{l = \frac{d(C-C')}{x\epsilon_0}}$$

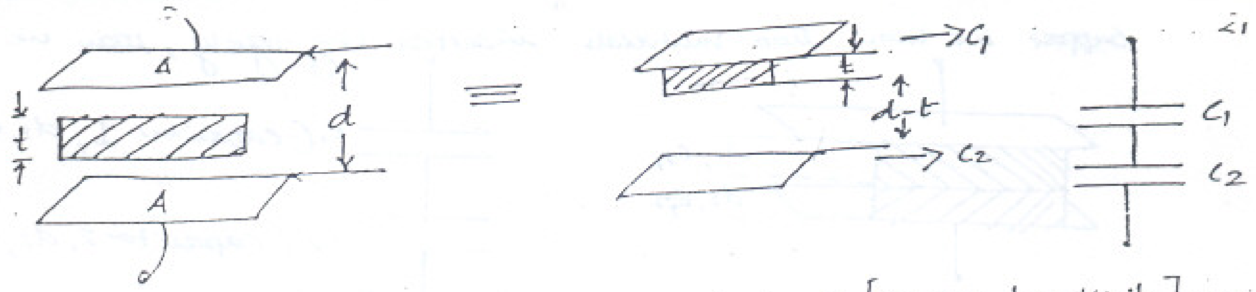
, accuracy of determining 'l' depends on how accurately we can measure C'.

[illegible] can be computed by measuring C in the [illegible]

All materials have relativity, should know before hand

Capacitance Tomography :- Composition of material can be measured by measuring the capacitances, and the change in capacitances produced due to the change in relative permittivity, when the dielectric medium changes. Capacitive probes are inserted at various points inside the tube carrying reactants and chemicals, and the capacitance is constantly monitored. When a chemical enters b/w the space plates the dielectric medium changes. If we can measure the capacitance accurately then we can precisely determine composition of the gas, fluid or solid (Tomography used for fluids and gases only)

∴ Thus calibration is very much a necessity.



If we introduce a material of dielectric constant ϵ_r [relative permittivity] and thickness 't' and assuming it to have the same area as the plates, then it is equivalent of two capacitors C_1 and C_2 in series connection.

C_1 = Capacitance between the upper plate and material.

C_2 = Capacitance between the material and lower plate.

Effective capacitance, $C = \frac{C_1 C_2}{C_1 + C_2}$. $\quad C_1 = \frac{A \epsilon_0 \epsilon_r}{t}, \quad C_2 = \frac{A \epsilon_0}{d-t}$.

For C_2 the medium is air and for C_1 the medium is the ... 'medium' means the 'Dielectric'.

$$\therefore \quad C = \frac{\frac{A\epsilon_0 \epsilon_r}{t} \cdot \frac{A\epsilon_0}{d-t}}{\frac{A\epsilon_0\epsilon_r}{t} + \frac{A\epsilon_0}{d-t}} \longrightarrow \text{Simplify} \qquad C = \frac{A\epsilon_0}{d + t\left[\frac{1}{\epsilon_r} - 1\right]} \quad \text{—} (1)$$

Change in dielectric medium to produce a change in capacitance is used in textile industry, paper industry, and capacitance tomographs. It is also used to measure the humidity. Proper correlation needs to be there between 'ϵ_r' and the humidity measured ($\because$ it needs to be calibrated).

'ϵ_r' changes with the quantity of moisture present and hence induces a change in capacitance.

taking partial derivative after taking log (it becomes series)

$$C = \frac{A\epsilon_0\epsilon_r}{d} \Rightarrow \frac{\Delta C}{C} = \frac{\Delta A}{A} + \frac{\Delta \epsilon_0}{\epsilon_0} - \frac{\Delta d}{d} + \frac{\Delta \epsilon_r}{\epsilon_r};$$

A, d, ϵ_0 remaining a constant

$\boxed{\Delta c \propto \Delta \epsilon_r}$

To study variation realistically with thickness ~~Partial differentiation~~ of (1) yields, $\frac{\Delta C}{\Delta t} = \frac{A\epsilon_0\left[\Delta \frac{1}{\epsilon_r} - 1\right]}{\left[d + t\left(\frac{1}{\epsilon_r} - 1\right)\right]^2} = \frac{C\left[1/\epsilon_r - 1\right]}{\{d + t\left[\frac{1}{\epsilon_r} - 1\right]}$

$\Rightarrow \frac{\Delta C}{} = \left[\Delta t \cdot (1/\epsilon_r - 1)\right]$

Suppose we have two materials inserted very tightly, then we have.

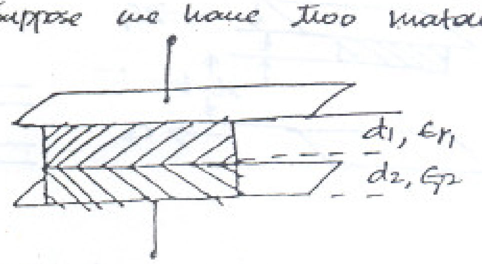


Fig.2

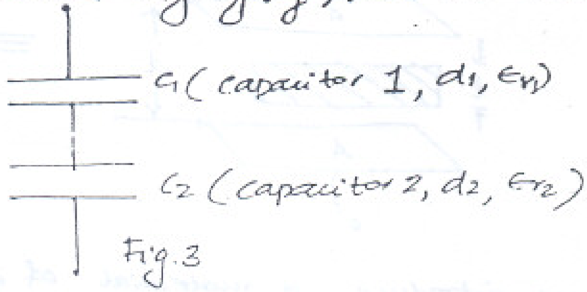


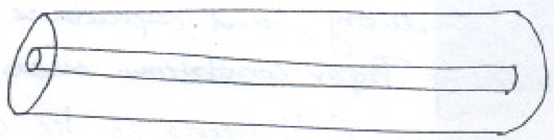
Fig.3

Material 1 has a ~~(dielectric constant)~~ relative permittivity of ϵ_{r_1} and a thickness of d_1 and Material 2 has a ~~di~~ relative permittivity of ϵ_{r_2} and thickness d_2 then if they are similar to Fig 2, then they can be thought of as two capacitors connected in series (Fig. 3).

$$\text{Effective capacitance} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{A\epsilon_0\epsilon_{r_1}}{d_1} \cdot \frac{A\epsilon_0\epsilon_{r_2}}{d_2}}{\frac{A\epsilon_0\epsilon_{r_1}}{d_1} + \frac{A\epsilon_0\epsilon_{r_1}}{d_2}} = \frac{\epsilon_{r_1}\epsilon_{r_2} A\epsilon_0}{d_1\epsilon_{r_2} + d_2\epsilon_{r_1}}$$

$$\boxed{C_{eff} = \frac{A\epsilon_0}{\frac{d_1}{\epsilon_{r_1}} + \frac{d_2}{\epsilon_{r_2}}}}$$

~~Parallel axis~~ <u>Cylindrical capacitors</u>



diameter of the inner ~~P~~ cylinder $= d_2$

diameter of the outer cylinder =.

How to derive the capacitance of a parallel plate cylindrical capacitor?

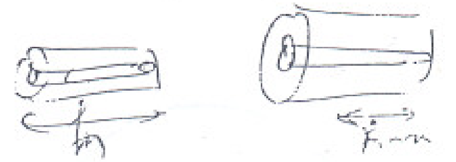
Solution :-

$$C = \frac{2\pi\epsilon_0\epsilon_r \cdot \cancel{\ell} \cdot h}{\ln(d_1/d_2)}.$$

~~& $d_1/2$ = radius of the outer cylinder~~

When the cylinder (maybe outer or inner) is moved away by a distance 'x', then we have

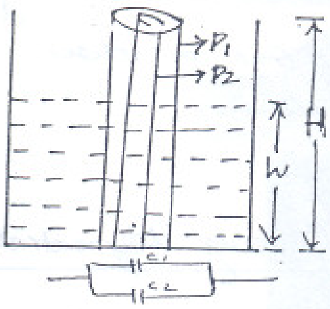
$$C' = \frac{2\pi \epsilon_0 \epsilon_r \cdot r(l-x)}{\log(d_1/d_2)}$$



$$\Delta C = C - C' = \frac{2\pi \epsilon_0 \epsilon_r r W}{\log(d_1/d_2)} - \frac{2\pi \epsilon_0 \epsilon_r (W-x) r}{\log(d_1/d_2)} = \frac{2\pi r x \epsilon_0 \epsilon_r}{\log(d_1/d_2)}$$

direct relationship

We can use this technique to measure liquid level in a tank.



H = total height of the tank.

h = height of liquid.

d_1 = inner diameter of pipe 1 or P_1 (outer pipe)

d_2 = outer diameter of pipe 2 or P_2 (inner pipe)

ϵ_r = relative permittivity of the liquid.

This configuration can be thought as two capacitors connected in parallel.

C_1 = capacitance formed due to the liquid column (Height 'h').

C_2 = capacitance formed due to air column (Height 'H-h').

$r = d_1/2$ = radius of outer pipe. P

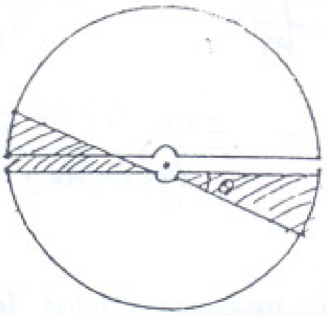
$$\therefore C = C_1 + C_2 = \frac{2\pi r (\epsilon_0 \epsilon_r) h}{\log(d_1/d_2)} + \frac{2\pi r \epsilon_0 (H-h)}{\log(d_1/d_2)}$$

$$= \frac{2\pi r \epsilon_0}{\log(d_1/d_2)} \left\{ \epsilon_r h + (H-h) \right\} = \frac{2\pi r \epsilon_0}{\log(d_1/d_2)} \left[h(\epsilon_r - 1) + H \right]$$

$$h = \left[\left\{ \frac{(C_1 + C_2)}{(2\pi r \epsilon_0)/\log(\frac{d_1}{d_2})} \right\} - H \right] \times \frac{1}{(\epsilon_r - 1)}$$

← final formula for calculating liquid level.

Measurement of Angle.



r = radius of the semicircular plates

d = distance of seperation.

Maximum capacitance obtained when $\theta = \pi$ or when they are totally on top of the other.

We use two hemispherical plates rotating on a common axis.

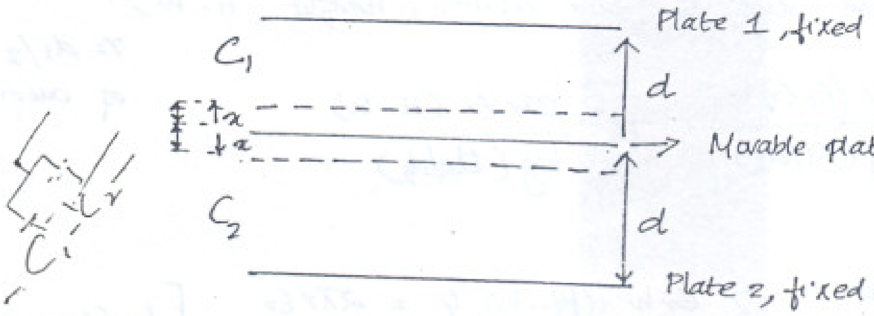
When they do not have any common area between them

$C = 0$.

When one ~~he~~ semicircular plate starts moving over the other, the the capacitance between them

$$C_\theta = \frac{\left\{\frac{\pi r^2}{2}\right\} \epsilon_0 \epsilon_r}{d} \cdot \frac{\theta}{180^\circ}, \text{ or } \frac{\theta}{\pi}$$

Measurement of $\underbrace{\text{distance moved}}_{\text{displacement}}$, using differential technique :-



$\uparrow x \rightarrow$ +ve 'x' movement

$\downarrow x \rightarrow$ -ve 'x' movement

All the plates have same area, A, m^2.

d = distance between movable plate and a fixed plate, m.

When the movable plate moves upwards by 'x'

C_1 = Capacitance between plate 1 and movable plate $= \frac{A\epsilon_0}{d-x}$

C_2 = Capacitance between plate 2 and movable plate $= \frac{A\epsilon_0}{d-x}$.

$$C_1 \sim C_2 = \frac{A\epsilon_0}{d-x} - \frac{A\epsilon_0}{d+x} = \frac{A\epsilon_0(d+x) - A\epsilon_0(d-x)}{d^2 - x^2}$$

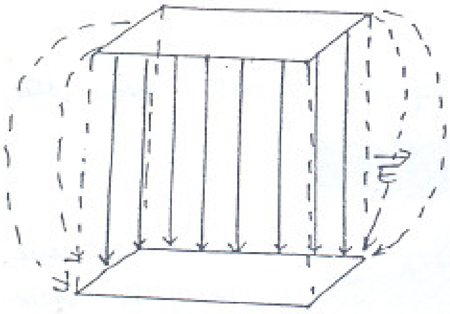
$$C_1 \sim C_2 = \frac{2A\epsilon_0 x}{d^2 - x^2} \simeq \frac{2A\epsilon_0 x}{d^2}.$$

$\underline{\underline{d^2 - x^2 \simeq d^2}}$

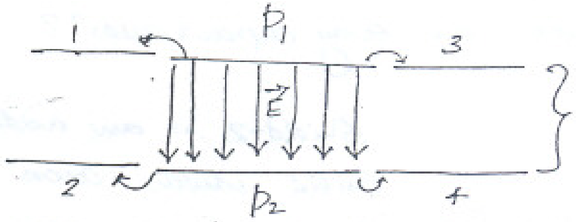
The differential capacitive measurement technique is used in accelerometers. $C_1 \sim C_2 = C \propto x$.

FRINGING IN CAPACITIVE TRANSDUCERS.

$\vec{E}$ - electric lines of force:



In a normal parallel plate capacitor, electrical lines of force (electrostatic) are perpendicular to the plate surface. But there will be curved lines of force from one edge of the plate to another and the bottom plate. This is known as Fringing effect. This is undesirable i.e, the curved lines of force need to be resisted. This can be eliminated by putting a guard ring.



1, 2, 3, 4. guard plates.

Fringing $\propto \log\sqrt{A}$.

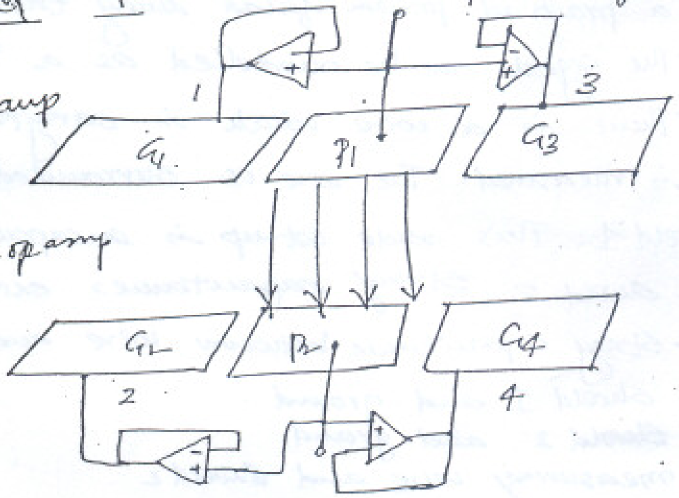
$$C_1 = \frac{A\epsilon_0}{d} + \underbrace{\alpha \log\sqrt{A}}_{\text{Not accounted}}$$

Now the guard ring has obstructed the curved lines of force. But a capacitance can be formed between the capacitor plate and ~~the~~ guard ring. This is undesirable. So we need to maintain the guard ring and the capacitor plate at same potential. No change in Voltage => No charge is accumulated ∴ no capacitance.

ckt loading:- The i/p V is in very low value ∴ loading of V.

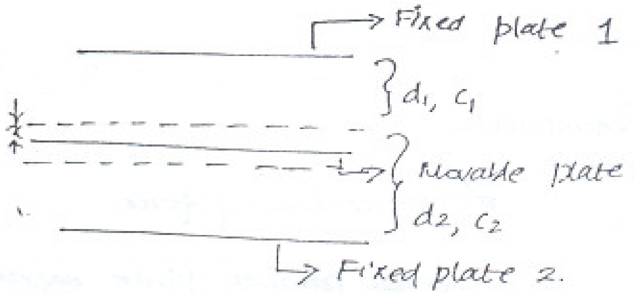
$Q = CV$ or $C = \frac{Q}{V}$. So the best way to maintain the same potential for both:-

Soln.:- at buffer amp it gets supplied by P.S. of op amp



The biggest advantage is that the guard rings are being powered by the voltage follower which has its own power supply. If the guard rings were simply attached to the plates then the guard ring potential is derived from the signal, which will attenuate the signal power, during processing.

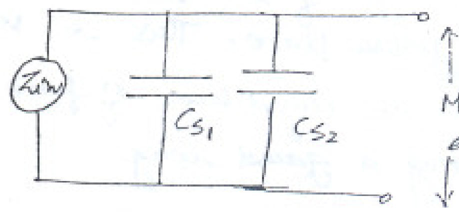
Stray capacitances :-



$$C_1 - C_2 = \frac{2A\varepsilon_0\varepsilon_r}{d^2}$$

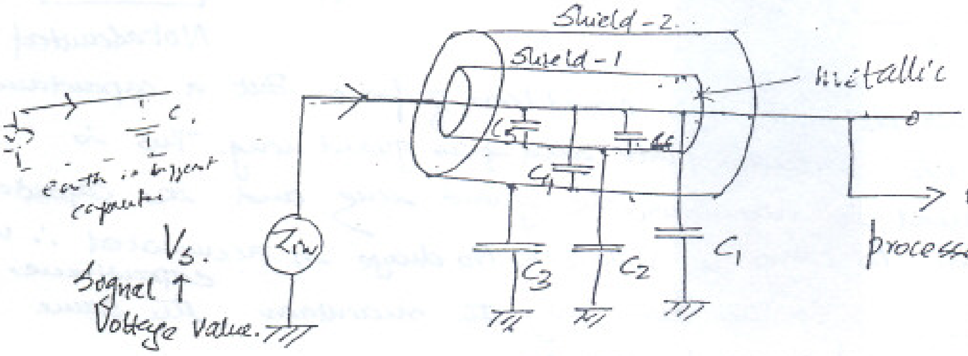
We have the capacitance as $\frac{A\varepsilon_0\varepsilon_r}{d}$ + a non-linear component.

If 'Zin' is input impedance of the signal. Then the signal along with its input impedance can be modelled as :-



C_{s_1} and C_{s_2} are stray capacitance which may be due to shield and ground or the signal wire and ground and hence forth, these are undesirable.

So the question is "How to eliminate the stray capacitances"?



Shield-2 is an additional shield which covers the shield and the wire carrying the signal.

→ wire carrying the signal to processed or measured.

<u>Situation</u> :- We have modelled a practical problem faced during transmission a signal during measurement. The signal can be modelled as a Voltage source having a source impedance. There is a wire which in carrying this signal to the place where it is measured. The wire is surrounded or encapsuled in a primary shield 1. This whole set-up is encapsuled again inside a secondary shield – shield 2. Stray capacitances exist at various locations, eg:- C_1 → Stray capacitance between wire and ground

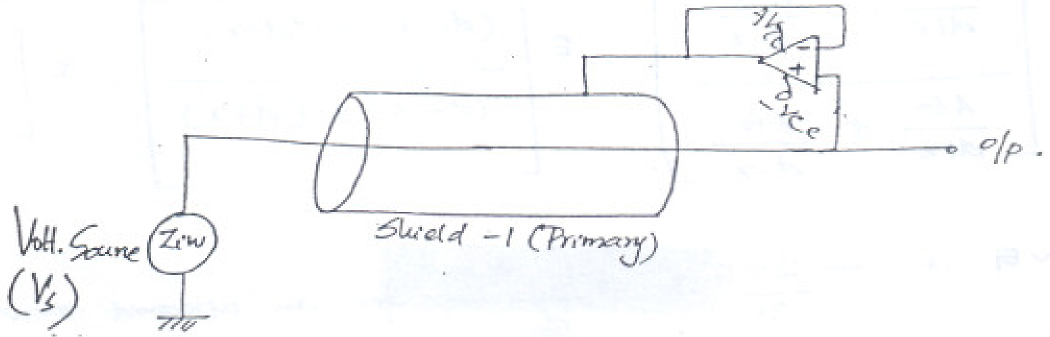
C_2 – Stray capacitance between shield 1 and ground.

C_3 – Stray capacitance between shield 2 and ground

C_4 – Stray capacitance between measuring wire and shield 2

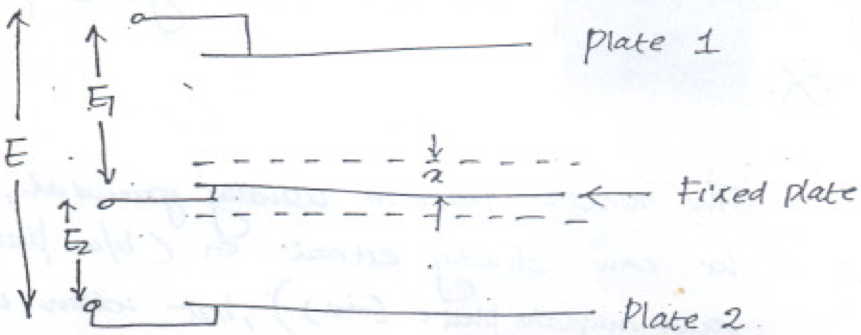
C_5 C_6 Stray capacitance between measuring wire and shield 1 measuring wire & shield 1

Except for C_5 and C_6, the others will not harm the measurement if it can be isolated. C_5 and C_6 can be taken care of, if both the shield and measuring wire can be kept at the same potential. Shield 2 can be left unconnected and hence the other capacitances can be over ru



Now shield 2 need not be connected (i.e., the secondary shield), hence the other capacitances do not appear in the picture.
Now, the first shield is at the same potential as of the signal and it made to stay at the same potential using a voltage source.

It is always advised to measure the differential capacitance rather than the absolute value of capacitance itself. This always eliminates one fac that can be conflicting and which is redundant.

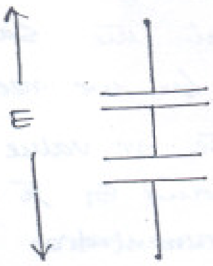


i) Plate 1 and moving plate capacitance = C_1 and Potential difference = E_1

ii) Plate 2 and moving plate capacitance = C_2 and Potential difference = E_2.

iii) Plate-1 and Plate-2 capacitance = C and Potential difference = E.

$Q = CV = CE$.



$C_1 = \frac{1}{j\omega C_1}$

$C_2 = \frac{1}{j\omega C_2}$

$$E_1 = E \cdot \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = E \cdot \frac{C_2}{C_1 + C_2}$$

$$E_2 = E \cdot \frac{\frac{1}{j\omega C_2}}{\ldots} = E \cdot \frac{C_1}{C_1 + C_2}$$

$$\therefore \Delta E = E_1 \sim E_2 = E\left[\frac{C_2}{C_1+C_2} - \frac{C_1}{C_1+C_2}\right] = E\left[\frac{-C_1^2 + C_2^2 + C_1C_2 - C_1C_2}{(C_1+C_2)^2}\right]$$

$$C_1 = \frac{A\epsilon_0}{d+x},\ C_2 = \frac{A\epsilon_0}{d-x};\ \Rightarrow E\left[\frac{C_2 - C_1}{C_1+C_2}\right]$$

For a movement in the upward direction :-

$$= E\left[\frac{\frac{A\epsilon_0}{d+x} - \frac{A\epsilon_0}{d-x}}{\frac{A\epsilon_0}{d+x} + \frac{A\epsilon_0}{d-x}}\right] = E\left[\frac{(d-x)-(d+x)}{(d-x)+(d+x)}\right] = E\left[-\frac{2x}{2d}\right]$$

$$\Delta E = E_2 \sim E_1 = -\frac{2xE}{2d} = \underline{\underline{-\frac{Ex}{d}}} \rightarrow$$ for an upward movement of say 'x'.

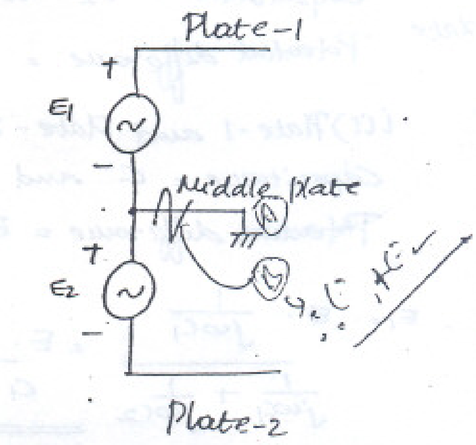
Now, if the movement is downwards

$$\Delta E = E_2 \sim E_1 = E\left[\frac{\frac{A\epsilon_0}{d-x} - \frac{A\epsilon_0}{d+x}}{\frac{A\epsilon_0}{d-x} + \frac{A\epsilon_0}{d+x}}\right] = E\left[\frac{d+x-(d-x)}{d+x+d-x}\right]$$

$$\Delta E = E_2 \sim E_1 = \frac{Ex}{d} \rightarrow$$ for a downward movement of say 'x'.

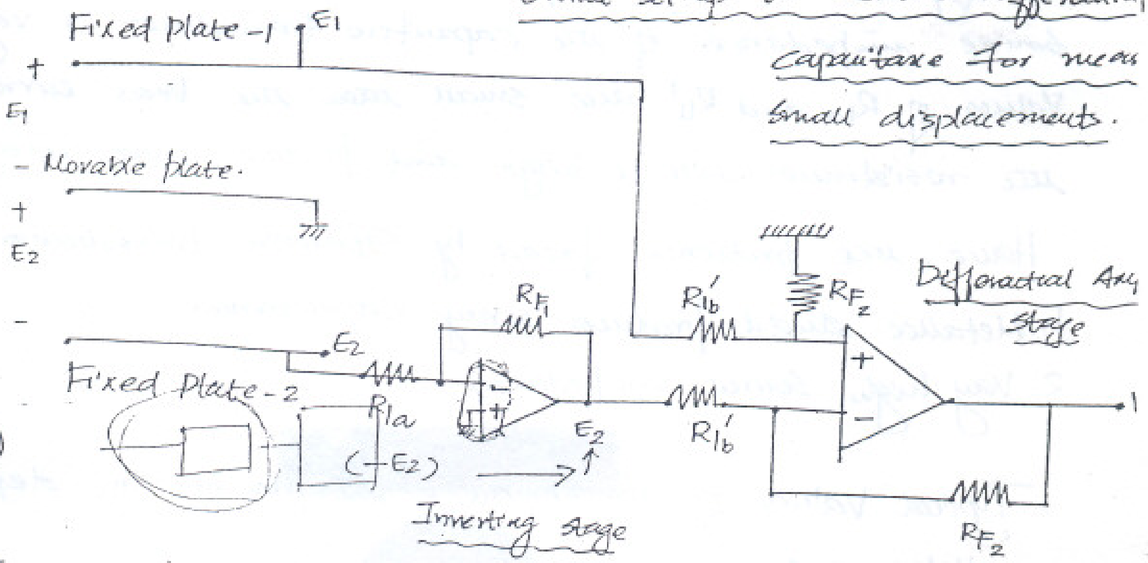
Now, since we are measuring the differential voltage we have drawn A, ϵ_0. We have gained additional knowledge regarding the direction of movement by measuring the value of ΔE and observing the polarity.

Now $\Delta E = |E_1| - |E_2|$. $\cancel{\cdot}$



The middle plate is usually grounded, hence we can directly extract E_1 (b/w plate 1 and middle plate (+ve)), but when we take E_2 in the same manner it is inverted. So we need to invert it again to get the +ve value of E_2 and then apply this E_2 and E_1 to a differential amplifier an instrumentation amplifier to get the differential value of capacitance.

Overall set-up to measure differential capacitance for measuring small displacements.



(Fig. 1)

$V_0 = (E_1 - E_2) \cdot \frac{R_F}{R_1}$. Hence the advantage is that E_2 is buff... E_1 can also be buffered if allowed to pass t... a unity gain follower.

How to measure very small capacitances in the range of pico Farad where the change itself is of the order of femto Farads (10^{-15}) or atto Farads (10^{-18}).

To measure such values with great precision we make use of frequency maybe of the order of 10×10^6 or a couple of tens of MHz. $X_c = \frac{1}{2\pi f C}$, $f \uparrow$ $X_c \downarrow$ ...

Subsequent stage ... 2 is l...

The main advantages of using capacitive transducers are :-

1. The size is very small
2. change in ΔC is pretty considerable for small changes in Δx, hence the sensitivity (o/p ÷ i/p) is very high.
3. No mechanical forces are involved or infact generated unlike an inductor where a current passing through it produces a magnetic f... which is attractive in nature.
4. Hence no magnetic fields are produced which interfere with the capa... tance.
5. capacitance transducers have very high source impedance or output impedance, Because of this we use silicon wafer in semiconductor ty... transducers, where ∥ plates are not involved.

Disadvantage :- In semiconductor type transducers the dielectric constant ... changes with temperatures and hence it affects the capacitance.

In fig. 1 → the value of R_{1a} and R_{1b}' have to be high because the source impedance of the capacitive transducer is very high. If the values of R_{1a} and R_{1b}' are small then the bias current flowing out of the resistances will be large and produce more voltage drop.

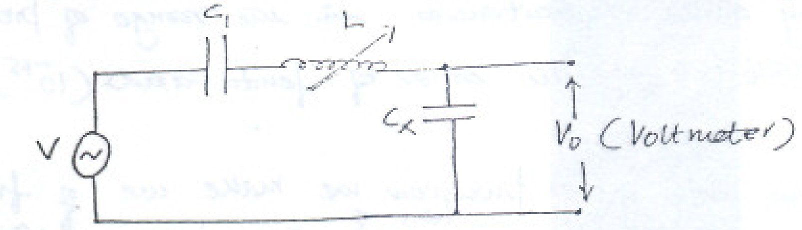
Hence the problems faced by capacitive transducers are :-

1. Metallic shield produces stray capacitances
2. Very high source impedances.

Typical values of displacement = in μm or mm depends on the value of ΔC.

Methods to measure "capacitance" Effectively :-

1.



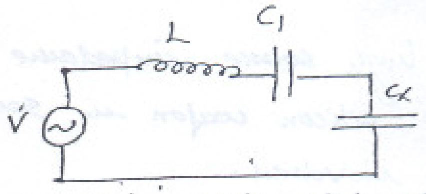
$$V_0 = V \cdot \frac{1/j\omega C_x}{\frac{1}{j\omega C_x} + \frac{1}{j\omega C_1} + j\omega L}$$

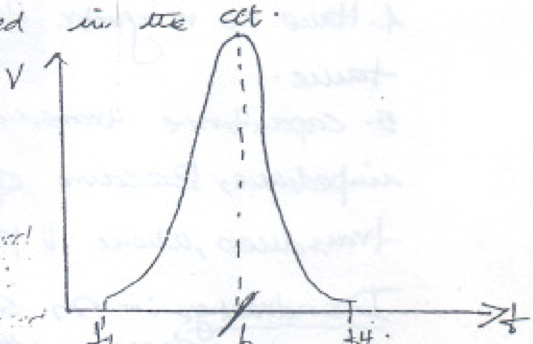
$$= V \cdot \cancel{\frac{j\omega C_1 \, j\omega C_x / j\omega C_x}{\frac{j\omega C_1 + j\omega C_x + j^3\omega^3 L C_1 C_x}{j\omega C_x \cdot j\omega C_1}}}$$

$$= V \cdot \frac{\cancel{j\omega C_x} \cdot j\omega C_1}{\cancel{j\omega C_x}\left[j\omega C_1 + j\omega C_x + j^3\omega^3 L C_1 C_x\right]}$$

$$= V \cdot \frac{C_1}{C_1 + C_x + (-\omega^2 L C_1 C_x)} = V \cdot \frac{C_1}{(C_1 + C_x) - \omega^2 L C_1 C_x} = V \cdot \frac{\frac{C_1}{C_1 C_x}}{\frac{C_1 + C_x}{C_1 C_x} - \omega^2 L}$$

$\frac{C_1 + C_x}{C_1 C_x} = 1/C_p$ C_p = effective capacitance when connected in the ckt.





Keep on varying L such that resonance is achieved at a particular frequency. [illegible] the value of L and the value of [illegible] to calculate the value of C_x.

at resonant frequency (f_0) $V_0 = \infty$ i.e., $\frac{C_1+C_2}{C_1 C_x} - \omega^2 L = 0$

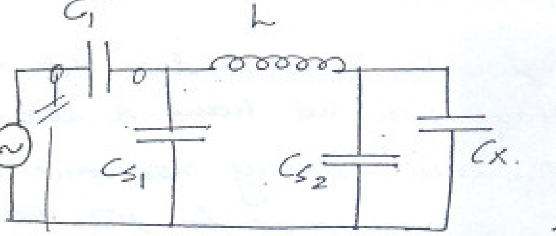
$\Rightarrow \frac{1}{C_p} - \omega^2 L = 0 \Rightarrow \omega^2 = \frac{1}{LC_p} \Rightarrow \omega = \frac{1}{\sqrt{LC_p}}$

$\therefore \omega = 2\pi f \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC_p}}$. C_p = series capacitance of C_x and C_1

So 'f_0' can be found out, L and C_1 is fixed. C_x is the unknown capacitance. So by finding out the resonant frequency and solving

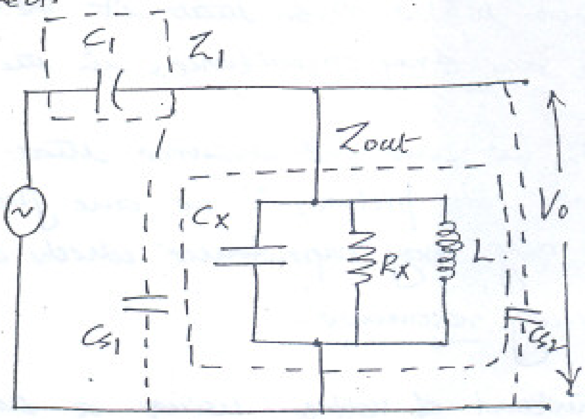
$\omega_0 = \frac{1}{\sqrt{LC_p}}$, gives the value of C_x.

Here the value of C_1 produces a problem, and C_1 needs to be strictly fixed. Stray capacitances can creep in the following forms :-



C_{S1} and C_{S2} are stray capacitances and they interfere in measurement and now the effective capacitances include C_{S1} and C_{S2} and the result be an erroneous 'f_0', thereby resulting in wrong value of C_x being measured.

2.



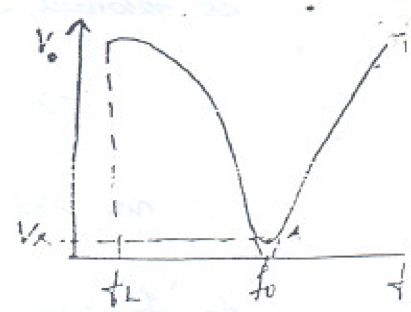
R_x = leakage resistance associated with the capacitor, it has to be very high, for a good capacitor.

$V_0 = V \cdot \frac{Z_{out}}{Z_1 + Z_{out}}$, $Z_1 = \frac{1}{j\omega C_1}$

$Z_{out} = X_L \| X_C = \frac{X_L X_{C_x}}{X_L + X_{C_x}} = \frac{j\omega L * \frac{1}{j\omega C_x}}{j\omega L + \frac{1}{j\omega C_x}} = \frac{j\omega L}{(j\omega)^2 L C_x + 1}$

$V_0 = V\left[\frac{(j\omega L) / 1+(j\omega)^2 LC_x}{\frac{1}{j\omega C_1} + \frac{j\omega L}{1+(j\omega)^2 LC_x}}\right] = V\left[\frac{j\omega L \cdot j\omega C_1}{1 + j^2\omega^2 LC_x + j^2\omega^2 LC_1}\right] = V\left[\frac{-\omega^2 L}{1-\omega^2(L}\right.$

$$V_0 = V\left[\frac{1}{\left[\frac{C_1+C_x}{C_1}\right] - \frac{1}{\omega^2 L C_1}}\right]$$ at f_0, $V_0 = \infty$



$$\Rightarrow \frac{C_1+C_x}{\cancel{C_1}} = \frac{1}{\omega^2 L \cancel{C_1}} \Rightarrow \omega = \frac{1}{\sqrt{L(C_1+C_x)}} \quad \text{—— (1)}$$

During resonance $X_L = X_{C_x}$, so opposing effect takes place and it behaves as a short circuit. Moreover we need to take the value of R_x into account.

i) If the value of $R_x = 0$, then the whole setup will act as a short circuit and the potential drop = 0, hence the curve A touches the zero point at $f = f_0$.

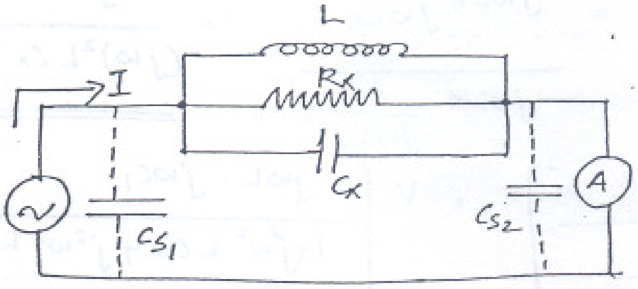
ii) If R_x has a finite value, then the curve A does not touch the zero point as the whole setup will not have a zero potential drop. The drop across the parallel tank circuit during resonance will be equal to $I_x \cdot R_x$. I_x = current passing during resonance. So this value will be ... and it appears as the offset.

So for a good ckt. R_x has to be ∞, so that it becomes an open circuit. (C_{S1} and C_{S2} are stray capacitances in the fig.)

But if we consider eqn (1), we can just assume that during resonance the value of C_1 does not come into picture, $\because$ we are just concerned about the parallel tank circuit. So any stray capacitance which changes the value of C_1 is unaffected during resonance.

Can we measure current instead of voltage using a small modification?

Yes

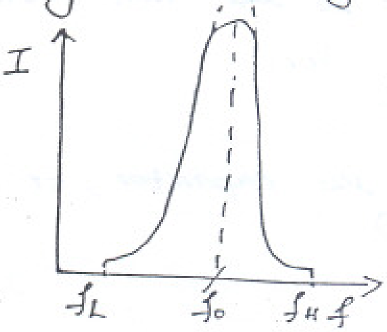


R_x = leakage capacitance of C_x

C_x = unknown capacitance

C_{S1} and C_{S2} = Stray capacitance

∴ During resonance the current through the circuit will be at its maximum and will flow only through R_x. Now the Ammeter (A) connected will have a very small resistance, hence the current will flow through the ammeter and it will bypass the stray capacitances C_{s1} and C_{s2}.

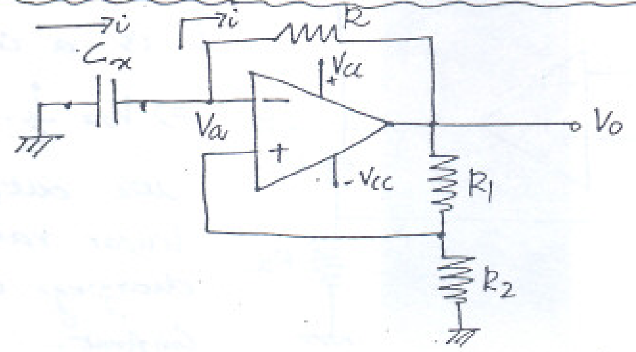


at $f_0 = \frac{1}{2\pi\sqrt{LC_x}}$ the value of I will be ∞ (ideally).

What are the parameters that should be certain in this kind of measurement using C_x, L, and C_1 (C_f used)?

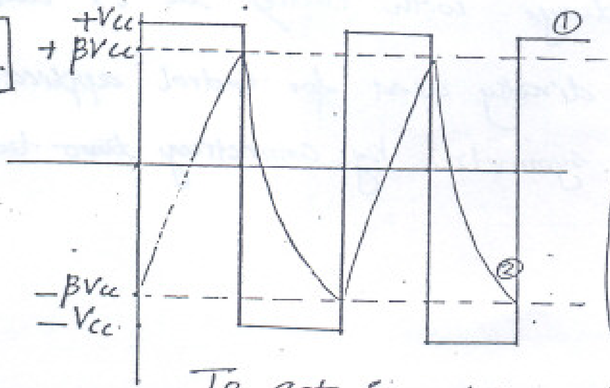
1. Uncertainty exists in precisely measuring the frequency, 'f'.
2. Uncertainty exists in precisely measuring the inductance, 'L'.

Multivibrator (op-amp based) technique to measure 'C'.



V_a = Voltage fed back from the output and it is the potential difference at the non-inverting terminal. It is not the differential potential appearing at the input terminals of the op-amp. Since the feedback is positive the output goes to V_{cc} and the voltage fed back is $\frac{R_1}{R_2+R_1} \cdot V_o$. Put $\frac{R_1}{R_1+R_2} = \beta$.

$[V_a = \beta . V_o]$



$V_a = \frac{R_1}{R_1+R_2} \cdot V_{cc}$.

$iR = V_o$.

$i = C_x \frac{dV_a}{dt} = C_x \left[\frac{R_1}{R_1+R_2}\right] \frac{V_{cc}}{T}$.

$\Rightarrow V_o = C_x \cdot R \cdot \frac{V_{cc}}{T} \left(\frac{R_1}{R_1+R_2}\right)$.

To get symmetric outputs we can use a pair of zener

The output of the circuit would have been I, image is not been the picture. The frequency of oscillations depend on the time constant. The free running astable multivibrator would have settled with oscillation of magnitudes $\pm V_{CCSAT}$.

Now a part of the o/p voltage is fed back to the non-inverting terminal (+ve feedback). $V_a = \beta V_{cc} = \frac{R_2}{R_1+R_2} \cdot V_{cc}$.

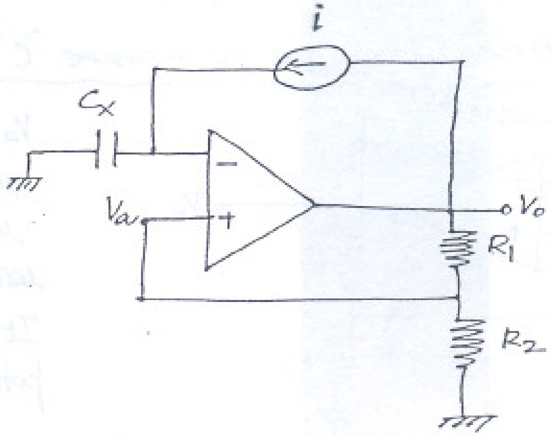
Now C_x charges to 'V_a' or 'βV_{cc}'.

So the output will be the voltage across the capacitor, or the ckt acts as a differentiator. The o/p = waveform ②.

$f_0 = \frac{1}{2\pi R C_x}$. R = fixed resistor.

If 'f_0' can be determined accurately then C_x can be calculated.

Now the capacitor is charging using the current flowing through the resistor. Hence the shape of the waveform. Instead of that if we use a constant current source then



$V_o = \frac{1}{C_x}\int i\,dt$

'i' is a constant.

$\Rightarrow V_o = \frac{i}{C_x} \cdot t$.

The output will be a linear ramp, because the charging current remains constant.

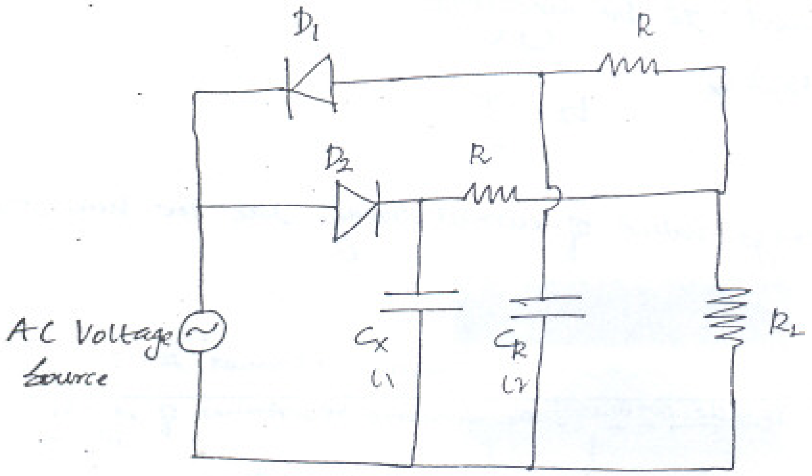
$\therefore \beta V_{cc} = \frac{i}{C_x} \cdot t$.

The value of $t = \frac{1}{f_0}$ will change with change in C_x and since the o/p of C_x is buffered, it can be directly used for control applications.

The output can be made symmetric by connecting ~~two~~ two zener diode in back to back fashion.

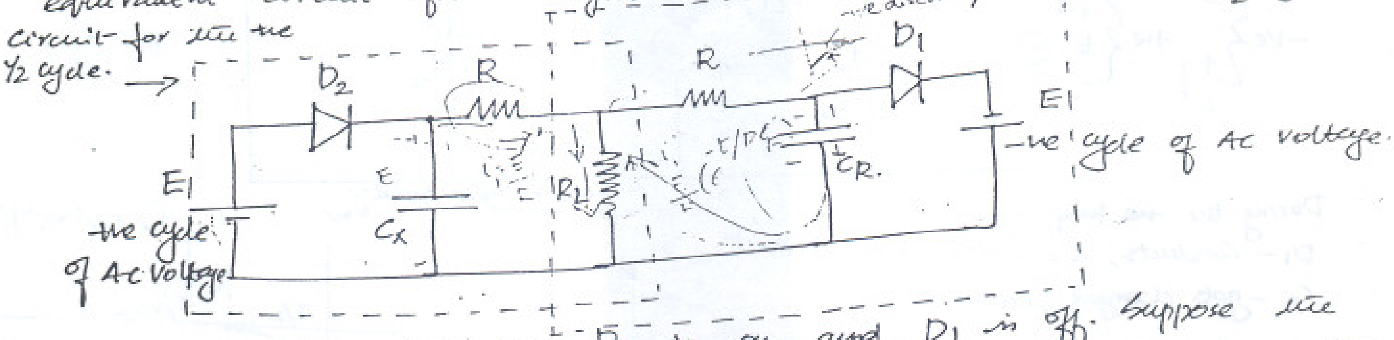
'Is it possible to measure the value of C_X using a fixed value of capacitance C_R?

For this we employ the twin-T circuit.



Circuit for the -ve $\frac{1}{2}$ cycle.

equivalent circuit for analysis:-



During the positive half cycle D_2 is on and D_1 is off. Suppose the value of voltage = E. C_X charges and C_R discharges through R_L during this time

Current through $R_L = \left[\frac{E}{R} - \frac{E}{R} e^{-t_1/RC_R}\right]$

Since the diode resistance is negligible, the capacitor charges instantaneously, the potential across C_X = E.

Now if we short circuit R_L, the current through that path ∴ of 'C_X' and $R = \frac{E}{R} = i_1$.

Assuming that C_R has already charged to 'E' and with the indicated polarity, during the +ve ½ cycle since D_1 is off, C_R discharge through R. $\therefore i_1' = -\frac{E}{R} e^{-t_1/RC_R}$. $\frac{T}{2} = t_1$.

$$\therefore i_1(t) = i_1 + i_1' = \frac{E}{R} - \frac{E}{R} e^{-t_1/RC_R}$$

$$\frac{1}{2}\int i\,dt$$

$$\frac{It}{C} = V_0$$

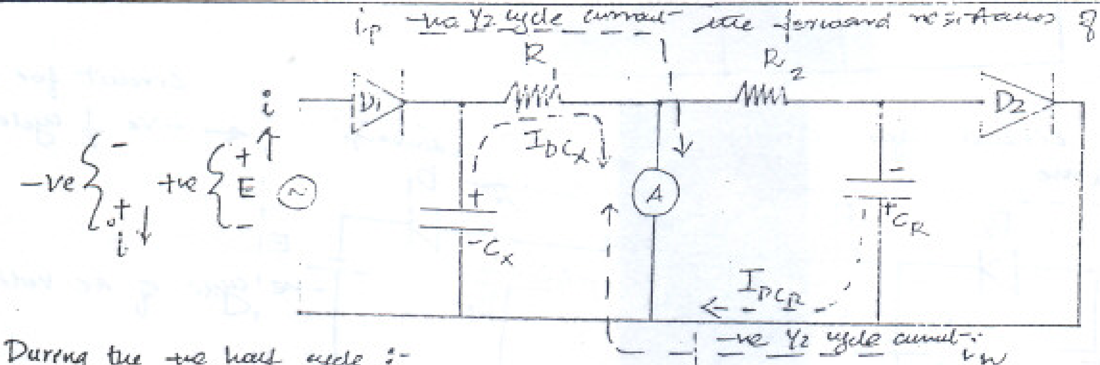
Similarly when the -ve 1/2 cycle is analyzed, D_1 is off and D_2 is on.
'C_x' is charged to 'E' and it tries to discharge through R.
'C_R' gets charged instantaneously because the forward bias resistance of diode D_1 is assumed to be negligible.

$$\therefore i_{2(-)} = \frac{E}{R} - \frac{E}{R} e^{-t_2/RC_x} \qquad t_2 = \frac{T}{2}.$$

now, $i_1(+)$ avg = average value of current during the +ve half cycle

$$i_{avg1} = \frac{E}{R}$$

Diodes assumed to [ideal], the forward resistance of diode



During the +ve half cycle :-
D_1 – Conducts, D_2 – does not conduct.
C_x – gets charged instantaneously.
now, C_R has been already charged to '$-E$' (initial condition) and discharges through R_2 (or R) and this current is in the opposite direction compared to the +ve 1/2 cycle current.

$$\therefore \frac{E}{R} - \left[\frac{E}{R}\left(-1 + e^{-t/RC_R}\right)\right] = i_{(+ve)}$$

$$i_{(+ve)} = \frac{E}{R} + \frac{E}{R} - \frac{E}{R} e^{-t/RC_R}$$

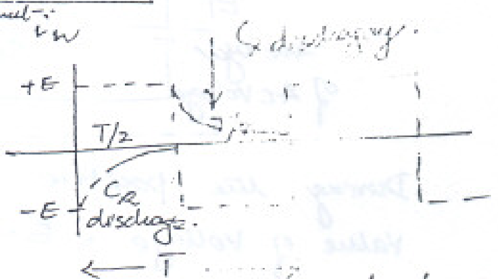
$$= \frac{2E}{R} - \frac{E}{R} e^{-t/RC_R}.$$

The whole process takes place for 'T/2' ← time but avg. calculated over time period → T.

$$i_{(+ve)}avg = \frac{1}{T}\left[\int_0^{T/2} \frac{2E}{R} dt - \int_0^{T/2} \frac{E}{R} e^{-t/RC_R} dt\right]$$

$$= \frac{1}{T}\left[\frac{2E}{R} \cdot t - \frac{E}{R} \cdot \frac{1}{-1/RC_R} \cdot e^{-t/RC_R}\right]_0^{T/2}$$

$$\Rightarrow \frac{E}{R} - \frac{EC_R}{T} = i_{(+ve)} avg$$ ← substitute and simplify.



During the -ve half cycle, C_x discharges through R_1 and the -ve half cycle current (in) is in the opposite side direction. C_R gets charged instantaneously.

$$\therefore i_{(-ve)} = \frac{E}{R} e^{-t/RC_x} - \frac{E}{R}$$

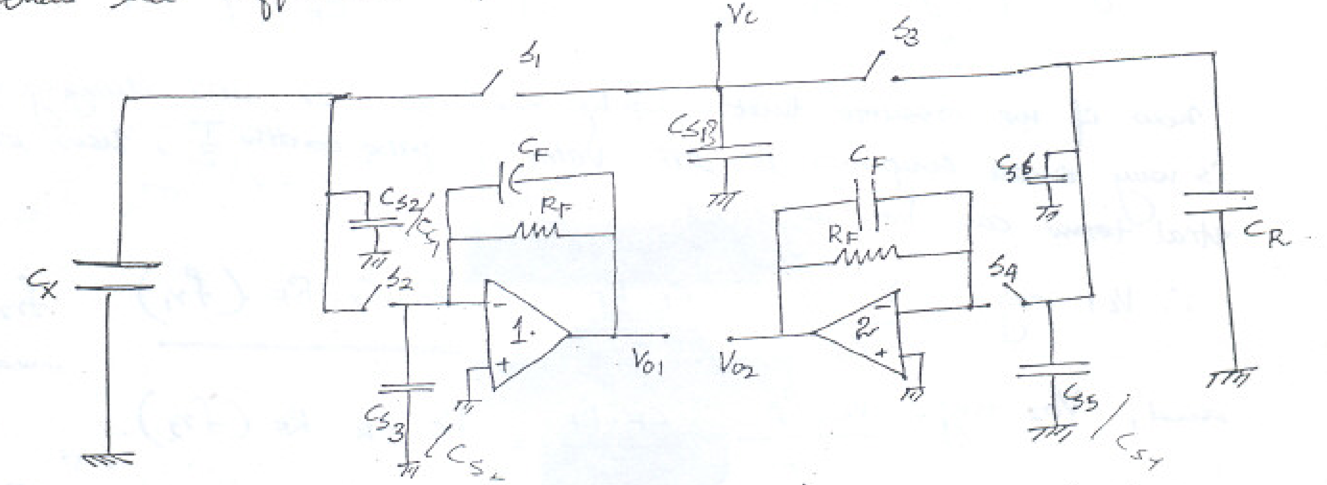
$$i_{(-ve)}avg. = \frac{1}{T}\int_{T/2}^{T}\left[\frac{E}{R} e^{-t/RC_x} - \frac{E}{R}\right] dt$$

$$= \frac{1}{T}\left[\frac{E}{R} \cdot \frac{1}{-1/RC_x} e^{-t/RC_x} - \frac{E}{R}t\right]_{T/2}^{T}$$

=

Switched capacitor technique or the charge Transfer technique.

This technique is used to find out the differential capacitance C_x and then gives a direct reading. Used mainly in accelerometers where the differential capacitance is used to measure acceleration.



C_x = unknown capacitance and C_R = Known reference capacitance.

i) S_1 - ON; S_2, S_3, S_4 - OFF.

'C_x' charges to V_c and the stored charge = $V_c \cdot C_x$.

ii) S_1 - OFF ; S_2 - ON ; S_3, S_4 - OFF.

The voltage across C_x is transferred to the ~~non~~ inverting terminal of the op-amp 1 and V_{01} is the resulting output voltage. It acts as a charge transferring device.

$$V_{01} = \frac{(V_c \cdot C_x)}{C_F} \cdot e^{-t/R_F C_F} \quad \text{here } t = \frac{T}{4}.$$

$$\boxed{V_{(1)} = \frac{V_c\, C_x}{C_F}}$$

S_1 ON $\rightarrow \frac{T}{4}$, S_2 ON $\rightarrow \frac{T}{4}$ } $\frac{T}{2}$

S_3 ON $\rightarrow \frac{T}{4}$, S_4 ON $\rightarrow \frac{T}{4}$ } $\frac{T}{2}$

iii) S_1, S_2 - OFF ; S_3 - ON ; S_4 - OFF.

The capacitor C_R gets charged to V_c and holds the charge, which is be equal to $V_c \cdot C_R$.

iv) S_1, S_2 - OFF; S_3 - OFF; S_4 - ON.

The charge stored by C_R is transferred into the inverting terminal of 2nd op-amp. V_{02} is the corresponding output.

$$V_{02} = \frac{(V_c \cdot C_R)}{C_F} \cdot e^{-t/R_F C_F}.$$

$$\boxed{V_{(2)} = \frac{V_c \cdot C_R}{C_F}}$$

V_{01} avg = average value of V_{01} over a period of 1

$$= \frac{1}{T}\int_0^{T/2} \frac{V_c \cdot C_x}{C_F}\left\{e^{-t/R_F C_F}\right\}dt = \frac{V_c \cdot C_x}{C_F} \cdot \left\{\frac{-C_F R_F}{T}\left(e^{\frac{-T}{2R_F C_F}} - 1\right)\right\}$$

$$V_{02}\ avg = \frac{1}{T}\int_0^{T/2} \frac{V_c \cdot C_R}{C_F}\left\{e^{-t/C_F R_F}\right\}dt = \frac{V_c \cdot C_R}{C_F}\left\{-\frac{R_F C_F}{T}\left(e^{\frac{-T}{2R_F C_F}} - 1\right)\right\}$$

now if we assume that $C_F R_F << T/2$ or the charging time const is very small compared to the value of pulse width $\frac{T}{2}$, then the exponential term can be neglected.

$$\therefore V_{01}\ avg = \frac{V_c \cdot C_x}{C_F} \cdot \frac{C_F \cdot R_F}{T} = \underline{V_c \cdot C_x \cdot R_F \cdot (fr_1)} \qquad fr_2 = fr_1 = \frac{1}{T}$$

and, V_{02} avg $= \frac{V_c \cdot C_R}{C_F} \cdot \frac{C_F \cdot R_F}{T} = V_c \cdot C_R \cdot R_F\ (fr_2)$.

fr_1 = switching frequency and it remains a constant

$$\therefore V_{01}\ avg \sim V_{02}\ avg = (C_x - C_R)\left[V_c \cdot R_F \cdot fr_1\right]$$

$\therefore$ <u>output voltage $\propto (C_x - C_R)$</u> $\leftarrow$ no-non linearities are present.

The differential output voltage can be fed to an instrumentation amplifier for further amplification.

Now if the stray capacitances are considered :-

C_{S2} and C_{S6} are connected to V_c and they can be ruled out.

C_{S3} and C_{S5} can be ruled out as it is connected to the inverting terminal of the op-amp by the principle of virtual short.

The switches used for these purposes are analog switches (MOSFET based) which have to operate at frequencies of about 1 MHz.

So at higher frequencies, the better the output.

$$V_0 = C_1 - C_2$$

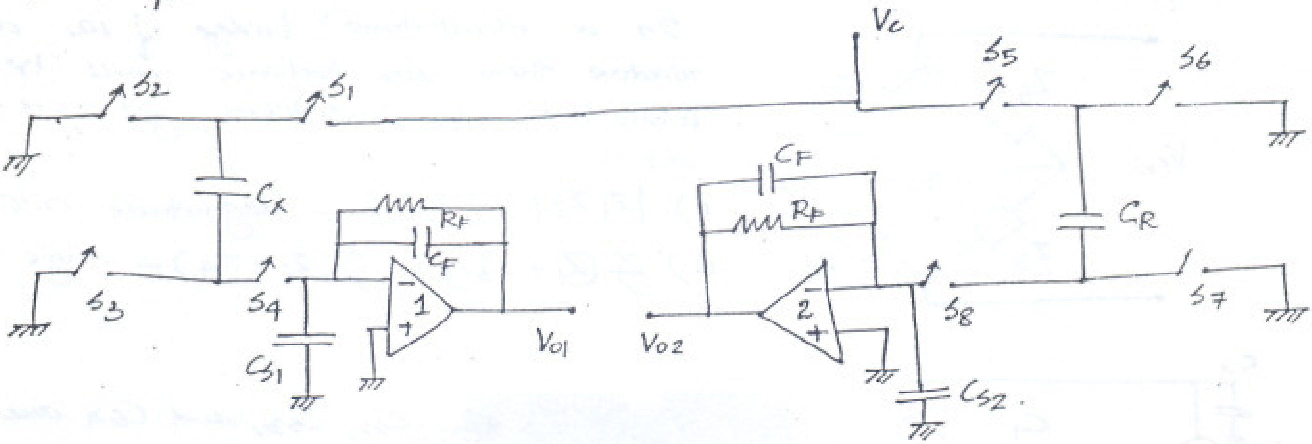
$C_1 = C_x + C_{S2}$ and $C_2 = C_R + C_{S4}$

even C_{S1} can be split since common for both S_1 & S_2

$$V_x = C_1 - C_2$$

$$= C_x + C_{S2} - C_R - C_{S4}$$

- What if the capacitance C_R and C_X are floating?
- To implement this we need 4 additional switches.



i) S_1, S_3 – ON ; S_2, S_4 – OFF ; S_5, S_6, S_7, S_8 – OFF.

C_X charges to V_C. Charge accumulated $= Q_1 = V_C \cdot C_X$

ii) S_1, S_3 – OFF ; S_2, S_4 – ON ; S_5, S_6, S_7, S_8 – OFF

'C_X' transfers the charge stored in it to the opamp's (1) inv. terminal and V_{o1} is produced.

iii) S_1, S_2, S_3, S_4 – OFF ; S_5, S_7 – ON ; S_6, S_8 – OFF.

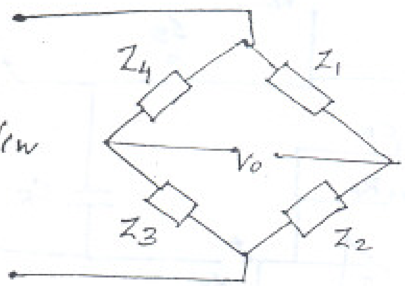
'C_R' charges to V_C. Charge accumulated $= Q_2 = V_C \cdot C_R$.

(v) S_1, S_2, S_3, S_4 – OFF ; S_5, S_7 – OFF ; S_6, S_8 – ON.

'C_R' transfers the charge stored in it to the non inverting terminal of (2) and V_{o2} is produced.

$$V_{o1} \sim V_{o2} = (C_X - C_R)\left[R_F \cdot V_C \cdot f_{r_1} \right]$$

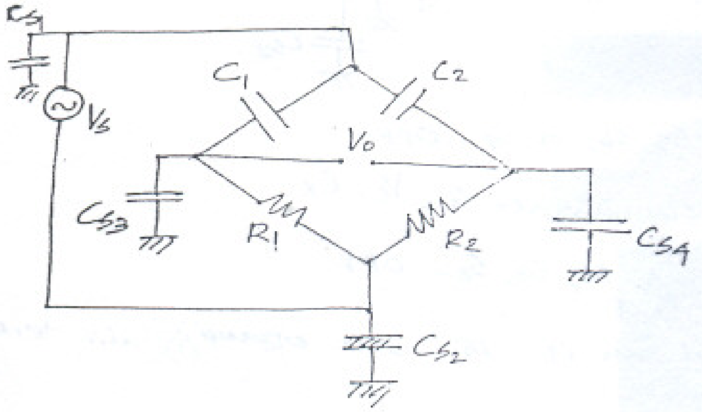
Bridge technique for the measurement of capacitance.



In a wheatstone's bridge if the arms a reactive then the balance will be achie when both the conditions get satisfied. T are :-

i) $|Z_1 Z_3| = |Z_2 Z_4|$ — Magnitude criterion

(ii) $\angle(Z_1 + Z_3) = \angle(Z_2 + Z_4)$ — Angle criterion

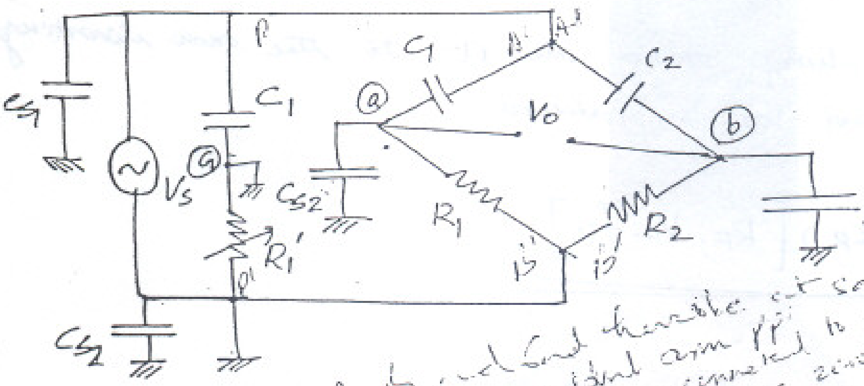


C_{S1}, C_{S2}, C_{S3}, and C_{S4} are stray capacitances.

C_{S3} and C_{S4} cause maximum problems and can make the br unbalanced even if the above cnt for balancing is met.

So how to balance the bridge under this condition ?

Solution :- Wagner's Earth technique.



$$Z_1 = \frac{1}{j\omega C_2}$$

$$Z_2 = R_2$$

$$Z_3 = R_1$$

$$Z_4 = \frac{1}{j\omega C_1}$$

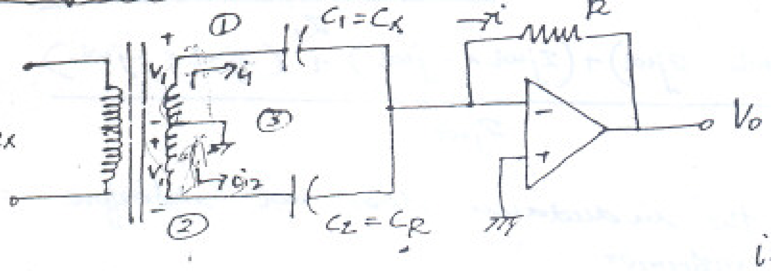
A, B and G and therefore at same potential ... $Z_1 Z_3 = Z_2 Z_4$

Hence, $\frac{R_1}{C_2} = \frac{R_2}{C_1}$ or $R_1 C_1 = R_2 C_2$

Now, under no change in the impedance values the value of V_0 ma not be equal to zero because of the stray capacitances. So another set up consisting of C_1 and R_1' (Variable) is connected. By adjusting the valu of R_1' we can balance the bridge. Points ⓐ and ⓑ are brought to same potential so that $V_0 = 0$ and both are maintained at earth poten

∴ C_{S3} and C_{S4} do not affect the measurement.

Transformer coupled ratio armed bridge :-



$i_1 = \frac{V_1}{X_{C_1}} = V_1 \cdot j\omega C_1$

$i_2 = -\frac{V_1}{X_{C_2}} = -V_1 j\omega C_2$

$i = i_1 + i_2$

$V_0 = i \cdot R = (i_1 + i_2) R = V_1 j\omega (C_1 - C_2) \cdot R = \underline{V_1 j\omega (C_X - C_R) R}$

$\therefore V_0 \propto (C_X - C_R)$ or $C_1 - C_2$.

Requirements :-

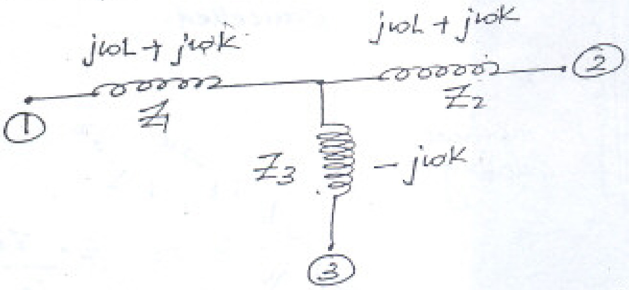
The mid point of the secondary should be grounded, since the potential V_1 and V_2 are measured with respect to that point.

If a loose coupling exists then the inductance of the transformer comes into picture and that is a stray component, because measure involve al.

So the coupling has to be tight. ie, $K = 1$.

To ensure this we use a bifilar winding at the secondary.

i.e., impedance between ③ and ①; and ② and ③ should be



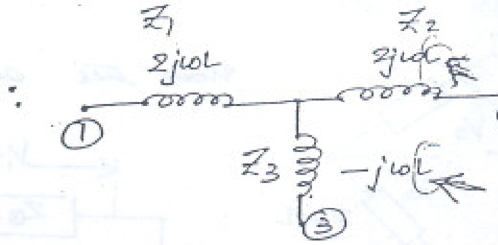
$K \sqrt{L_1 L_2} = M.$

if $L_1 = L_2$ and $K = 1$ then

$j\omega L =$ Self inductance

$j\omega K =$ Mutual inductance

$\therefore$



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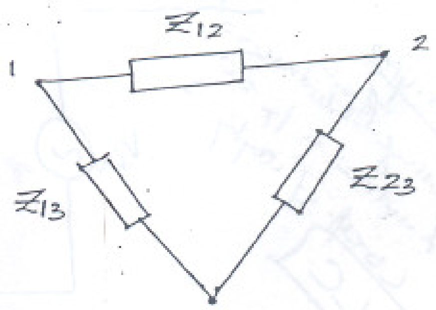


Our objective is to make Z_{13} and $Z_{23} = 0$.

now $Z_{13} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} = \frac{(2j\omega L \times 2j\omega L) + (-j\omega L \cdot 2j\omega L) + (-j\omega L}{2j\omega L}$

$= 0.$

In the same manner $Z_{33} = \dfrac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$

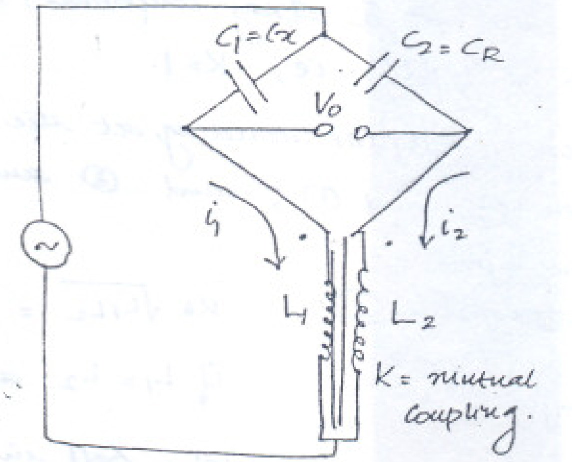
$$= \frac{(2j\omega L \cdot 2j\omega L) + (2j\omega L \times -j\omega L) + (2j\omega L \times -j\omega L)}{2j\omega L} = 0.$$

Since $Z_{13} = Z_{23} = 0$, the inductance does not interfere with the c flowing in the transformer.

We made the source impedance to be zero so that the ap. Voltage to the secondary will be independant of L, else the impedance seen will be $\left\{\frac{1}{j\omega C_1} + j\omega L\right\}$. $L = 0$ for best results

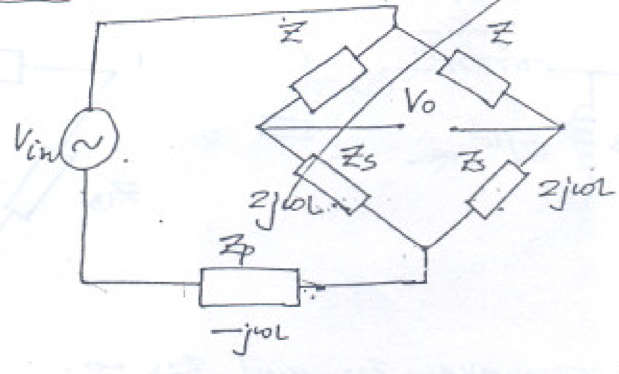
Normally high permeability materials are used, eg. pot cores.

Alternate method ~~[BLU~~ BLUMLEIN BRIDGE



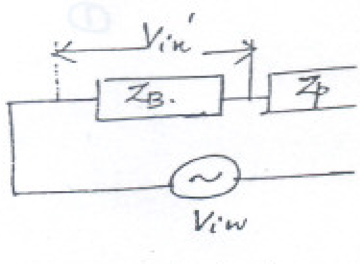
Bifilar winding exists. Have currents are flowing in the direction. So the flux gets cancelled.

eqvt. cct.



Assuming leakage inductance ... Coeff of coupling KLC

now the eqvt. of the



Z_B = impedance of the bridge, Z_P = mutual impedance of the inductance

Now the voltage appearing across the bridge is V_{in}' instead of V_{in}. The mutual inductance Z_P has come into scene and a drop of V_{in}

Idea is to remove Z_P and maximize V_{in}

$$V_{in}' = \frac{(Z+Z_s)/2}{Z_p + (Z+Z_s)/2} V_{in}$$

$z_{eq} = (Z+Z_s)/2$ = equivalent impedance of the bridge.

$Z = \frac{1}{j\omega C}$ and $Z_s = 2j\omega L$ and $Z_p = -j\omega L$

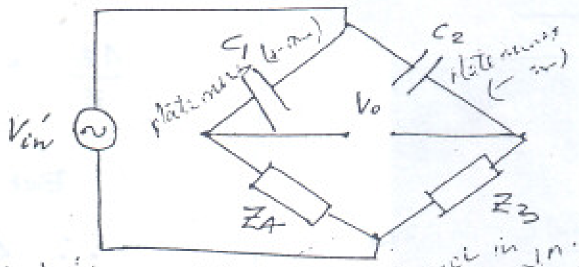
So $$V_{in}' = \left\{\frac{Z+Z_s}{2Z_p + Z + Z_s}\right\} V_{in} = V_{in}\left[\frac{\frac{1}{j\omega C} + 2j\omega L}{-2j\omega L + \frac{1}{j\omega C} + 2j\omega L}\right] = \frac{1+2j^2\omega^2 LC}{\cancel{1-j^2\omega^2 LC}}$$

$$V_{in}' = \underline{(1-2j^2\omega^2 LC)} \cdot V_{in}$$

To measure diff when both $C_1, C_2 \sim C_x$ and C_R changes

Now $Z = C_x$ or C_R.

low $\overline{\quad}$ $x+\Delta x$



$Z_1 = \frac{1}{j\omega C_1}$, $Z_2 = \frac{1}{j\omega C_2}$

$C_2 = C_R$ and $C_1 = C_x$.

$Z_3 = j\omega L$ and $Z_4 = j\omega L_s$.

$Z_4 = Z_3 = Z'$

now if $Z_1 \to Z + \Delta Z$ and $Z_2 \to Z_2 - \Delta Z$. i.e. change takes place

$Z_1 = Z_2 = Z$

Actually could write $V_o = V_{in}\frac{Z_1}{Z_1+Z_2+Z_4}$ But to get jeck in the numerator for V_o we have Z_4

$$V_o = V_{in}'\left\{\frac{Z_4}{Z_1+\Delta Z+Z_4} - \frac{Z_3}{Z_3+Z_2-\Delta Z}\right\}$$

now, $Z_1 = Z_2$ and $Z_4 = Z_3$, during balanced condition

$$\therefore V_o = V_{in}'\left\{\frac{Z'}{Z'+Z+\Delta Z} - \frac{Z'}{Z'+Z-\Delta Z}\right\} = V_{in}'\frac{\left\{Z'^2 + ZZ' - Z'\Delta Z - \cancel{Z'^2} - \cancel{Z'Z} - Z'\Delta Z\right.}{(Z'+Z)^2 - (\Delta Z)^2}$$

$$V_o = \left[\frac{-2Z'\Delta Z}{(Z'+Z)^2 - (\Delta Z)^2}\right] V_{in}'$$

, now multiplying and dividing both numerator and denominator by (ΔZ is a very small quantity so $\frac{(\Delta Z)^2}{ZZ'}$ will be a negligible quantity.

$$= \frac{\frac{-2Z'\Delta Z}{ZZ'}}{\frac{Z'^2}{ZZ'} + \frac{Z^2}{ZZ'} + \frac{2ZZ'}{ZZ'} - \frac{\Delta Z^2}{ZZ'}} = \frac{\frac{-2\Delta Z}{Z}}{\frac{Z}{Z'} + \frac{Z'}{Z} + 2}$$

now $Z = \frac{1}{j\omega C}$ and $Z' = 2j\omega L$.

$$V_0 = V_{in}' \left\{ \frac{\frac{2\Delta Z}{Z}}{\frac{Z'}{Z} + \frac{Z}{Z'} + 2} \right\} =$$

$$= \frac{V_{in}(1 - 2\omega^2 LC) \cdot 2\frac{\Delta C}{C}}{\frac{2j\omega L}{1/j\omega C} + \frac{1/j\omega C}{2j\omega L} + 2}$$

$$= \frac{V_{in}(1 - 2\omega^2 LC) \cdot \frac{2\Delta C}{C}}{-\cancel{4}\omega^2 LC - \frac{1}{2\omega^2 LC} + 2.}$$

$$= \frac{(V_{in})(1 - 2\omega^2 LC)\frac{2\Delta C}{C} \times 2\omega^2 LC}{-[4\omega^4 L^2 C^2 + 1 - \cancel{4}\omega^2 LC]}$$

$$= \frac{[-V_{in}][1 - 2\omega^2 LC](2\frac{\Delta C}{C})[2\omega^2 LC]}{[1 - 2\omega^2 LC]^2}$$

$$V_0 = [-V_{in}][1 - 2\omega^2 LC]^{-1}\left[\frac{2\Delta C}{C}\right][2\omega^2 LC] = \frac{[-V_{in}]\left[\frac{2\Delta C}{C}\right][2 \ldots}{[1 - 2\omega^2 LC]}$$

$[1 - 2\Delta C \ldots]^{-1}$

$Z = \frac{1}{j\omega C}$

$\Delta Z = \frac{1}{j\omega C + \Delta C}$

$\Delta Z = \frac{1}{j\omega C} \quad \frac{\Delta C}{C + \Delta C}$

$\frac{\Delta Z}{Z} = \frac{\cancel{\frac{1}{j\omega C}}\left(\frac{\Delta C}{C + \Delta C}\right)}{\cancel{1/j\omega C}}$

$\frac{\Delta Z}{Z} = \frac{\Delta C}{C + \Delta C}$

But $C + \Delta C \simeq C$

$\therefore \frac{\Delta Z}{Z} = \frac{\Delta C}{C}$

3 Conditions :-

i) $2\omega^2 LC >> 1 \quad \therefore V_0 = V_{in} \cdot \frac{2\Delta C}{C}$ Sns

$\therefore 1 - 2\omega^2 LC \simeq -2\omega^2 LC$.

C is small
C is frm

ii) $2\omega^2 LC << 1$

$\therefore 1 - 2\omega^2 LC \simeq 1 \quad \therefore V_0 = -V_{in}\left(\frac{2\Delta C}{C}\right) \cdot 2\omega^2 LC \simeq 0$.

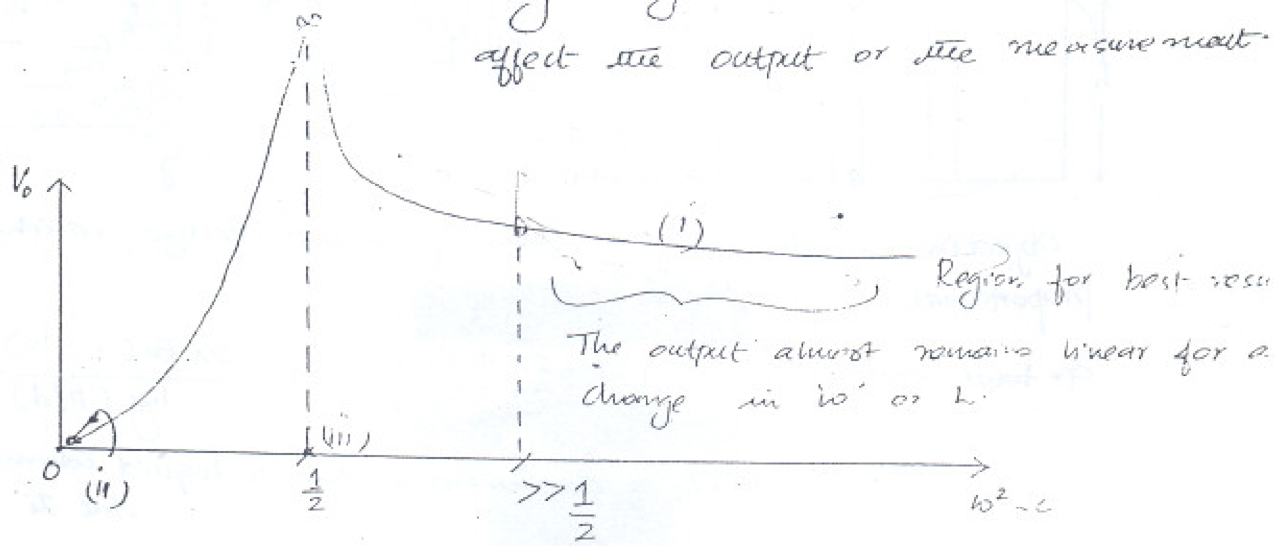
whole of
+ depe
Hence it we
close
for

iii) $2\omega^2 LC = 1$ or $\omega = \frac{1}{\quad}$. $V_0 = \infty$ (Resonance)

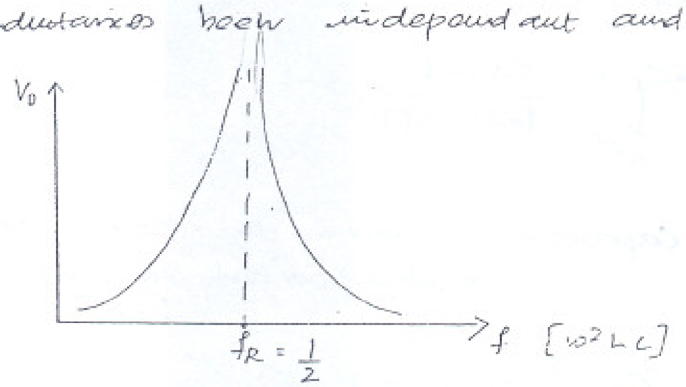
Now, under condition i) if $2\omega^2 LC >> 1$

$$\boxed{V_0 = V_{in} \cdot \frac{2\Delta C}{C}}$$

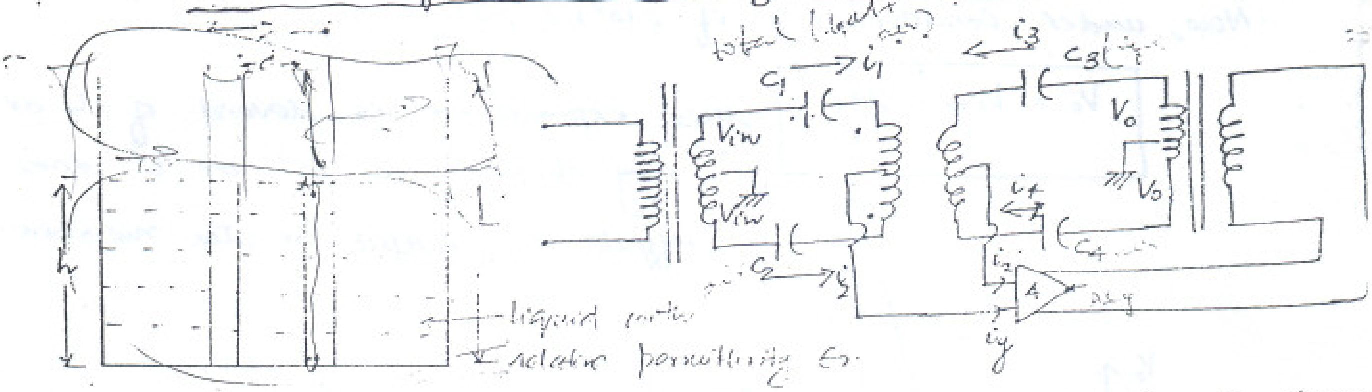
this expression is devoid of L or ω, ∴ any change in 'ω' or 'L' does not affect the output or the measurement.



Had the two inductances been independent and not mutual wound then



Measurement of liquid level using ...



Objective is to get a value of output voltage which is direct proportional to height 'h'.

C_T = ~~total~~ capacitance = $C_1 = \dfrac{2\pi\epsilon_0\epsilon_r h}{\log(D/d)} + \dfrac{2\pi\epsilon_0(L-h)}{\log(D/d)}$.

$C_1 = C_T$ = Capacitance due to liquid column + capacit due to air column

$C_2 = C_4$ = capacitance when the whole set up is filled with air

$C_2 = C_4 = \dfrac{2\pi\epsilon_0 L}{\log(D/d)}$

C_3 = capacitance when the whole set up is filled with liquid relative permittivity = ϵ_r

$C_3 = \dfrac{2\pi\epsilon_0\epsilon_r L}{\log(D/d)}$

'A' = feed back amplifier with gain = A.

'i_x' has to be made equal to 'i_y' so that there is no .

$i_x = i_1 + i_2 = V_{in}(j\omega C_1) + (-V_{in})(j\omega C_2) = V_{in}\, j\omega(C_1 - C_2)$ — ①

$i_y = i_3 + i_4 = V_{out}(j\omega C_3) + (-V_{out})(j\omega C_4) = V_{out}\, j\omega(C_3 - C_4)$. ②

$i_x = i_y$ ∴ V_{out}/V_{in}

∴ $\dfrac{V_{out}}{V_{in}} = \dfrac{C_1 - C_2}{C_3 - C_4} = \dfrac{\dfrac{2\pi\epsilon_0\epsilon_r h}{\log D/d} + \dfrac{2\pi\epsilon_0(L-h)}{\log D/d} - \dfrac{2\pi\epsilon_0 L}{\log D/d}}{\dfrac{2\pi\epsilon_0\epsilon_r L}{\log D/d} - \dfrac{2\pi\epsilon_0 L}{\log D/d}}$

equating ① and ② and taking the subsequent ratio.

$$\frac{V_{out}}{V_{in}} = \frac{\epsilon_r h + \cancel{l} - h - \cancel{l}}{\epsilon_r L - L} = \frac{\epsilon_r h - h}{\epsilon_r L - L} = \frac{h}{L}$$

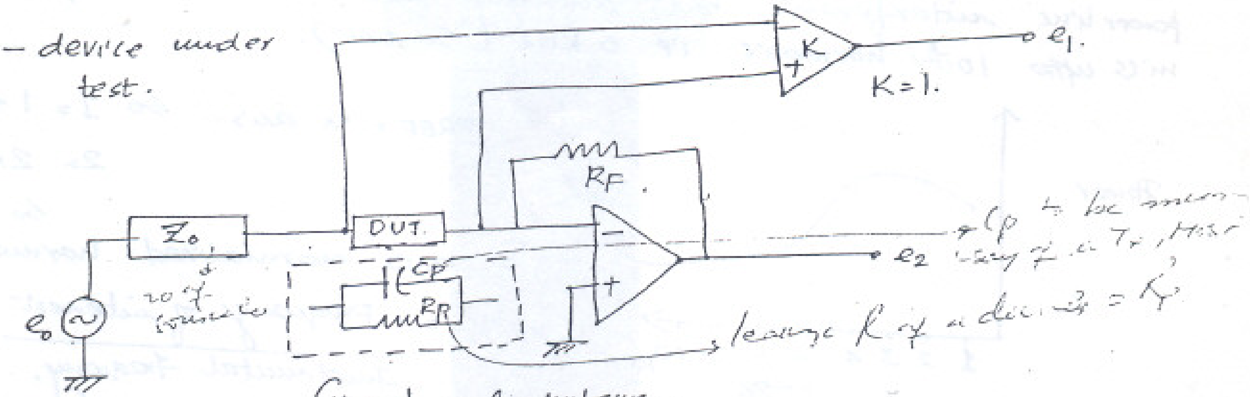
$\therefore V_{out} = V_{in} \cdot \frac{h}{L}$ or $\boxed{V_{out} \propto h}$ Objective accomplished.

Conditions to be met :- i_x has to be equal to i_y to stabilise 'A' has to be very high. So we go for an op-amp

Measurement of capacitances in semiconductor devices

Suppose we need to measure the impedance of a semiconductor device we need to take care of the leakage resistance of the device.

DUT – device under test.



$G_p = \frac{1}{R_p}$ = Conductance.

$$Z_p = X_{cp} \| R_p = \frac{\frac{1}{j\omega C_p} \cdot R_p}{\frac{1}{j\omega C_p} + R_p} = \frac{\frac{1}{j\omega C_p} \cdot \frac{1}{G_p}}{\frac{1}{G_p} + \frac{1}{j\omega C_p}} = \frac{1}{j\omega C_p + G_p}$$

Z_0 = maybe the output impedance of the source e_0.

So (1) $e_2 = \frac{(-e_0) R_F}{Z_0 + Z_p} = \frac{(-e_0) R_F}{Z_0 + \frac{1}{G_p + j\omega C_p}}$, we are concerned only about the magnitude, so polarity can be omitted.

(2) $e_1 = \frac{e_0 \cdot Z_p}{Z_0 + Z_p} = \frac{e_0 \cdot \frac{1}{j\omega C_p + G_p}}{Z_0 + \frac{1}{G_p + j\omega C_p}} = \frac{e_0}{Z_0(G_p + j\omega C_p)}$

$\frac{e_2}{e_1} = \frac{R_F(G_p + j\omega C_p)}{} = -R_F(Z_p)$ so $(G_p + j\omega C_p) = Z_p = \left(\frac{e_2}{e_1}\right) \cdot \frac{1}{R_F}$.

The main problem associated with this technique is the corruption.

(1) corruption of the signal due to the fact that the o/p signals are of very low level

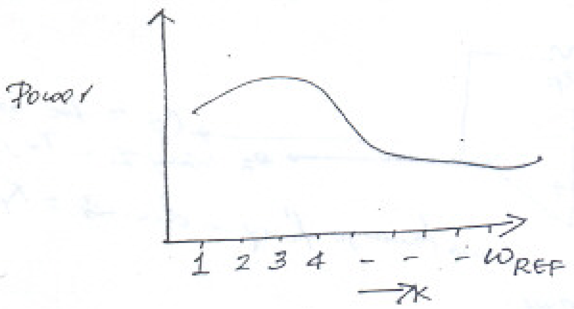
(2) Noise can easily creep into the circuit.

Phase sensitive detection → has been explained in page no. 7 - 10.

Continuation from page 10.

Spectrum Analyzer

This is an instrument for studying the power content in the harmonics of the fundamental frequency we are interested in. When we are measuring devices (dc), we try to block the 50 Hz power line interference, but actually the power line produces harmonics upto 100th harmonic, i.e 5 KHz (50 × 100).



ω_{REF} = x axis. so 1 = 1 ω_{ref}.

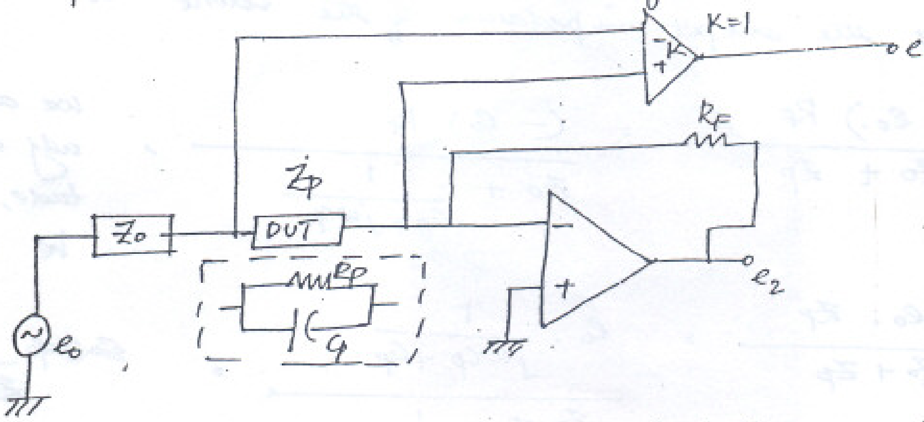
2 = 2 × ω_{ref} and so on

x - axis normalized harmonic frequency

$$K = \frac{\text{Frequency of interest}}{\text{Fundamental frequency}}.$$

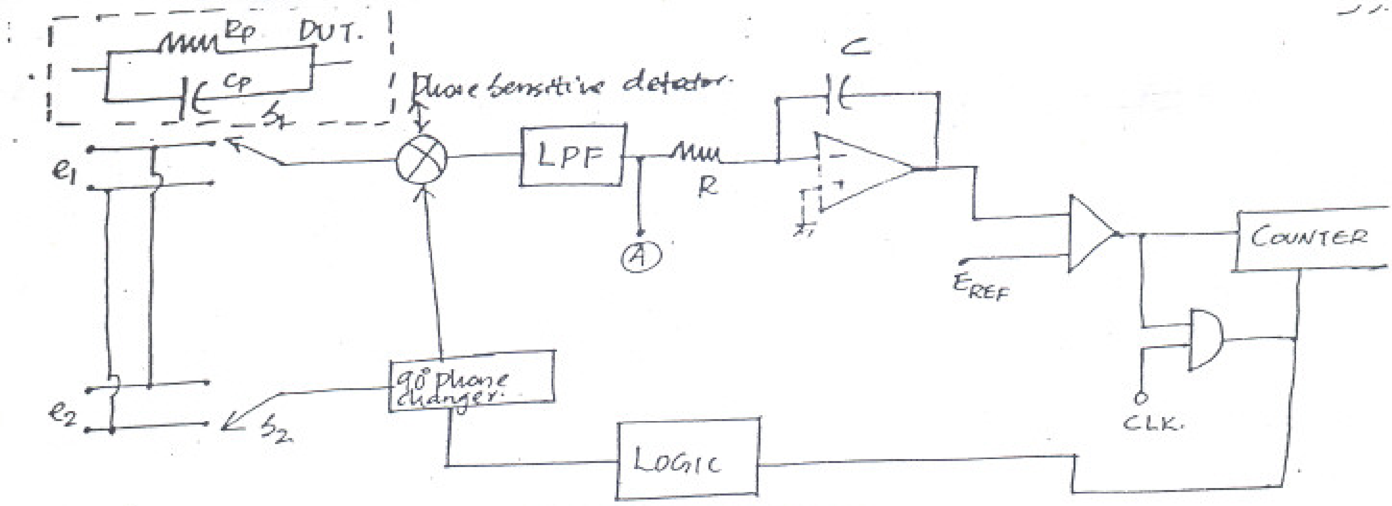
If ω_{ref} is changed then, the spectrum analyzer will give the power content in various harmonics. For this we should be able to change the fundamental harmonic frequency

Is it possible to utilise the DUT for this ?



$$\frac{e_2}{e_1} = R_F(G_P + j\omega C_P)$$

Can a phase sensitive detector + a dual slope ADC be used to synthesise this ?



Logic is used to change the phase by 90°.