Linear Algebra, Optimization and Solving Ordinary Differential Equations Using Scilab

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System of Linear Equations

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  $3x_1 + x_2 + 2x_3 = 5$
  $x_1 + x_2 - x_3 = 1.$

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  where $A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$
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- Number of Equations may or may not be equal to number of unknowns.

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- To find Kernel(nullspace) of a system separately use
  `ker=kernel(A)`

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- `[Q,R]=qr(A)       //QR-Decomposition`

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- Use `linsolve`
- Try this for previously obtained solution
  `A*x  A*(x+ker)   //In this case kernel is a line`

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  Solve $x^2 + 3x + 2 = 0$
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- One can also define function $f : R^n \rightarrow R^n$ and solve it for zero locations.

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- `[f,xopt]=optim(list(NDcost,myf),x0)`

For Example

- Minimize:
  $f(x, y) = (x + y)^2 + x + y + 2$
- Gradient of the Function f
  $\nabla f = \begin{pmatrix} 2(x + y) + 1 & 2(x + y) + 1 \end{pmatrix}$

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- If $f : R^n \to R$, then g is gradient of f at x.
- If $f : R^n \to R^m$, then g is Jacobian a $m \times n$ Matrix.

Hessian

- `[g,H]=derivative(f,x)` is the calling sequence
- for a function $f : R^n \rightarrow R$
  g is the gradient of f
  and H is Hessian matrix of f

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  t is the time instants at which solution is needed.
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- Higher Order Equations must be made into first order equations of form $\dot{x} = Ax + Bu$.

Examples

- Solve the differential equation $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L} sin(\theta) = 0$
- Take $g = 9.8\ m/s^2\ L = 1\ m$
- Check the plot of solution against time using `plot2d(t,x(1,:)` and `plot2d(t,x(2,:))`
- Also obtain the phase plane plot using `plot2d(x(1,:),x(2,:))`

Thank You!

- www.scilab.org

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- "Modeling And Simulation in Scilab/Scicos",by S.L.Campbell, J. Chancelier, R. Nikoukah.