Performance Analysis of Maximum-Likelihood Multiuser Detection in Space-Time Coded CDMA

G. V. V. Sharma and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore 560012

***Abstract*—In this paper, we analyze the performance of maximum-likelihood (ML) multiuser detection in space-time coded CDMA systems. A K-user synchronous CDMA system which employs orthogonal space-time block coding with M, $1 \leq M \leq 8$ transmit antennas and N receive antennas is considered. Using the characteristic function of the decision variable, we derive an exact expression, in closed-form, for the pairwise error probability (PEP) of the joint data vector of bits from different users. Using this exact PEP expression, we obtain an upper bound on the average bit error rate (BER). The analytical BER bounds are compared with the BER obtained through simulations. The BER bounds are shown to be increasingly tight for large SNR values.**

Keywords – ML multiuser detection, space-time codes, CDMA, pairwise error probability.

I. Introduction

Space-time coded transmission using multiple transmit antennas can offer the benefits of transmit diversity and high data rate transmission on fading channels [1],[2],[3]. Space-time coding applied to code division multiple access (CDMA) systems has been of interest [4]. Multiuser detection schemes, which can significantly enhance the receiver performance and increase the capacity of CDMA systems, have been extensively studied in the literature, mainly for single transmit antenna systems [5]. Multiuser detection schemes and their performances in space-time coded CDMA systems with multiple transmit antennas has been a topic of recent investigations [6],[7],[8],[9]. In [6], Huang *et al* studied a decorrelating decision feedback multiuser detector for a space-time CDMA system with multiple transmit antennas. In [7], Reynolds *et al* developed blind adaptive multiuser detector implementations for synchronous/asynchronous CDMA systems with two transmit/two receive antennas on Rayleigh fading channels. In [8], the convergence behaviour and bit error performance of various adaptive multiuser detectors under near-far conditions in space-time coded CDMA systems using orthogonal space time block codes were studied. The performance of the systems considered in [6]-[8] were evaluated mainly through simulations. In [9], Uysal and Georghiades derived an exact analytical expression for the pairwise error probability (PEP) and obtained approximate bit error probability for a space-time coded CDMA system. However, the detector considered in [9] is not a multiuser detector. In this paper, we are interested in the analytical evaluation of the error performance of maximum-likelihood (ML) multiuser detection in space-time coded CDMA.

This work was supported in part by the Swarnajayanti Fellowship from the Department of Science and Technology, Government of India, New Delhi, under scheme Ref: NO. 6/3/2002 - S.F

In [10],[11], Taricco and Biglieri obtained an expression for the pairwise error probability (PEP) of space-time codes in a single user system, assuming perfect channel estimation at the receiver. Using this PEP, they obtained bounds on the probability of error for maximum-likelihood (ML) detection. In [12], Garg *et al* extended the work in [10] by incorporating imperfect channel estimation in the system model, again for the single user system. In this paper, using a similar approach as in [10],[12], we extend the analysis to a space-time coded CDMA system which uses maximum-likelihood (ML) multiuser detection. Using a discrete-time vector model of the received signal in a space-time coded CDMA system with M transmit and N receive antennas, and the characteristic function of the decision variable, we derive an exact expression, in closed-form, for the pairwise error probability (PEP) of the joint data vector of bits from different users. Using this exact PEP expression, we then obtain an upper bound on the average bit error rate (BER). We compare the analytical BER bounds with the BER obtained through simulations, and show that the BER bounds are increasingly tight for large SNR values.

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we present the performance analysis. Section IV presents the results and discussions. Conclusions are given in Section V.

II. System Model

Consider a K-user synchronous CDMA system with M transmit antennas per user. Users transmit data blocks with Q bits per data block. Let b_{iq}, $i \in \{1, 2, ..., K\}$, $q \in \{1, 2, ..., Q\}$, be the q^{th} bit of the i^{th} user, transmitted in a time interval of length T. The bits in a data block are mapped on to the M transmit antennas using orthogonal space-time block codes (STBC). We assume that the channel fading is quasi-static and the quasi-static interval is QT time units, where $Q = 2^r$, r being the smallest integer satisfying $Q \geq M$ [4]. For square real orthogonal STBC with $M = Q = 8$, the transmission matrix $\mathbf{X}$ is given by [3]

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_2 & -x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & -x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & -x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & -x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & -x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & -x_1 & x_2 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & -x_1 \end{bmatrix}. \quad (1)$$

In the above transmission matrix, the columns represent the transmit antenna index and the rows represent the bit interval index. For BPSK modulation, x_i's can take $+1$ or -1. The transmission matrix $\mathbf{X}$ for other real orthogonal designs for $M, Q < 8$ can be obtained to be the upper leftmost submatrix of $\mathbf{X}$ of order $Q \times M$. In the following, we illustrate the received signal model for $M = Q = 2$ (extension of the model for other values of $M, Q \leq 8$ is straightforward). For $M = Q = 2$, the received signal on a receive antenna can be written using (1) as

$$y(t) = y_1(t) + y_2(t) + z(t), \tag{2}$$

$$y_1(t) = \sum_{i=1}^{K} A_{i1} h_{i1} \left\{ b_{i1} s_{i1} + b_{i2} s_{i2} \right\}, \tag{3}$$

$$y_2(t) = \sum_{i=1}^{K} A_{i2} h_{i2} \left\{ b_{i2} s_{i1} - b_{i1} s_{i2} \right\}. \tag{4}$$

In the above, $y_p(t)$, $p \in \{1, 2\}$ is the signal component due to the p^{th} transmit antenna, A_{ip} is the transmit amplitude on the p^{th} transmit antenna of the i^{th} user, h_{ip} is the complex channel gain from the p^{th} transmit antenna of the i^{th} user, and s_{iq} represents the signature waveform of the i^{th} user for the q^{th} bit in a data block, $q \in \{1, 2\}$, given by $s_{iq} = s_i\big(t - \overline{q-1}T\big)$, where $s_i(t)$ is a unit energy signature waveform time limited in the interval $[0, T]$. Also, $z(t)$ is a zero mean complex Gaussian noise process with variance $2\sigma^2$.

The demodulator on each receive antenna uses a bank of K matched filters, each matched to a different user's signature waveform. The received signal at the output of the matched filters can be written as

$$y_{jq} = \int_0^{QT} y(t) s_{jq}(t) dt, \tag{5}$$

where j=$1, 2, ..., K$, and $q \in \{1, 2\}$. We define matrix, $\mathbf{R}$, as

$$\mathbf{R} = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1K} \\ \rho_{12} & 1 & \dots & \rho_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{1K} & \rho_{2K} & \dots & 1 \end{bmatrix}, \tag{6}$$

where $\rho_{jk} = \int_0^T s_j(t) s_k(t) dt$. Also, define the *channel matrix*, $\mathbf{H}$, as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2 & \mathbf{H}_1 \end{bmatrix}, \tag{7}$$

where $\mathbf{H}_q = diag\big[A_{1q}h_{1q}, \dots, A_{Kq}h_{Kq}\big]$. Defining $\mathbf{y}_q = \big[y_{1q}, \dots, y_{Kq}\big]^T$ and $\mathbf{y} = \big[\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_Q^T\big]^T$, and $\mathbf{b}_q = \big[b_{1q}, \dots, b_{Kq}\big]^T$ and $\mathbf{b} = \big[\mathbf{b}_1^T, \dots, \mathbf{b}_Q^T\big]^T$, the received signal vector $\mathbf{y}$ can be written in the form

$$\mathbf{y} = \mathbf{C}\mathbf{H}\mathbf{b} + \boldsymbol{\eta}, \tag{8}$$

where the *correlation matrix*, $\mathbf{C}$, is given by

$$\mathbf{C} = \begin{bmatrix} \mathbf{R} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{R} & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \mathbf{R} & \mathbf{0} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}, \tag{9}$$

and $E\big[\boldsymbol{\eta}\boldsymbol{\eta}^\dagger\big] = 2\sigma^2\mathbf{C}$, where $\boldsymbol{\eta}^\dagger$ denotes the Hermitian of the complex vector $\boldsymbol{\eta}$. The vector model in (8) can be valid for other values of M, provided the matrices, $\mathbf{H}$ (of order $QK \times QK$) are defined appropriately. For example, if $M = Q = 8$,

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_3 & \mathbf{H}_4 & \mathbf{H}_5 & \mathbf{H}_6 & \mathbf{H}_7 & \mathbf{H}_8 \\ -\mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_4 & -\mathbf{H}_3 & \mathbf{H}_6 & -\mathbf{H}_5 & -\mathbf{H}_8 & \mathbf{H}_7 \\ -\mathbf{H}_3 & -\mathbf{H}_4 & \mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_7 & \mathbf{H}_8 & -\mathbf{H}_5 & -\mathbf{H}_6 \\ -\mathbf{H}_4 & \mathbf{H}_3 & -\mathbf{H}_2 & \mathbf{H}_1 & \mathbf{H}_8 & -\mathbf{H}_7 & \mathbf{H}_6 & -\mathbf{H}_5 \\ -\mathbf{H}_5 & -\mathbf{H}_6 & -\mathbf{H}_7 & -\mathbf{H}_8 & \mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_3 & \mathbf{H}_4 \\ -\mathbf{H}_6 & \mathbf{H}_5 & -\mathbf{H}_8 & \mathbf{H}_7 & -\mathbf{H}_2 & \mathbf{H}_1 & -\mathbf{H}_4 & \mathbf{H}_3 \\ -\mathbf{H}_7 & \mathbf{H}_8 & \mathbf{H}_5 & -\mathbf{H}_6 & -\mathbf{H}_3 & \mathbf{H}_4 & \mathbf{H}_1 & -\mathbf{H}_2 \\ -\mathbf{H}_8 & -\mathbf{H}_7 & \mathbf{H}_6 & \mathbf{H}_5 & -\mathbf{H}_4 & -\mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \end{bmatrix},$$

It is noted that the structure of the square matrix $\mathbf{C}$ (of order QK) remains same. For values of M and Q other than 8 ($M, Q < 8$), $\mathbf{H}$ is given by the upper leftmost submatrix of order $QK \times QK$ in (10). For the case of $M \notin \{1, 2, 4, 8\}$, $M < Q$. Therefore, only the elements $\mathbf{H}_q$, $q = 1, 2, ..., M$, are non-zero, i.e., $\mathbf{H}_q = \mathbf{0}$ for $M < q \leq Q$. The entries of the channel matrix $\mathbf{H}$ are assumed to be i.i.d, zero-mean complex circular Gaussian r.v's (Rayleigh fading).

Assuming the correlation matrix $\mathbf{C}$ to be positive definite, we do the Cholesky decomposition of $\mathbf{C}$

$$\mathbf{C} = \mathbf{F}^T\mathbf{F}. \tag{10}$$

Then

$$\hat{\mathbf{y}} = (\mathbf{F}^T)^{-1}\mathbf{y} \;=\; \mathbf{F}\mathbf{H}\mathbf{b} + \mathbf{n} \tag{11}$$

where $E[\mathbf{n}] = \mathbf{0}_{QK\times 1}$, $E[\mathbf{n}\mathbf{n}^\dagger] = 2\sigma^2\mathbf{I}_{QK}$, where $(.)^\dagger$ represents the Hermitian operation and $\mathbf{I}$ is the identity matrix.

A. ML Criterion

Using the vector representation of the multiuser received signal in (11), the maximum-likelihood (ML) multiuser detection criterion can be written as follows. Assume that perfect estimates of the channel gains are available at the receiver. The ML estimate of the transmitted bit vector, $\mathbf{b}$, (comprising the bits from all users) is given by

$$\tilde{\mathbf{b}} = arg\left\{\min_{\mathbf{x}} \sum_{j=1}^{N} \|\hat{\mathbf{y}}^{(j)} - \mathbf{F}\mathbf{H}^{(j)}\mathbf{x}\|^2\right\}, \tag{12}$$

where the superscript (j) in $\mathbf{y}$ and $\mathbf{H}$ denote the receive antenna index, and the $\min_{\mathbf{x}}$ is over all possible bit vectors of length QK. Substituting (11) in (12)

$$\tilde{\mathbf{b}} = arg\left\{\min_{\mathbf{x}} \sum_{j=1}^{N} \|\mathbf{F}\mathbf{H}^{(j)}(\mathbf{b} - \mathbf{x}) + \mathbf{n}^{(j)}\|^2\right\} \tag{13}$$

III. Performance Analysis

In this section, we analyze the bit error performance of the ML multiuser detection scheme in (13). We first derive an expression for the pairwise error probability (PEP), $P(\mathbf{b} \rightarrow \tilde{\mathbf{b}})$, and then obtain a bound on the bit error probability. The PEP is given by

$$P(\mathbf{b} \rightarrow \tilde{\mathbf{b}}) = \Pr\left\{\sum_{j=1}^{N} \|\mathbf{F}\mathbf{H}^{(j)}(\mathbf{b} - \tilde{\mathbf{b}}) + \mathbf{n}^{(j)}\|^2 - \|\mathbf{n}^{(j)}\|^2 < 0\right\}. \tag{14}$$

Define the metric D as

$$D = \sum_{j=1}^{N} \|\mathbf{m}^{(j)}\|^2 - \|\mathbf{n}^{(j)}\|^2, \tag{15}$$

where $\mathbf{m}^{(j)} = \mathbf{F}\mathbf{H}^{(j)}(\mathbf{b} - \tilde{\mathbf{b}}) + \mathbf{n}^{(j)}$. Eqn. (15) can be written in the form

$$D = \mathbf{V}^{\dagger}\mathbf{S}\mathbf{V}, \tag{16}$$

where

$$\mathbf{V} = \begin{pmatrix} \mathbf{m}^{(1)} \\ \vdots \\ \mathbf{m}^{(N)} \\ \mathbf{n}^{(1)} \\ \vdots \\ \mathbf{n}^{(N)} \end{pmatrix}, \tag{17}$$

$$\mathbf{S} = \left[\begin{array}{cc} \mathbf{I}_{QKN} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_{QKN}. \end{array} \right]. \tag{18}$$

The decision variable D in (16) is in Hermitian quadratic form in the complex Gaussian random vector $\mathbf{V}$. This form, from a result in [13], allows us to write the characteristic function of D, $\Phi_D(j\omega)$, in closed-form. In order to do that, let

$$\mathbf{T} = E[\mathbf{V}\mathbf{V}^{\dagger}]. \tag{19}$$

To evaluate $\mathbf{T}$ in the above, we write $\mathbf{H}^{(j)}\mathbf{b}$ in an alternate form [4]

$$\mathbf{H}^{(j)}\mathbf{b} = \mathbf{B}\mathbf{h}^{(j)}, \tag{20}$$

where $\mathbf{B}$ is a $QK \times QK$ matrix, which for $M = Q = 8$ is defined as

$$\mathbf{B} = \left[\begin{array}{cccccccc} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_4 & \mathbf{B}_5 & \mathbf{B}_6 & \mathbf{B}_7 & \mathbf{B}_8 \\ \mathbf{B}_2 & -\mathbf{B}_1 & -\mathbf{B}_4 & \mathbf{B}_3 & -\mathbf{B}_6 & \mathbf{B}_5 & \mathbf{B}_8 & -\mathbf{B}_7 \\ \mathbf{B}_3 & \mathbf{B}_4 & -\mathbf{B}_1 & -\mathbf{B}_2 & -\mathbf{B}_7 & -\mathbf{B}_8 & \mathbf{B}_5 & \mathbf{B}_6 \\ \mathbf{B}_4 & -\mathbf{B}_3 & \mathbf{B}_2 & -\mathbf{B}_1 & -\mathbf{B}_8 & \mathbf{B}_7 & -\mathbf{B}_6 & \mathbf{B}_5 \\ \mathbf{B}_5 & \mathbf{B}_6 & \mathbf{B}_7 & \mathbf{B}_8 & -\mathbf{B}_1 & -\mathbf{B}_2 & -\mathbf{B}_3 & -\mathbf{B}_4 \\ \mathbf{B}_6 & -\mathbf{B}_5 & \mathbf{B}_8 & -\mathbf{B}_7 & \mathbf{B}_2 & -\mathbf{B}_1 & \mathbf{B}_4 & -\mathbf{B}_3 \\ \mathbf{B}_7 & -\mathbf{B}_8 & -\mathbf{B}_5 & \mathbf{B}_6 & \mathbf{B}_3 & -\mathbf{B}_4 & -\mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_8 & \mathbf{B}_7 & -\mathbf{B}_6 & -\mathbf{B}_5 & \mathbf{B}_4 & \mathbf{B}_3 & -\mathbf{B}_2 & -\mathbf{B}_1 \end{array} \right]. \tag{21}$$

where $\mathbf{B}_q = \mathbf{A}_q diag\{\mathbf{b}_q\}$, $\mathbf{A}_q = diag\{A_{1q}, A_{2q}, \cdots, A_{Kq}\}$, $q = 1, 2, \cdots, Q$. For values of M and Q other than 8, $(M, Q < 8)$ $\mathbf{B}$ is obtained as follows. For $M = Q \in \{1, 2, 4\}$, $\mathbf{B}$ is given by the upper leftmost submatrix of order $QK \times QK$ in (21). For $M \notin \{1, 2, 4, 8\}$, $M < Q$. In this case, $\mathbf{B}$ is given by the $QK \times QK$ upper leftmost submatrix in (21) with all the entries in the q^{th} column ($M < q \leq Q$) as zeros. The vector $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \cdots, \mathbf{h}_Q^T]^T$, and $\mathbf{h}_q = [h_{1q}, h_{2q}, \cdots, h_{Kq}]^T$ such that $E[\mathbf{h}] = \mathbf{0}_{QK \times 1}$ and $E[\mathbf{h}\mathbf{h}^{\dagger}] = \Omega\mathbf{I}_{QK}$. With the above definitions, we obtain

$$E\left[\mathbf{m}^{(i)}\mathbf{m}^{(j)\dagger}\right] = \left\{ \begin{array}{ll} \mathbf{0} & i \neq j \\ \Omega\mathbf{F}(\mathbf{B} - \tilde{\mathbf{B}})(\mathbf{B} - \tilde{\mathbf{B}})^T\mathbf{F}^T & \\ +2\sigma^2\mathbf{I}_{QK} & i = j \end{array} \right. \tag{22}$$

$$E[\mathbf{m}^{(i)}\mathbf{n}^{(j)\dagger}] = E[\mathbf{n}^{(i)}\mathbf{m}^{(j)\dagger}] = E[\mathbf{n}^{(i)}\mathbf{n}^{(j)\dagger}] = \left\{ \begin{array}{ll} \mathbf{0} & i \neq j \\ 2\sigma^2\mathbf{I}_{QK} & i = j \end{array} \right. \tag{23}$$

from which $\mathbf{T}$ can be evaluated.

Now, the the characteristic function of D, $\Phi_D(j\omega)$ can be written as (Ref. [13], Eqn. (4.a))

$$\Phi_D(j\omega) = \frac{1}{|\mathbf{I}_{2NQK} - 2j\omega\sigma^2\mathbf{G}|}, \tag{24}$$

where $\mathbf{G} = \mathbf{T}\mathbf{S}$. From (18), (19), (22), (23), we can write $\mathbf{G}$ as

$$\mathbf{G} = \left[\begin{array}{cc} \mathbf{I}_N \otimes (\frac{\Omega}{2\sigma^2}\mathbf{F}(\mathbf{B} - \tilde{\mathbf{B}})(\mathbf{B} - \tilde{\mathbf{B}})^T\mathbf{F}^T + \mathbf{I}_{QK}) & \mathbf{I}_N \otimes -\mathbf{I}_{QK} \\ \mathbf{I}_N \otimes \mathbf{I}_{QK} & \mathbf{I}_N \otimes -\mathbf{I}_{QK} \end{array} \right]. \tag{25}$$

Defining $\hat{\mathbf{G}}$ as

$$\hat{\mathbf{G}} = \left[\begin{array}{cc} (\frac{\Omega}{2\sigma^2}\mathbf{F}(\mathbf{B} - \tilde{\mathbf{B}})(\mathbf{B} - \tilde{\mathbf{B}})^T\mathbf{F}^T + \mathbf{I}_{QK}) & -\mathbf{I}_{QK} \\ \mathbf{I}_{QK} & -\mathbf{I}_{QK} \end{array} \right], \tag{26}$$

(24) can be written as

$$\begin{aligned} \Phi_D(j\omega) &= \frac{1}{|\mathbf{I}_{2QK} - 2j\omega\sigma^2\hat{\mathbf{G}}|^N}. \\ &= \prod_{i=1}^{2QK} \frac{1}{|1 - 2j\omega\sigma^2\hat{\lambda}_i|^N}. \end{aligned} \tag{27}$$

where $\hat{\lambda}_1, \cdots, \hat{\lambda}_{2QK}$ are the eigenvalues of $\hat{\mathbf{G}}$.

For the case when the amplitudes of all bits from all the users are the same, i.e., $A_{iq} = A_{jq} = A$, $i, j = 1, 2, \cdots, K$, $q = 1, 2, \cdots, Q$, and $M = Q$ (27) can be written in the form

$$\begin{aligned} \Phi_D(j\omega) &= \frac{1}{|\mathbf{I}_{2K} - 2j\omega\sigma^2\tilde{\mathbf{G}}|^{MN}} \\ &= \prod_{i=1}^{2K} \frac{1}{|1 - 2j\omega\sigma^2\lambda_i|^{MN}}, \end{aligned} \tag{28}$$

where $\tilde{\mathbf{G}}$ is given by

$$\tilde{G} = \left[\begin{array}{cc} \frac{\Omega A^2}{2\sigma^2}\mathbf{P}\mathbf{\Lambda}\mathbf{P}^T + \mathbf{I}_K & -\mathbf{I}_K \\ \mathbf{I}_K & -\mathbf{I}_K \end{array} \right], \tag{29}$$

where $\mathbf{P}$ is the Cholesky decomposition of the $\mathbf{R}$ matrix (i.e., $\mathbf{R} = \mathbf{P}^T\mathbf{P}$), $\mathbf{\Lambda}$ is given by

$$\mathbf{\Lambda} = \frac{1}{A^2}\sum_{i=1}^{Q}(\mathbf{B}_i - \tilde{\mathbf{B}}_i)^2, \tag{30}$$

and $\lambda_1, \cdots, \lambda_{2K}$ are the eigenvalues $\tilde{\mathbf{G}}$. Substituting $z = 2j\omega\sigma^2$, we have

$$\Phi_D(z) = \prod_{i=1}^{2K} \frac{1}{(1 - z\lambda_i)^{MN}}. \tag{31}$$

From the above characteristic function of D, the PEP in (14) can be obtained as [16], [12]

$$P(\mathbf{b} \to \tilde{\mathbf{b}}) = -\sum_k \frac{1}{(p_k - 1)!}\frac{d^{p_k - 1}}{dz^{p_k - 1}}\left\{(z - \lambda_k)^{p_k}\frac{\Phi_D(z)}{z}\right\}, \tag{32}$$

where λ_k are the negative eigenvalues of $\tilde{G}$, $Re(\lambda_k) < 0$, and p_k is the multiplicity of λ_k. We obtain (32) in closed-form as follows. The characteristic equation of $\tilde{\mathbf{G}}$ is given by

$$det|\lambda\mathbf{I}_{2K} - \tilde{\mathbf{G}}| = det\left| \begin{array}{cc} (\lambda - 1)\mathbf{I}_K - \gamma\mathbf{J} & \mathbf{I}_K \\ -\mathbf{I}_K & (\lambda + 1)\mathbf{I}_K \end{array} \right| = 0 \tag{33}$$

where $\gamma = \frac{\Omega A^2}{2\sigma^2}$ is the average SNR and $\mathbf{J} = \mathbf{P}\mathbf{\Lambda}\mathbf{P}^T$. Eqn. (33) can be shown to reduce to the form [14]

$$det|\lambda^2\mathbf{I}_K - \gamma(\lambda + 1)\mathbf{J}| = 0. \tag{34}$$

If $\mu_1, \cdots, \mu_L$ are the L distinct eigenvalues of $\mathbf{J}$, each with multiplicity v_i, i.e. $\sum_{i=1}^{L} v_i = 2K$, then (34) reduces to

$$\prod_{i=1}^{L} (\lambda^2 - \gamma\mu_i\lambda - \gamma\mu_i)^{v_i} = 0. \tag{35}$$

Denote the roots of $\lambda^2 - \gamma\lambda\mu_i - \gamma\mu_i = 0$ as λ_{i1} and λ_{i2}. From Sylvester's Law of Inertia [15], the eigenvalues of $\mathbf{J}$ are non-negative (i.e., $\mu_i \geq 0$). Hence, λ_{i1} and λ_{i2} are real, and they can both be either zero or otherwise. If they are not zero, then one will be strictly positive and the other strictly negative. In the following, we let λ_{i1} to denote the non-negative roots (i.e., ≥ 0) and λ_{i2} to denote the negative roots (i.e, < 0). With this, we can now follow the steps similar to the ones in [12], and obtain the closed-form expression for the PEP as

$$\begin{aligned} P(\mathbf{b} \to \tilde{\mathbf{b}}) &= \sum_j \frac{(-\lambda_{j2})^{MN(2K-v_j)}}{\prod_i (\lambda_{i1} - \lambda_{j2})^{MNv_i} \prod_{i \neq j} (\lambda_{i2} - \lambda_{j2})^{MNv_i}} \\ &\quad \cdot \sum_{\substack{(l_1, \cdots, l_{MNv_j-1}) \\ 0 \leq l_1, \cdots, l_{MNv_j-1} \leq MNv_j - 1 \\ l_1 + 2l_2 + \cdots + (MNv_j - 1) l_{(MNv_j-1)} = (MNv_j - 1)}} \prod_{m=1}^{MNv_j-1} \frac{1}{l_m!} \\ &\quad \cdot \left[\frac{1}{m} + \frac{MN}{m} \left(\sum_i \frac{v_i \lambda_{i1}^m}{(\lambda_{i1} - \lambda_{j2})^m} + \sum_{i \neq j} \frac{v_i \lambda_{i2}^m}{(\lambda_{i2} - \lambda_{j2})^m} \right) \right]^{l_m}, \end{aligned} \tag{36}$$

where K is the number of users, M is the number of transmit antennas per user, and N is the number of antennas at the receiver.

Bound on Probability of Bit Error

Using the expression for PEP in the above, we obtain an upper bound on the bit error probability as follows. Let $\mathbf{b}^{(j)}, 1 \leq j \leq 2^{QK}$ be the set of QK-bit vectors comprising of Q bits from each of the K users. Suppose $\mathbf{b}^{(k)}$ was the transmitted vector. Define

$$D_m = \sum_{j=1}^{N} \|\hat{\mathbf{y}}^{(j)} - \mathbf{F}\mathbf{H}^{(j)}\mathbf{b}^{(m)}\|^2, \quad m = 1, 2, \cdots, 2^{QK} \tag{37}$$

where $\hat{\mathbf{y}}$, $\mathbf{F}$ and $\mathbf{H}$ are as defined in (12). If $\mathbf{b}^{(l)}$ is the received vector, define

$$P_{exact}\left(\mathbf{b}^{(k)} \to \mathbf{b}^{(l)}\right) = \Pr\left(\bigcap_{\substack{m=1 \\ m \neq l}}^{2^{QK}} (D_l < D_m) \right). \tag{38}$$

It is noted that the PEP in (36) is nothing but

$$P\left(\mathbf{b}^{(k)} \to \mathbf{b}^{(l)}\right) = \Pr\left(D_l < D_k\right). \tag{39}$$

It is clear that

$$P_{exact}\left(\mathbf{b}^{(k)} \to \mathbf{b}^{(l)}\right) \leq P\left(\mathbf{b}^{(k)} \to \mathbf{b}^{(l)}\right). \tag{40}$$

Let $P(e_{iq})$ denote the probability of error for the q^{th} bit of the i^{th} user, $q = 1, 2, \cdots, Q$ and $i = 1, 2, \cdots, K$. $P(e_{iq})$ is then given by

$$\begin{aligned} P(e_{iq}) = &\sum_{j=1}^{2^{QK-1}} P(e_{iq}|\mathbf{b}^{(j)}, b_{iq}^{(j)} = 1) P(\mathbf{b}^{(j)}, b_{iq}^{(j)} = 1) \\ &+ \sum_{k=1}^{2^{QK-1}} P(e_{iq}|\mathbf{b}^{(k)}, b_{iq}^{(k)} = -1) P(\mathbf{b}^{(k)}, b_{iq}^{(k)} = -1). \end{aligned} \tag{41}$$

$P(e_{iq}|\mathbf{b}^{(j)}, b_{iq}^{(j)} = \pm 1)$ and $P(\mathbf{b}^{(j)}, b_{iq}^{(j)} = \pm 1)$ are then given by

$$P\left(e_{iq}|\mathbf{b}^{(j)}, b_{iq}^{(j)} = 1\right) = \sum_{k=1}^{2^{QK-1}} P_{exact}\left(\mathbf{b}^{(j)} \to \mathbf{b}^{(k)} | b_{iq}^{(j)} = 1, b_{iq}^{(k)} = -1\right), \tag{42}$$

$$P\left(e_{iq}|\mathbf{b}^{(k)}, b_{iq}^{(k)} = -1\right) = \sum_{j=1}^{2^{QK-1}} P_{exact}\left(\mathbf{b}^{(k)} \to \mathbf{b}^{(j)} | b_{iq}^{(j)} = 1, b_{iq}^{(k)} = -1\right), \tag{43}$$

$$P(\mathbf{b}^{(j)}, b_{iq}^{(j)} = 1) = P(\mathbf{b}^{(k)}, b_{iq}^{(k)} = -1) = \frac{1}{2^{QK}}. \tag{44}$$

From (40) (41),(42),(43) and(44), an upper bound on the bit error probability $P(e_{iq})$ is obtained as

$$\begin{aligned} P(e_{iq}) \leq &\frac{1}{2^{QK}} \left[\sum_{j=1}^{2^{QK-1}} \sum_{k=1}^{2^{QK-1}} P(\mathbf{b}^{(j)} \to \mathbf{b}^{(k)} | b_{iq}^{(j)} = 1, b_{iq}^{(k)} = -1) \right. \\ &\left. + \sum_{k=1}^{2^{QK-1}} \sum_{j=1}^{2^{QK-1}} P(\mathbf{b}^{(k)} \to \mathbf{b}^{(j)} | b_{iq}^{(k)} = 1, b_{iq}^{(j)} = -1) \right]. \\ = &\frac{1}{2^{QK-1}} \left[\sum_{j=1}^{2^{QK-1}} \sum_{k=1}^{2^{QK-1}} P(\mathbf{b}^{(j)} \to \mathbf{b}^{(k)} | b_{iq}^{(j)} = 1, b_{iq}^{(k)} = -1) \right] \end{aligned} \tag{45}$$

IV. RESULTS AND DISCUSSION

In this section, we present the numerical results of the error performance of the ML multiuser detection scheme. Fig. 1 shows the PEP and the bit error probability plots for a two user system ($K = 2$), with two transmit antennas ($M = 2$) at each user, and one antenna at the receiver ($N = 1$). The correlation coefficient between the two users' signature waveforms, $\rho = 0.2$. The power imbalance between the two users is characterized by the near-far ratio (NFR)[1]. In Fig. 1, the NFR is taken to be 0 dB (i.e., equal power users). The average PEP results obtained through analysis (Eqn. 36) and the bit error probability results obtained through analysis (Eqns. 45) as well as simulations are shown in Fig. 1. It is observed that the BER bound is loose at low SNRs, but gets increasingly tight for larger SNRs. Fig. 2 shows the bit error performance as a function of the NFR at an average SNR of 10 dB, $K = 2$, $\rho = 0.2$, $M = 2$, $N = 1$. It is seen that, as expected, the ML multiuser detector is near-far resistant (whereas the conventional matched filter detector is not near-far resistant as seen in the Figure), and the analysis predicts the BER quite close to the simulation results.

[1]We define near-far ratio as 10 log$\frac{\sum_{p=1}^{M} A_{2p}^2}{\sum_{p=1}^{M} A_{1p}^2}$ assuming $\Omega = E|h_{iq}|^2 = 1$.

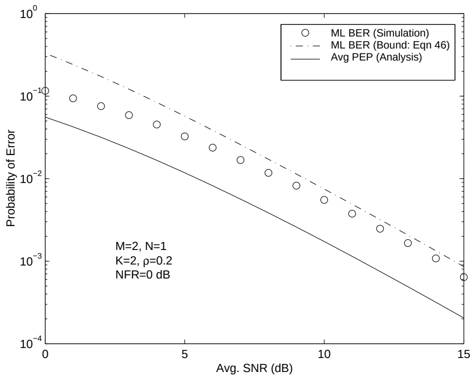

Fig. 1. Error performance as a function of average SNR for $K = 2$,$M = 2$, $N = 1$, NFR = 0 dB.

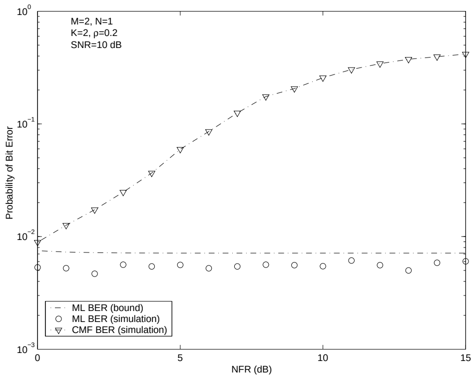

Fig. 2. Bit error performance as a function of NFR. $K = 2$,$M = 2$, $N = 1$, SNR = 10 dB.

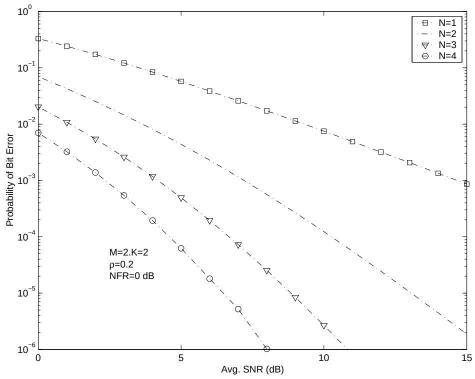

Fig. 3. Bit error performance as a function of average SNR for different number of Rx antennas, $N = 1, 2, 3, 4$. $K = 2$, $M = 2$, NFR = 0 dB.

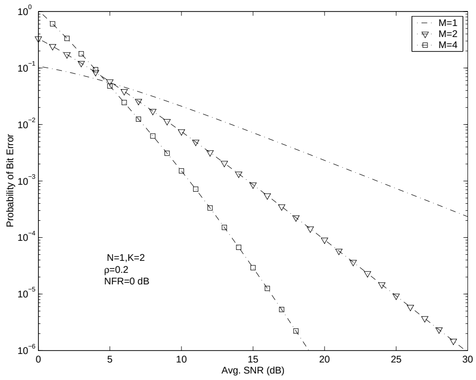

Fig. 4. Bit error performance as a function of average SNR for different number of Tx antennas, $M = 1, 2, 4$. $K = 2$, $N = 1$, NFR = 0 dB.

Fig. 3 shows the bit error probability performance predicted by the analysis for different number of receive antennas, $N = 1, 2, 3, 4$ for $M = 2$, $K = 2$, NFR = 0 dB. It is seen that the bit error performance improves as the receive diversity order is increased. Similarly, Fig. 4 shows the bit error performance when the number of transmit antennas is changed ($M = 1, 2, 4$), which illustrates the bit error performance improvement due to transmit diversity.

V. CONCLUSION

We analyzed the bit error performance of maximum-likelihood (ML) multiuser detection in space-time coded CDMA systems. We considered a K-user synchronous CDMA system which employs orthogonal space-time block coding with M transmit antennas and N receive antennas. We derived a closed-form exact expression for the pairwise error probability, using which we obtained an upper bound on the bit error probability. We showed that the analytical BER bounds are increasingly tight for large SNR values.

REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744-765, March 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE Jl. Sel. Areas in Commun.*, vol. 17, pp. 451-460, March 1999.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory,* vol. 45, no. 5, pp. 1456-1467, July 1999.

[4] B. Hochwald, T. L. Marzetta and C. B. Papadias, "A transmitter diversity scheme for wideband CDMA systems based on space-time spreading," *IEEE Jl. Sel. Areas Commun.*, vol. 19, no. 1, pp. 48-60, January 2001.

[5] S. Verdu, *Multiuser Detection,* Cambridge University Press, 1998.

[6] H. Huang, H. Viswanathan, and G. J. Foschini, "Multiple antennas in cellular CDMA systems: Transmission, detection, and spectral efficiency," *IEEE Trans. Wireless Commun.,* vol. 1, no. 3, pp. 383-392, July 2002.

[7] D. Reynolds, X. Wang, and H. V. Poor, "Blind adaptive space-time multiuser detection with multiple transmitter and receiver antennas," *IEEE Trans. Sig. Proc.,* 50(6), pp. 1261-1276, June 2002.

[8] M. M. Alam and A. Chockalingam, "Performance of adaptive multiuser detectors in space-time coded CDMA systems," *IEEE PIMRC'2003*, September 2003.

[9] M. Uysal and C. N. Georghiades, "Performance of space-time coded CDMA systems for wireless communication," *Electronic Letters,* vol. 36, no. 14, pp. 1231-1232, July 2000.

[10] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Trans. Inform. Theory,* vol. 48, iss. 2, pp. 510-513, February 2002.

[11] G. Taricco and E. Biglieri, "Correction to 'Exact pairwise error probability of space-time codes'," *IEEE Trans. Inform. Theory,* vol. 49, iss. 3, pp. 766-766, March 2003.

[12] P. Garg, R. K. Mallik and H. M. Gupta, "Performance analysis of space-time coding with imperfect channel estimation," *IEEE ICPWC'2002,* pp. 71 -75, December 2002.

[13] G. L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables," *Biometrika,* vol. 47, pp. 199-201, 1960.

[14] D. C. Lay, *Linear Algebra and its Applications,* Addison Wesley, 2000.

[15] G. Strang, *Linear Algebra and its Applications*, Harcourt, Brace, Jovanovich, 1988.

[16] J. Gil-Pelacz,"Note on the inversion theorem," *Biometrika* , vol.38 , pp.481-482, 1951.