# Distraction Free Evolution of Active Contours

V. Srikrishnan, Subhasis Chaudhuri
V.I.P Lab, Electrical Engineering Department
Indian Institute of Technology Bombay, Mumbai-400076, India
krishnan@ee.iitb.ac.in

## Abstract

*We propose a novel energy term to make the curve evolution quite robust to spurious edges. The physical intuition behind the formulation is that an object edge is generally continuous, but it could be composed of weak and strong segments. We then formulate the energy term which depends on a second order measure defined on the contour. Minimisation of this energy term yields a space varying curvature based curve evolution equation. An added advantage of the formulation is that this term also acts as the regularising term for smoothing the curve evolution. The proposed term can therefore be used in conjunction with any of the numerous gradient based active contour models. For our experimentation purpose, we have used the well-known gradient vector force model as the external force. We have performed a number of experiments on images and obtained good results.*

## 1. Introduction

Active contours or "snakes" are a very popular class of models used in computer vision for segmentation and tracking. Contour evolution is obtained by deriving the Euler-Lagrange equations corresponding to an energy functional defined on the curve. The energy term consists of an image based term and a smoothness or a regularising term. The image force driving the active contours can be region or edge based models or some combination of both depending on the energy functional. Region based contours[3][15] use a probabilistic description of the target appearance. On the other hand, gradient based contours[2][4][7][10][12] are attracted to the image areas with higher magnitudes of gradient. Region based models have generally better capture range but the segmentation provided the edge based model is more accurate in capturing the object boundary.

It is well known in computer vision literature that active contours are quite sensitive to initialisation and easily distracted by clutter. This is especially true for the case of gradient based active contours, which are distracted by the presence of stronger edges near the target. The contribution of this work is the definition of a new curve energy term which makes the curve evolution robust to clutter and at the same time plays the important role of the regularising term. The proposed energy term depends on a second order measure defined on the curve and uses the image gradient information. However, as we shall explain later, the proposed term is not an external energy term in the conventional sense.

For the sake of completion, we mention here that depending on the implementation scheme used, active contours have been classified as geometric or parametric active contours[5]. Geometric active contours use the level set methods[11][13] for implementation. In this work, use cubic B Splines for implementation. It is to be noted that the ideas presented in this work is quite independent of the implementation scheme used.

**Notation**: We assume that the curve, denoted by $C(p,t) = (x(p,t), y(p,t))$ is closed and planar. The curve is parametrised by $p$ and $t$ is the artificial time parameter of evolution. Thus, $C(p,0)$ is the initial contour. The curvature is denoted by $\kappa$ and the *inward* normal and tangents are denoted by $\mathbf{N}$ and $\mathbf{T}$, respectively. The image is denoted by $I$. The curve speed parameter is $g = |C_p|$. The general curve evolution equation, consisting of both tangential and normal term is,

$$C_t = \eta \mathbf{N} + \rho \mathbf{T} \tag{1}$$

where $\eta$ and $\rho$ denote the normal and the tangential evolution term respectively.

## 2. Gradient Based Contour Models: Overview

In this section, we provide a overview of the gradient based models and discuss the shortcoming common to all the models. This will provide the motivation for the work presented in the paper.

## 2.1. Energy Models

The generic energy functional defined on the contour is as follows,

$$E(C) = E_{ext}(C) + \alpha E_{int}(C) \tag{2}$$

For the gradient based models, the external energy term $E_{ext}(C)$ attracts the curve towards high gradient in the image. The internal energy $E_{int}(C)$ term is used as a regularising term for stable evolution and smoothness of the evolved curve. The relative strengths of each term is controlled by $\alpha$.

The first active contour model was proposed by Kass, Witkin and Terzopoulos[7] in their seminal work. The energy term defined by Kass et al. is

$$\begin{aligned} E(C) = \int_0^1 f(C(p))dp + \int_0^1 \gamma(p)|C_p|^2 dp \\ + \int_0^1 \frac{\beta(p)}{2}|C_{pp}|^2 dp \end{aligned} \tag{3}$$

where $f(C) = -|\nabla I(C)|^2$. The first term on the RHS moves the curve towards the nearest local edge. The next two terms, weighted by $\gamma$ and $\beta$ are called the membrane term and the thin plate term respectively. In this model, both the membrane and thin plate terms are modelled as space variant. However, there is no specification of their exact functional form. In the implementation, it is assumed that these terms are constant. The Euler Lagrange equations obtained by minimising equation(3) provided the velocity field for evolving the contour. The geodesic active contour[2] model proposed by Caselles et al. basically reformulates the problem as that of finding a geodesic on a Riemannian space defined by the image. Their energy functional leads to the following evolution equation

$$C_t = h(I)\kappa\mathbf{N} - (\nabla h \,.\, \mathbf{N})\mathbf{N}, \tag{4}$$

where $h = \frac{1}{1+|\nabla I|^2}$.

Prince et al.[4] proposed an formulation for computation of the motion field for contour evolution, called the gradient vector force(GVF) model. The force field $\mathbf{F} = (u, v)$, is obtained as the minimisation of the following functional,

$$\begin{aligned} E_{ext} = E_{GVF} = \int\int (\mathbf{F} - \nabla I)^2 \\ + \gamma(u_x^2 + u_y^2 + v_x^2 + v_y^2)dxdy \end{aligned} \tag{5}$$

where $\gamma$, with an overuse of notation, is the term controlling the spatial extent of the force field. The calculated force field is equal to the local gradient in regions of strong edges and is extended to the other regions by penalising rapid variations, controlled by $\gamma$. The GVF model increased the capture range and produced better segmentation in concave regions than the earlier models. However, the motion field obtained from this model often attracts the curve toward stronger edges. In real life scenario, such edges could be present near the object of interest and hence led to incorrect segmentation. Li *et al.*[8] present an useful extension to the GVF model to solve the problem associated with the GVF model, called the edge preserving gradient vector force(EPGVF) model. The EPGVF model adds one more term to the GVF model. This is the Jacobian of the motion field, $\int\int |\mathbf{J_v p}|^2 dxdy$, and is called the edge preserving term. This term penalises the deviation of the motion field from the local image gradient direction. This prevents the stronger gradient from overwhelming the weaker gradient.

In [9], the authors have proposed a higher order active contour model(HOAC). They have used this model for segmenting out road networks in satellite images where the target boundaries are elongated and parallel in structure. Imposing the condition of Euclidean invariance, the energy functional used by the authors was,

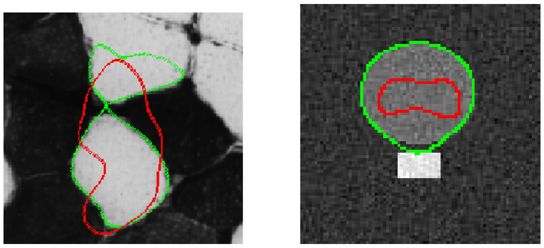
$$E(C) = E_{int}(C) - \frac{\lambda}{2}\int\int [\mathbf{T}(p)\cdot\mathbf{T}(p')]\psi(R(p,p'))dpdp'$$

Here $\psi$ is an interaction function which controls the extent of the interaction and $R(p,p')$ is the Euclidean distance between the points $C(p)$ and $C(p')$. Although interesting, the curve evolution equations are extremely dependent on the values of different parameters.

Our work is inspired by this work, however there are some crucial differences. The physical significance of our model is that it conforms to the notion that object boundaries are continuous entities and penalise disjoint sections of the curve. There is no such intuition in the HOAC model. Also, a crucial point to note is that the energy functional proposed in [9] leads to structural elongations. This is potentially disastrous for most practical segmentation problems.

## 2.2. Motivation

In figure 1(a), we illustrate the general problem associated with the gradient based edge models. As we have mentioned earlier, we use the GVF model to generate the motion field and the following problem is quite independent of the energy functional used to attract the curve toward the object boundaries. The curve in red shows a general initialisation for segmentation and the final converged curve is shown in green. As can be seen in the figure, the final curve segments out the cell of interest only partially and the segmentation "spills" into the next cell. This is because the edges of the adjacent cells act as the sources of attraction for the snake. Here, clearly the poor segmentation is due to the initialisation because a significant portion of the red curve lies in the



(a) Due to initialisation, final curve traces the border of the adjacent cell.

(b) Stronger edge of the white square overwhelms the weaker gray boundary of the circle.Image taken from [8]

**Figure 1. Results of GVF[4] segmentation. Initial curve in red, final curve in green.**

adjoining cell. Therefore, it is not perhaps too surprising that the final curve also leaks into the adjacent cell.

Figure 1(b) shows another example where the traditional gradient based method fails. In this case, the initialisation is made inside the circle to be segmented and therefore we expect an accurate segmentation. However, we see in the figure that the stronger edge of the square near the bottom portion of the circle has attracted the contour. Summarising, the gradient based model fails in many cases practically, either due to incorrect initialisation or due to the presence of stronger but distracting edge near the target boundary.

## 3. Structurally Salient Snake

In computer vision, high level knowledge about the problem is incorporated as some kind of prior. For example, energy terms based on shape priors are frequently used in segmentation problems to provide robustness to occlusion. For a gradient based segmentation task, specifying a prior has its own merits in the form of a goal directed evolution of the contour and demerit in the form of having to define and enumerate and learn the prior. Here we use structural saliency, a feature that can be easily computed, to provide robustness during curve evolution.

To get a intuitive feel of prior term, we re-examine the terms of the generic curve evolution equation(2). The first term, which is the external energy term, forces the curve towards the local edge, which could be the proper edge or a spurious one. The smoothness or the regularisation term maintains the coherence of the curve. Examining figures 1(a) and 1(b), we see that in the converged contour there are parts of it which do not lie along the edge. However, these segments form a link between two sections of the contour which themselves lie along the edges. Therefore, the segmentation does not make sense visually because of these "breakages". Of course, it is not necessary nor is it possible that the whole of the curve lie along one continuous edge; minor deviations are to be expected. The drawback

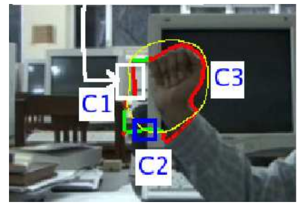



**Figure 2. Illustration of contour segment**

in these models is that there is no feedback from those portions of contour which are "sitting" on the true edges to the rest of the curve. Keeping these points in mind, therefore our aim is to design a model wherein a contour is made aware of the edge it is following and to prevent it from leaving that edge in search of a nearby stronger edge, providing the much needed robustness against clutter. We must penalise those excursions of the contour which are small in comparison to the longer section of the contour.We therefore encourage the contour to evolve towards the *relevant local edge rather than the strongest local edge*.

The key idea of this work is to define a smoothness term which varies along the curve *dynamically during evolution* to guide the curve to the proper edge as discussed earlier. If a section of the curve lies on a long edge section, then the deviation of the curve away from that section should be penalised, even if the deviation itself is toward a stronger but a shorter edge, thus emphasising the importance of edge saliency further. The mathematical details of the model is given next.

### 3.1. Energy Functional

The proposed energy term has the following form,

$$E_{int}(C) = \int_0^1 \alpha(p) f(C) dp. \tag{6}$$

This energy term consists of the space varying weighing term $\alpha(.)$ and the energy functional defined on the contour $f(C)$. We now define the function $\alpha(.)$. Before that,let us assume that the curve is divided into $N$ segments; we shall describe how to divide the curve later. The parameter values corresponding to the segments are denoted by $p_i, i = 0, .., N-1$. The factor $\alpha(p)$ for any point lying within the $i-$th segment weighting the smoothness function $f(C)$ is defined as follows

$$\alpha(p) = \omega_i \theta(p), \tag{7}$$

Here $g(p) = ||C_p(p)||$ is the curve speed parameter. Equation 7 which governs the weight of the smoothness term, is a product of two terms. The first term, $\omega_i$ is constant for a segment and is defined as

$$\omega_i = 1 - \frac{\int_{p_i}^{p_{i+1}} |\nabla I(C(p'))| g(p') dp'}{\int_0^1 |\nabla I(C(p'))| g(p') dp'} \tag{8}$$

From the expression, it can be seen that $\omega$ is depends both on the curve length and the strength of the edge along which it lies. It is nothing but a normalised weighted length of the curve segment, where the weight is the gradient magnitude. Therefore, for a curve segment lying along a strong edge, it will be a small number. For example, in figure 2, $\omega_3 < \omega_1$. To make the concept clearer, let us consider two curve segments of unequal lengths. Let the shorter segment lie along a strong edge while the longer segment lie along a relatively weaker edge. Therefore, there is a distinct possibility that $\omega$ for the shorter edge would be greater. If we now view the strong edge to be part of clutter and the weaker edge to be the part of the curve, we have that the smoothness term $f(C)$, whatever be its functional form, would have a higher weight for the shorter curve segment lying on the distracting edge. It is worth noting here that adjacent segments would have completely different value of $\omega$. Therefore, at the junction of two successive segments, the weight would vary drastically. This could potentially lead to instabilities in curve evolution.

The potential instability leads us to define a smoothing term in equation 7. One requirement of the term is that at the boundaries, it should ensure that $\alpha(p)$ changes smoothly. Therefore, this means reducing the value $\omega$ to that of its neighbour or rather, to the minimum value of all the segments. However, we also need to maintain the $\alpha(p)$ as close to $\omega_I$ for the segment for the $i-$th segment. Keeping these in mind, $\theta$ is defined as follows

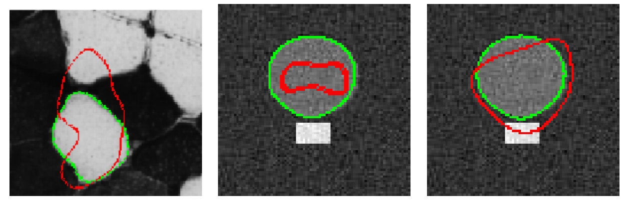
$$\theta(p) = \exp\left\{-\frac{D^2(p)}{\sigma_i^2}\right\} \tag{9}$$

where $D(p)$ is defined as,

$$D(p) = \int_{p_i}^{p} g(p')dp' - \frac{1}{2}\int_{p_i}^{p_{i+1}} g(p')dp' \tag{10}$$

The value $D(p)$ is nothing but the Euclidean distance of the point $C(p)$ from the centre of the segment. Therefore, $\theta$ is symmetric about the centre of the segment. The maximum value of $\alpha$ for the $i-$th segment occurs at the centre of the segment and is equal to $\omega_i$. It gradually reduces as we move toward either end of the segment. There remains the problem of selection of $\sigma_i$. We can fix the value of $\sigma$ as follows. Let us take the minimum of $\omega$ for all the segments and denote it as $\omega_{min}$. We now choose a value $v < \omega_{min}$ and set $\alpha(p_i) = v$ for $i = 0...N-1$. Now, we can determine $\sigma_i$, because we know the boundary value $\alpha(p_i)$ and we know the value of $D$.

We now discuss the choice of the smoothness functional $f(C)$ in equation 6. In [5], the authors have discussed a wide variety of smoothness functional. We set $f(C) = |C_p|^2$. This is the length based regularising term occurring in the Kass *et al.* model. There are two reasons for this choice of $f(C)$. Firstly, it has been shown by



(a) Segmentation results for the image in fig. 1(a) (b) Results for image in fig. 1(b) internal initialisation (c) external initialisation of the contour.

**Figure 3. Results using the proposed model.**

Grayson[6] that the evolution $C_t = \kappa\mathbf{N}$, obtained by minimising $E(C) = \int_0^1 |C_p|^2 dp$, is guaranteed to be smooth. Secondly and more importantly, all planar simple curves shrink to a round point by this evolution. During evolution, concave regions become convex and then this convex region shrinks to a point.

Substituting $\alpha$ and $f(C)$ in equation 6, the Euler-Lagrange equation minimising this energy functional lead to the following curve evolution equation,

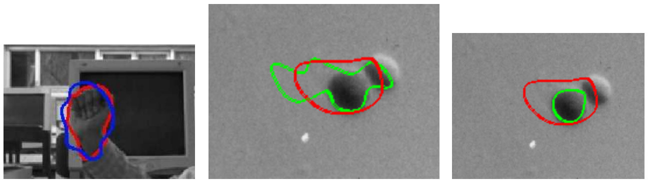
$$C_t(p) = \alpha(p)\kappa(p)\mathbf{N} \tag{11}$$

This constitutes the position varying smoothness term of the model. Since we use parametric B splines for implementation, we add a tangential reparametrisation term as suggested in [14]. The need for reparametrisation is discussed in this work and the references therein. The complete evolution equation is

$$C_t(p,t) = (\kappa(p)\alpha(p) + \mathbf{F}(C(p)) \cdot \mathbf{N})\mathbf{N} + \rho\mathbf{T} \tag{12}$$

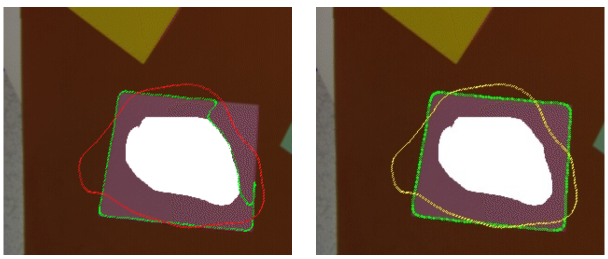
# 4. Results and Discussion

For implementation, we have used the OpenCV computer vision library[1]. In figure 3 we show the results obtained using our model for the images in figure 1. It is to be noted that the segmentation is as desired. To show the robustness of the proposed model with respect to initialisation, for the image in figure 1(b), we initialise the curve such that it cuts through the square, see figure 3(c). We again notice the the segmentation is quite accurate. We note that though for the internal initialisation of the curve in figure 3(b), the EPGVF model[8] has comparable results, for the external initialisation the method would completely fail. This is because of the fact that the EPGVF model would force the curve toward the stronger edge of the square. Figures 2 and 4(a) show the segmentation results obtained using the GVF and proposed model respectively for another image. In this image, the GVF snake is distracted by the presence of the stronger edge of the monitor. Again, we note that the proposed model is able to segment the hand correctly. Figure 4 shows another example of segmentation



(a) Result using proposed model for the image in figure 2 (b) Edge leakage using the GVF[4] model. (c) Result using the proposed model

**Figure 4.**

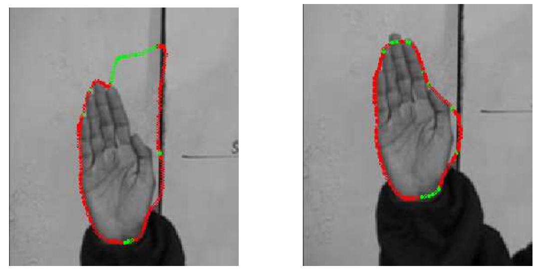


(a) GVF Segmentation:the GVF model gets attracted to the strong edge of the white region inside the quadrilateral. (b) Segmentation using the proposed model

**Figure 5. Results on a synthetic image.**

using the proposed model. Figure 4 again demonstrates the efficiency of our model in a segmentation task.

Next, we show the results of experimentation using both the GVF and the proposed model on a synthetic image as shown in figure 5. We use this example to demonstrate the robustness of the proposed model to spurious strong edges. The edge formed by the white path at the centre of the pinkish quadrilateral is much stronger than the edge if the quadrilateral with the background. The evolution using the GVF model therefore latches onto this edge. Due to the varying internal force, the proposed model is able to segment out the quadrilateral properly. Here we see the importance of the curvature flow as the smoothing term. Since the distracting edge is located within the actual target, we need a smoothness term which moves the curve in the outward direction. This is contrast to the other cases where the distracting edge is locate outside the true object. In such a case, the smoothness term should pull the curve inward toward the object. Motion by curvature takes care of both these cases because of its property mentioned earlier. Finally, we show some results on a real world example where the GVF model is completely overwhelmed by the stronger edges. In figure 6, the contour is distracted by the strong edge of the blackboard. Using our model, the segmentation is much better.



(a) GVF Segmentation (b) Segmentation using the proposed model

**Figure 6. Result on the hand image.**

## 5. Conclusion

We proposed a model wherein the weight of the smoothness term is not constant along the contour but varies along it. This enables distraction free evolution of active contours. Although the results are shown on the GVF gradient based active contour model, this new energy term can be used with any other gradient based active contour model in literature. We demonstrated the effectiveness of our model on different test images with complex initialisations and show good results.

## References


[1] `http://sourceforge.net/projects/opencvlibrary/`.

[2] V. Caselles, R. Kimmel, and G. Sapiro. Geodesic active contours. *IJCV*, 22(1):61–79, 1997.

[3] T. F. Chan and L. A. Vese. Active contours without edges. *IEEE Trans. on Image Proc.*, 10(2):266–277, 2001.

[4] C.Xu and J.L.Prince. Snakes, shapes, and gradient vector flow. *IEEE Trans. on Image Proc.*, 7(3):359–369, 1998.

[5] H. Delingette and J. Montagnat. Shape and topology constraints on parametric active contours. *CVIU*, pages 140–171, 2001.

[6] M. Grayson. The heat equation shrinks embedded plane curves to round points. *J. of Diff. Geom.*, 23:69–96, 1987.

[7] M. Kass, A. Witkin, and D. Terzopoulos. Snakes: Active contour models. *IJCV*, pages 321–331, 1988.

[8] C. Li, J. Liu, and M. D. Fox. Segmentation of edge preserving gradient vector flow: An approach toward automatically initializing and splitting of snakes. In *CVPR '05: - Volume 1*, pages 162–167, 2005.

[9] M. Rochery, I. H. Jermyn, and J. Zerubia. Higher order active contours. *IJCV*, 69(1):27–42, 2006.

[10] G. Sapiro. Color snakes. *CVIU*, 68(2):247–253, 1997.

[11] J. A. Sethian. *Level Set Methods and Fast Marching Methods*. Cambridge University Press, 1999.

[12] S.Kichenessamy, A. Kumar, P. Olver, A.Tannenbaum, and A.Yezzi. Gradient flows and geometric active contour models. In *Proc. IEEE ICCV*, pages 810–815, 1995.

[13] S.Osher and R.Fedkiw. *Level Set Method and Dynamic Implicit Surfaces*. Springer, 2003.

[14] V. Srikrishnan, S. Chaudhuri, S. D. Roy, and D. Sevcovic. On stabilisation of parametric active contours. In *CVPR*, 2007.

[15] S. C. Zhu and A. Yuille. Region competition: Unifying snakes, region growing, and bayes/mdl for multiband image segmentation. *IEEE Trans.PAMI*, 18(9):884–900, 1996.