EE608: Adaptive Signal Processing
Course Instructor: Prof. U.B.Desai
Course Project Report

Adaptive Beamforming for Multi-path Mitigation in GPS

By

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Abstract: An adaptive approach for multipath mitigation would be studied in this course project. Adaptive beamforming techniques that will be studied include two well known beamformers: Frost beamformer and Griffiths-Jim beamformer. The study includes the algorithm implementation details and analysis of the two beamformer results with main emphasis on null rejection and complexity of the algorithm.

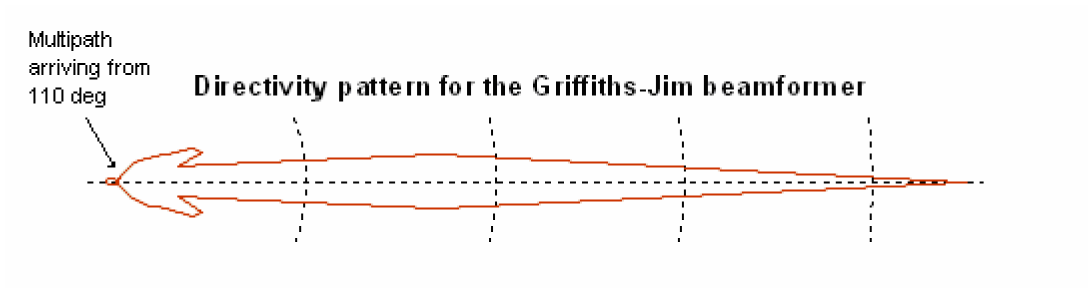
I INTRODUCTION

The major sources of error in Global Positioning System (GPS) are the Multiple Access Interference (MAI) from different GPS satellites all employing DSS spread spectrum technique and the multipath interference due to reflecting surface around the receiver antenna. MAI can affect proper acquisition and tracking of these GPS satellites resulting in measurement errors or reduced sensitivity. On the other hand multi-path interference is due to the extra path length the signal takes on its way from the satellite to the user. The effect of multipath is twofold: signal taking the direct path (specular component) and the multipath signal can interfere destructively at the antenna or the receiver can interpret the extra path length of the multipath itself as the actual transit delay leading to erroneous measurements.

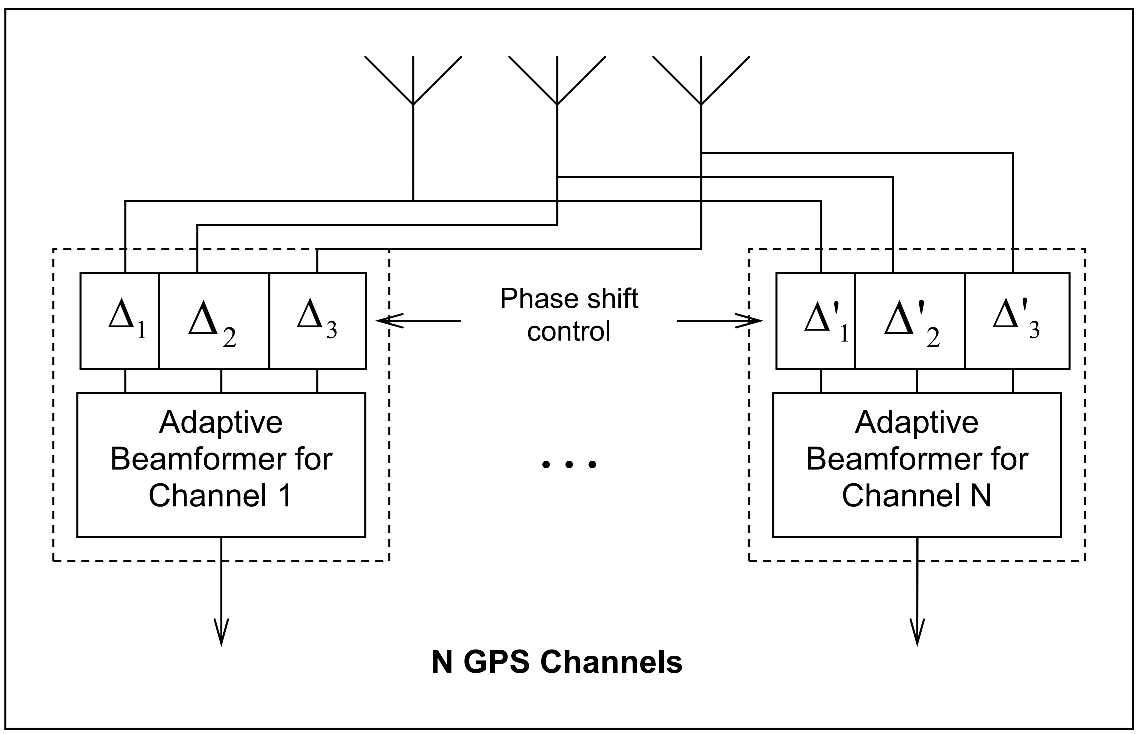
In our course project, we will investigate an adaptive approach to mitigate multipath interference. The adaptive beamformer is such an approach where an array of sensors will create a null (where the sensor array has minimum gain) in a particular direction with the signal arriving from look direction being intact. We also observed that the same adaptive beamformer will work effectively for reducing the MAI as well.

II ADAPTIVE BEAMFORMERS

As already mentioned adaptive beamformers create nulls in the direction of interfering signals. The important point to be noted here is that the adaptive beamformer can have nulls in several other directions along with the null in the required direction. The other null present can be attributed to the radiation pattern which naturally occurs in any antenna array. A typical radiation pattern of an adaptive beamformer is shown below with interference arriving at 110^{o}.



The structure of a beamformer is as shown below. The beamformer consists of an array of sensors. The array is fed to *N* adaptive beamformers after passing it through the phase shifters. The phase shifters of each adaptive beamformer will be so adjusted as to steer the main lobe of the array towards the direction of the GPS satellite to be acquired.



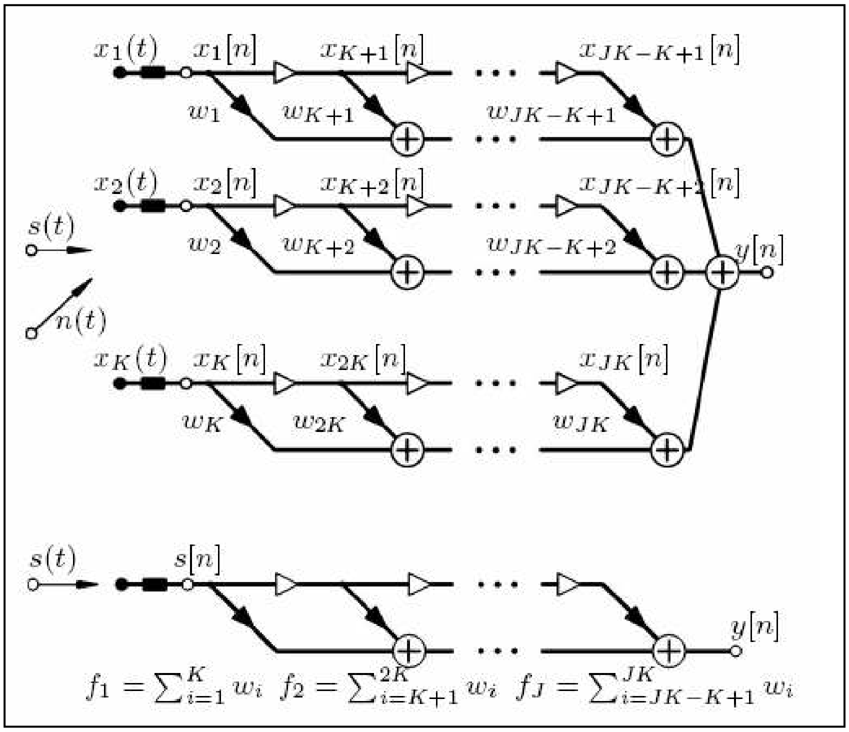
The output of each beamformer will then be given to the N channels of the GPS receiver. If the adaptive beamformer was able to learn and create null in the direction of the interference, the output of the beamformer will be a free of any MAI and multipath interference.

We will now study two adaptive beamformers which creates a main lobe in the look direction and, at the same time, creates null in the direction of interference. Both the adaptive beamformers have a constrained minimization approach which ensures the required frequency response in the look direction.

A. Frost Beamformer

The Frost beamformer, as already mentioned, imposes a hard constraint on the minimization of the cost function. With this hard constraint, the response of the beamformer is fixed in the look direction.

The structure of the Frost beamformer is as shown in figure below. The beamformer comprises *K* tapped delay line filers with *J* weights. These J weights are then adaptively varied according to the constrained minimization of the output power. The constraints are implemented as an equivalent TDL whose weights are equal to the sum of the corresponding weights of the adaptive processor. The parameters of the equivalent processor can be chosen to obtain the required frequency response in the look direction.



The signal arriving from the look direction is the desired signal while any signal arriving from any other angle while be incoherent is considered as interference. Minimizing output power subject to Frost constraint causes the system output to be a minimum variance estimate of the filtered target signal. The impulse response of the target filter was chose as a simple impulse thus providing a flat frequency response over the entire frequency band.

The objective of the algorithm can be outlined as

$$\underset{\mathrm{W}}{\text{minimize}}\, \mathbf{W}^T \mathbf{R}_{\mathbf{XX}} \mathbf{W}$$

$$\text{subject to } \mathbf{C}^{\mathbf{T}} \mathbf{W} = \Im$$

where $\mathbf{C} \underline{\underline{\Delta}} [c_1 ... c_j ... c_J]$ and

$$c_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and $\Im$ is the impulse response of the equivalent tapped delay line filter

$$\Im = [100\ldots0]:1\times M$$

M being the number of LMS taps. The final Frost algorithm proceeds as below

$$\mathbf{F} = \mathbf{C}(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\,\Im$$

$$\mathbf{P} = \mathbf{I} - \mathbf{C}(\mathbf{C}^{\mathrm{T}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{T}}$$

$$\mathbf{W(0)} = \mathbf{F}$$

$$\mathbf{W}(k+1) = \mathbf{P}[\mathbf{W}(k) - \boldsymbol{\mu} y(k)\mathbf{X}(\mathrm{k})] + \mathbf{F}$$

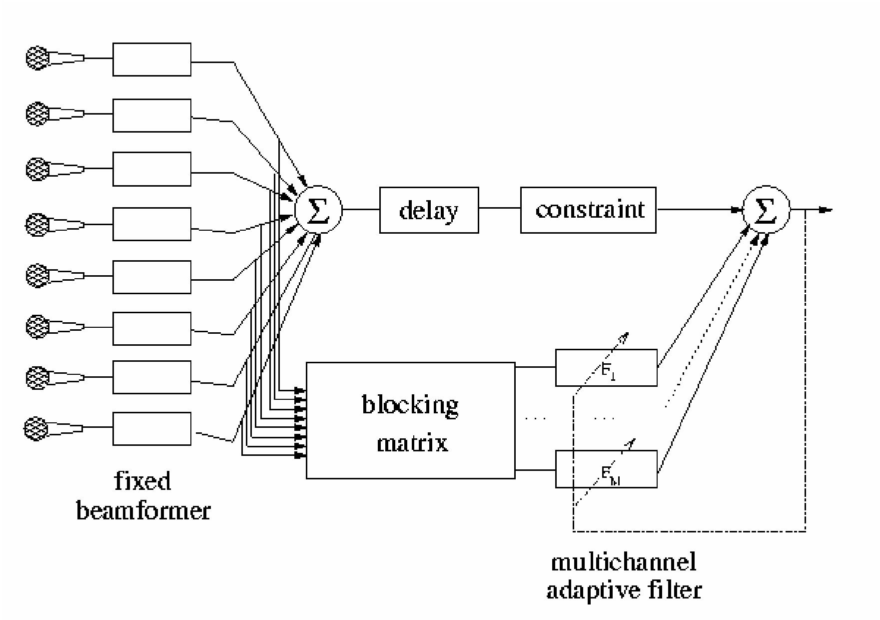
$$y(k) = \mathbf{W}^{\mathrm{T}}(k)\mathbf{X}(k)$$

B. Griffiths-Jim Beamformer

The main assumption in Frost beamformer is that the signal and the interference are highly uncorrelated. When the interference considered is multipath signals, then the assumption of uncorrelated signal and interference does not hold good. Griffiths-Jim Beamformer works well in these cases.

Griffiths-Jim beamformer achieves the Frost constraint but by using an unconstrained least squares algorithm. The typical block diagram of the Griffiths-Jim algorithm is as shown in figure below. The outputs from all sensors are first summed up and then given to a "fixed target signal filter" which, acts as $\Im$ in Frost beamformer.

The sensor outputs are also pair-wise subtracted and fed to a bank of TDL filters. The flexibility of Griffith-Jim algorithm is that any adaptive algorithm can be used for this adaptive tapped delay line filter. If there are *K* sensors, then there will be a total of *(K-1)* adaptive filters. This pair-wise differencing is represented in the form of a blocking matrix **B** given by $\mathbf{B} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$.



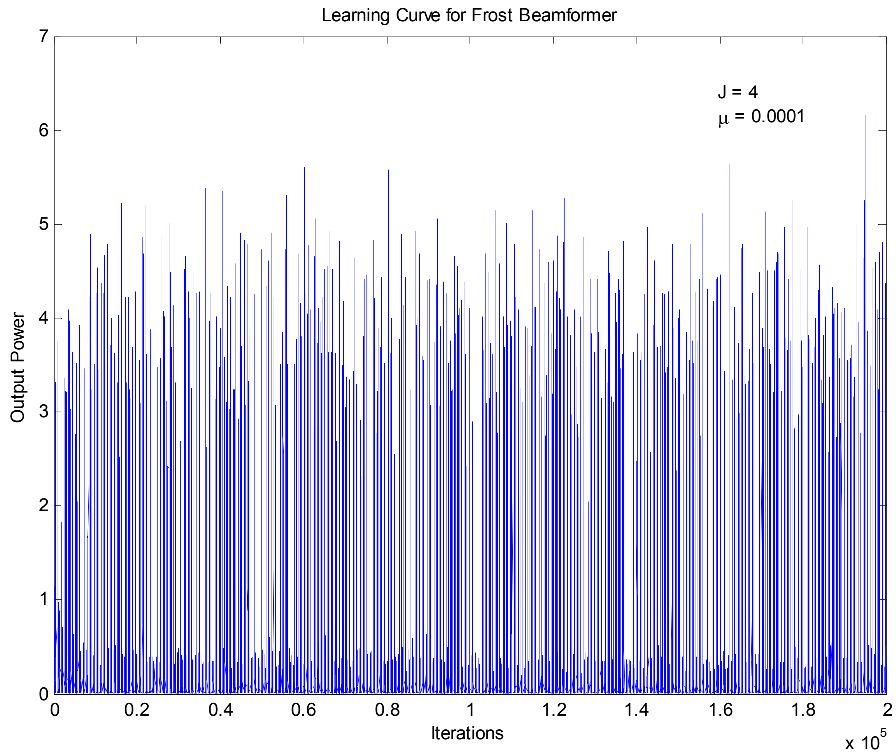
It has to be observed that the degree of freedom in both Frost and Griffiths-Jim beamformer is (KJ-J) where J is the number of taps in the TDL.

III SIMULATION

All the simulations were done using MATLAB. GPS data for 1ms was generated for various satellites and this is done in the function `siggen.m`. The sampling frequency was chosen as 200 MHz, this particular frequency was chosen so that different angles of arrival can be conveniently simulated by shifting the data samples by samples. For simulating multipath arriving from different angles, the original look direction signals were shifted sample by sample equivalent to the DOA according to the equation

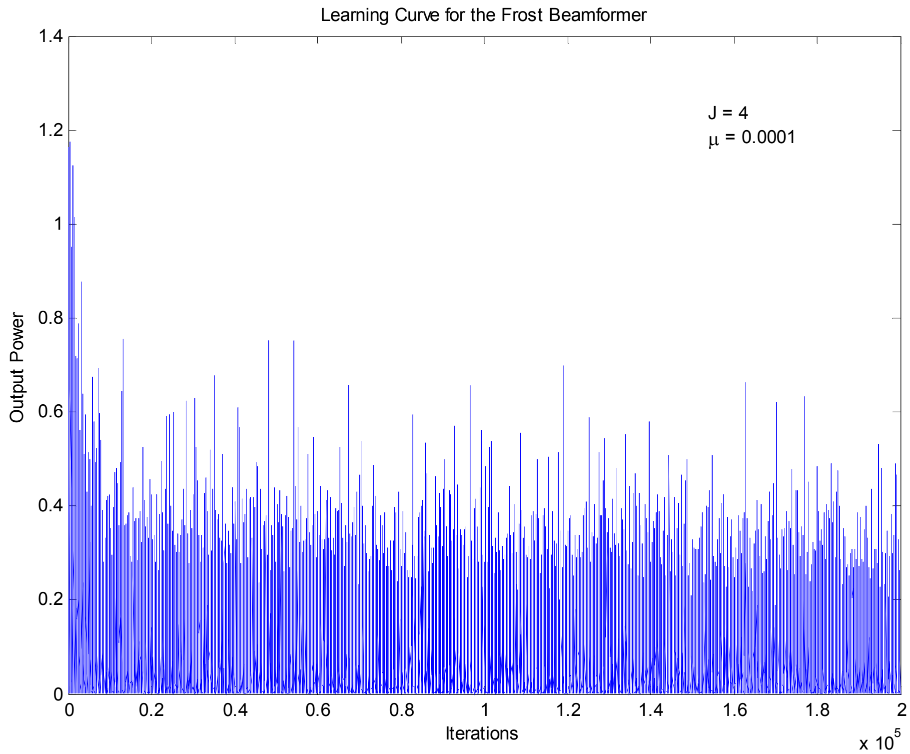
$$shift = \frac{l \sin\theta}{cT_s}$$

Where l is the spatial separation between the sensor elements θ is the angle or arrival w.r.t to the array axis, c is the velocity of light and T_s is the sampling interval. Samples of Rayleigh random numbers were generated and multiplied with the above shifted signal. This was done to simulate a Rician fading in which there is a specular component and other indirect paths are Rayleigh faded. These faded samples were then corrupted with AWGN with noise power as -10dB. The data generation method remains same for both Frost and Griffiths-Jim simulations. The Frost algorithm has been implemented in the file '`FrostAlgo.m`'. The algorithm was run for 200000 samples. The beamformer had three sensors and each sensor had a TDL of 4 taps. The cost function for the Frost beamformer is the output power $\mathbf{W}^{\mathrm{T}}\mathbf{R}_{\mathrm{XX}}\mathbf{W}$. The learning curve was obtained by plotting the output power across iterations. Learning curve is as shown below



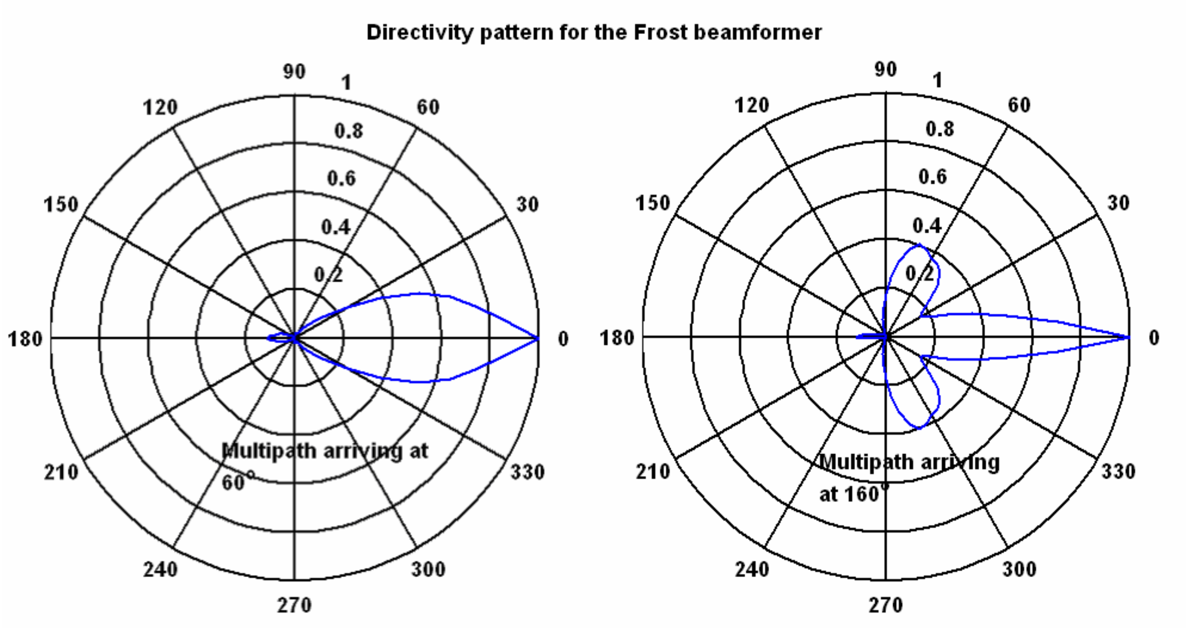
The Learning curve clearly shows a sloping output power but overlapped with huge spikes which were observed only if the Pseudo Random Sequence for the GPS sequence was turned ON. To ensure that the phenomenon observed was only due to the PRN sequence. The GPS data was generated without the PRN sequence and the Frost algorithm was ran for 200000 iterations and the learning curve was obtained, shown below.

Once the Frost beamformer converged, the converged weights were then used to obtain the radiation pattern. This has been implemented in `'Plotfrost.m'`. As already mentioned the equivalent TDL impulse response was chosen as $\Im = [1,0,0,0]$. The same mechanism used to generate data for Frost algorithm was used to generate data to simulate multipath arriving at different angles. This data was then fed to a FIR filter whose impulse response was the converged weights of the Frost algorithm. At each angle the input and the output power was measured and the normalized gain was used. To get the final radiation pattern the values of the normalized gain was multiplied with the array factor of the physical dipole antenna array. For our simulation a 3-element vertical dipole antenna array was used whose array factor (A) in terms of power is given by



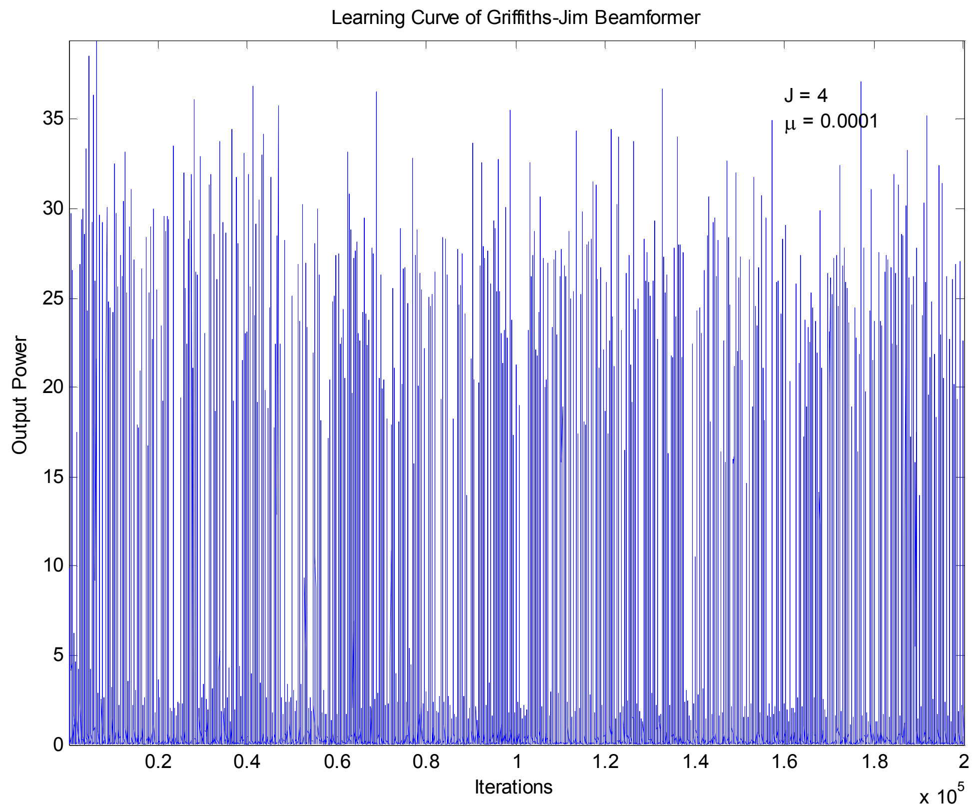
$$A = \left(\frac{\sin\left(\frac{n\psi}{2}\right)}{n \sin\left(\frac{\psi}{2}\right)} \right)^2$$

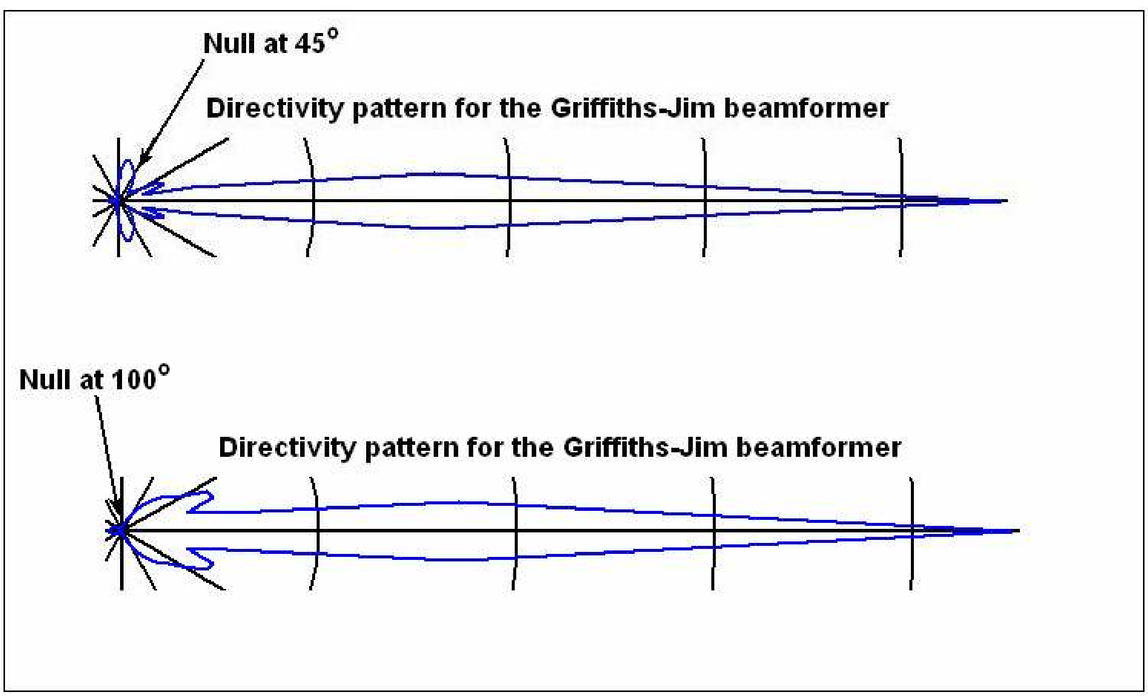
Radiation pattern of the Frost beamformer for multipath arriving at 60^o and 140^o is shown in the figure below. These angles were chosen to avoid natural nulls of the physical antenna array



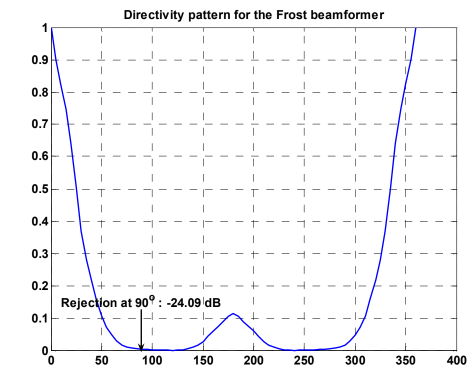
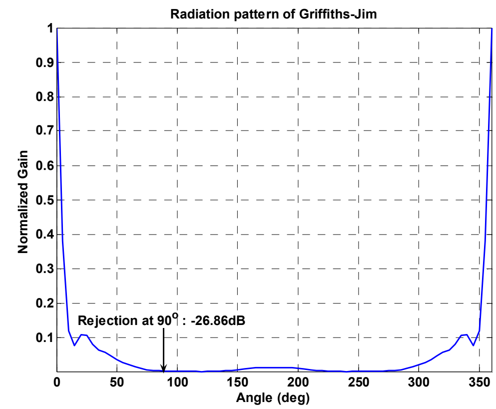
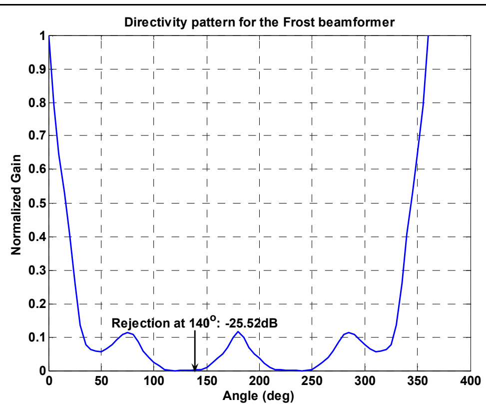
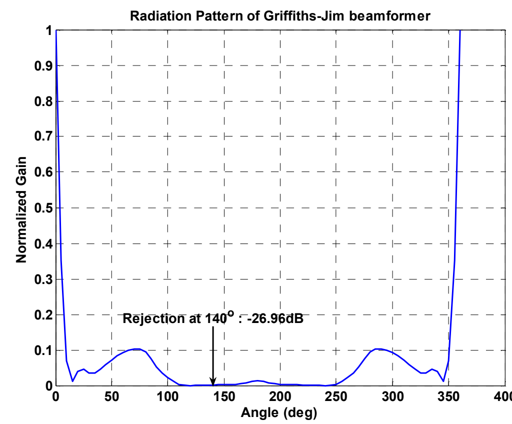
Griffiths-Jim Beamformer was also implemented to operate on the same data set. The file `'GriffithJim.m'` does this. The structure of the algorithm was as mentioned in the previous section. LMS algorithm was used for the TDL filters. The converged weights were then used to obtain the radiation pattern, which has been implemented in `'PlotGriffith.m'`.

Figure shows the learning curve for Griffiths-Jim algorithm with multipath arriving from 100^o. It showed the same noisy behavior as was observed in the case of Frost beamformer. Radiation pattern for the arrivals from 100^o and 45^o are also plotted below.


Learning Curve of Griffiths-Jim Beamformer
J = 4
μ = 0.0001
Output Power
Iterations
x 10^5
0
5
10
15
20
25
30
35
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

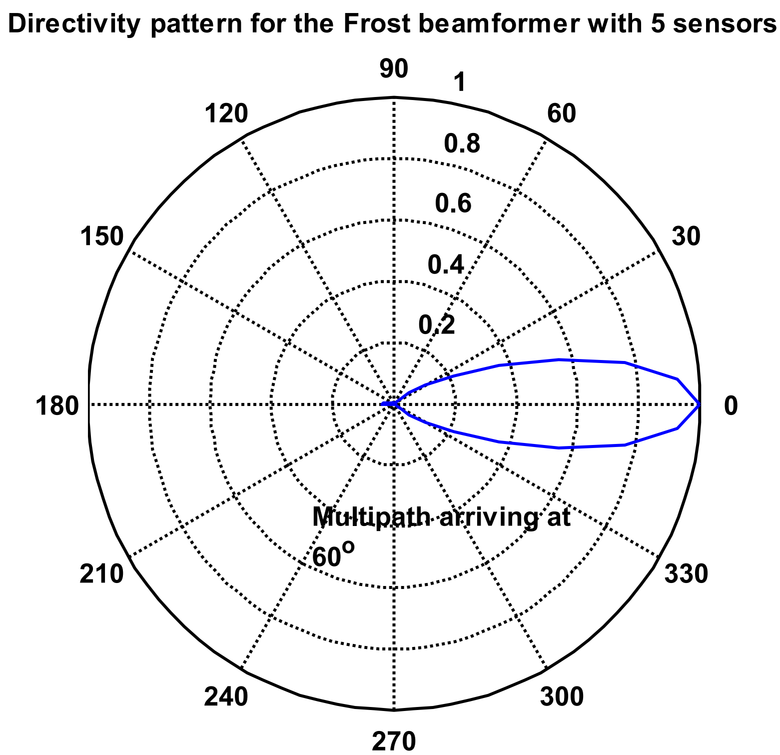

Null at 45°
Directivity pattern for the Griffiths-Jim beamformer
Null at 100°
Directivity pattern for the Griffiths-Jim beamformer

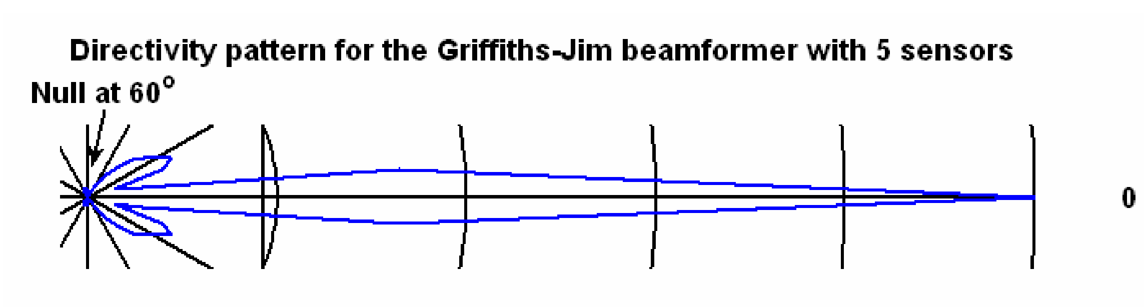
IV CONCLUSION

DOA of multi path	Frost	Null rejection	Griffiths-Jim	Null rejection
90°		-24.09dB		-26.86 dB
140°		-25.52dB		-26.96dB

By comparing the above rejection ratios, we can conclude that Griffiths-Jim gives an improvement in interference rejection by at least 3dB. This can be attributed to the nature of Frost beamformer, in which the look direction signal and the interference are assumed to uncorrelated, which is not so in our case. Griffiths-Jim, on the other hand, does not make this assumption. The blocking matrix before the LMS filters will pass only the interference (acting as a high pass filter).

Both the beamformers were iterated with 5 sensor array and the radiation pattern was obtained and is shown below. We can observe that no considerable improvement was observed with increased number of sensors. However, when more nulls are needed in the radiation pattern then the beamformer should have more degrees of freedom. We know that both the beamformers have exactly *(KJ-J)* degrees of freedom. So increasing sensor elements will only be useful if we want more nulls.





REFERENCES

1. O.L.Frost, III, “An algorithm for linearly-constrained adaptive array processing”, Proc. IEEE, vol. 60, no. 8, pp. 926-935, Aug 1972
2. L.J.Griffiths, C.W.Jim, “An alternative approach to linearly-constrained adaptive beamforming”, IEEE Trans. Antennas Propogat., vol. AP-30, no. 1, pp. 27-34, Jan 1982
3. B.Widrow, S.D.Stearns, Adaptive Signal Processing, Singapore: Pearson Education Asia, 2002
4. J.D.Kraus *et al*, Antennas for all Applications, New Delhi: Tata McGraw-Hill Publishing Company Ltd., 2003