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Seminar Report

on

Static and Dispersion Analysis of Strip-like Structures

Submitted by

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CONTENTS

Static and Dispersion Analysis of Strip-like Structures

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Abstract— **Different models for the static and dispersion analysis of strip-like structures will be studied. We will be studying, in particular, Tri-plate stripline, Micro stripline, Coupled micro stripline, Inverted and suspended micro striplines, Slot lines, and Coplanar waveguide. We delve into the analysis of the losses involved in some of these transmission structures. We will also look at the design equations for these transmission structures**

I. INTRODUCTION

Millimeter waves are extensively being used for Radar and Wireless communication. Wireless communication includes wireless computer networks, voice and data networks etc., Transmission lines forms an integral part of these Microwave Integrated Circuits (MICs) and Monolithic Microwave Integrated Circuits (MMICs). One major constraint on the transmission lines for their use in MICs is that they have to be *planar*. Microstrip lines, slot lines and coplanar structures are used as the fundamental blocks in building these circuits. All these structures are planar, their characteristics being controlled by their dimensions in that plane.

Over the years various static and dispersion models have been developed for the analysis of striplines. Various models for the analysis of these structures has been consolidated here. Different methods adopted for Static and dispersion analysis of microstrips are shown in Fig 1 (next page). We shall begin with a brief introduction on the classic methods employed for the static and dispersion analysis of the various strip line configurations. We shall then proceed with the static and dispersion analysis of the individual striplines mentioned above. We shall conclude with a section on the various design equations that can be used during the stripline design process.

II. STATIC ANALYSIS

Static analysis of striplines involves the analysis of the transmission structure at frequency, $f = 0$. The analysis is carried out to find the vital parameters of the transmission lines viz., Characterisitc impedance(Z_0), Effective dielectric permittivity (ϵ_{eff}), and the phase velocity (v_p). These three parameters are related to the Capacitance (C) of the structure [2]. We know that the wave propagates through a medium with velocity $v = 1/\sqrt{\mu\epsilon}$. If the medium is not free space but a uniform dielectric, then the velocity of propagation is given by (1).

$$v = v_0/\sqrt{\epsilon_r} \tag{1}$$

where $v_0 = 1/\sqrt{\mu_0\epsilon_0}$ is the free-space velocity and ϵ_r is the relative permit

$$Z = \frac{V_0}{I_0} = \frac{V_0}{Qv} = \frac{1}{Cv} \tag{2}$$

where C is the capacitance between the conductors per unit length. The electrostatic capacitance used in (43) is independant of the operating frequency and depends only on the static field configuration in the transmission line [3]. If the substrate is thin in terms of wavelength and the strip width is also narrow compared to the wavelength, and high dielectric substrates are used, then static analysis itslef can be enough [4], [5].Substituting the dielectric by air ($\epsilon_r = 1$) in (43), we have

$$Z_a = \frac{1}{C_a v_a} \tag{3}$$

where C_a is the capacitance per unit length of the transmission line with dielectric replaced by air. Dividing (3) by (43) and putting $\epsilon_r = C/C_a$ we get

$$Z = \frac{Z_a}{\sqrt{\epsilon_r}} \tag{4}$$

[2]Open transmission lines like Microstrip lines are examples of mixed dielectric problem and as such cant support TEM waves. To aid in the analysis of these structures, a new quantity called *'effective dielectric constant'*is defined under *quasi-static* approximation. This quantity is defined as

$$\epsilon_{eff} = \frac{C}{C_a} \tag{5}$$

The expressions for the phase velocity and characteristic impedance Z then follows

$$v = v_a/\sqrt{\epsilon_{eff}} \tag{6}$$

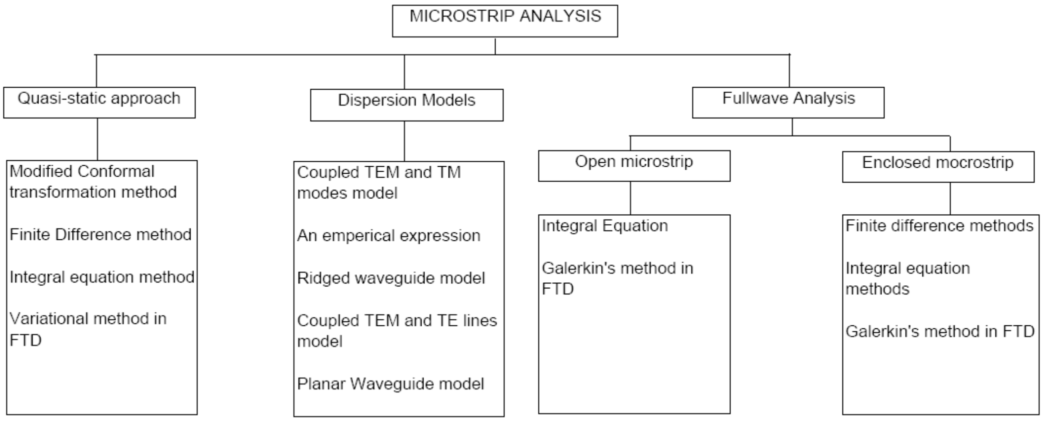

Fig. 1. Various techniques for Microstrip Analysis [1]

$$Z = Z_a/\sqrt{\epsilon_{eff}} \tag{7}$$

Propagation constants calculated using (5) - (7) give results accurate enough for most of the practical cases [2].

The electrostatic capacitance is found by the solution of a two dimensional Laplace Equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{8}$$

The solution of (8) would then give us the field within the structure. The total charge can be found out by using Gauss law

$$Q = \epsilon_0\epsilon_r \int\int E \cdot \mathbf{d}S \tag{9}$$

The integration in (9) being carried out over the entire surface of the transmission structure.

The determination of the characteristic impedance Z_0 proceeds with first finding the capacitance C of the transmission line and then using that in (5) to find the effective dielectric constant and then putting it into (6) and (7) to find out phase velocity and characteristic impedance respectively.

We will now discuss the theoretical aspects of the static analysis techniques namely, The Conformal mapping method, The Variational method, and The Finite Difference method.

A. The Conformal Mapping method

[6]A mapping $w = f(z)$ defined on a domain D is called conformal at $z = z_0$ if the angle between any two curves in D intersecting at z_0 is preserved by f. Such a mapping is known as *Conformal mapping (a.k.a Angle-preserving mapping)*. If $f(z)$ is analytic in domain D and $f^{'}(z_0) \neq 0$, then f is conformal at $z = z_0$. The criterion for analyticity is that: if $u(x,y)$ and $v(x,y)$ satisfy the Cauchy-Reimann equations, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ at all points in domain D, then the function $f(z) = u(x,y) + jv(x,y)$ is analytic everywhere in D provided $u(x,y)$ and $v(x,y)$ are continious and has first-order partial derivatives.

The reason we can apply conformal mapping to solve Laplace equations follow directly from a result in Complex analysis, which states that: If f is an analytic function that maps from a domain D to $D^{'}$ and if W is harmonic in $D^{'}$, then the real valued function $w(x,y) = W(f(z)$ is harmonic in in D [6].

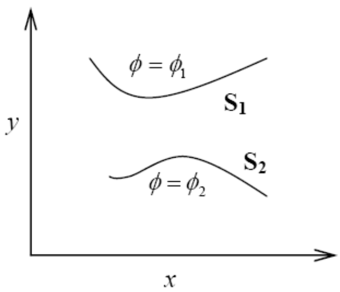

Fig. 2. A system of curves

[3], [2] Consider the solution of a two dimensional laplace equation $\nabla^2\phi = 0$ for the system of conductors shown in Fig 2 with the boundary conditions as $\phi = \phi_1$ on S_1 and $\phi = \phi_2$ on S_2. The principle behind conformal mapping approach is to transform the given system of conductors to a different complex plane where it may be easier to solve the given laplace equation. This technique gives an upper bound on Z_0. Let us consider a conformal transformation W given by (10)

$$W = F(z) = F(x + jy) = u + jv \tag{10}$$

where,

$$u = u(x, y)$$

and

$$v = v(x, y)$$

Assuming a Transverse Electromagnetic Mode (TEM), we shall define the gradient operator as ∇_t

$$\nabla_t u = \frac{\partial u}{\partial x}\frac{\mathbf{a}_x}{h_1} + \frac{\partial u}{\partial y}\frac{\mathbf{a}_y}{h_2} \tag{11}$$

and

$$\nabla_t v = \frac{\partial v}{\partial x}\frac{\mathbf{a}_x}{h_1} + \frac{\partial v}{\partial y}\frac{\mathbf{a}_y}{h_2} \tag{12}$$

The scale factors h_1 and h_2 are given by

$$\frac{1}{h_1^2} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

$$\frac{1}{h_2^2} = \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2$$

$$\frac{1}{h_1^2} = \frac{1}{h_2^2} = \frac{1}{h^2} = \left|\frac{dW}{dz}\right|^2$$

where the last step directly follows from the Cauchy-Reimann equations. Laplace equations in uv plane are given by

$$\frac{\partial}{\partial u}\frac{h_2}{h_1}\frac{\partial \phi}{\partial u} + \frac{\partial}{\partial v}\frac{h_1}{h_2}\frac{\partial \phi}{\partial v} = \frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2}{\partial v^2} = 0 \tag{13}$$

The above result shows that the potential function ϕ satisfies the same Laplace equations in uv co-ordinate systems. The same result has been shown in a slightly different way in [7].

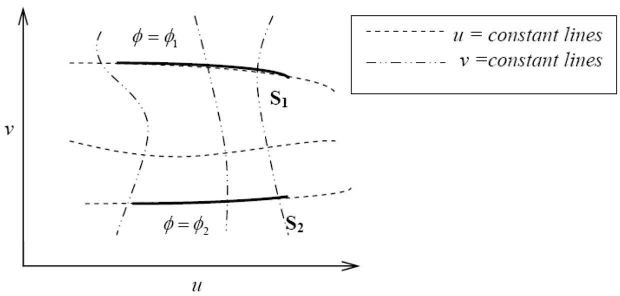

Fig. 3. The system of curves after conformal mapping

If the curves S_1 and S_2 can be represented in terms of constant co-ordinate curves as shown in Fig 3, then the solution of Laplace equation will be simpler than the original Laplace equation.

The energy stored in the electrostatic field is given by

$$\begin{aligned}
W_e &= \frac{1}{2\epsilon}\int\int\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right] dxdy \\
&= \frac{1}{2\epsilon}\int\int |\partial_t \phi|^2\, dxdy \\
&= \frac{1}{2\epsilon}\int\int\left[\frac{1}{h_1^2}\left(\frac{\partial \phi}{\partial u}\right)^2 + \frac{1}{h_2^2}\left(\frac{\partial \phi}{\partial v}\right)^2\right] h_1 dx h_2 dy \\
&= \frac{1}{2\epsilon}\int\int\left[\left(\frac{\partial \phi}{\partial u}\right)^2 + \left(\frac{\partial \phi}{\partial v}\right)^2\right] dudv \\
&= 1/2C(\phi_2 - \phi_1)^2
\end{aligned} \tag{14}$$

The last equation shows that capacitance C is same in the conformal mapped domain.

Situations may sometimes arise where a particular type of a stripline can be considered as a polygon. This polygon can then be conformal mapped to the upper half of a plane by the use of *Schwarz-Christoffel Transformation*. The transformation is formulated as [6]: *Let $f(z)$ be a function that is analytic in the plane $y > 0$ having the derivative*

$$f^{'}(z) = A(z-x_1)^{\alpha_1/\pi-1}(z-x_2)^{\alpha_2/\pi-1}\cdots(z-x_n)^{\alpha_n/\pi-1} \tag{15}$$

where $x_1 < x_2 < \cdots < x_n$ and each α_i satisfies $0 < \alpha_i < 2\pi$. Then f(z) maps the upper half of the plane $y > 0$ to a polygon with its interior angles $\alpha_1, \alpha_2, \cdots, \alpha_n$ Successive applications of this transformations would sometimes be needed to end up with the desired configuration. Detailed explanation on Schwarz-Christoffel Transformation and the method to obtain the functions for such a mapping is outlined in [6].The steps followed while using the conformal mapping approach shall be outlined during the analysis of Micro stripline.

B. Variational approach

The principle is based on finding the maxima or the minima of an integral involving a *functional*[1].The aim is to determine the stationary condition of a functional. This is usually defined in terms of a differential equation with the boundary conditions on the required function [8].Consider a problem of finding a function $y(x)$ such that the function

$$I(y) = \int_a^b F(x, y, y^{'})dx$$

[1]functional is defined as function of functions.Examples include distance between two points, which is a function of the path taken, which again is another function. Other example is the inner product of two vectors $\langle u, v\rangle$

subject to the boundary conditions $y(a) = A$ and $y(b) = B$ is rendered stationary.$F(x, y, y')$ is the given function of x, y, y' and $I(y)$ is the *functional* or *variational principle*.The necessary condition for $I(y)$ to have an extremum is

$$\delta I = 0$$

It can be shown that the necessary condition for a function $y(x)$ to yield an extremum for $I(y)$ is that $y(x)$ should satisfy the Euler-Lagrange equation [8].

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)$$

The solution of the problem involves the construction of a functional from a given Parial Differential Equation (P.D.E) and the extremising the resulting variational principle.

Name of Equation	Partial Differential Equation (PDE)	Variational Principle
Inhomogenous wave Equation	$\nabla^2\phi + k^2\phi = g$	$I(\phi) = \frac{1}{2}\int_v \left[\lvert\nabla\phi\rvert^2 - k^2\phi^2 + 2g\phi\right]dv$
Homogenous wave equation	$\nabla^2\phi + k^2\phi = 0$	$I(\phi) = \frac{1}{2}\int_v \left[\lvert\nabla\phi\rvert^2 - k^2\phi^2\right]dv$
	or	
	$\nabla^2\phi - \frac{1}{u^2}\phi_{tt} = 0$	$I(\phi) = \frac{1}{2}\int^{t_0}\int_v \left[\lvert\nabla\phi\rvert^2 - \frac{1}{u^2}\phi_t^2\right]dvdt$
Diffusion equation	$\nabla^2\phi - k\phi_t = 0$	$I(\phi) = \frac{1}{2}\int^{t_0}\int_v \left[\lvert\nabla\phi\rvert^2 - k\phi\phi_t\right]dvdt$
Poisson's equation	$\nabla^2\phi = g$	$I(\phi) = \frac{1}{2}\int_v \left[\lvert\nabla\phi\rvert^2 + 2g\phi\right]dv$
Laplace's equation	$\nabla^2\phi = 0$	$I(\phi) = \frac{1}{2}\int_v \left[\lvert\nabla\phi\rvert^2\right]dv$

Table 1: Variational principles associated with Common P.D.Es [8]

Table 1 gives the variational principle associated with some of the P.D.Es commonly encountered in Electromagnetic waves.There are several methods of solving the Variational principle which are

1) Rayleigh-Ritz Method
2) The Weighted Residual Method
3) Collocation Method
4) Subdomain Method
5) Galerkin Method
6) Least Squares Method

This principle is used for systems whose function attains either of the extremes. This is true for electrostatic systems where the energy function attains a minimum value stated as Thomson's theorem, which says "Charges residing on conducting surfaces giving rise to an electric field $\mathbf{E}$ will distribute themselves such that the energy function is minimized".

Noting that the electric field $\mathbf{E}$ is the gradiant of ϕ, we can write the electrostatic energy stored as [2]

$$W_e = \frac{\epsilon}{2}\int\int \nabla\phi\cdot\nabla\phi dV \tag{16}$$

We can readily recognize that W_e in (16) is in fact a variational principle. We can write the change in W_e, δW_e for a small change in ϕ to $\phi + \delta\phi$ as

$$\delta W_e = \frac{\epsilon}{2}\left[\int_{vol}\nabla(\phi+\delta\phi)\cdot\nabla(\phi+\delta\phi)dV - \int_{vol}\nabla\phi\cdot\nabla\phi dV\right] \tag{17}$$

By using the linearity property of ∇ operator, (17) can be re-written as

$$\delta W_e = \frac{\epsilon}{2}\left[\int_{vol}2\nabla\phi\cdot\nabla\delta\phi dV + \int_{vol}\nabla\delta\phi\cdot\nabla\delta\phi dV\right] \tag{18}$$

Using Green's first identity[2], we can write the first integral as

$$2\int_{vol}\nabla\phi\cdot\nabla\delta\phi dV = 2\left(\int_{surf}\delta\phi\nabla\phi dS - \int_{vol}\delta\phi\nabla^2\phi dV\right)$$

Using Gauss divergence theorem for the second integral in the above equation, we can show that

$$2\int_{vol}\nabla\phi\cdot\nabla\delta\phi dV = 0$$

So (18) can be written as

$$\delta W_e = \frac{\epsilon}{2}\int_{vol}\nabla\delta\phi\cdot\nabla\delta\phi dV \tag{19}$$

The important point to note here is that change in W_e is proportional to $(\delta\phi^2)$. A first order change in ϕ will lead to a second order change in W_e.

Suppose we have a two conductor transmission line S_1, S_2 with one being held at a potential V_0 and the other at the ground potential, the energy stored is

$$W_e = \frac{\epsilon}{2}\int\int\left|\partial_t\phi\right|^2 dxdy = \frac{CV_0^2}{2}$$

The above integral being over the transverse plane. The capacitance per unit length C can then be written as

$$C = \frac{\epsilon}{V_0^2}\int\int\left|\partial_t\phi\right|^2 dxdy = \frac{\int\int\left|\partial_t\phi\right|^2 dxdy}{\left(\int_{S_1}^{S_2}\nabla_t\phi\cdot dl\right)^2} \tag{20}$$

We can see that (20) is a variational principle. The value of C is determined by employing a trial function in several variables. If these variables are $\beta_i, i = 1, 2, \cdots, N$, then the capacitance will then be a function of all

[2] $\int_{vol}\left(U\nabla^2 V + \nabla U\cdot\nabla V\right)dV = \oint U\nabla V\cdot dS$

these βs so that $C = C(\beta_1, \beta_2, \cdots, \beta_N)$. Extremizing the variational principle then involves solving the N simultaneous equations $\partial C/\partial \beta_i, i = 1, 2, \cdots, N$. The right choice of a trial function will lead to the extremum for the capacitance C, but any other trial function will give a value of C, which is greater than the actual capacitance. Hence this approach will always yield an upper bound on the capacitance. The Variational can also be solved in a different way by transforming the given Poisson's or Laplace equation to the Fourier domain [9], [10].

Let us consider a system of conductors as shown in Fig 4. The two conductors S_1 and S_2 are infinetly long

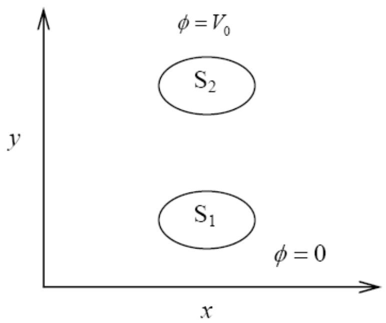

Fig. 4. A system of two conductors comprising the problem

conductors along the $z-direction$. The potential on the two surfaces are as shown in Fig 5. For the conductor S_2 to be at potential V_0 there should be a charge distribution ρ on its surface. The magnitude of the charge is of course given by $-\epsilon E_n = -\epsilon \partial\phi/\partial n$ where E_n is the normal component of the electric field. So the system should obey the Poisson's equation subject to the boundary condition [2], [3]

$$\nabla_t \phi^2 = -\frac{\rho(x_0, y_0)}{\epsilon} \tag{21}$$

The method followed to solve such problems is to use a trial function in the form of a unit charge at the point (x_0, y_0) replacing the given conductor S_2. The unit charge is represented in terms of the Dirac-delta function. So (21) can be written as

$$\nabla_t \phi^2 = -\frac{\delta(x - x_0)\delta(y - y_0)}{\epsilon} \tag{22}$$

The solution for the potential ϕ can be written in terms of Green's function, which gives the potential at a point (x, y) for a unit charge at (x_0, y_0) as [3]

$$\phi(x, y) = \oint_{S_2} G(x, y|x_0, y_0)\rho(x_0, y_0)dl_0 \tag{23}$$

[3]The integral in (23) can be compared to the convolution integral in systems theory involving the system impulse response and the input function

where the integration is carried out over the contour of S_2. The integral should then be equal to V_0 on the surface S_2. So, (23) can be written as

$$V_0 = \oint_{S_2} G(x, y|x_0, y_0)\rho(x_0, y_0)dl_0 \tag{24}$$

considering the charge distribution $\rho(x, y)$ as the trial function, the integral is converted to functional

$$V_0 \oint_{S_2} \rho(x, y)dl = \oint_{S_2}\oint_{S_2} G(x, y|x_0, y_0)\rho(x_0, y_0)\rho(x, y)dl_0 dl \tag{25}$$

by recognising that $\oint_{S_2} \rho(x, y)dl$ gives the total charge Q. So,(25) can be written as

$$\frac{1}{C} = \frac{1}{Q^2}\oint_{S_2}\oint_{S_2} G(x, y|x_0, y_0)\rho(x_0, y_0)\rho(x, y)dl_0 dl \tag{26}$$

Using (23) we can write the final variational form for the capacitance C as

$$\frac{1}{C} = \frac{1}{Q^2}\oint_{S_2} \phi(x, y)\rho(x, y)dl \tag{27}$$

For any trial function $\rho(x, y)$ the variation principle always takes a value greater then its true extremum. So the solution for the variational principle (27) will give us a lower bound on capacitance C and hence an upper bound for Z_0. This method of solving for ϕ in terms of the integral involving the Green's function is sometimes referred to as the *Integral equation method* [1]. However the crux of this problem is to determine the form of the Green's function. A method to determine this for the microstrip lines has been outlined in [1].

C. Finite Difference Method (FDM)

[8], [11] As we now know that the fundamental problem in finding Z_0 for a given transmission line is the solution of either the Laplace equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{28}$$

or the Poisson's equation

$$\nabla^2\phi = -\rho/\epsilon$$

An approximate solution for these equations can be obtained by using FDM. Well known basic techniques being the Newton's forward, Newton's backward difference formula and the central difference formula. For numerical computations, We divide the whole region of analysis into small discrete regions (a.k.a *meshes*), each intersection of horizantal and vertical lines representing a *node* at which the value of the potential is determined.

The difference between the potential at a node A and the node P can be represented as a infinite series using Taylor's theorem

$$\phi_B - \phi_P = \Delta x \frac{\partial \phi}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 \phi}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3\ phi}{\partial x^3} + \cdots$$

and similarly between another node B and P

$$\phi_A - \phi_P = -\Delta x \frac{\partial \phi}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 \phi}{\partial x^2} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 \phi}{\partial x^3} + \cdots$$

Neglecting the higher order terms,

$$\frac{\partial^2 \phi}{\partial x^2} \approx \frac{\phi_A + \phi_B - 2\phi_P}{(\Delta x)^2} \tag{29}$$

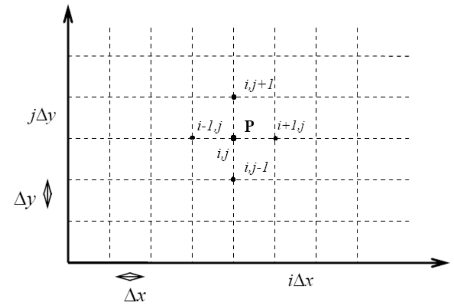

Fig. 5. The region of analysis being divided into meshes

(29) can be re-written using the notation shown in Fig 5

$$\frac{\partial^2 \phi}{\partial x^2} \approx \frac{\phi(i+1,j) + \phi(i-1,j) - 2\phi(i,j)}{(\Delta x)^2} \tag{30}$$

Using the same argument,

$$\frac{\partial^2 \phi}{\partial y^2} \approx \frac{\phi(i,j+1) + \phi(i,j-1) - 2\phi(i,j)}{(\Delta y)^2} \tag{31}$$

Taking $\Delta x = \Delta y = h$ and putting (30) and (31) in (28)

$$[\phi(i+1,j) + \phi(i-1,j) + \phi(i,j-1) + \phi(i,j+1)] \\ -4\phi(i,j) = 0 \tag{32}$$

or

$$\phi(i,j) = \frac{[\phi(i+1,j) + \phi(i-1,j) + \phi(i,j-1) + \phi(i,j+1)]}{4} \tag{33}$$

Looking at (33), we can infer that the potential ϕ at a point is being approximated in terms of the potentials at its four neighbouring points.The original Laplace equation has now been approximated by a set of linear equations. The bottleneck in FDM is the solution of the large set of simultaneous equations.

There are two popular approaches to solve the simulataneous equations in FDM.

1) **Band Matrix Method** : (33) is applied to all the free nodes in the solution region. The set of simulataneous equations are then formulated as a matrix equation

$$\mathbf{AX} = \mathbf{B}$$

where $\mathbf{A}$ is the sparse matrix representing the relationship between the nodal voltages, $\mathbf{X}$ is a coloumn vector containing the variables representing the unknown nodal voltages, and $\mathbf{B}$ being the Right hand side constants, which are obtained from the given boundary and initial conditions. The solution of the linear equation is then obtained by the use of Gauss-elimination method

$$\mathbf{X} = \mathbf{A^{-1}B} \tag{34}$$

2) **Iterative method**: One of the iterative method is the *successive over-relaxation method*[4]. Here we define the residual $R(i,j)$ at the node (i,j) denoting the error by which the value of $\phi(i,j)$ deviates from (33).

$$\begin{aligned} R(i,j) &= \phi(i+1,j) + \phi(i-1,j) + \phi(i,j-1) \\ &\quad +\phi(i,j+1) - 4\phi(i,j) \end{aligned} \tag{35}$$

The value of the Residual at the k^{th} iteration is then propagated to the next iteration using

$$\phi^{k+1}(i,j) = \phi^k(i,j) + \frac{\Omega}{4} R^k(i,j) \tag{36}$$

The method is convergent for values $0 < \Omega < 2$, and repaidly convergent for $1 < \Omega < 2$ [11]. The optimum value of Ω is usually found out on trial-and-error basis.

Any of these two methods can be used to determine the value of the potential ϕ. The value of ϕ will then be used to determine the value of the charge on the conductor.

$$Q = \epsilon_0 \epsilon_r \sum\sum \left(\frac{\partial \phi}{\partial n}\right) \tag{37}$$

The double summation in (37) covers the entire cross section of the transmission line. $\frac{\partial \phi}{\partial n}$ is approximated as

$$\frac{\partial \phi}{\partial n} = \frac{\phi(i+i,j) - \phi(i,j)}{\Delta x}$$

The value of the capacitance is then $C = Q/V_t$, where the V_t is the voltage applied between the plates of a stripline. The air Capacitance C_a can be found out by putting $\epsilon_r = 1$ in (37).

The accuracy of FDM relies on the fineness of the mesh. Finer the mesh better will be accuracy of the

[4] the others being the Jacobi and the Gauss-Siedel methods [8]

solution. The algorithm usually starts with a coarse mesh and then advances to a finer net [11]. Coupled lines usually need finer mesh as the field variations are significant at the edges of the strip [2].

D. Losses

Two kinds of losses are associated with striplines, Conductor and dielectric losses. The total loss of the stripline in question is then the combination of these two i.e., the total loss α is the sum of Dielectric loss (α_d) and the conductor loss (α_c)

$$\alpha = \alpha_c + \alpha_d$$

The loss is attributed to the finite conductivity of the strip and the ground planes and lossy dielectric as substrates.

The techniques used to analyze losses assume a $low-loss$ line, which is used to simplify the expressions for the propagation constants and Z_0. The expression for the propagation constant γ is given by [12]

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{38}$$

where $R, G, L, andC$ are the values of the distributed elements of the transmission line

$$\begin{aligned} \gamma &= \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)} \\ &= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}} \end{aligned} \tag{39}$$

using Taylor's expansion for $\sqrt{1+x}$ [5]we can write (38) as

$$\gamma \approx j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] \tag{40}$$

separating the real and imaginary parts we have,

$$\alpha \approx \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \tag{41}$$

and

$$\beta \approx \omega\sqrt{LC} \tag{42}$$

Characteristic impedance, Z_0 is approximated as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}} \tag{43}$$

[5]which is also a standard binomial expansion in the form of

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} \pm \frac{n(n-1)x^2}{2!} \pm \cdots$$

There are two main techniques used to analyze the two types of losses mentioned above

1) **Perturbation method [12]:** The technique avoids the determination of the transmission line parameters $L, C, R,$ and G. It uses the field equations to determine the losses assuming that there will be little changes between the field of a lossy line and that of a loss-less line, thus justifying its name. The method considers the power flow along a transmission line with attenuation α (in the absence of reflections), which is given by

$$P(z) = P_0 e^{-2\alpha z} \tag{44}$$

where P_0 is the power at $z = 0$ plane. The power loss per unit length can be represented as a derivative

$$P_l = \frac{-\partial P}{\partial z} = 2\alpha P_0 e^{-2\alpha z} = 2\alpha P(z) \tag{45}$$

Attenuation constant α is now defined using (44) and (45) as

$$\alpha = \frac{P_l(z)}{2P(z)} = \frac{P_l(z=0)}{2P_0} \tag{46}$$

The crux of the problem, as we can see, is the computation of the power (P_0) and the power loss (P_l). Due to the generality of this technique, this method is applicable for both conductor and dielectric losses. P_0 is in general given by the Poynting theorem

$$P_0 = \frac{1}{2} Re \int_s \mathbf{E} \times \mathbf{H}^* \mathbf{ds} \tag{47}$$

The Ohmic conductor loss can be written as

$$P_l = \frac{R_s}{2}\int_s |\mathbf{J}_s|^2 \mathbf{ds} = \frac{R_s}{2}\int_s |\mathbf{H}_t|^2 \mathbf{ds} \tag{48}$$

The Dielectric power loss is

$$P_l = \frac{\omega\epsilon''}{2}\int_V |\mathbf{E}|^2 \mathbf{dv} \tag{49}$$

where ϵ'' is imaginary part of the dielectric permittivity

$$\epsilon = \epsilon' + j\epsilon'' = \epsilon(1 + jtan\ \delta)$$

where $tan\ \delta$ is the loss tangent of the dielectric. Depending on the expression used for P_l in (46), the attenuation constant will refer to either the conductor loss or the dielectric loss.

2) **Wheeler's incremental inductance rule [12], [13] :**The rule gives the effective resistance due to the skin effect and begins with inductance calculations. The method holds good for all types

of metallic surfaces for which skin effect plays a significant role. The only constraint being that the thickness of the conductor should be great compared to the skin depth (at least twice). The incremental rule can be stated as: the effective resistance in any circuit is equal to the change of reactance due to the penetration of magnetic field into the metal surfaces as would be caused by the surface receeding to a depth of $\delta/2$. As we can see that the field cannot penetrate a perfect conductor and hence the losses in the conductor is due to their *non-infinite* conductivity. The power loss into a cross section S of a perfect conductor is

$$P_l = \frac{R_s}{2}\int_s |\mathbf{J}_s|^2 \mathbf{ds} = \frac{R_s}{2}\int_C |\mathbf{H}_t|^2 \mathbf{dl} W/m^2 \tag{50}$$

The above integral refers to the power loss per unit length, the contour integral being carried out across the two conductors. The inductance per unit length is given by

$$L = \frac{\mu}{|I|^2}\int_S |\mathbf{H}|^2 ds \tag{51}$$

The expression for L assumes a lossless conductor. However there will be penetration of the field and thus $\mathbf{H}$ will bo no longer 0 inside the surface. This will add an *incremental inductance* ΔL to L in (51). Knowing that the mean depth of current inside the conductor is $\delta/2$ we have

$$\Delta L = \frac{\mu_0 \delta}{2|I|^2}\int_C |\mathbf{H}_t|^2 ds \tag{52}$$

Putting (52) in (50), we have

$$P_l = \frac{|I|^2 \omega \Delta L}{2} W/m \tag{53}$$

Using (46) and proceeding further, we have the final expression for α as [12]

$$\alpha_c = \frac{R_s}{2Z_0 \eta}\frac{dZ_0}{dl} \tag{54}$$

where $\eta = \sqrt{\mu_0/\epsilon}$ is the intrinsic impedance of the dielectric. As can be observed from our argument, that this rule is applicable only for the evaluation of conductor losses.

III. Dispersion analysis

Microstrip lines due to presence of two different dielectric boundaries does not support a pure TEM wave. It is assumed that only the fundamental mode will propagate, but the propagation constant, γ, is a non-linear function of frequency. This non-linearity causes dispersion [14]. [15] Due to the presence of two different dielectrics, the fringing fields experience an inhomegenous dielectric leading to a discontinuity on the field. A parameter called *effective permittivity (ϵ_{eff})* is introduced, which is always lesser than the permittivity of the substrate as the fields exists both in air and the substrate. Due to the non-TEM nature of the fields, the effective permittivity is dependent on the frequency. This is due to the fact that more field lines will penetrate the substrate with increasing frequency thus increasing the effective permittivity [6].

Quasi-static method used for low-frequency analysis doesn't predict the frequency dependence of the micro stripline transmission parameters.The deviation is due to the fact that hybrid modes get excited as frequency increases. The Quasi-static analysis does not take the higher order modes into account. The number of higer order propogating modes increases with frequency [16]. ϵ_{eff} varies significantly over frequency thus leading to distortion in pulse shapes.A typical variation is shown in Fig 6

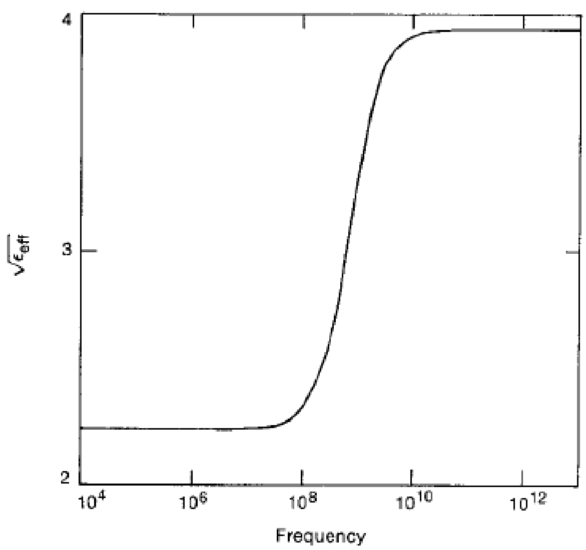

Fig. 6. Variation of ϵ_{eff} with frequency for a microstrip of $w = h = 1$ and $\epsilon_r = 80$ (water) [15]

The velocity of propagation varies with ϵ_{eff} as $v = c/\sqrt{\epsilon_{eff}(f)}$ where c is the velocity of light in vacuum. If a pulse is made of different spectral components, then each frequency component will travel with different velocities thus leading to the pulse shape distortion. Large variations in ϵ_{eff} are observed at wavelenghts comparable to the transverse dimensions of the micro stripline. The variation in ϵ_{eff} has a direct bearing on

[6]This can be understood by observing the Maxwell's equation in the absence of conduction current as $\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t}$

the Z_0 as well, making even Z_0 a frequency dependent parameter. As the frequency increases higher modes start to contribute significantly. The cutoff frequency above which the first longitudinal mode (TE) begins to contribute significantly is approximately given by [15]

$$f_{TE} = c/(4h\sqrt{\epsilon_r - 1}) \tag{55}$$

Operating the micro stripline above this frequency leads to what is known as *modal dispersion*. 55 shows that the effective range of quasi-static analysis reduces with increasing ϵ_r. This also means that the frequency range of operation decreases with increasing ϵ_r.

Various analytical, empirical and semi-empirical methods are employed to study the dispersion effects. Some of them are

1) Spectral domain immitance method
2) The integral equation method
3) Finite difference techniques
4) Modal analysis
5) Method of lines
6) Planar waveguide model

A. Spectral Domain Analysis

In this method, a set of algebraic functions is formulated relating the Fourier transform of the currents on the strip conductors to that of the fields at the dielectric interface in the plane of the conductor [2].

B. Method of Lines

[17], [18] For a Partial Differential Equation, the principle lies in the discretization of all but one of the indpenant variables to obtain a set of ordinary differential equations or difference equations. The procedure is also known as "semidiscretization by the Method of Lines". Consider the scalar potential $\phi^{(e)}$ and $\phi^{(h)}$ satisfying the Helmoltz' equation

$$\frac{\partial^2\phi^{(e,h)}}{\partial x^2} + \frac{\partial^2\phi^{(e,h)}}{\partial y^2} + (k^2 - \beta^2)\phi^{(e,h)} = 0 \tag{56}$$

We begin with discretizing the $x-axis$, which is done by drawing N parallel lines along $y-axis$. Let the spacing between the lines be same and be equal to h. Now due to the discretization, the potential ϕ is now divided into a set of N values $(\phi_1, \phi_2, \cdots, \phi_N)$ at the lines $x_i = x_0 + ih, i = 1, 2, \cdots, N$. The discretization then yields a set of N coupled ordinary differential equations

$$\frac{\partial^2\phi_i}{\partial y^2} + \frac{1}{h^2}\left[\phi_{i-1}(y) - 2\phi_i(y) + \phi_{i+1}(y)\right]$$
$$+(k^2 - \beta^2)\phi_i(y) = 0, i = 0, 1, \cdots, N \tag{57}$$

writing ϕ as

$$\vec{\phi} = [\phi_1(y), \phi_2(y), \cdots, \phi_N(y)]^t$$

and the matrix $\mathbf{P}$ as

$$\mathbf{P} = \begin{bmatrix} p_1 & -1 & \cdots & & \\ -1 & 2 & -1\cdots & & \\ \cdots & -1 & 2 & -1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \\ \cdots & \cdots & \cdots & -1 & p_2 \end{bmatrix}$$

with the constants p_1 and p_2 representing the boundary conditions. we have (57) as

$$h^2\frac{\partial^2\vec{\phi}}{\partial y^2} - [\mathbf{P} - h^2(k^2 - \beta^2)\mathbf{I}]\vec{\phi} = \mathbf{0} \tag{58}$$

where $\mathbf{I}$ is the identity matrix. The potential vector $\vec{\phi}$ is then transformed using the orthogonalizing vector $\mathbf{T}^t$ such that

$$\mathbf{T}^t\vec{\phi} = \vec{U}$$

Using this (57) will then be

$$h^2\frac{\partial^2\phi_i}{\partial y^2} - \left[\lambda_i - h^2(k^2 - \beta^2)U_i = 0, i = 0, 1, \cdots, N\right]$$

where λ_is are the eigen values of $\mathbf{P}$. The resulting uncoupled differential equations are then solved for ϕ.

Due to the presence of the edge signularities in the strip conductors, there is usually a discretization error associated with it. This can however be minimized by ensuring that the edge conditions are met, which states that the strip should exceed the last $\phi^{(h)}$ line by $\frac{3h}{4}$ and the last $\phi^{(e)}$ line by $\frac{h}{4}$ [17].These edge conditions are difficult to be met in the case of multiple conductor lines and strips with small strip dimensions. A modified method of lines has been suggested which transforms the given strip configuration to another dimension where the edge conditions can possibly be met. This is achieved by the use of some transfomration functions [19].

IV. New Techniques for Microwave CAD

The conventional approach used in Microwave CAD, as already outlined, is either an accurate but mathematically rigorous analytical model that might be diffult to obtain for new devices, a computationally intensive numerical techniques or a semi-emperical design equations. Semi-emperical relations have a definite range only over which the relationship actually models the device at hand.

One of the emerging trends in Microwave CAD, which can even be used to model and analyse the interconnects (the various strip transmission lines) is the Artificial Neural Network approach (ANN) [20]. The approach consists of a training phase where the ANN is trained with the data pertaining to the input-output relationship of the device. ANN has an edge over the conventional approaches, where the analytic approach can be tidious for newer devices and the emperical curve-fitting techniques has limited accuracy and input ranges. Once ANN is trained with accurate data about the device, then ANN acts as an accurate and speedy model for that device.The training data for the ANN is obtained by EM simulations. The reason ANN ca represent the RF device is due to the universal approxiamtion theorem, which states that *" a three leveled Multi Level Perceptron (MLP) network can simulate any arbitrary continious non-linear multidimensional function to any desired accuracy "* .

A technique call diakoptics has been suggested for millimeter-wave CAD [21]. Diakoptics (called domain-decomposition method) can be used to analyse the Quasi-static and frequency dependent charecteristics of a microwave structure. The analysis divides the transmission structure into different homogenous regions and these individual regions are then analysed using Boundary Element Method (BEM). The individual results are then combined to yield the final results. The analysis, due to its segmentation, can be helpful for local modification where only those segments have to be re-analysed where the modifications were done.

V. Static Analysis for Various Microstrip line Configurations

Various static analysis techniques employed for the determination of Z_0 for different micro striplines will be studied along with the closed form expressions for Z_0 and ϵ_{eff}.

A. *Tri-plate®*

It is also called as a Stripline or a Sandwich Line. Typical structure of a stripline is as shown in Fig 7 The technique used for analysing striplines is the Conformal technique. Conformal mapping is applied twice to one quarter of the stripline to map it into a form similar to that of a parallel plate capacitor. Schwarz-Christoffel transformation is used for conformal mapping. The final expression for the distance between the two plates of a capacitor is given by [2]

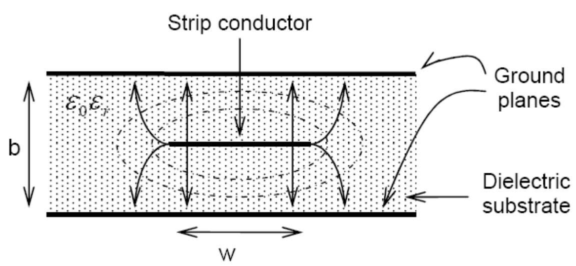

Fig. 7. Typical configuration of a stripine

$$d = \frac{K}{K'} = \frac{K(k)}{K(k')} \tag{59}$$

where K and K' are the complex elliptic integrals of the first kind given by

$$K(k) = \int_0^1 \frac{d\zeta}{[(1-\zeta^2)(1-k^2\zeta^2)]^{1/2}}$$

and

$$K(k') = \int_0^1 \frac{d\zeta}{[(1-\zeta^2)(1-\zeta^2+k^2\zeta^2)]^{1/2}}$$

k is given by

$$k = sech\left(\frac{\pi w}{2b}\right) \tag{60}$$

and

$$k' = \sqrt{1-k^2} = tanh\left(\frac{\pi w}{2b}\right) \tag{61}$$

finally noting that the capacitance $C = (4\epsilon_0\epsilon_r/d)$ we have the expression for Z_0 as

$$Z_0 = \frac{\sqrt{\mu_0\epsilon_0\epsilon_r}}{C} = \frac{29.976\pi}{\sqrt{\epsilon_r}} \cdot \frac{K(k)}{K(k')} \tag{62}$$

Approximations are used for $K(k)/K(k')$ to obtain closed-form expressions for Z_0 [22]

For $0.5 \leq k^2 \leq 1$

$$\frac{K(k)}{K(k')} = \frac{1}{\pi} ln2\left(\frac{1+\sqrt{k}}{1-\sqrt{k}}\right) = F_2(k) \tag{63}$$

For $0 \leq k^2 \leq 0.5$

$$\frac{K(k)}{K(k')} = \frac{1}{F_2(k')} \tag{64}$$

Closed form expressions are finally obtained by putting (63) and (64) in (62).

$$\alpha = \alpha_c + \alpha_d$$

α_d is given by [23]

$$\alpha_d = \frac{27.3\sqrt{\epsilon_r} tan\delta}{\lambda} \ dB/unit\ length$$

α_c can be determined by using Wheeler's incremental inductance rule. This has been done for various cases in [23] for a shielded stripline. Similar analysis can be carried out by noting that $\partial Z_0/\partial w$ used in [23] is 0 for the case of stripline.

B. Microstrip Lines

Microstrip are similar to a two conductor transmission line in principle. The field configuration is very similar to a two conductor line except for the presence of the fringing fields due to the inhomogeneity in the dielectric. Fig 8 shows the field configuration in a Microstrip line. Due to the inhomogeneity in the dielectric, Microstrips cannot support pure TEM waves. Consider a microstrip as shown in Fig 8 aligned so that y-axis is normal to the surface of the strip and z-axis is the axis of propagation.Applying continuity condition across the dielectric boundary, we have

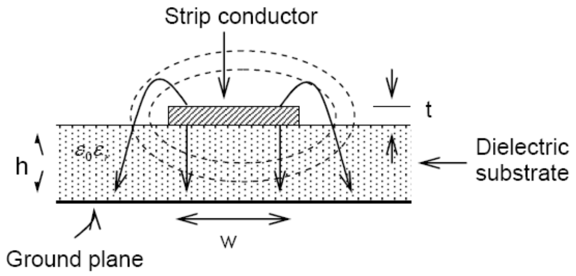

Fig. 8. Field lines in a Micro stripline

$$E_{x_{diel}} = E_{x_{air}} \tag{65}$$

Applying the continuity condition for **H** we have

$$(\nabla \times H)_{x_{diel}} = \epsilon_r (\nabla \times H)_{x_{air}} \tag{66}$$

Expanding the curl operation and taking only its i component, we have

$$(\epsilon_r \partial \mathbf{H}_z/\partial y)_{air} - (\partial \mathbf{H}_z/\partial y)_{diel} = (\epsilon_r - 1)\partial \mathbf{H}_y/\partial z \tag{67}$$

Since $\epsilon_r \neq 1$, (67) means that H_z is non-zero making it non TEM. Quasi-static analysis assumes that the deviation from the pure TEM behavior is very small. Conformal mapping is applied for the microstrip line configuration to map it into a parallel plate configuration. For the evaluation of C, the concept of effective dielectric is introduced. The transformation used for mapping from $z - plane$ to $z' - plane$ for $(W/h > 2)$ is [1]

$$z = j\pi + dtanh(z'/2) - z' \tag{68}$$

A filling fraction q is used to define the effective dielectric constant

$$\epsilon_{eff} = (1 - q) + q\epsilon_r \tag{69}$$

Different expressions for ϵ_{eff} is used for wide strips($W/h > 2$) and narrow strips($W/h < 2$)

$$\begin{aligned} q &= 1 - \frac{1}{d} ln\frac{d+c}{d-c} + \frac{0.732}{d\epsilon_r}\left[ln\frac{d+c}{d-c}\right] \\ &\quad - \frac{0.732}{d\epsilon_r}\left[cosh^{-1}(0.358d + 0.595)\right] \\ &\quad + \frac{\epsilon_r - 1}{d\epsilon}\left[0.386 - \frac{1}{2(d-1)}\right] \end{aligned} \tag{70}$$

where $d = 1 + \sqrt{1 + c^2}$ where c is the solution for the equation

$$\frac{\pi W}{2h} = c - sinh^{-1}c$$

For narrow strips

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{ln\frac{\pi}{2} + \frac{1}{\epsilon_r}ln\frac{4}{\pi}}{ln\frac{8h}{W}} \tag{71}$$

The final design equations are presented below [1].

$$Z_0 = \frac{\eta}{2\pi\sqrt{\epsilon_{eff}}} ln\left(\frac{8h}{W} + 0.25\frac{W}{h}\right) \quad (W/h \leq 1) \tag{72}$$

For $(W/h \geq 1)$

$$Z_0 = \frac{\eta}{\sqrt{\epsilon_{eff}}}\left[\frac{W}{h} + 1.393 + 0.667ln\left(\frac{W}{h} + 1.444\right)\right]^{-1} \tag{73}$$

where $\eta = 120\pi \ \Omega$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}F(W/h)$$

where

$$F(W/h) = \begin{cases} (1 + 12\frac{h}{W})^{-1/2} + 0.04(1 - \frac{W}{h})^2 & (W/h \leq 1) \\ (1 + 12\frac{h}{W})^{-1/2} & (W/h \geq 1) \end{cases}$$

The synthesis equations are

for $W/h \leq 2$

$$W/h = \frac{8e^A}{e^{2A} - 2}$$

for $W/h \geq 2$

$$\begin{aligned} \frac{W}{h} &= \frac{2}{\pi}\left[B - 1 - ln(2B - 1)\right] \\ &\quad + \frac{2}{\pi}\left[\frac{\epsilon_r - 1}{2\epsilon_r}\left(ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r}\right)\right] \end{aligned} \tag{74}$$

where

$$A = \frac{Z_o}{60}\left(\frac{\epsilon_r + 1}{2}\right)^{1/2} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{60\pi^2}{Z_0\sqrt{\epsilon_r}}$$

Finite Difference method with relaxation technique has also been employed for the analysis of Microstrip lines [1], [11], [5]. Microstrips have also been analysed using the integral equation approach, where the potential ϕ is formulated as an integral using the Green's function as

$$\phi(x,y) = \int G(x,y|x_0,y_0)\rho(x_0,y_0)dx_0 \qquad (75)$$

The problem proceeds with the formulation of the Green's function and then solving (75) in the form of a matrix equation

$$\mathbf{V} = \mathbf{P}\cdot\mathbf{Q}$$

where $\mathbf{V}$ and $\mathbf{Q}$ represent potential and charge respectively. The solution involves the inversion of $\mathbf{P}$. The capacitance C is then given by

$$C = Q/v = \sum_j \sum_k (p^{-1})_{jk}$$

where p^{-1} are the terms of $\mathbf{P}^{-1}$. The corresponding Green's function is given by [1]

$$\begin{aligned} G(h,x) &= \frac{1}{2\pi(\epsilon_r+1)\epsilon_0}\sum_{n=1}^{\infty} K^{n-1} \\ & ln\left\{ \frac{\left[4n^2+\left(\frac{x-x_0}{h}\right)^2\right]}{\left[4(n-1)^2+\left(\frac{x-x_0}{h}\right)^2\right]}\times \right. \\ & \left. \frac{\left[4n^2+\left(\frac{x+x_0}{h}\right)^2\right]}{\left[4(n-1)^2+\left(\frac{x+x_0}{h}\right)^2\right]}\right\} \qquad (76) \end{aligned}$$

Variational method in Fourier transfer domain has also been employed for Microstrip lines [10]. A similar approach has been used by formulating Green's function as a sum of sinusoids [24].

A Spectral Domain Approach(SDA) is also used to solve the Laplace equation using the Galerkin method. The technique outlined in [16] is quiet similar to the one in [10]. The closed form expressions are then obtained by using the curve fitting techniques to match the results obtained by the SDA [16] valid for $1 \le \epsilon_r \le 500$.

$$\epsilon_{eff} = \begin{cases} Au^3+Bu^2+Cu+D & if \quad 0.05 \le u \le 2 \\ Eln(u)+F & if \quad 2 \le u \le 10 \end{cases}$$

where

$$A = 0.008149\epsilon_r - 0.003118$$

$$B = -0.0397\epsilon_r + 0.01726$$

$$C = 0.113\epsilon_r - 0.0814$$

$$D = 0.546\epsilon_r + 0.4764$$

$$E = 0.0968\epsilon_r - 0.10851$$

$$F = 0.609\epsilon_r + 0.4366$$

Z_0 is given by [16]

$$Z_0 = \begin{cases} 57.839ln(1/u)+127.415 & 0.05 \le u \\ 136.6131-30.88u+3.343u^2-0.13384u^3 & 2 \le u \le \end{cases}$$

$u = (W/d)$.

The dielectric loss α_d is given by [25]

$$\alpha_d = 27.3\left(\frac{q\epsilon_r}{\epsilon_{eff}}\right)\frac{tan\delta}{\lambda_g}\ dB/cm$$

or

$$\alpha_d = 4.34\frac{q}{\sqrt{\epsilon_{eff}}}\sqrt{\frac{\mu_0}{\epsilon_0}}\sigma\ dB/cm$$

where q is the filling factor given by (70). The choice of expressions to be used depends on whether the substrate is non-conducting or not.The expressions for α_c has been derived by the use of Wheeler's incremental inductance rule as [25]

$$W/h \le 1/2\pi :$$

$$\begin{aligned} \frac{\alpha_c Z_0 h}{R_s} &= \frac{8.68}{2\pi}\left[1-\left(\frac{w'}{4h}\right)^2\right]\left\{1+\frac{h}{w'}+\frac{h}{\pi w'}\right. \\ & \left.\left[ln\left(\frac{4\pi w}{t}+1\right)-\frac{1-t/w}{1+t/4\pi w}\right]\right\} \end{aligned}$$

$$1/2\pi \le W/h \le 2 :$$

$$\begin{aligned} \frac{\alpha_c Z_0 h}{R_s} &= \frac{8.68}{2\pi}\left[1-\left(\frac{w'}{4h}\right)^2\right]\left\{1+\frac{h}{w'}+\frac{h}{\pi w'}\right. \\ & \left.\left[ln\left(\frac{2h}{t}+1\right)-\frac{1-t/h}{1+t/2h}\right]\right\} \end{aligned}$$

$$2 \le W/h :$$

$$\begin{aligned} \frac{\alpha_c Z_0 h}{R_s} &= \frac{8.68}{\left\{\frac{w'}{h}+\frac{2}{\pi}ln\left[2\pi e\left(\frac{w'}{2h}+0.94\right)\right]\right\}^2} \\ & \left[\frac{w'}{h}+\frac{w'/\pi h}{\frac{w'}{2h}+0.94}\right]\times \\ & \left\{1+\frac{h}{w'}+\frac{h}{\pi w'}\left[ln\left(\frac{2h}{t}+1\right)-\frac{1-t/h}{1+t/2h}\right]\right\} \end{aligned}$$

where

$$w' = w + \Delta w$$

$$\begin{aligned} \Delta w &= \frac{t}{\pi}ln\left(\frac{4\pi w}{t}+1\right);\ \ w/h \le 1/2\pi \\ & (2t/h < w/h, 1/2\pi) \\ &= \frac{t}{\pi}ln\left(\frac{2h}{t}+1\right);\ \ w/h \ge 1/2\pi \end{aligned}$$

C. Coupled Microstrip line

Coupled microstrip line consists of two microstrips placed side by side on a substrate as shown in Fig 9. Because of the two conductors present, two dominant modes exist called even and odd modes. An even mode exists when both the strips are exited with inphase signals. An Odd mode exists when strip excitations are opposite in phase. Z_0 and ϵ_{eff} will not be same for the two cases as there is field penetration even through air in the case of Odd mode excitation. Due to this, the two cases has to be separately discussed.

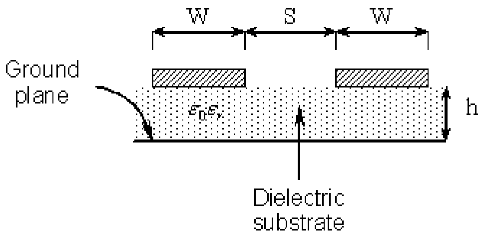

Fig. 9. A Coupled Microstrip line

Several techniques have been employed to analyse the coupled microstrip lines. Variational method in Space domain is used to represent the unknown potential functions in terms of Fourier coefficients [1]. The method proceeds with finding the fourier coefficients. These potential functions are then used to find the stored energy in a region

$$W = \frac{\epsilon}{2}\int\int(E_x^2 + E_y^2)dxdy$$

Capacitance C is then found out by

$$C = \frac{2}{V^2}W$$

Variational method in the Fourier transform domain involves the Fourier transformed variables for the variational principle involving C, written as [1]

$$\frac{\epsilon_0}{C_e} = \frac{1}{\pi}\int_0^\infty G(\beta)\tilde{\rho}_e(\beta)d\beta$$

and

$$\frac{\epsilon_0}{C_o} = \frac{1}{\pi}\int_0^\infty G(\beta)\tilde{\rho}_o(\beta)d\beta$$

where

$$G(\beta) = \frac{1}{\beta[\epsilon_r coth(\beta h) + coth(\beta h')]}$$

Analysis has also been carried out using the expressions for fringing capacitance of a single microstrip line for even mode fringing capacitances. Odd mode fringing capacitances are determined by using an equivalent geometry for coplanar strips and coupled lines. Fig 10 and Fig 11 shows the various fringing capacitances associated with odd and even modes of a coupled microstrip.This analysis has yielded closed form expressions for Z_0 and ϵ_{eff} thus facilitating in its design [26]

$$C_e = C_p + C_f + C'_f \tag{77}$$

$$C_o = C_p + C_f + C_{ga} + C_{gd} \tag{78}$$

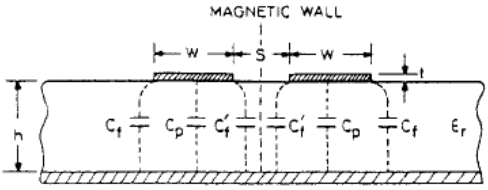

Fig. 10. Even mode fringing capacitances in a Coupled Microstrip line [26]

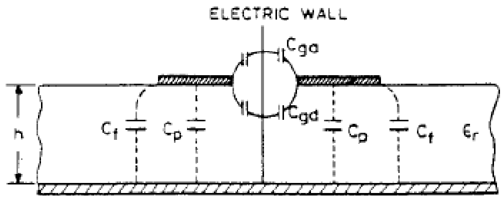

Fig. 11. Odd mode fringing capacitances in a Coupled Microstrip line [26]

C_p is the parallel plate capacitance given by $C_p = \epsilon_0\epsilon_r W/h$. C_f is the fringing capacitance of a single microstrip line with parameters Z_0 and ϵ_{eff} given by

$$2C_f = \sqrt{\epsilon_{eff}}/cZ_0 - C_p \tag{79}$$

where $c = 3\times10^8 m/s$ is the velocity of light in vacuum. An empirical relationship for C'_f is given by

$$C'_f = \frac{c_f}{1 + A(h/s)tanh(8S/h)} \tag{80}$$

where

$$A = exp[-0.1exp(2.33 - 2.53W/h)]$$

C_{ga} is the fringing field in odd mode across the air gap given by

$$C_{ga} = \epsilon_0\frac{K(k')}{K(k)} \tag{81}$$

where

$$k = \frac{S/h}{S/h + 2W/h}$$

(63) and (64) can be used for approximating $K(k)/K(k')$.

Expression for odd mode capacitance across the gap through the dielectric (C_{gd}) is given by

$$\begin{aligned} C_{gd} &= \frac{\epsilon_0 \epsilon_r}{\pi} ln \left\{ coth \left(\frac{\pi S}{4h} \right) \right\} \\ &+ 0.65 C_f \left\{ \frac{0.02}{S/h} \sqrt{\epsilon_r} + 1 - \epsilon_r^{-2} \right\} \quad (82) \end{aligned}$$

putting (80) - (82) in (77) and (78), C_e and C_o are obtained. Charecterisitic impedance is got by

$$Z_{0i} = c \left[\sqrt{C_i^a C_i} \right]^{-1}$$

and

$$\epsilon_r^i = C_i / C_i^a$$

where i refers to either even or odd mode and C_a is the capacitance with the dielectric replaced by air.

following are the width corrections due to thick strips [26]

$$\frac{W_t^e}{h} = \frac{W}{h} + \frac{\Delta W}{h} [1 - 0.5 exp(-0.69 \Delta W / \Delta t)]$$

$$\frac{W_t^o}{h} = \frac{W_t^e}{h} + \frac{\Delta t}{h}$$

where

$$\frac{\Delta t}{h} = \frac{1}{\epsilon_r} \frac{t/h}{S/h}$$

Ohmic conductor losses are given by [26]

$$\begin{aligned} \alpha_C^i &= \frac{8.686 R_s}{240 \pi Z_{0i}} \cdot \frac{2}{hc \left(C_i^{at} \right)^2} \times \\ & \left[\frac{\partial C_i^{at}}{\partial (W/h)} \left(1 + \delta \frac{W}{2h} \right) - \frac{\partial C_i^{at}}{\partial (S/h)} \left(1 + \delta \frac{S}{2h} \right) \right. \\ & \left. + \frac{\partial C_i^{at}}{\partial (t/h)} \left(1 + \delta \frac{t}{2h} \right) \right] \end{aligned}$$

where C_i^{at} represents odd or even mode line capacitances with air as dielectric and strips of finite thickness and

$$\delta = \begin{cases} 1, & for \ strips \ only \\ 2, & for \ strips \ and \ ground \ plane \end{cases}$$

Dielectric Losses are given by [26]

$$\alpha_d^i = 27.3 \frac{\epsilon_r}{\sqrt{\epsilon_{eff}^i}} \frac{\epsilon_{eff}^i - 1}{\epsilon_r - 1} \frac{tan\delta}{\lambda_0}$$

where $tan\delta$ is the loss tangent.

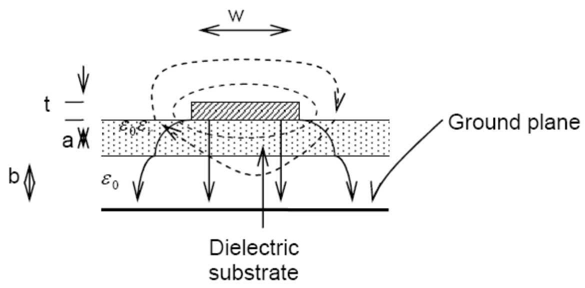

Fig. 12. Field lines in a suspended Microstrip line

D. Suspended Microstrip Lines

Suspended microstrip lines are inhomogeneous microstrip lines in which the substrate is separated by the ground plane by an air gap as shown in Fig. 12.

With the addition of air gap ϵ_{eff} is reduced thus increasing the bandwidth. Reduction of ϵ_{eff} also brings down the guide wavelength, thus allowing for wider strip dimensions and less stringent tolerances. Air gap also reduces the field present near the ground plane thus reducing the conductor losses [27].

An approximate CAD model has been derived in [28] for $\epsilon_r \leq 20$, $0.5 \leq W/b \leq 10$, and $0.05 \leq a/b \leq 1.5$

$$\epsilon_{eff} = (1 - f_1 f_2)^{-1}$$

where

$$f_1 = 1 - \frac{1}{\sqrt{\epsilon_r}}$$

and

$$f_2^{-1} = \sum_{i=0}^{3} c_i (w/b)^i$$

where c_is are given by

$$c_i = \sum_{j=0}^{3} d_{ij} \left(\frac{b}{a} \right)^j$$

expressions for d_{ij}s are given in [28]. After determining ϵ_{eff}, Z_0 is calculated as

$$Z_0 = Z_{air} / \sqrt{\epsilon_{eff}}$$

where Z_{air} is an identical air-filled suspended microstrip line.

$$Z_{air} = 60 ln \left[\frac{f(u)}{u} + \sqrt{1 + \frac{4}{u^2}} \right]$$

where

$$u = \frac{w/b}{1 + (a/b)}$$

and

$$f(u) = 6 + (2\pi - 6) exp \left[-(30.666/u)^{0.7528} \right]$$

A slightly different set of expressions are present in [29].

E. Inverted Microstrip lines

Inverted Microstrip Line is a twin structure of the suspended microstrip line studied earlier. A typical structure is as shown in Fig 13 A shielded variation of this is also used which will then be identical to a shielded suspended microstrip line.Due to presence of the air gap, greater part of the electromagnetic waves pass through the air gap thus making the effective dielectric constant very small. A small effective dielectric constant also increases the phase velocity in the microstrip line. Its low dispersion characteristics makes it useful in broadband applications. It is also used in electromagnetically coupled microstrip antenna [30].

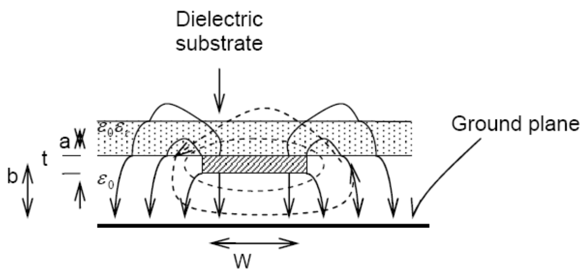

Fig. 13. Field lines in a Inverted Microstrip line

A technique similar to the one used for suspended lines are used. A curve fitting technique is used to match the theoretical data, which has been contained by variational principle in Fourier Transform Domain [28].

$$\epsilon_{eff} = 1 + f_1 f_2$$

where

$$f_1 = \sqrt{\epsilon_r} - 1$$

and

$$f_2^{-1} = \sum_{i=0}^{3} c_i (w/b)^i$$

where c_is are given by

$$c_i = \sum_{j=0}^{3} d_{ij} \left(\frac{b}{a}\right)^j \tag{83}$$

expressions for d_{ij}s are given in [28]. Z_0 is calculated as

$$Z_0 = Z_{air}/\sqrt{\epsilon_{eff}}$$

where Z_{air} is an identical air-filled suspended microstrip line.

$$Z_{air} = 60ln\left[\frac{f(u)}{u} + \sqrt{1 + \frac{4}{u^2}}\right]$$

where

$$u = w/b$$

and

$$f(u) = 6 + (2\pi - 6)exp\left[-(30.666/u)^{0.7528}\right]$$

F. Slot Lines

Slot line was first proposed by Cohn [31]. It consists of a conducting plane on a dielectric with a small gap in them as shown in Fig 14. To minimize radiation, high permittivity dielectrics are used. This makes the slot wavelength very small compared to the free space wavelength, thus confining the waves [31].The field configuration in a slot line is shown Fig 15

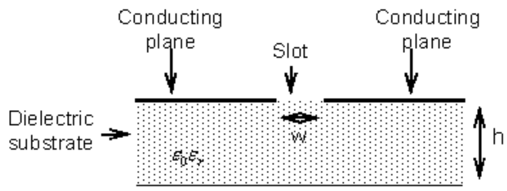

Fig. 14. Slot Line

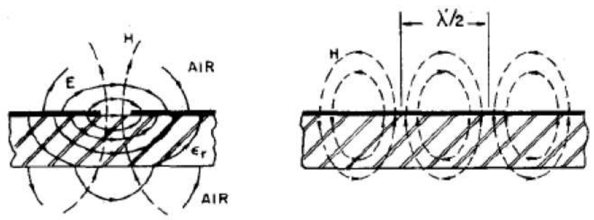

Fig. 15. E and H field in an slot line [32]

Various methods used for Slot lines are [1]

1) Approximate analysis
2) The Transverse resonance method
3) Galerkin's method in FTD
4) Analysis in elliptical coordinates

Approximate analysis has been used for a zeroth order value for ϵ_{eff} as [1]

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2}$$

and slot wavelength λ_s is given by

$$\frac{\lambda_s}{\lambda_0} = \sqrt{\frac{2}{\epsilon_r + 1}}$$

A transverse resonance method is used to determine Z_0 of the slot line in which the slot line is modeled as a waveguide with a capacitive iris. Slot line impedance is given by the power-voltage relationship

$$Z_0 = V^2/2P$$

where V is the peak voltage across the slot. The average power P is expressed in terms of energy stored W_t, which is related to the rate of change of total suseptance B_t with frequency

$$W_t = (V^2/4)(dB_t/d\omega)$$

but

$$W_t = \frac{2\pi P}{\omega}\frac{v}{v_g}$$

we have

$$Z_0 = (v/v_g)\pi/(\omega \partial B_t/\partial\omega)$$

Z_0 is finally given by

$$Z_0 = \eta\frac{v}{v_g}\frac{\pi}{p}\cdot\frac{\Delta p}{-\Delta(\eta B_t)}$$

A detailed method for the evaluation of Slot impedance is given in [1].Galerkin's method in Fourier Transfer Domain has also been applied [1].

G. Coplanar Waveguide

It consists of a strip at the center placed on a dielectric substrate with two ground planes placed on either side of it parallely as shown in Fig 16. A conformal mapping

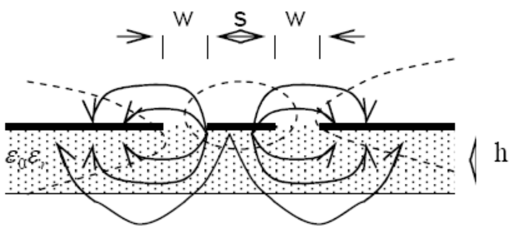

Fig. 16. Field lines in a Coplanar waveguide

approach has been used to calculate Z_0 under Quasi-static assumption.A closed form expression for Z_0 has been obtained through this method [1]

$$Z_0 = \frac{1}{Cv_{cp}} = \frac{30\pi}{\sqrt{\frac{\epsilon_r+1}{2}}}\frac{K(k')}{K(k)}$$

where

$$k = \frac{s}{s+2w}$$

and $k' = \sqrt{1-k^2}$. A Finite Difference Method Approach and a Fullwave analysis using the Galerkin's method has also been applied to the case of coplanar waveguides [1].

Though a detailed analysis of the losses were not done for all of the different class of microstrip lines, a qualitative idea can be obtained by fig 17

	MS	SS	SL	CPW	CBCPW	FL	
Effective Diel. Const.	M	L	L	M	M	L	M: Moderate
Dispersion	M	L	H	M	L	H	L: Low
Impedance	M	H	H	M	M	VH	H: High
Attenuation	H	L	H	H	H	M	VH: Very high
Radiation	L	L	M	M	L	L	

MS: Microstrip line
SS: Suspended Stripline
CPW: Coplanar Waveguide
CBCPW: Conductor backed Coplanar Waveguide
FL: Fin lines

Fig. 17. Comparison of various parameters among different types of microstrip lines [30]

VI. Dispersion Analysis for Various Microstrip line Configurations

The frequency dependence of the transmission parameters will be studied. Closed form expressions shall be provided wherever available for the dispersive effects on Z_0 and ϵ_{eff}

A. Microstrip lines

The Dispersion analysis is done in two ways. In the first category, an equivalent parallel plate model of the waveguide is used which is then used to analyse the frequency dependence. The following observations are made regarding the dispersive effects in the Microstrip [33]

1) The normalized phase velocity v_p/v is a monotonically decreasing function of frequency
2) The normalized phase velocity and its first derivative at $f = 0$ are given by

$$\tilde{v_p} = v_p/v|_{f=0} = \frac{1}{\sqrt{\epsilon_{eff}}}$$

and

$$\left.\frac{d\tilde{v_p}}{df}\right|_{f=0} = 0$$

3) The normalized phase velocity and its first derivative as $f \to \infty$ are given by

$$\tilde{v_p} = v_p/v|_{f\to\infty} = \frac{1}{\sqrt{\epsilon_r}}$$

and

$$\left.\frac{d\tilde{v_p}}{df}\right|_{f\to\infty} = 0$$

The second being the solution of field problem in spectral domain and then using the power-current definition for Z_0 .The Dispersion model is based on considering the microstrip as a Longitudinal-section Electric (LSE) [34].The procedure begins with modeling the given microstrip as an LSE with a modified structure to aid in analysis. The modified structure then is made to take the zero-frequency electrical parameters. A transverse resonance analysis of the model relates ϵ_{eff} to the frequency. A closed form expression for frequency dependence of ϵ_{eff} is then obtained as [34]

$$\epsilon_{eff}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{eff}(0)}{1 + G(f/fp)^2} \tag{84}$$

where

$$f_p = \frac{Z_0}{2\mu_0 b}$$

[34] has quoted that G approaches unity according to the experimental results, but the modified expressions are [1]

$$Z_0(f) = Z_{0T} - \frac{Z_{0T} - Z_0(0)}{1 + G(f/fp)^2} \tag{85}$$

where Z_{0T} is twice the characteristic impedance of a stripline of width W and height $2h$ and

$$G = \sqrt{\frac{Z_0 - 5}{60}} + 0.004Z_0$$

and

$$f_p(GHz) = 15.66Z_0/h$$

An alternative set of expressions have been derived based on the coupling of the surface wave and the LSE modes. The expressions are [35]

$$\epsilon_{eff}(f) = \epsilon_r - \frac{K_1(\epsilon_r - \epsilon_{eff}(0))}{1 + K_2(f/f_p)^2} \tag{86}$$

where

$$f_p = \frac{Z_0}{2\mu_0 b}$$

$$K_1 = \frac{\epsilon_r - \epsilon_{II}}{\epsilon_r - 1}$$

and

$$K_2 = \frac{\pi^2}{12}\frac{(\epsilon_r - \epsilon_{eff}(0))(\epsilon_{eff}(0) - 1)(\epsilon_r - \epsilon_{II})}{(\epsilon_r - 1)^2\epsilon_{eff}(0)}$$

and

$$\epsilon_{II} = \epsilon_r + (s_1 + s_2 - a_2/3)/k_0^2$$

$$s_1 = \sqrt[3]{\eta_2 + \sqrt{\eta_1^3 + \eta_2^2}}$$

$$s_1 = \sqrt[3]{\eta_2 - \sqrt{\eta_1^3 + \eta_2^2}}$$

and

$$\eta_1 = a_1/3 - a_2^2/9$$

$$\eta_2 = (a_1a_2 - 3a_0)/6 - a_2^3/27$$

a_i s are given by

$$a_2 = (2p + qp^2 - r)/p^2$$

$$a_1 = (2pq + 1)/p^2$$

$$a_0 = q/p^2$$

where

$$p = b/3$$

$$q = (\epsilon_r - 1)k_0^2$$

$$r = (b/\epsilon_r)^2$$

Another approach for dispersion analysis has been used in [36] using the so-called Logistic Dispersion Model(LDM) which makes use of the basic statement that *The rate of increase of effective dielectric constant with frequency* $\propto$ *[Effective relative permittivity at the given frequency]* $\times$ *[Remaining fractional relative permittivity of the substrate]*

B. Coupled Microstrip Lines

A semi-empirical dispersion model has been used for the modeling. The model consists of an equivalent coupled parallel-plate waveguide filled with the corresponding dielectric. This structure is then analysed in terms of the quasi-static line impedances and capacitances. The frequency dependence of these parameters are then assumed to be similar to that of a microstrip.

Expressions similar to Gentsinger's relations are give as [26]

$$\epsilon^i_{eff}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon^i_{eff}(0)}{1 + G(f/f_p)^2}$$

where

$$G = \begin{cases} 0.6 + 0.018Z_{0o} & odd\ \ mode \\ 0.6 + 0.045Z_{Oe} & even\ \ mode \end{cases}$$

and

$$f_p = \begin{cases} 31.32Z_{0o}/h & odd\ \ mode \\ 7.83Z_{Oe}/h & even\ \ mode \end{cases}$$

Similar equations holds for Z_{0i}, which are [26]

$$Z_{0i}(f) = Z_{Ti} - \frac{Z_{Ti} - Z_{0i}(o)}{1 + G(f/f_p)^2}$$

where Z_{Ti} is the impedance of a coupled stripline with gap s and width of the strip w and spacing between the ground planes $2h$.

$$Z_{Ti} = \frac{60\pi}{\sqrt{\epsilon_r}}\frac{K(k_i)}{K(k_i'}$$

where

$$k_i = \begin{cases} tanh\left(\frac{\pi W}{4h}\right)/tanh\left(\frac{\pi(W+s)}{4h}\right) & odd\ \ mode \\ tanh\left(\frac{\pi W}{4h}\right)tanh\left(\frac{\pi(W+s)}{4h}\right) & even\ \ mode \end{cases}$$

C. Suspended Microstrip lines

Suspended Microstrip lines are relatively low-dispersion lines, but the flowing observations hold [37]

1) For a given W/b and a/b, the effects of dispersion is more pronounced as ϵ_r increases
2) For a given ϵ_r and a/b, the effects of dispersion decreases with W/b
3) For a given ϵ_r and W/b, no simple relationship exists between the dispersion effects and a/b
4) dependence of the effects of dispersion on t is negligibly small

The following design equations are developed based on the modeling of the frequency dependence to match the Spectral-domain full-wave analysis [38]

$$\epsilon_{eff}(f) = \frac{\epsilon_{eff}(0) + K_\epsilon \epsilon_r}{1 + K_\epsilon}$$

where

$$K_\epsilon = \sum_{i=0}^{4} (c_i(\epsilon_r, a/b, w/b))(f/f_p)^i$$

$$f_p = \frac{Z_0}{2\mu_0(a+b)} \frac{a}{0.064}$$

c_i s are given by (83). Dispersive effects of Z_0 is given by [38]

$$Z_0(f) = \frac{120\pi(a+b)}{W_e(f)\sqrt{\epsilon_{eff}(f)}} \frac{a}{0.074}$$

where $W_e(f)$ is the width of the equivalent planar waveguide given by the solution of the equation

$$\sum_{i=0}^{4} F_i (W_e(f)/\lambda)^i = 0$$

where λ is the free-space wavelength in cm. All the dimensions in cm.

$$F_0 = d_0 W/\lambda$$

$$F_1 = -d_0 - 2\left(\frac{W_e(0)}{\lambda} - d_1 \frac{W}{\lambda}\right)\sqrt{\epsilon_{eff}}$$

$$F_2 = 2(1 - d_1)\sqrt{\epsilon_{eff}} + 4d_2 \frac{W}{\lambda}\epsilon_{eff}$$

$$F_3 = -4\left(d_2 - 2d_3 \frac{W}{\lambda}\sqrt{\epsilon_{eff}}\right)\epsilon_{eff}$$

$$F_4 = -8d_3 \epsilon_{eff}\sqrt{\epsilon_{eff}}$$

and

$$W_e(0) = \frac{120\pi(a+b)}{Z_0\sqrt{\epsilon_0}} \frac{a}{0.064}$$

expressions for d s are given in [28].

D. Inverted Microstrip Line

[38] suggests the same set of expressions used in the previous subsection holds good for inverted microstrip lines as well.

VII. Conclusions

A brief survey of the various techniques used in Static and Dispersion analysis of various Microstrip line configurations has been done. Wherever possible closed-form expressions has been provided to aid in CAD of these microstrip lines. Dispersion analysis for several microstrip lines are absent as closed-form expressions were not available for these. Dispersive effects on Dielectric and Conductor losses has also been omitted in the discussion. All the static equations given here were tested against various other transmission line calculators available. The equations were implemented as a part of the Microwave transmission line tool written by me [7]. An excellent reference for the various analysis equations is [39].

Appendix

Surface Impedance

The concept of surface impedance arises from the phenomenon called *"skin effect"*, which is the tendancy of high frequency currents to penetrate into the surface of a conductor only to a limited depth. The depth of penetration (or skin depth), δ, is defined as the depth in which the wave has been attenuated to $1/e$ times its original value (the value at the surface). For a good conductor ($\sigma/\omega\epsilon >> 1$), the depth of penetration is given by [8]

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Due to this skin effect, the transmission of high frequency currents through conductors becomes a surface phenomenon if the thickness of the conductor is large in comparison to the skin depth. Due to this a parameter called *"surface impedance"* is associated with conductors, defined as the resistance of a conducting surface of equal width and length. For a circular conductor, the width of the surface is its circumference [13]. The

[7] a pre-beta version of this tool - LineMeter is available

[8] Skin depth (in m) as function of frequency, f, for some metals at 300K are [40]: Silver: $0.0642f^{-1/2}$, Alluminium: $0.0826f^{-1/2}$, Brass: $0.127f^{-1/2}$, Copper: $0.066f^{-1/2}$.

surface impedance (a.k.a internal impedance), Z_s, is defined as [41]

$$Z_s = \frac{E_{tan}}{J_s} \tag{87}$$

where E_{tan} is the tangential component of the electric field and J_s is the surface current density due to the tangential electric field.The total conduction current will then be

$$J_s = \int_0^\infty J dy \tag{88}$$

assuming that the conductor is placed such that the y-axis is normal to its surface. Putting $J = J_0 e^{-\gamma y}$ in (88), we have

$$\begin{aligned} J_s &= J_0 \int_0^\infty e^{-\gamma y} dy \\ &= -\frac{J_0}{\gamma}\left[e^{-\gamma y}\right]_0^\infty \\ &= \frac{J_0}{\gamma} \end{aligned} \tag{89}$$

putting (89) in (87) and using Ohm's law for J_0, we have

$$Z_s = \frac{\gamma}{\sigma}$$

putting $\gamma = \sqrt{j\omega\mu\sigma}$ we have

$$Z_s = \sqrt{\frac{j\omega\mu}{\sigma}} = \eta \tag{90}$$

(90) shows that for a good conductor whose thickness is much greater than the skin depth, the surface impedance is equal to its internal impedance. splitting Z_s into resistance and reactance we have

$$R_s = X_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \frac{1}{\sigma\delta} \tag{91}$$

We can infer from (91) that a conductor having thickness very great compared to δ and having an exponential current distribution has the same resistance as would a conductor with thickness equal to δ and having a uniform current distribution.

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