Generation of Pink Noise using Pseudo Random Binary Sequence

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Abstract

The course project presents a true discrete time filter to produce pink noise from white noise using a Pseudo Random Binary Sequence (PRBS) generator. The PRBS is generated using linear feedback shift registers. The PRBS output spectrum approximates a white noise spectrum. A self starter circuit is employed in the design to avoid the possibility of the PRBS generator getting "stuck" at the 0 state. The filter coefficients, of an FIR filter to convert the white noise (PRBS output) to Pink noise, are obtained using MATLAB. The transfer function of this filter is given by $H(s) = \frac{K}{\sqrt{s}}$. The filter is realised using a set of resistors and an opamp summer. The spectrum of the PRBS and the pink noise generated are analysed using a Spectrum Analyser and this matches the expected outcome.

1 Introduction

White noise has a frequency characteristic which raises the power level by 3dB with each increasing octave. By combining a 3dB/octave filter and a white noise source, we can get a very good approximation to a "perfect" pink noise, where the power in each octave is exactly the same. White Noise is easily generated from PRBS by using maximal length shift registers. This white noise is to be filtered out to produce Pink noise. The problem is that most of the basic filters roll off at 6dB/ octave, so to create a 3dB/ octave filter we have to use multiple filter sections. The number of sections determines how flat the filter will be, and the more the better. So design of the filter should be accurate for proper pink noise generation. We present here an FIR filter implementation that filters white noise to produce pink noise.

2 Problem Statement

To design a true discrete time filter to produce pink noise from white noise using a Pseudo Random Binary Sequence (PRBS) Generator with not more than 8 bit shift registers.

3 Literature Review

3.1 Pseudo Random Binary Sequence

The chapter `Digital meets Analog` in [1] mentions the generation of white noise using PRBS and filtering the output using resistors and opamps. The project implementation is based on this idea. Figure 1 shows a PRBS generator. The most popular and the simplest PRBS generator is the feedback shift register. A shift register of length m bits is clocked at some fixed rate, f_o. An exclusive OR gate generates the serial input signal from the exclusive OR combination of the n^{th} bit and the last (m^{th}) bit

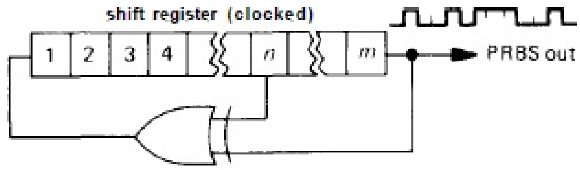

Figure 1: Block Diagram of PRBS Generator [1]

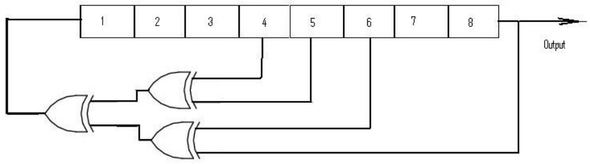

Figure 2: 8 bit PRBS Generator

of the shift register. Such a circuit goes through a set of states eventually repeating itself after L clock pulses; i.e., it is cyclic with period L (L = 2^m).

The maximum number of conceivable states of an m-bit register is L = 2^m. However the state of all 0's would get "stuck" in this circuit, since the EX-OR would regenerate a 0 at the input. Thus the maximum length sequence you can possibly generate with this scheme is $2^m - 1$. It turns out that you can make such "maximal-length shift register sequences" if m and n are chosen correctly, and the resultant bit sequence is pseudo-random.

For some values of m, a maximal-length register can only be made with more than two taps. In the case where m = 8, the taps required are 4, 5 and 6 and the maximal length of the sequence is 255. 8 bit PRBS generator is shown in Figure 2.

3.2 Analog noise generation from maximal-length sequences

3.2.1 Power Spectrum of shift register sequences

The output spectrum generated by maximal-length shift registers consists of noise extending from the repeat frequency of the entire sequence, f_{clock}/L, to the clock frequency and beyond. It is flat within ±0.1dB up to 12% of the clock frequency (f_{clock}) , dropping rather rapidly beyond its -3dB point of 44% f_{clock}. The power spectrum is a set of equally spaced series of spikes (delta functions), beginning at the frequency at which the whole sequence repeats, f_{clock}/L, going up in frequency by equal intervals f_{clock}/L. The fact that the spectrum consists of a set of discrete spectral lines reflects the fact that the shift register sequence eventually (and periodically) repeats itself. It will look continuous for any measurement or application that takes less time than the cycle time of the register. The envelope of the spectrum of the unfiltered output is shown in Figure 3. The envelope is proportional to the square of (sin x)/x. Note the peculiar property that there is no noise power at the clock frequency or its harmonics.

3.2.2 Digital Filtering

A disadvantage of analog filtering is the need to readjust the filter cutoff if the clock frequency is changed by large factors. In situations where that is desirable, an elegant solution is provided by digital filtering, in this case performed by taking an analog weighted sum of successive output bits. In this way the effective filter cutoff frequency changes to match changes in the clock frequency. In addition, digital filtering lets you go to extremely low cutoff frequencies (fractions of a hertz) where analog filtering becomes awkward. In order to perform a weighted sum of successive output bits simultaneously, you

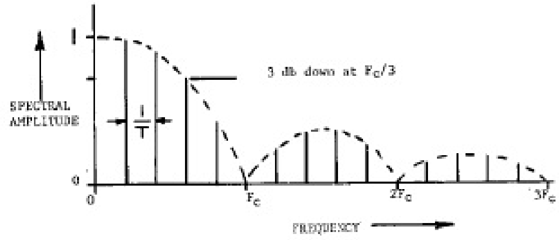

Figure 3: Spectrum of unfiltered PRBS output [2]

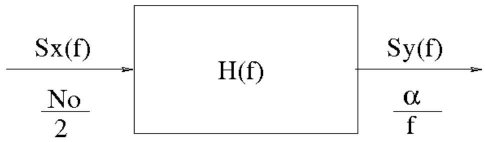

Figure 4: Block Diagram of the White Noise to Pink Noise Filter

can simply look at the various parallel outputs of successive shift register bits, using resistors of various values into an op-amp summing junction.

For a low-pass filter the weights should be proportional to (sin x)/x. Since no capacitors are used in this scheme, the output waveform consists of a set of discrete output voltages. The approximation to Gaussian noise is improved by using a weighting function over many bits of the sequence. In addition, the analog output then becomes essentially a continuous waveform. For this reason it is desirable to use as many shift register stages as possible, adding additional shift register stages outside the exclusive-OR feedback if necessary.

In order to generate Pink noise or 1/f noise, the White noise obtained at the shift register output needs to be filtered using a 3dB/octave filter.

3.2.3 White and Pink Noise

White Noise White noise is the name given to random signals that contain constant power per unit bandwidth for all frequencies (or at least for the audio frequency range). White noise can be considered to be a wide-sense stationary ergodic random process with absolutely no periodic components and exhibiting a flat power-density spectrum [2].

Pink Noise Pink noise is the name given to random signals that contain constant power per percentage bandwidth for all frequencies. The Power Spectral Density of Pink noise is given by $S(f) = \alpha/f$. Pink noise is usually obtained by passing white noise through a filter that has a transfer characteristic as follows [2]:

$$H(s) = \frac{K}{\sqrt{s}} \tag{1}$$

or

$$H(\omega) = \frac{K}{\sqrt{j\omega}} = \frac{K}{\sqrt{\omega}}\, e^{-j(\frac{\pi}{4})} \tag{2}$$

where K is an arbitrary constant. The filtering action is represented by the block diagram given in Figure 4. More details on this are given in Section 4.2.

This transfer function has a decreasing amplitude characteristic of -3 dB per octave and a constant lagging phase shift of $\pi/4$ or 45°. The filter is usually synthesized with an RC network which approximates the transfer characteristic with a finite set of real poles and zeros. Pink noise has a power-density spectrum that falls at the rate of -3dB per octave.

As stated in section 3.2.2, the above filter could be realised using various resistors and an opamp summer. The implementation is given in Figure 5. (As the coefficients we got are positive, we have used two inverting amplifiers in series.)

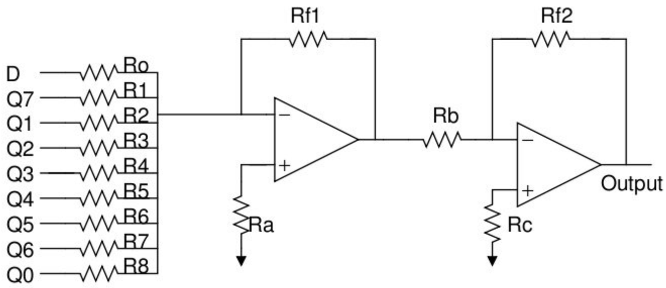

Figure 5: Opamp Summing block for realising the FIR filter

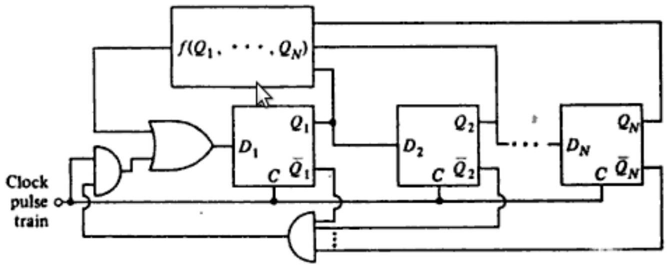

Figure 6: How to start a maximal sequence generator [3]

4 Investigations and Design of the Circuit

The state of all 0's could get "stuck" into the PRBS generator as mentioned in section 3.1. Therefore it is necessary to avoid this situation by employing a self starter circuit into the design of PRBS generator. The basic idea behind the self starter circuit is that the first clock pulse will take the system to the state 10000...0 , if all the Flip-flops are in state 0 [3]. The block diagram of the self starter circuit is given in Figure 6.

The shift registers commonly used come with only the Q outputs and so the above design was modified as shown in the Figure 7. The two 4-input NOR gates and the AND gate produce the desired feedback input, i.e., $\bar{Q_0}\bar{Q_1}....\bar{Q_7}$. 4015 is a dual 4 bit shift register, 4030 is a quad 2-input Ex-OR gate, 4002 is a dual 4-input NOR gate, 7432 is quad 2-input OR gate and 7408 is a quad 2-input AND gate.

4.1 FIR Filters

An FIR filter with input $\mathbf{x(n)}$ and output $\mathbf{y(n)}$ is described by the difference equation:

$$y(n) = b_0x(n) + b_1x(n-1) + ... + b_{M-1}x(n-M+1) = \sum_{k=0}^{M-1} b_kx(n-k) \tag{3}$$

where b_k is the set of filter coefficients [4].

We can also express the output sequence as the convolution of the unit sample response $\mathbf{h(n)}$ of the system with the input signal. Thus we have:

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k) \tag{4}$$

Comparing Equations 3 and 4, it is clear that in the case of FIR filters, $b_k = h(k)$.

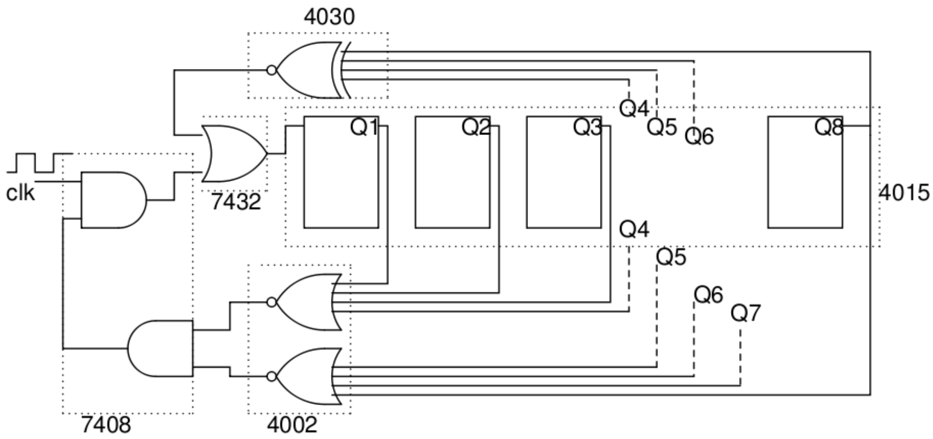

Figure 7: Modified self starter circuit

4.2 Design of the White Noise to Pink Noise Filter

Consider the block diagram of the filter that converts White noise to Pink (1/f) noise given in Figure 4. The output Power Spectral Density (PSD) is given by:

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{5}$$

where $S_Y(f)$ is the PSD of the output, $S_X(f)$ is the PSD of the input, $|H(f)|$ is the transfer function of the filter and f is the frequency.

In this case, $S_X(f) = N_0/2$ as the input is White noise. $S_Y(f) = \alpha/f$ as the output is Pink noise. Let $\alpha = 1$ and $N_0/2 = 0.1$. Substituting these values in Equation 5, we get:

$$|H(f)|^2 = \frac{10}{f} \therefore |H(f)| = \frac{3.1623}{\sqrt{f}} \tag{6}$$

Applying the rectangular windowing method for FIR filter design, we get

$$H(k) = H(\omega)|_{\omega = \frac{2\pi}{M} k f_s} \tag{7}$$

where M is the order of the filter and f_s is the sampling frequency. The order of the filter is equal to the number of delay elements. Therefore the order of the filter in our case is 8. Therefore Equation 6 becomes,

$$|H(k)| = \sqrt{\frac{10 M f_s}{k}} = \frac{3.78}{\sqrt{k}} \tag{8}$$

after substituting M = 8, f_s = 10kHz.

The filter coefficients are obtained by taking the inverse DFT (Discrete Fourier Transform) of

$$H(k) = |H(k)| e^{-j(\pi/4)} \tag{9}$$

We have used MATLAB to get the coefficients for the above FIR filter. The MATLAB function to do the same is given below:

```
function coeff = pink_coeff(N) % N point DFT

% to find FIR filter coeff for White noise to pink noise filter

for_phase = 0.707 - 0.707*1i; % the phase component
for i = 1:N+1
```

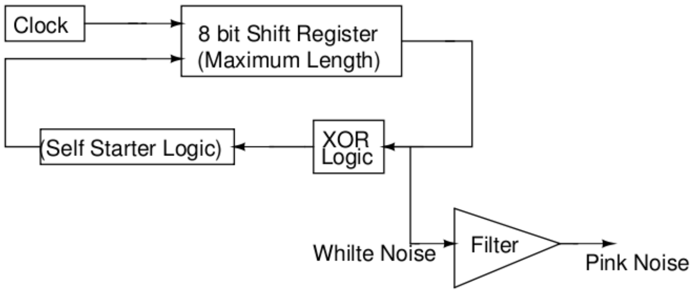

Figure 8: The final block diagram of the design

```
    H(i) = (3.7847/sqrt(i))*for_phase; % magnitude and phase
end

% finding inverse fft

coeff = ifft(H,N);
coeff = abs(coeff);  % absolute value
end
```

The coefficients thus obtained are given below:
$\mathbf{h(n)} = [1.9785 \quad 0.3873 \quad 0.2562 \quad 0.2071 \quad 0.1884 \quad 0.1884 \quad 0.2071 \quad 0.2562 \quad 0.3873]$.
There are nine coefficients as the filter order is eight.

4.3 Design of resistors for the op amp summer

Refer to the op amp summer block diagram given in Figure 5. As the coefficients are positive, we have used two inverting amplifiers in series. Choosing $R_{f1} = 10k\Omega$ and using the values for $\mathbf{h(n)}$, we get the various values for the resistors as follows:
$R_0 = 5.054k\Omega$
$R_1 = 25.820k\Omega = R_8$
$R_2 = 39.032k\Omega = R_7$
$R_3 = 48.29k\Omega = R_6$
$R_4 = 53.08k\Omega = R_5$
Choose $R_a = 10k\Omega||3.3k\Omega = 2.2k\Omega$, $R_b = R_{f2} = 10k\Omega$, $R_c = 4.7k\Omega$. R_a and R_c has been added for offset compensation for the Opamp amplifier circuit.

5 Final Block Diagram and Circuit

The final block diagram is given in Figure 8. The final circuit diagram is given in Figure 9.

6 Test Procedure

The procedure is as follows:

1. Generate Pseudo Random Binary Sequence with the help of shift registers and a self starter circuit.
2. Obtain the spectrum of the PRBS generated above using Spectrum Analyser and Digital Storage Oscilloscope (DSO).
3. Implement the filter to convert the above output (White Noise) to Pink Noise.
4. Obtain the spectrum of the Pink Noise output using Spectrum Analyser and DSO.

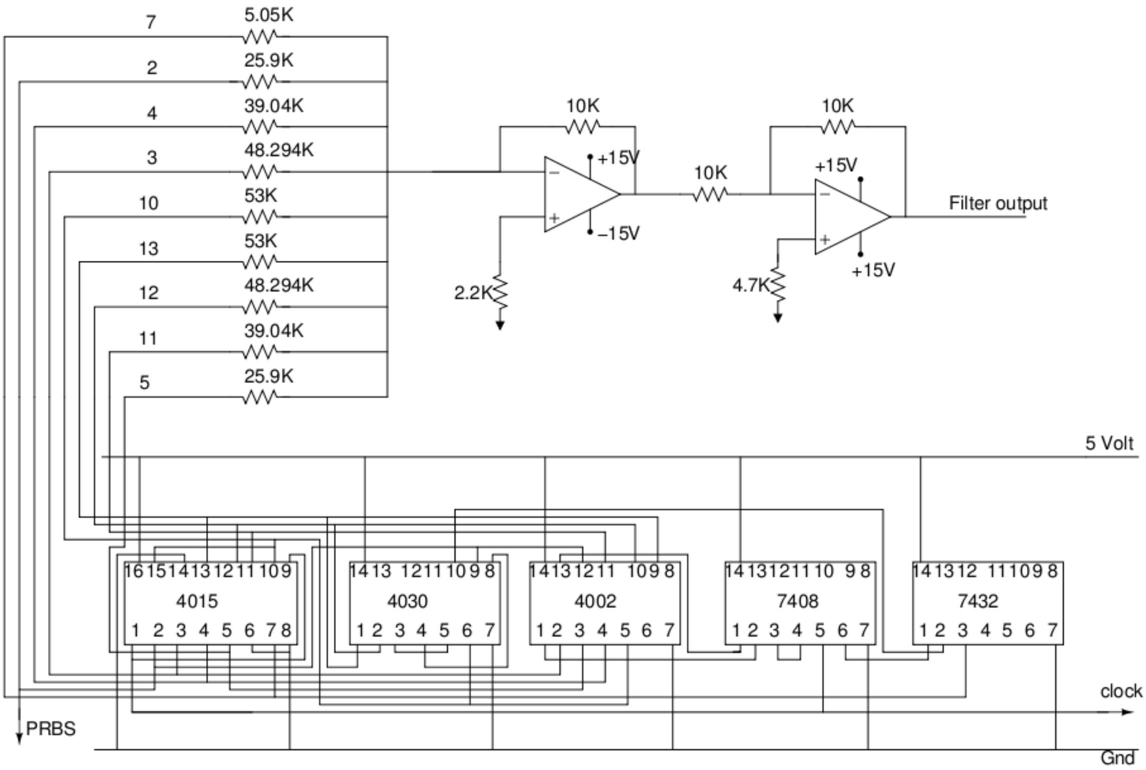

Figure 9: The final circuit diagram

7 Results and Discussion

The results obtained are summarised below:

7.1 Pink Noise Spectrum

- PRBS output along with the clock is given in Figure 10
- Filtered Output along with PRBS output is given in Figure 11
- Spectrum of White Noise (PRBS output) obtained on Spectrum Analyser is given in Figure 12
- Pink Noise spectrum obtained on Spectrum Analyser is given in Figure 13
- Spectrum of White Noise (PRBS output) obtained on DSO in FFT mode is given in Figure 14
- Pink Noise spectrum obtained on DSO in FFT mode is given in Figure 15
- **Using MATLAB**: We captured the filtered output data through the sound card of a Laptop using Audacity. This was done after attenuating the output to about 0.4V inorder to feed it to the sound card. The data was captured as a .wav file. The spectrum of the filtered output was plotted in MATLAB using the command spectrogram. Figure 16 shows the filtered output spectrum.

The above results show that spectrum of the PRBS output and filtered output matches the expected outcome. The spectrum of the filtered output shifts (to the right or to the left) with change in Clock frequency.

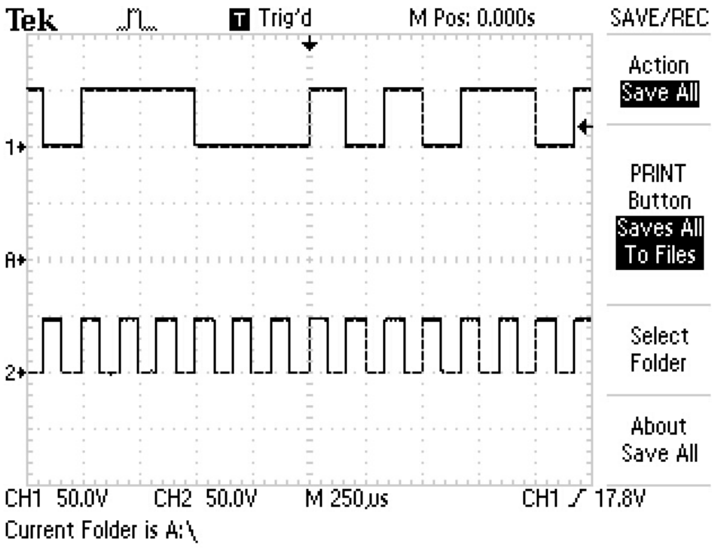

Figure 10: PRBS output along with Clock: First figure is the PRBS, second is the clock

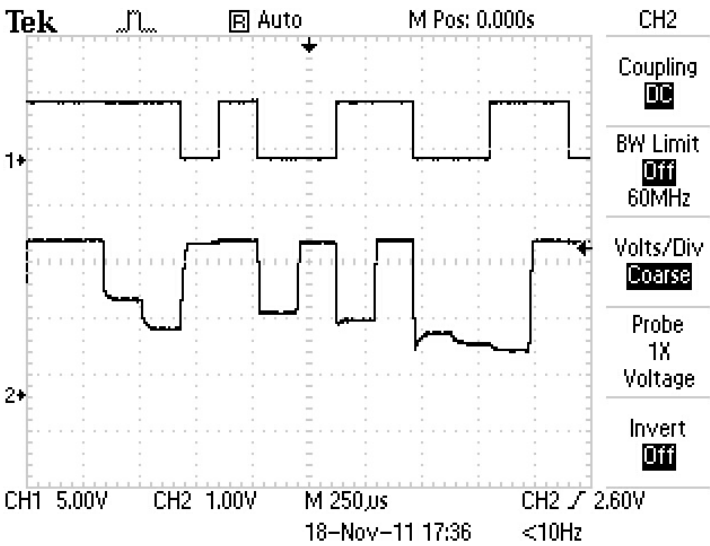

Figure 11: Filtered output along with PRBS output: First Figure - PRBS, second - Filtered output

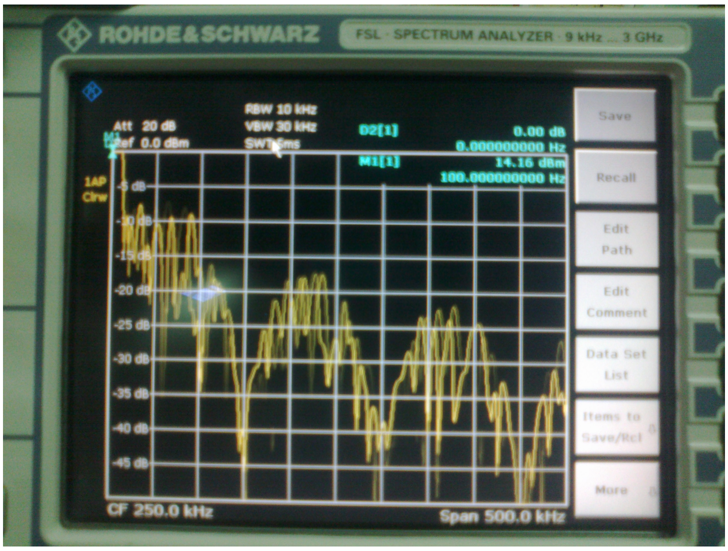

Figure 12: Spectrum of PRBS output in Spectrum analyser

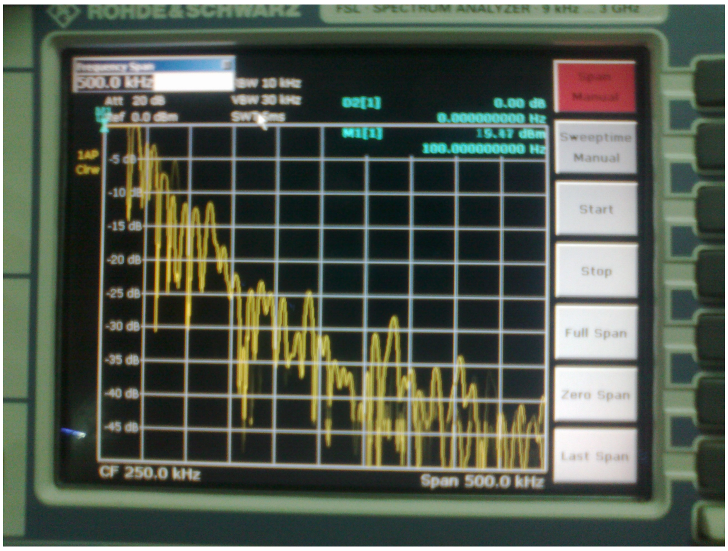

Figure 13: Spectrum of Pink Noise output in Spectrum analyser

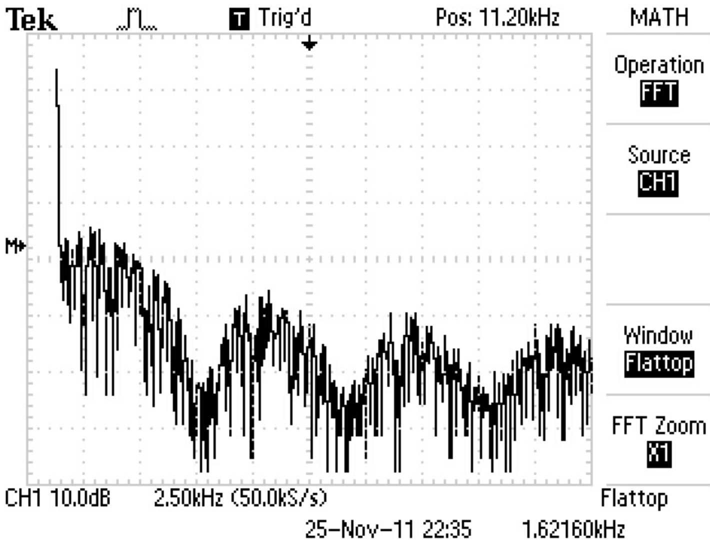

Figure 14: Spectrum of PRBS output in DSO

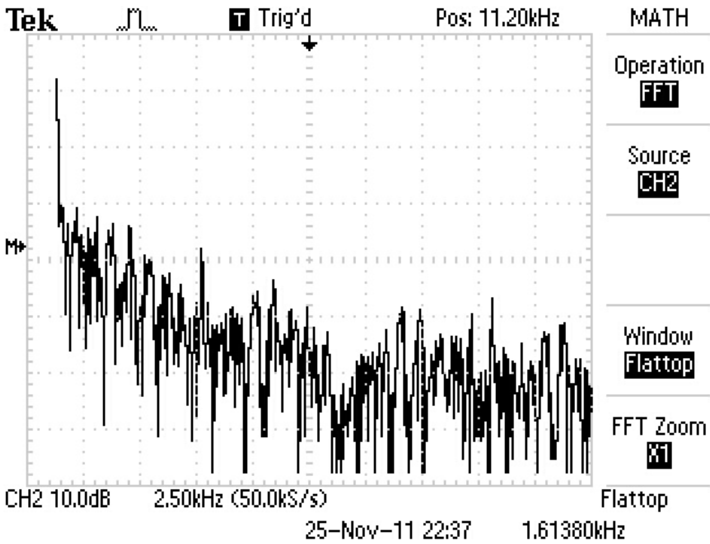

Figure 15: Spectrum of Pink Noise output in DSO

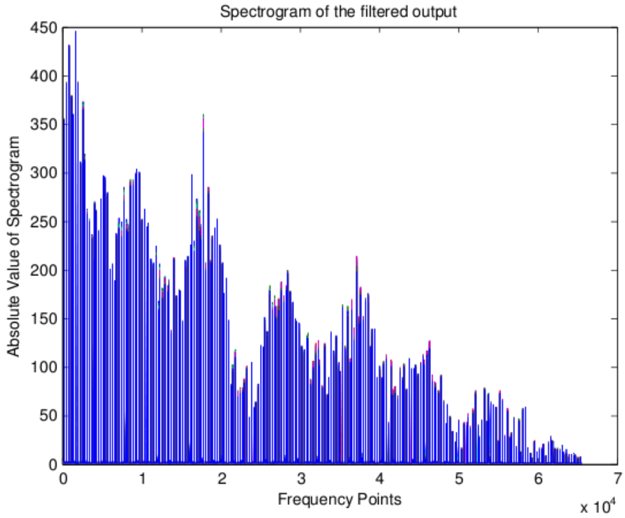

Figure 16: Spectrum of Filtered Output (obtained with the help of sound card and plotted using MATLAB)

8 Conclusion

We have obtained White noise using maximal length Pseudo Random Binary Sequence generator. The White noise output is filtered by using an FIR filter. This filter has been realised using the PRBS output, a set of resistors and an Opamp summer. The spectrum of PRBS output and the filtered output have been analysed by using a Spectrum Analyser and the FFT mode of DSO. The filtered output data is also captured using the sound card of a Laptop and its spectrogram has been plotted using MATLAB.

It has been verified that the envelope of the Spectrum of the PRBS output is proportional to the square of $(\sin x)/x$. The spectrum of the filtered output (Pink Noise) is proportional to 1/f; where f is the frequency.

References

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