2-D Simulation of Laminating Stresses and Strains in MEMS Structures

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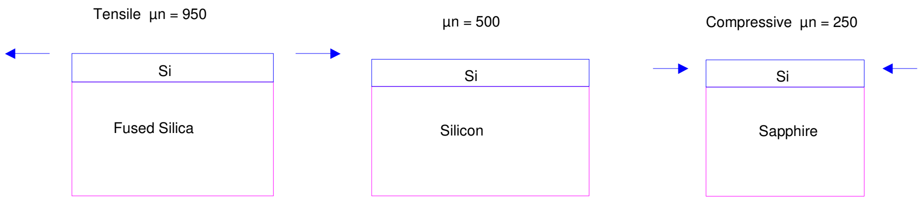
OUTLINE

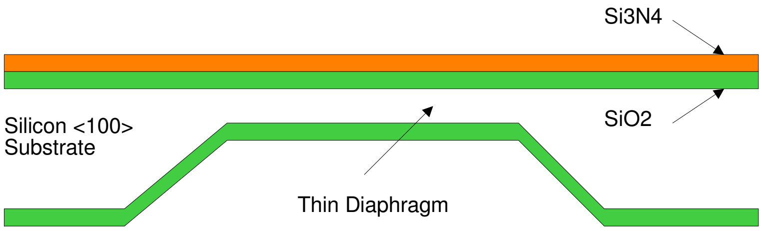
- Laminated Structures in MEMS
- 2-D Stress-Strain Analysis for Laminae
- Equilibrium and Compatibility Equations
- 4^{th} Order Derivatives and 13-Point Finite Differences
- Boundary Conditions : Fixed, Simply-Supported
- Program LAMINA
- Simulation of Laminated Diaphragms : having Si, SiO_2 and Si_3N_4 Layers
- Conclusions

2-Layer Laminated Structures



From : Fan, Tsaur, Geis, APL, 40, pp 322 (1982)

Multi-Layer Laminated Structures in MEMS



Equilibrium and Compatibility Equations

$M_{xx} + 2\,M_{xy} + M_{yy} = -q_{eff}$ - - - Equi. Eqn.

where M_{xy} are the bending moments

q_{eff} = Transverse Loads + In-plane Loads + Packaging Loads

$= q_a + \mathbf{N}_x\,\omega_{xx} + 2\mathbf{N}_{xy}\,\omega_{xy} + \mathbf{N}_y\,\omega_{yy} + q_{pack}$

where ω is the deflection in the transverse direction

N are the in-plane stresses

Equilibrium and ***Compatibility*** Equations

$\varepsilon_{10,yy} - \gamma_{120,xy} + \varepsilon_{20,xx} = 0$ - - - Comp. Eqn.

where $\begin{Bmatrix} \varepsilon_{20,xx} \\ M \end{Bmatrix} = \begin{bmatrix} a & b \\ b^T & d \end{bmatrix} \begin{Bmatrix} N \\ \chi \end{Bmatrix}$

where [a] , [b] , [d] are
3 x 3 matrices of functions of
elastic, thermal and lattice constants
of lamina materials

Equilibrium and ***Compatibility*** Equations

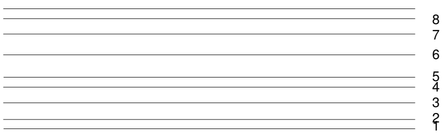
Equilibrium Equations

$$b_{21}\,U_{xxxx} + 2\,(b_{11} - b_{33})U_{xxyy} + b_{12}\,U_{yyyy} + d_{11}\,\omega_{xxxx} + 2\,(d_{12} - 2d_{33})\,\omega_{xxyy} + d_{22}\,\omega_{yyyy}$$
$$= -\,[\,q + U_{yy}\,\omega_{xx} + U_{xx}\,\omega_{yy} - 2\,U_{xy}\,\omega_{xy}\,]$$

Compatibility Equations

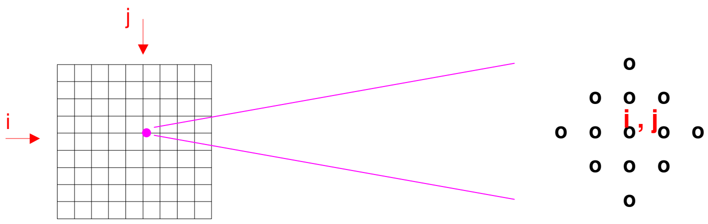
$$a_{22}\,U_{xxxx} + (2a_{12} - a_{33})U_{xxyy} + a_{11}\,U_{yyyy} + b_{21}\,\omega_{xxxx} + 2\,(b_{11} - b_{33})\,\omega_{xxyy} + b_{12}\,\omega_{yyyy} = 0$$

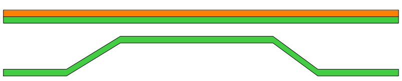
Stress- Strain Analysis of Laminated structures



LAMINA with 8 layers

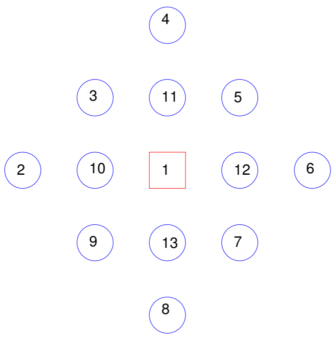
Diaphragm
with
Mesh or Grid





13 - point Formulation
of Finite Differences
for 4th order derivatives

4^{th} order derivatives in
U, Airy Stress function and ω, deflection



13 - point Formulation
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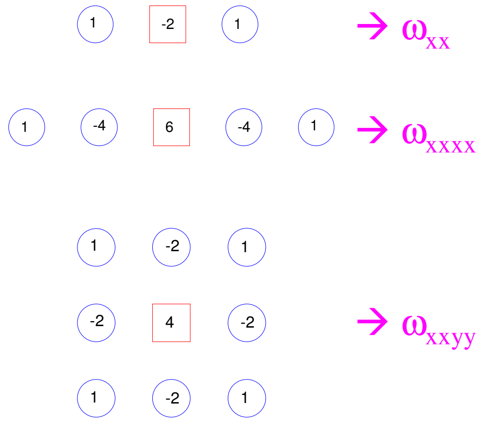
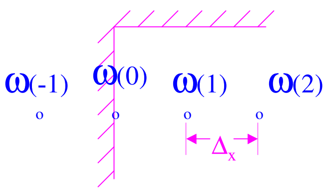
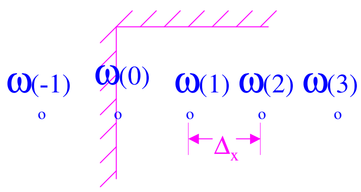


Entries in box/circles are weights

Boundary Conditions



4-Point Formulae



5-Point Formulae

Boundary Conditions: 4-Point Formulae

1st order derivative $\omega_x = [\,-1/3\ \omega(-1) - 1/2\ \omega(0) + \omega(1) - 1/6\ \omega(2)]\ /\Delta_x$

which gives $\omega(-1) = [\,-3/2\ \omega(0) + 3\ \omega(1) - 1/2\ \omega(2) - 3\Delta_x\ \omega_x\ (0)$

Similarly,

2nd order derivative $\omega(-1) = [\ 2\ \omega(0) - \omega(1) + 3\ \Delta^2_x\ \omega_{xx}\ (0)$

Fixed or Built-in Edge : at $x = a$, then $\omega(a) = 0$ and $\omega_x\ (a) = 0$

$\therefore\ \omega(-1) = 3\ \omega(1) - 1/2\ \omega(2)$

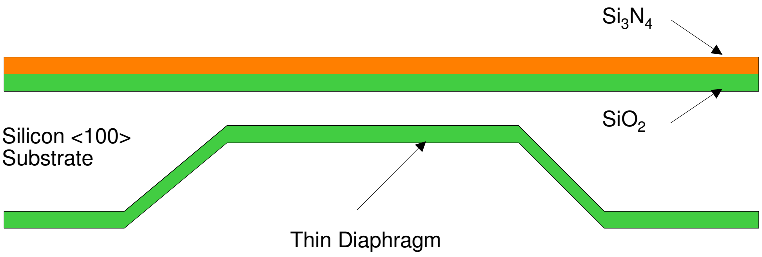
Simply-Supported Edge : at $x = a$, then $\omega_x\ (a) = 0$ and $M_x = 0 \rightarrow \omega_{xx}\ (a) = 0$

$\therefore\ \omega(-1) = \omega(1)$

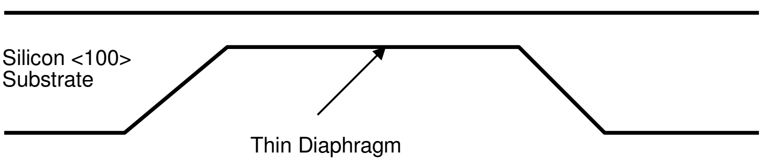
PROGRAM LAMINA

INIT	Initialization
READIN	Read Input Data
SUMMAR	Print Input Summary
ZQMAT	Setup **[Z]** and **[Q]** matrices
SOLVE	Solve for **U** and ω simultaneously
OUTPUT	Print deflection w and stress at each mesh point

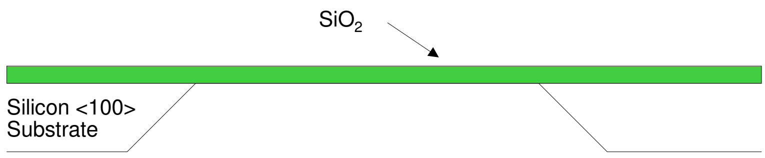
Typical multi-layer Diaphragm



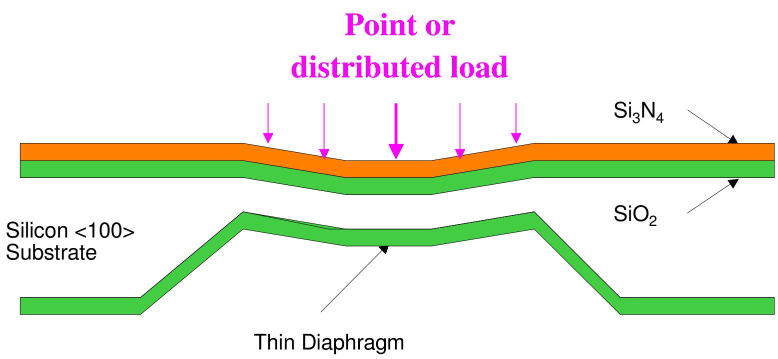
A Silicon Diaphragm



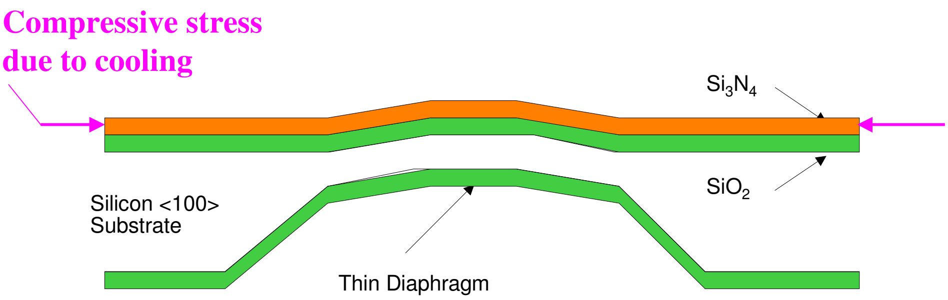
Silicon-dioxide Diaphragm



A multi-layer Diaphragm bent concave by point or distributed loads



A multi-layer Diaphragm bent convex by temperature stresses



Deflection at center ‘ω’ for lateral loads ‘q’

Table 1 Deflection at center `w ' for lateral loads `q'
(Reference for known results is Timoshenko [1])

serial no.	Boundary conditions	known results	LAMINA simulation
1	X-edges fixed Y-edges free	0.260E-2 (TIM pp 202)	0.261E-2
2	X-edges simply supp. Y-edges free	0.130E-1 (TIM pp 120)	0.130E-1
3	All edges fixed	0.126E-2 (TIM pp 202)	0.126E-2
4	All edges simply supported	0.406E-2 (TIM pp 120)	0.406E-2

Deflection at center for lateral and in-plane loads

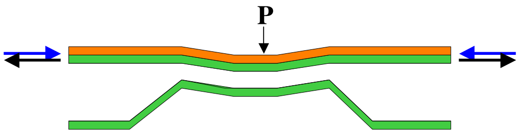


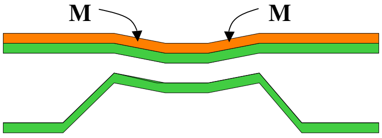
Table 2 Deflection at center for lateral and in-plane loads
(Reference is Timoshenko [1], pp 381)

serial no.	Normalized in-plane load	known results	LAMINA simulation
1	Tensile N_x=N_y=N_o=1	0.386E-2	0.384E-2
2	Tensile N_o = 19	0.203E-2	0.202E-2
3	Compressive N_o=-1	0.427E-2	0.426E-2
4	Compressive N_o=-10	0.822E-2	0.830E-2
5	Compressive N_o=-20	(indefinite)	(-0.246E+0)

Deflection at center for uniform bending moment

Table 3 Deflection at center for uniform bending moment
(Reference is Timoshenko [1], pp 183)

serial no.	condition	known results	LAMINA simulation
1	M_o = 1 on all edges	0.736E-1	0.732E-1



Diaphragm consisting of Si, SiO_2 and Si_3N_4 layers

Table 4 Diaphragm consisting of Si, SiO_2 and Si_3N_4 layers

serial no.	lamina layers	Deflection at center	In-plane resultant	stress couple
(a)	SiO_2+ Si+ SiO_2	0	-0.216E+2	0
(b)	Si + SiO_2	-0.583E-2	-0.381E+1	-0.628E-1
(c)	SiO_2+ Si	+0.583E-2	-0.381E+1	+0.628E-1
(d)	Si+SiO_2+ Si_3N_4	+0.583E-3	+0.191E+2	+0.168E-1
(e)	SiO_2+ Si+ SiO_2 + Si_3N_4	+0.515E-2	+0.643E+0	+0.727E-1

Conclusions

- LAMINA takes into account
  - → Single Crystal anisotropy in elastic constants
  - → Temperature effects – growth and ambient
  - → Heteroepitaxy, lattice constant mismatch induced strains
- LAMINA accepts
  - → Square or rectangular diaphragm/beam/cantilever
  - → non-uniform grid spacing in x- and y-directions
  - → non-uniform thickness at each grid point – taper
- LAMINA can be used as design tool
  - → to minimize deflections (at the center)
  - → minimize in-plane stresses in Si, SiO_2 and Si_3N_4
  - → make the diaphragm insensitive to temperature or packaging parameters

Thank You

Web-page http://www.tifr.res.in/~apte/LAMINA.htm

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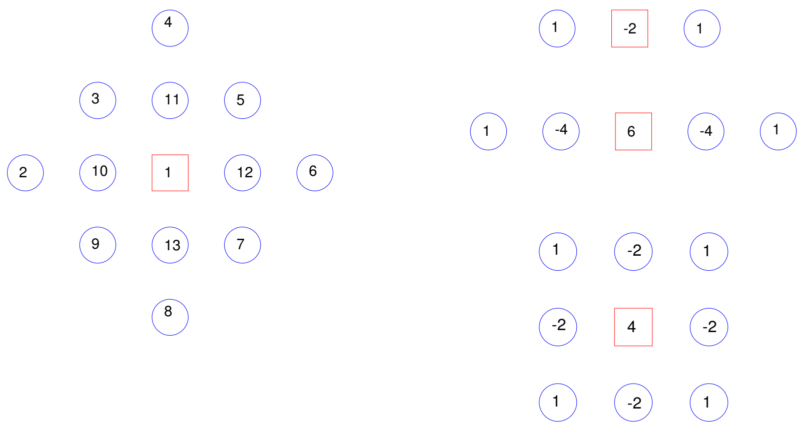

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