Midsem Exam: EE 752 Model Order Reduction.
27^{th} Feb 2015

Q-1 Consider the stable system $\dot{x} = Ax + Bu$, $y = Cx$ with A Hurwitz, (A,B) controllable and (C,A) observable.
(a) Write the equation of which the controllability & observability grammians P & Q are solutions.
(b). Show that the eigenvalues of the product of PQ is invariant w.r.t. coordinate transformations $x = Tz$ (T nonsingular) and also show that in general eigenvalues of P are not invariant (under $x = Tz$).
(c) Suggest a procedure that yields $\tilde{P}\tilde{Q} = D^2$, D diagonal. and $\tilde{P} = D = \tilde{Q}$
Show that this procedure indeed gives $\tilde{P} = D$
(d) Show that if P & Q are symmetric & +ve definite, then eigenvalues of PQ are positive, and comment on symmetry of PQ.

Q-2: For the state space system in Q-1 (with D = feedthrough term = 0), obtain LMI conditions for H_∞ norm to be $\leq \gamma$, $\gamma > 0$.
Suppose the H_∞ norm $< \gamma$, then obtain the corresponding Hamiltonian matrix and show that ~~each~~ an ARE solution X can be obtained from a suitable n-dimensional H-invariant subspace of $\mathbb{R}^{2n}$.

Q-3 Find $\underset{u \in C^\infty(\mathbb{R},\mathbb{R})}{\text{minimum}} \int_0^\infty (x^2 + u^2)\,dt$, $x(0) = 8$

using a suitable Riccati equation and its solution.

Note: ~~All~~ Q-1,2: 10 marks each.
Q-3 — 5 marks each.
weightage 35%.

Make suitable assumptions yourself in case of ambiguity & state assumption explicitly.