Tutorial Sheet EE 752 Model Order Reduction 15th Jan 2015

1. Consider the Hankel matrix H constructed from the Markov parameters.
 - Show that $H = \mathcal{O} R$ ← reachability matrix,
 ↖ observability matrix

 - Show that when the state space is changed (using a similarity transf on A), H remains unchanged.

2. Check the above for continuous & discrete time both.

3. Show that the Lyapunov operator $L_A : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ given by $L_A(P) = AP + PA^T$ is nonsingular iff no two eigenvalues of A add to zero.
 (Use eigenvalues of A & eigenvectors of A to construct "eigenvectors" of P: assume (for simplicity) A has real & distinct eigenvalues).

4. Show that the controllability & observability grammians P & Q satisfy $AP + PA = -BB^*$, $A^TQ + QA = -C^TC$
 (assume A is Hurwitz)

5. Consider A Hurwitz and the LQR problem (look this up).
 (infinite horizon)
 Assume control input is very very expensive (compared to deviations x) ⟺ $Q = 0$, $R = I$ (suppose).
 Relate that Riccati eqn with Lyapunov eqn & controllability grammian.

6. Consider the Lyapunov operator
 $L_A(P) = Q$.
 Look up for a result that relates inertia of A, P & Q
 (P & Q symmetric
 A need not be.)

7. Use LaSalle's invariance principle (or Lyapunov thm) to
 - show that $A^TQ + QA = -C^TC$ has soln $Q > 0$ iff A is Hurwitz. (Assume (C, A) is observable)
 - Relax observability & check truth of "iff".

Prob 8: Continued from Prob 5

Look up finite time LQ control and obtain controllability Grammian as optimal solution (with inverse perhaps).

Assume controllability, etc.

Prob 9: Verify that the Lyapunov operator $A^TQ + QA =: L_Q$ is linear.

Use a basis for $Q \in \mathbb{R}^{n\times n}$ using left/right eigenvectors of A and thus suggest a procedure to solve for Q in $A^TQ + QA = CC^T$

Prob 10: (By differentiating)

check $\int_0^\infty e^{At}BB^Te^{A^Tt}\,dt$ satisfies $AP + PA^T = -BB^T$ (A is Hurwitz).

Prob 11: $P(t)$ was defined for +ve time t.

Interpret $x_0 P(t)^{-1} x_0$ as minimum energy required to change $\left(\int_{-\infty}^{0} u^Tu\,dt\right)$ from 0 at $t = -\infty$ to x_0 at $t = 0$.)

(Assume A is Hurwitz $\frac{\text{negative time}}{\text{\& controllable}}$).

12 For constant matrix $X \in \mathbb{R}^{m\times n}$,

show that k^{th} ~~...~~ rank approximation using SVD is nearest (in induced 2-norm)

13 For $X \in \mathbb{R}^{n\times n}$, nonsingular,

show (using a family of nearest) nonuniqueness of singular "nearest" (in induced 2-norm)

14. Consider product of observability & controllability grammians (infinite time $T=\infty$)

Show eigenvalues of product is independent of state space basis.