Behavioral preliminaries

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Outline

- **Behavioral view**
- **Input/output models**
- **Kernel representation**

State space, transfer functions, behavioral approach

- **We have had input/output models (transfer function)**
- **Then, we have state space**
- **And now, behavioral approach**

Are they 'competing'?

Multiple views can only help

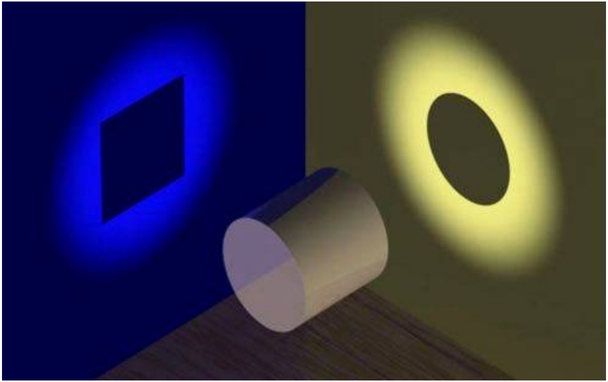

Figure: Source unknown, shared by Waghulde

- **Input/output classification of variables often un-natural. (Resistor, capacitor, spring, mass, damper)**
- **System $\equiv$ signal processor: input/output ideal**
- **Causality also helps classify**

Key advantages of behavioral viewpoint

- **Causality: relevant when time is the independent variable**
- **For control of PDEs, behavioral viewpoint yielded controllability definition**

Key advantages of i/o viewpoint

- **Low-pass filter, band-pass filter (Bode plots)**
- **Bode plot helps <u>calculate</u> gain/phase margins**
- **Recall: Nyquist plot helps <u>understand</u> gain/phase margins**

- **Energy exchange not necessarily linked to input/output classification**
- **Dissipativity studies: since early 1970s**
- **Behavioral approach: $\sim$ 1987**
- **Riccati equations: easier to follow**
- **Key work by Megretski and Rantzer on Integral Quadratic Constraints**

Throughout these lectures:

$\mathtt{m}$: **number of inputs,**	$\mathtt{p}$: **number of outputs**
$\mathtt{w}$: **number of 'manifest' variables:**	**typically** $\mathtt{m} + \mathtt{p}$
$\mathtt{n}$: **(minimum) number of states**	**(McMillan degree)**

$$G(s) \in \mathbb{R}^{\mathtt{p}\times\mathtt{m}}(s), \quad G(s) = P(s)^{-1}Q(s) = V(s)U(s)^{-1}$$

with $P, Q, U, V \in \mathbb{R}^{\bullet\times\bullet}[s]$**. More precisely,** $P, Q \in \mathbb{R}^{\mathtt{p}\times\bullet}[s]$ **and** $U, V \in \mathbb{R}^{\bullet\times\mathtt{m}}[s]$**.**

- **A 'system' is nothing but the set of trajectories that the system allows.**
- **The system 'behavior' is the set of allowed trajectories, i.e. those that the system laws allow. Suppose the system variables are w.**

$$\mathfrak{B} := \{w \in \mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^{\mathtt{w}}) \mid w \text{ satisfies the system laws } \}.$$

- **$\mathfrak{C}^\infty$: trajectory is infinitely often differentiable: primarily for convenience.**
- **Some notions do depend on the signal space used. $\mathfrak{L}^1_{\text{loc}}$ is another frequently used space: this includes step, ramp and other such signals.**
- **For dissipativity-preserving model order reduction, $\mathfrak{C}^\infty$ is fine.**

Consider again a set of differential equations $R(\frac{d}{dt})w = 0$. 'Elementary row operations' on R do not change the set of solutions.

- **Interchange two rows of R: premultiply R by (more generally) a permutation matrix**
- **Multiply an equation by a nonzero constant: premultiply R by diagonal nonsingular constant matrix**
- **Differentiate an equation and add to another equation: premultiply by matrix P which has just one entry (say (i,j)-th entry) different from the identity matrix: $P(i,j) = p(\xi)$, $i \neq j$.**

Any number and sequence of above three types of operations do not change the set of solutions/system-behavior.
'Sequence' here refers to product of the corresponding matrices.
A matrix $U(\xi) \in \mathbb{R}^{p \times p}[\xi]$ is called unimodular if its determinant is a nonzero constant.

Theorem

Let $U \in \mathbb{R}^{p \times p}[\xi]$. Then the following are equivalent.

- *U is unimodular.*
- *U can be written as a (non-unique) product of elementary matrices.*
- *The inverse of U exists and is (not just rational, but in fact) polynomial*

We also saw

- **the Smith canonical form of a polynomial matrix**
- **(normal) rank of a polynomial matrix**
- **unimodular completion (of a wide, nonsquare matrix) and its special case: Bezout identity**
- **Input/output partitions**