Active Noise Cancellation using H_∞ Control Techniques

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Abstract

The noise cancellation problem has been studied extensively [4], [2]. Low frequency (acoustic) noise attenuation has faced attention in industrial environments and in automobile applications. The modeling of the one-dimensional case (where sound travels in a duct and hence between 'wave guides') is relatively straightforward for example [1], [6] & [3]. In this paper, a laboratory experimentation of canceling noise using the H_∞ control algorithm is presented. The setup consists of a rectangular wooden duct fitted with speakers (for actuation and noise) and microphones (for sensing). The system is interfaced to a stand-alone DSP board (dSPACE) with a TMS 320C31 chip and the control algorithm is implemented digitally on the DSP. [1]

1 Experimental Setup

A block diagram of the noise cancellation setup is as shown in Fig.1.

P1=Acoustic path from speaker 1 to microphone 1
P2=Acoustic path from microphone 1 to microphone 2
P3=Acoustic path from speaker 2 to microphone 2
P4=Acoustic path from speaker 2 to microphone 1
w =Digital signal input to D/A converter
y =Digital signal input to DSP
u =Digital signal output of DSP
z =Digital signal, error signal
S1=Electrical path from digital signal 'w' to speaker 1
S2=Electrical path from microphone 1 to DSP input 'y'
S3=Electrical path from digital signal 'w' to speaker 2
S4=Electrical path from microphone 2 to signal 'z'
C =Controller

The transfer functions P1, P2, P3 & P4 relate to acoustic signals. It may be noted that the transfer functions $S1$ & $S3$ include the transfer functions of speakers Spk1 & Spk2 respectively. Similarly $S2$ & $S4$ include the transfer functions of the microphone Mic1 & Mic2 respectively. For this block diagram and the transfer functions shown as above, the equations between the signals w, u, y & z are related as in equation below.

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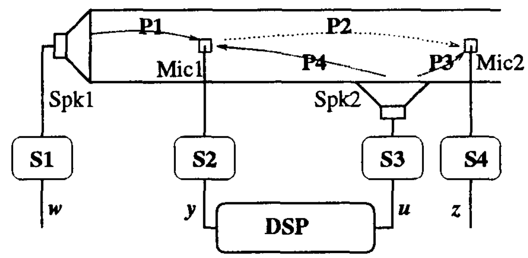

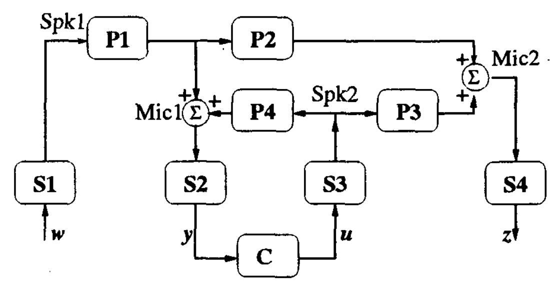

Figure 1: The noise control setup - block diagram

$$y = S_2(P_1S_1w + P_4S_3u) = S_2P_1S_1w + S_2P_4S_3u$$
$$z = S_4(P_2P_1S_1w + P_3S_3u) = S_4P_2P_1S_1w + S_4P_4S_3u \quad (1)$$

2 A Linear Fractional Setup

Define

$$G_{yw} := S_2P_1S_1, \quad G_{yu} := S_2P_4S_3,$$
$$G_{zw} := S_4P_2P_1S_1, \quad G_{zu} := S_4P_3S_3$$

and rewrite the equations in (1) as

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \quad (2)$$

The above equation and Fig.2 are more commonly referred to as the generalized plant [8], because most control problems can be formulated in this form. The signals w is the exogenous input, u is the control signals, y is the measured output and z is the output to be controlled. These signals, in general, could be vector valued. With this convention, the control problem is to find a controller

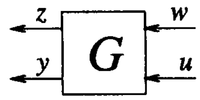

Figure 2: The generalized plant - block diagram

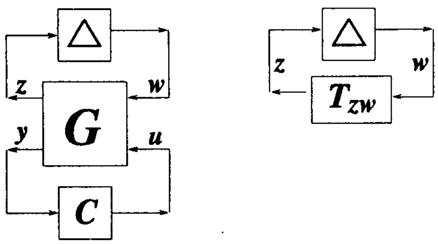

Figure 3: The uncertainty in closed loop

C (which computes u using y) that stabilizes the closed loop system (as in Fig 4) and minimizes the *effect* of w on z. The minimization criterion could be either the ∞ norm or the 2 norm or some other performance measures. The problem of finding a suitable controller to the generalized plant is referred to as the standard problem.

Our objective here is to find a controller C such that the closed loop is stable and

$$\|T_{zw}\|_\infty < \gamma \tag{3}$$

where $T_{zw} := G_{zw} + G_{zu}C(I - G_{yu}C)^{-1}G_{yw}$ is the closed loop transfer function from w to z. The performance index γ is an indication of the robustness of the controller design. In other words, lower the γ value, greater the controller's tolerance to changes in the plant model parameters. This tolerance to uncertainty (see Fig 3) can be expressed mathematically as

$$\| \triangle \|_\infty \leq \frac{1}{\gamma} \tag{4}$$

In our situation two major sources of error in the model of the plant are the identification scheme and the model reduction scheme that are used to obtain a model of the system. Identification with noisy measurements brings in uncertainty; model reduction that helps us design the controller with a lower order (more tractable) model is obviously an approximation to original behavior. So we desire that the controller be sufficiently robust to plant perturbations, and hence that γ be sufficiently low.

Further it may be necessary to emphasize certain frequencies at which the noise is more dominant during controller synthesis. There could also be a constraint that the control action can be taken only at certain frequencies. The original plant, then has to be replaced by a modified plant with weighting functions that incorporate such constraints (see Fig 5).

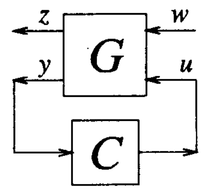

Figure 4: The Standard Problem - block diagram

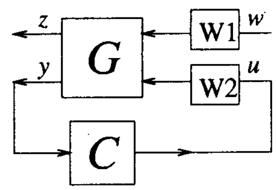

Figure 5: The standard problem with plant modified

Standard programs for the controller synthesis using the performance measures described above are available in the commercially available control software, *MATLAB*. An optimal controller was synthesized using the *MATLAB* routine ***hinfsyn*** which uses an iterative method in the state-space domain to find a controller that satisfies a specified performance index [8]. The optimal controller was implemented after a bilinear transformation into the discrete domain. Comparing the cases without cancellation and with cancellation, the reduction in sound, as sensed at the error microphone, was then plotted against various frequencies of the noise.

3 Results

The transfer functions G_{zw}, G_{zu}, G_{yw} & G_{yu} were identified as follows. A pseudo random binary sequence (PRBS) was given as the input to excite all frequencies upto 1000Hz [7], and the measured output was sampled at 3000Hz. Then an ARMAX model was fitted between the input/output data values using the method of least squares. This results in an IIR model of each transfer function in the discrete domain [5]. Then using a bilinear transformation, the continuous time transfer function was found. The order of this model was reduced before using the transfer functions for controller synthesis. Each input-output model was reduced to order 4. Then the following weighting functions were incorporated into the plant as shown in Fig. 5.

$$W1 = W2 = \frac{(0.0001s + 1)^2}{(0.001s + 1)^2} \tag{5}$$

The second order weights emphasize frequencies below 1000kHz, and the reduced order model of the generalized plant has four transfer functions of order 4. The controller was computed using the result of 1988 by Glover and Doyle, with real schur decomposition. The pole-zero

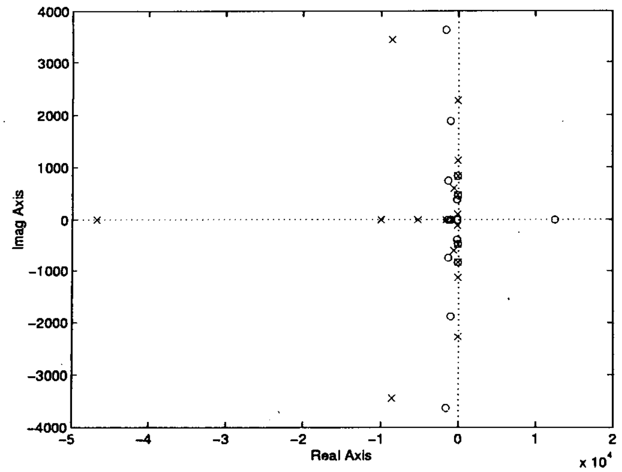

Figure 6: The poles (×) & zeros (○) of the controller

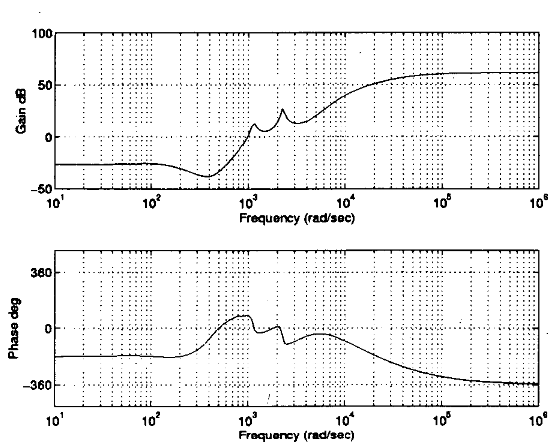

Figure 7: The frequency response of the controller

plots of this controller are as in Fig. 6. The frequency response of this controller is shown in Fig. 7.

This controller was implemented in the discrete domain on the DSP TMS320C31. The noise levels after the controller implementation, has been compared to that before implementation and the reduction has been plotted against various frequencies.

The difference in the two shows the reduction. Figures. 8 to 10 are bodeplots of these measured values.

The softwares *MATLAB* alongwith the toolbox *Identification* were used for the model fitting and controller synthesis. The coefficients of the controller obtained (from *MATLAB*) were implemented by incorporating these coefficients into a *C* code and forming an object file which was executed on the DSP.

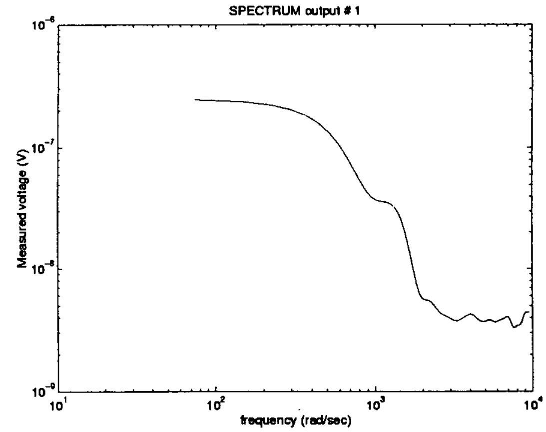

Figure 8: The measured voltage without cancellation

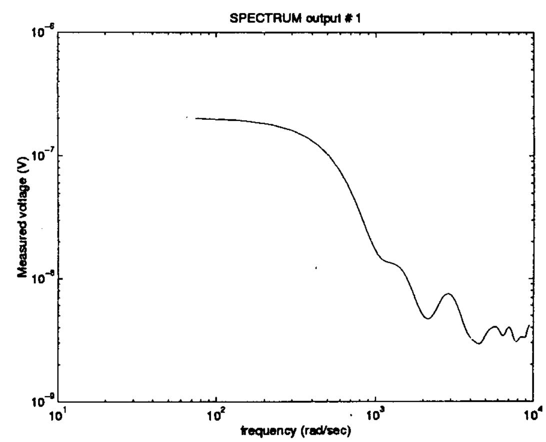

Figure 9: The measured voltage with cancellation, i.e. with control action

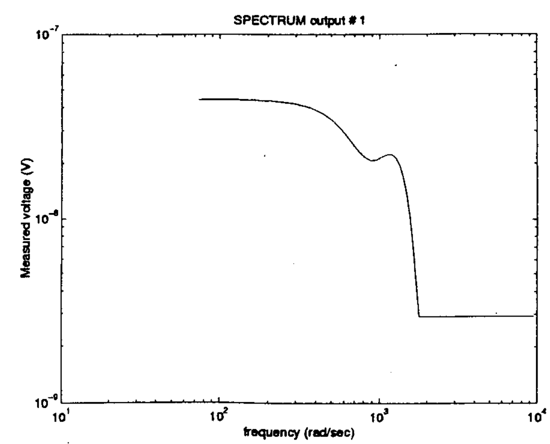

Figure 10: The difference, ***higher*** difference means ***more*** cancellation

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