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Decentralized Control of a Line Interactive Uninterruptible Power Supply (UPS)

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Abstract— **The decentralized control of a line interactive UPS through p-ω and q-V droop control requires measurement only of variables local to the UPS and does not require a communication link. The droop control strategy has been known to result in unstable systems for large gain constants. This paper begins with a behavioral approach to system representation and presents the transfer function of the controlled system. By determining the location of the poles and zeros of the open loop transfer function, comments are made about the stability of the controlled system for various gains. The value of control gain that causes instability can be determined analytically. Simulations in Scilab are used to show the change in the poles of the controlled system with changes in control gain. Experimental results have been presented to show the operation of the line interactive UPS in transient state.**

Index Terms— **Decentralized control, behavioral theory of systems, Uninterruptible Power Supply (UPS), Digital Signal Processor (DSP).**

I. INTRODUCTION

A line interactive Uninterrupted Power Supply (UPS) is an excellent solution to improve the reliability of electric power to a critical load [1], [2]. A line interactive UPS is connected to the distribution system (which will be called as ac grid) and the load through a single interface without an additional rectifier. When the ac grid is healthy, the UPS shares the power demanded by the load with the grid, while during grid outages, the load is supplied by the UPS. In this paper, the decentralized control of a line interactive UPS will be examined where the UPS is controlled by measurement of variables local to the UPS. For this purpose, the droop control strategy is analyzed and implemented.

In the droop control strategy [3]–[9] , the frequency ω and the magnitude V of the UPS output voltages are not constants but are made to vary with respect to the power supplied by the UPS. The frequency ω is varied with respect to the active power p supplied by the UPS while the magnitude V is varied with respect to the reactive power q supplied. Previous studies have presented stability analysis of the UPS with droop control strategy [4], [5], [7]–[9]. The study has been focused on the dominant poles of the controlled system that are closest to the imaginary axis of the complex s plane. Several studies have shown through simulations how the poles of the closed loop system become unstable for large droops [4], [7], [9]. However, a conclusive theoretical proof of stability of the controlled system has not been presented.

In this paper, a behavioral approach has been used to model the UPS connected to the ac grid. The system is shown to be controllable. The transfer function of the plant has been derived with respect to the droop controller and the poles and zeros of the open-loop transfer function have been plotted in the complex s plane. Using these plots, the movement of poles of the closed loop system with changes in the droop controller gains can be studied using conventional root locus techniques. Simulations performed in Scilab show the poles of the controlled system for changes in the droop controller gains and verify the conclusions obtained from the pole-zero plot of the plant transfer function. The value of droop controller gain for which the controlled system becomes unstable can be determined analytically. Experimental results have been presented showing the performance of the line interactive UPS.

The outline of the paper is as follows. Section II contains preliminaries on control theory from a behavioral approach and the power electronic inverter. Section III describes the mathematical model of the line interactive UPS connected to the grid and defines the behavior. Section IV shows that the system is controllable and presents the droop controller in a mathematical form. Section V derives the transfer function of the plant and the controlled system and plots the poles and zeroes of the plant on the complex s plane. Section VI presents simulation results in Scilab where the stability of the controlled system for variations in the controller gains. Experimental results of the droop control strategy are provided to verify the droop control strategy.

II. PRELIMINARIES

This section serves to provide a brief background of the behavioral approach to linear dynamical systems and to power electronics converters. The later sections will use the facts stated in this section for analysis and proofs.

A. Control theory

This subsection begins by introducing a few basic concepts of behavioral theory applied to control systems. The notations used in this paper are as follows. The set $\mathbb{C}^n$ is used for the n dimensional complex vector space. Vectors belonging to the complex vector space $\mathbb{C}^n$ are denoted by lowercase boldfaced letters, *e.g.* a vector $\mathbf{w} \in \mathbb{C}^n$. The ring of single variable polynomials with complex coefficients in the indeterminate s is denoted by $\mathbb{C}[s]$. The set $\mathbb{C}^{n_1 \times n_2}[s]$ denotes the set of matrices with n_1 rows and n_2 columns in which each entry is an element of $\mathbb{C}[s]$. A matrix is denoted by uppercase boldfaced letters *e.g.* a matrix $\mathbf{A} \in \mathbb{C}^{n_1 \times n_2}[s]$.

The behavior $\mathfrak{B}$ of the system is defined as the solution set of a system of linear constant coefficient ordinary differential

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equations given by $\mathbf{A}\left(\frac{d}{dt}\right)\mathbf{w}=0$, where $\mathbf{w}\in C^{\infty}(\mathbb{R},\mathbb{C}^n)$ is a vector containing complex valued variables of the system with components w_1, w_2, …, w_n. Such a representation is also called a kernel representation of a behavior. The matrix $\mathbf{A}$ is a complex polynomial matrix and is denoted by $\mathbf{A}\in\mathbb{C}^{\bullet\times n}[s]$ where $\bullet$ indicates that the number of rows may vary with the system formulation. The solution set is assumed to consist of infinitely often differentiable functions. Hence, the behaviour is defined as

$$\mathfrak{B}=\left\{\mathbf{w}\in C^{\infty}(\mathbb{R},\mathbb{C}^n)\mid \mathbf{A}\left(\frac{d}{dt}\right)\mathbf{w}=0\right\}. \tag{1}$$

A polynomial matrix $\mathbf{U}\in\mathbb{C}^{g\times g}[s]$ is called unimodular if $\mathbf{U}(s)$ has a non-zero constant determinant. $\mathbf{A_1},\mathbf{A_2}\in\mathbb{C}^{g\times n}[s]$ represent the same behavior $\mathfrak{B}$ minimally if and only if there exists a unimodular matrix $\mathbf{U}(s)$ such that $\mathbf{A_1}=\mathbf{U}\mathbf{A_2}$ [10].

Consider the behavior $\mathfrak{B}\in C^{\infty}(\mathbb{R},\mathbb{C}^n)$ with a kernel representation $\mathbf{A}(\frac{d}{dt})\mathbf{w}=0$ with $\operatorname{rank}(\mathbf{A})<n$. Therefore, some of the components of $\mathbf{w}=(w_1,w_2,\ldots,w_n)$ are not constrained by the requirements of $\mathbf{w}\in\mathfrak{B}$. These components are called inputs. After a permutation of components if required, the variable $\mathbf{w}$ can be partitioned in a possibly non-unique manner into $\mathbf{w}=(\mathbf{u},\mathbf{y})$ where $\mathbf{u}$ is the input and $\mathbf{y}$ is the output. The number of components in the output $\mathbf{y}$ turns out to be $\operatorname{rank}(\mathbf{A})$. If the kernel representation $\mathbf{A}(\frac{d}{dt})\mathbf{w}=0$ is minimal and $\mathbf{A}$ is partitioned as $\mathbf{A}=[\mathbf{A_1}\ \ \mathbf{A_2}]$ corresponding to the input-output partition $\mathbf{w}=(\mathbf{u},\mathbf{y})$, then y is the output if and only if $\det(\mathbf{A_2})\neq 0$.

Suppose a plant behavior $\mathscr{P}$ is required to be restricted to a subsystem $\mathscr{K}\subset\mathscr{P}$. In order to do so, the number of equations that the plant variables have to satisfy are increased. The additional equations result in another behavior termed as the controller behavior $\mathscr{C}$. The interconnection of these two systems ($\mathscr{P}$ and $\mathscr{C}$) results in $\mathscr{K}=\mathscr{P}\cap\mathscr{C}$ where both the plant and controller equations are satisfied. If $\mathbf{A}(\frac{d}{dt})\mathbf{w}=0$ and $\mathbf{C}(\frac{d}{dt})\mathbf{w}=0$ are kernel representations of the plant and the controller respectively, then the controlled system behavior is the solution set of $\left[\mathbf{A}^T\ \ \mathbf{C}^T\right]^T\mathbf{w}=0$. The interconnection of $\mathscr{P}$ and $\mathscr{C}$ is called regular if $\left[\mathbf{A}^T\ \ \mathbf{C}^T\right]^T\mathbf{w}=0$ is also a minimal kernel representation of $\mathscr{P}\cap\mathscr{C}$ assuming $\mathbf{A}$ and $\mathbf{C}$ are minimal kernel representations. When the controlled system is autonomous, the poles of the controlled system are the roots of determinant of $\left[\mathbf{A}^T\ \ \mathbf{C}^T\right]^T$. We require the important property of controllability for being able to ensure that transients go to zero at prescribed rates. A behavior $\mathfrak{B}$ is controllable if and only if $\mathbf{A}$ of equation (1) above satisfies full row rank property of $\mathbf{A}(\lambda)$ for every $\lambda\in\mathbb{C}$ (see [10]).

B. Power Electronics

Fig. 1 shows the topology of a standard Uninterrupted Power Supply (UPS). Inverters are used to convert the dc output voltage V_{dc} across the dc capacitor to a three phase ac output voltage. The voltage V_{dc} across the dc capacitor is assumed to remain constant in this paper. Switching devices S_1 to S_6 are Insulated Gate Bipolar Transistors (IGBTs) with their associated anti-parallel diodes that are used to produce a switched voltage waveform. The switches operate at a frequency of approximately 5 kHz. The inductor L_f and the capacitor C_f form a low pass *L-C* filter that removes the high frequency switching harmonics generated by the inverter. The voltages v_{fa}, v_{fb}, v_{fc} across the filter capacitor C_f bank are the output voltages of the inverter while the currents i_{ca}, i_{cb}, i_{cc} are the output currents of the UPS. [11] contains a detailed comparison of the control strategies that are commonly used to control a three phase UPS of the topology of Fig. 1.

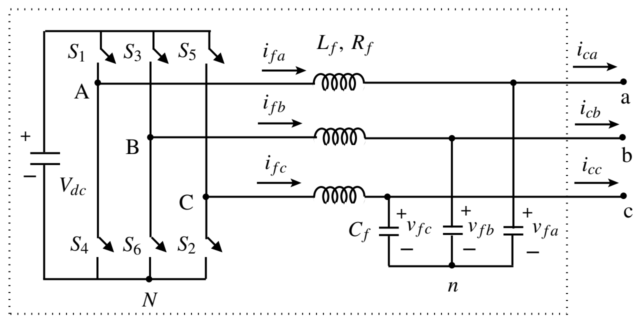


Fig. 1. Topology of the Uninterrupted Power Supply (UPS)

In Fig. 1, the ac part of the circuit is a three phase circuit. The output voltage v_f, output current i_c and the current i_f through the inductor L_f are three phase variables having three components "a", "b" and "c". For example, output voltages v_{fa}, v_{fb}, v_{fc} are the "a", "b" and "c" phase components of the output voltage v_f with the phase subscript added at the end. In all the analysis that follows where three phase ac networks are considered, the three phase variables are transformed using a method called the Clarke's transformation to obtain complex variables. For example, $v_f=v_{fd}+jv_{fq}$ will be the transformed output voltage. The subscripts "d" and "q" are the real and imaginary components of the complex variable. Similarly, all three phase variables transformed using Clarke's transformation will be expressed as complex variables with "d" and "q" components. In the following section, three phase variables will be written as complex valued variables in their transformed form without d and q components. This would reduce the number of variables thereby simplifying the analysis of three phase circuits.

III. Mathematical Model of the System

Fig. 2 shows the single line diagram of a line interactive UPS consisting of a UPS in Fig. 1 connected to a three phase ac grid. The UPS has a load connected to it locally that it will continue to supply if the ac grid fails. The topology of the UPS has been described in the previous section. L_{c1} is an inductance used to interface the UPS to the grid and the load. L_{1g} is the inductance of the cable that connects the UPS to the ac grid with R_{1g} being its parasitic resistance. v_g is the three phase ac grid voltage.

The mathematical model of the microgrid is written using the system variable $\mathbf{w}$

$$\mathbf{w}=\left[\begin{array}{ccccccc} v_{f1} & i_{c1} & v_{p1} & i_{l1} & u_1 & v_g & i_{1g}\end{array}\right]. \tag{2}$$

From (2), $(v_{f1},i_{c1},v_{p1},i_{l1})$ are the three phase variables shown in Fig. 2 in complex *d-q* form: $v_{f1}=v_{f1d}+jv_{f1q}$,

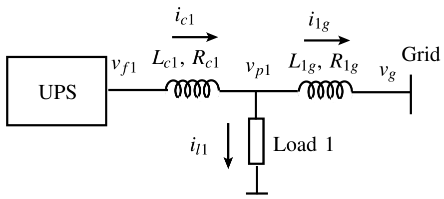


Fig. 2. Line interactive UPS with a load local to the UPS

$i_{c1} = i_{c1d} + ji_{c1q}$, $v_{p1} = v_{p1d} + jv_{p1q}$ and $i_{l1} = i_{l1d} + ji_{l1q}$. The variable $u_1 = \delta_1 - jV_1$ is also local to the UPS: δ_1 is defined as the phase angle of the complex variable v_{f1} while V_1 is defined to be the magnitude [4]. *i.e*,

$$\delta_1 = \tan^{-1}\left(\frac{v_{f1q}}{v_{f1d}}\right) \quad \text{and} \quad V_1 = \sqrt{\frac{v_{f1d}^2 + v_{f1q}^2}{V_{f1d}^2 + V_{f1q}^2}} \tag{3}$$

where V_{f1d} and V_{f1q} are the values of v_{f1d} and v_{f1q} at the equilibrium point about which the equations are linearized as will be discussed next. δ_1 and V_1 are the inputs to the system as will be described later in the section and further in the next section.

$i_{1g} = i_{1gd} + ji_{1gq}$ is the current flowing in the cable interconnecting the UPS and the ac grid as shown in Fig. 2. $v_g = v_{gd} + jv_{gq}$ is the voltage of the ac grid. In a line interactive UPS of Fig. 2, the UPS and the ac grid are two independent entities. Therefore, the grid voltage v_g is an external input to the system and this will be evident in the input-output partition of the behavior $\mathbf{w}$.

With the above description of the variables in the vector $\mathbf{w}$, the behavior of the system of Fig. 2 can be defined. However, from (3), the relations of δ_1 and V_1 are non-linear. The other variables of $\mathbf{w}$ are related by linear differential equations obtained from network laws such as Kirchoff's Voltage Law and Kirchoff's Current Law. Therefore, in order to define the behavior of the system as the solution set of equations, the entire system is linearized about an equilibrium point and all the components of the vector $\mathbf{w}$ are expressed as deviations about the equilibrium point. The symbol "Δ" denotes the deviation of the variables from the equilibrium point in the equations that follow. Therefore, the deviation in the system vector is written as

$$\Delta\mathbf{w} = \begin{bmatrix} \Delta v_{f1} & \Delta i_{c1} & \Delta v_{p1} & \Delta i_{l1} & \Delta u_1 & \Delta v_g & \Delta i_{1g} \end{bmatrix}^T. \tag{4}$$

The derivation of the mathematical equations describing the system is omitted and the final matrix equations are as below. The derivations can be found in detail in pages 4-14 of [12].

$$\mathbf{A_{total}}\ \Delta\mathbf{w} = 0 \tag{5}$$

where $\mathbf{A_{total}} \in \mathbb{C}^{5\times 7}[s]$ is written as

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & -1 \\ 1 & -Z_{c1}(s) & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -Z_{l1}(s) & 0 & 0 & 0 \\ k_{h1} & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & -Z_{c1}(s) & 0 & 0 & 0 & -1 & -Z_{12}(s) \end{bmatrix} \Delta\mathbf{w} = 0 \tag{6}$$

where $Z_{c1}(s) = (R_{c1} + sL_{c1}) + j\omega L_{c1}$, $Z_{l1}(s) = (R_{l1} + sL_{l1}) + j\omega L_{l1}$ and $Z_{1g}(s) = (R_{1g} + sL_{1g}) + j\omega L_{1g}$. ω is the angular frequency of the ac grid and is a constant.

$$k_{h1} = \frac{-V_{f1q} - jV_{f1d}}{V_{f1d}^2 + V_{f1q}^2}$$

V_{f1d} and V_{f1q} are the values of the variables v_{f1d} and v_{f1q} at the equilibrium point about which the nonlinear system is linearized. In the next section, the values of the variables at the equilibrium point will be listed.

With respect to the above presentation of the system equations, the behavior of the linearized system can be defined as $\mathfrak{B} = \text{kernel}\ \mathbf{A_{total}}\left(\frac{d}{dt}\right)$ The next section will use this behavior of the system to analyze controllability of the system. Furthermore, for a given controller, the stability of the system will be examined.

IV. Controllability and Controller Model

In this section we use suitable elementary row operations on the matrix $\mathbf{A_{total}}$ which allows calculation of the controlled system's poles. The system variables $\Delta\mathbf{w}$ can be partitioned in an input-output manner. The control input to the system is Δu_1 while Δv_g is the external input to the system combined together as the system inputs $\Delta\mathbf{w_1} = \Delta(u_1, v_g)$. The remaining variables are outputs of the system and combined together as $\Delta\mathbf{w_2} = \Delta(v_{f1}, i_{c1}, v_{p1}, i_{l1}, i_{12})$. The behavior of the system can be written with respect to this input-output partition as

$$\mathfrak{B} = \left\{ \begin{bmatrix} \Delta\mathbf{w_1} \\ \Delta\mathbf{w_2} \end{bmatrix} \in C^\infty(\mathbb{R}, \mathbb{C}^n) | \mathbf{A_{total}}\left(\frac{d}{dt}\right) \begin{bmatrix} \Delta\mathbf{w_1} \\ \Delta\mathbf{w_2} \end{bmatrix} = 0 \right\} \tag{7}$$

where

$$[\mathbf{A_1} \,|\, \mathbf{A_2}] = \left[\begin{array}{cc|ccccc} 0 & 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & -Z_{c1}(s) & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -Z_{l1}(s) & 0 \\ -1 & 0 & k_{h1} & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -Z_{c1}(s) & 0 & 0 & -Z_{12}(s) \end{array}\right]. \tag{8}$$

Following the derivation of the kernel representation of the behavior in input-output form, the controller can now be described. (8) shows that the output variables of the plant are v_{f1}, i_{c1}, v_{p1}, i_{l1}, i_{12}. However, during practical implementation it would be desirable if the control variable u_1 were dependent on measurement of variables at the vicinity of the UPS. From Fig. 1 and Fig. 2, the variables at the vicinity of the UPS and that interact with the ac grid are v_{f1} and i_{c1}. This form of a controller is called a decentralized controller.

In the analysis, the three phase ac variables of the system were transformed into single complex variables. The advantage of such a transformation was that a variable with three real components was transformed into a single complex variable with a real and imaginary part. This led to a reduction in the number of variables which simplified analysis. However, in the actual implementation of the system and the controller, only real variables are dealt with. Therefore, the following discussion describes the mathematical formulation of the controller and considers the complex variables v_{f1} and i_{c1} as real variables v_{f1d}, v_{f1q} and i_{c1d}, i_{c1q}. Similarly, the components δ_1 and V_1 of the control variable u_1 will be considered separately.

In this section, the controller equation will be written directly. The controller equations are written in the following matrix form

$$\begin{bmatrix}\Delta\delta_1\\ \Delta V_1\end{bmatrix} = -\begin{bmatrix} m_1\frac{I_{c1d}}{s} & m_1\frac{I_{c1q}}{s} & m_1\frac{V_{f1d}}{s} & m_1\frac{V_{f1q}}{s}\\ -n_1 I_{c1q} & n_1 I_{c1d} & n_1 V_{f1q} & -n_1 V_{f1d}\end{bmatrix}\begin{bmatrix}\Delta v_{f1d}\\ \Delta v_{f1q}\\ \Delta i_{c1d}\\ \Delta i_{c1q}\end{bmatrix}. \quad (9)$$

In the above equation, I_{c1d} and I_{c1q} are the values of i_{c1d} and i_{c1q} at the equilibrium point respectively. The values of the variables at the equilibrium point are $V_{f1d} = 1$, $V_{f1q} = 0$, $I_{c1d} = 0.776$, $I_{c1q} = -0.9$. The values have been obtained by a transient simulation of the non-linear system. From the control law of (9), the deviation in angle $\Delta\delta_1$ is an integral function of the variables Δv_{f1d}, Δv_{f1q} and Δi_{c1d}, Δi_{c1q} while the deviation in voltage magnitude ΔV_1 is a proportional function of the variables Δv_{f1d}, Δv_{f1q} and Δi_{c1d}, Δi_{c1q}.

V. Stability Analysis

The transfer function will be derived to understand the properties of the plant. The transfer function of the plant is derived with respect to the controller in (9). The kernel representation of the behavior $[\mathbf{A_1}\ \ \mathbf{A_2}](\frac{d}{dt})\Delta\mathbf{w} = 0$ with $\mathbf{A_{total}}$ used in (8) upon simplification using elementary row operations results in a block matrix of the form

$$\left[\begin{array}{c|cc} 0 & \mathbf{M_1} & \mathbf{A_{o2}}\\ -I & 0 & \mathbf{A_{o1}}\end{array}\right]\begin{bmatrix}\Delta\mathbf{w_1}\\ \Delta\mathbf{w_{2nc}}\\ \Delta\mathbf{w_{2c}}\end{bmatrix} = 0. \quad (10)$$

In the above equation, $\mathbf{w_1} = (u_1, v_g)$, $\mathbf{w_{2nc}} = (v_{p1}, i_{l1}, i_{1g})$, $\mathbf{w_{2c}} = (v_{f1}, i_{c1})$. The matrix $\mathbf{M_1} \in \mathbb{R}^{3\times 3}$ turns out to be a non-singular constant matrix. It is evident from the above matrix equation, that the plant transfer function can be obtained from the following equation

$$\mathbf{A_{o1}}\Delta\mathbf{w_{2c}} = \Delta\mathbf{w_1}. \quad (11)$$

Consider the transfer function matrix from input $\Delta\mathbf{w_1}$ to output $\Delta\mathbf{w_{2c}}$. It contains complex entries with the inputs and outputs also being complex. In order to analyze the stability of the controlled system, the open-loop plant equations have to be combined with the controller equations of (9). Therefore, the plant transfer function will have to be written in terms of real variables with real valued polynomials. The matrix $\mathbf{A_{o1}^{-1}}$ is written as,

$$\mathbf{A_{o1}^{-1}} = \begin{bmatrix} a_1 + ja_2 & 0\\ b_1 + jb_2 & c_1 + jc_2\end{bmatrix} \quad (12)$$

where a_1, a_2, b_1, b_2, c_1, c_2 are real rationals in the indeterminate s. From (11) and (12), the plant transfer function matrix with real valued variables and real valued rationals is written in terms of the matrix $\mathbf{T_p}(s)$ as follows

$$\begin{bmatrix}\Delta v_{f1d}\\ \Delta v_{f1q}\\ \Delta i_{c1d}\\ \Delta i_{c1q}\end{bmatrix} = \mathbf{T_p}(s)\begin{bmatrix}\Delta\delta_1\\ \Delta V_1\\ \Delta v_{gd}\\ \Delta v_{gq}\end{bmatrix} = \begin{bmatrix} a_1 & a_2 & 0 & 0\\ a_2 & -a_1 & 0 & 0\\ b_1 & b_2 & c_1 & -c_2\\ b_2 & -b_1 & c_2 & c_1\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \Delta V_1\\ \Delta v_{gd}\\ \Delta v_{gq}\end{bmatrix}. \quad (13)$$

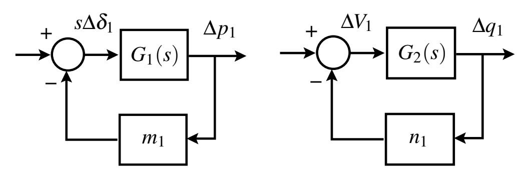
Equations (13) and (9) provide in terms of real valued polynomials the open-loop plant transfer function matrices and the controller equations respectively. In (9), the control input $u_1 = \delta_1 - jV_1$ has been related to the output variables v_{f1}, i_{c1}. The grid voltage v_g as has been described before is an external input. The grid voltage can change arbitrarily and independently with respect to the UPS. Hence, in order to analyze the stabilization performance of the controller, the perturbations in the grid voltage Δv_g are assumed to be zero. The complete controller equations are

$$\begin{bmatrix}\Delta\delta_1\\ \Delta V_1\\ \Delta v_{gd}\\ \Delta v_{gq}\end{bmatrix} = \begin{bmatrix} -m_1\frac{I_{c1d}}{s} & -m_1\frac{I_{c1q}}{s} & -m_1\frac{V_{f1d}}{s} & -m_1\frac{V_{f1q}}{s}\\ n_1 I_{c1q} & -n_1 I_{c1d} & -n_1 V_{f1q} & n_1 V_{f1d}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_{f1d}\\ \Delta v_{f1q}\\ \Delta i_{c1d}\\ \Delta i_{c1q}\end{bmatrix}$$

Denoting the above matrix by $\mathbf{T_c}(s)$, the controlled system is written as

$$\begin{bmatrix} -I & \mathbf{T_p}(s)\\ \mathbf{T_c}(s) & -I\end{bmatrix}\begin{bmatrix}\Delta\mathbf{w_{2c}}\\ \Delta\mathbf{w_1}\end{bmatrix} = 0 \quad (14)$$

The original mathematical model of (6) has been transformed into the plant model of (13) with respect to the controller model equation preceeding (14). These are shown in Fig. 3. The stability of the line interactive UPS connected to the grid will be analyzed using these transformed systems. The plant transfer function $G_1(s)$ with respect to the controller gain m_1 and $G_2(s)$ with respect to the controller gain n_1 will be derived below.



(a) Plant with p-ω droop (b) Plant with q-V droop

Fig. 3. Block diagram of the transformed system of (13) and (14)

By performing elementary row operations to the matrix of (14), the matrix can be converted to an upper triangular form having the determinant as the product of two rationals in s. One rational contains just the gain m_1 and the other rational contains the gain n_1 of equation preceeding (14). Upon simplification, the numerators of the rationals turns out to be the characteristic polynomial of the closed loop system as follows

$$[\mathrm{den}_1(s) + m_1\ \mathrm{num}_1(s)][\mathrm{den}_2(s) + n_1\ \mathrm{num}_2(s)] = 0 \quad (15)$$

with $\mathrm{den}_1(s)$ and $\mathrm{num}_1(s)$ being the terms corresponding to zeroth order in m_1 and first order in m_1 respectively. Similarly, $\mathrm{den}_2(s)$ and $\mathrm{num}_2(s)$ being the terms corresponding to zeroth order in n_1 and first order in n_1 respectively. With respect to Fig. 3, we define $G_1(s) = \frac{\mathrm{num}_1(s)}{\mathrm{den}_1(s)}$ and $G_2(s) = \frac{\mathrm{num}_2(s)}{\mathrm{den}_2(s)}$.

The parameters making up the transfer functions G_1 and G_2 are functions of the resistances R_{c1}, R_{l1}, R_{1g} and inductances L_{c1}, L_{l1}, L_{1g} of the circuit in Fig. 2 and the gain

TABLE I

INDUCTANCE AND RESISTANCE VALUES IN PER UNIT

Inductance	Resistance
$L_{c1} = 0.0039$	$R_{c1} = 0.00156$
$L_{1g} = 0.00147$	$R_{1g} = 3.125 \times 10^{-4}$
$L_{l1} = 1.178$	$R_{l1} = 0.937$

TABLE II

OPEN LOOP PLANT POLES AND ZEROS

Poles	$-0.35 \pm j$	$-0.35 \pm j$	0
Zeros	13.95	-14.65	$-0.35 \pm j14.37$

constants m_1 and n_1. The resistances and inductances of the circuit in Fig. 2 are listed in Table I. The numerators of the rationals are written in terms of the parameters as

$$\begin{aligned}&\mathrm{den}_1(s) + m_1\ \mathrm{num}_1(s)\\ &= 1.109s(s^4+2.285s^3+3.857s^2+2.915s+1.829)\\ &+ m_1(-s^4-2.285s^3+201.82s^2+324.13s+333.85)\end{aligned} \tag{16}$$

and

$$\begin{aligned}&\mathrm{den}_2(s) + n_1\ \mathrm{num}_2(s)\\ &= 1.109(s^4+2.285s^3+3.857s^2+2.915s+1.829)\\ &+ n_1(s^4+2.285s^3+209.53s^2+329.96s+337.51)\ .\end{aligned} \tag{17}$$

The poles and zeros of the plant are obtained by calculating the roots of the polynomials to which m_1 and n_1 in (16) and (17) are multiplied.

The poles and zeros of the plant transfer function are listed in Table. II. From Table. II, the equations (16), (17) the following observations can be made about the plant.

1) It is evident from the table that there is a real zero in the right half of the complex s plane at 13.95. This indicates that the plant is a non-minimum phase system with respect to the gain constant m_1.
2) Due to the system being non-minimum phase with respect to the gain constant m_1, the controlled system will become unstable as m_1 is increased to large values.

The next section presents simulation results where the poles of the controlled system will be plotted for variations in the gain m_1.

VI. SIMULATION AND EXPERIMENTAL RESULTS

This section will begin by showing the stable operation of the line interactive UPS using simulations in Scilab. The next subsection will then show the experimental results of the line interactive UPS.

A. Simulation Results

To examine the stability of the controlled system consisting of the plant and the controller, the poles of the controlled system are plotted for variations in the control gains m_1. The stability analysis has analyzed as follows. The controller gain n_1 is kept constant at a nominal value of $n_1 = 0.00125$ while the gain m_1 is varied from a low gain of $m_1 = 1.5 \times 10^{-5}$ to a high gain of $m_1 = 9.54 \times 10^{-3}$. The poles of the controlled system are plotted on the complex plane.

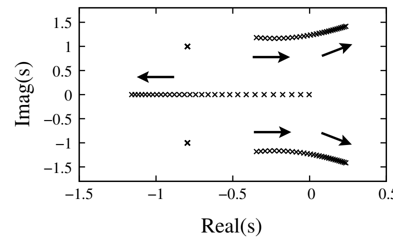


Fig. 4. Variation of closed loop poles for $n_1 = 0.00125$ and m_1 varying between 1.5×10^{-5} and 0.00954.

TABLE III

DETAILS OF EXPERIMENTAL SETUP

Grid
Three phase 140 V line-line R.M.S from an auto-transformer.
UPS
Dc bus voltage = 250 V, Switching frequency = 5 kHz
$L_{f1} = 3$ mH, 25A, $L_{c1} = 400\mu$H, 25A, $C_{f1} = 200\mu$F, 440V
Load
Resistive load of six 250V, 200W incandescent lamps.
Tunable inductor load of 22 mH, 25A
Delta connected bank of single phase diode rectifiers with 250V, 100W incandescent lamp load.

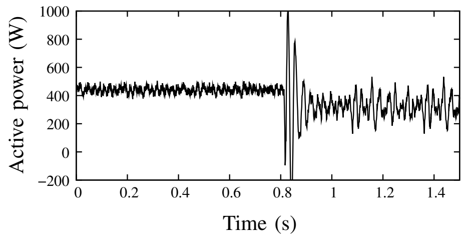
Fig. 4 shows the variation of the poles of the controlled system in the first case when n_1 is kept constant and m_1 is varied. The arrows show the direction of movement of the poles with increase in m_1. As can be seen from the figure, the controlled system becomes unstable for large values of gain m_1. The value of m_1 when the controlled system becomes unstable is approximately 3.66×10^{-3}.

B. Experimental Results

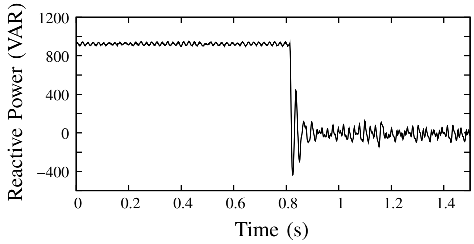
The experimental results have been obtained from a scaled down laboratory hardware of a line interactive UPS. Table III lists the ratings of the equipment used for the experimental setup. The control algorithms have been executed using the Texas Instruments TMS320VC33 floating point Digital Signal Processor (DSP). The DSP also has the capability of storing measured data in real-time that can be uploaded onto a personal computer and plotted. The waveforms presented in this section have been generated from data saved by the DSP.

The UPS is initially supplying the load in isolation to the ac grid. After 0.8s, the ac grid is connected to the UPS using an ac contactor and associated synchronizing circuits. The plots below show the transition of the operation of the UPS from a standalone mode in which it supplies the entire load to a line interactive mode where it shares the load power demand with the ac grid. Fig. 5 shows the active and reactive power supplied by the UPS. As can be seen both the active power in Fig. 5(a) and reactive power in Fig. 5(b) fall sharply when the ac grid is connected.

Fig. 6 shows the phase a output of the UPS. Fig. 6(a) shows the output voltage v_{fa} of the UPS. Upon interconnection, the magnitude of the voltage increases towards the magnitude of the ac grid voltage. Fig. 6(b) shows the output

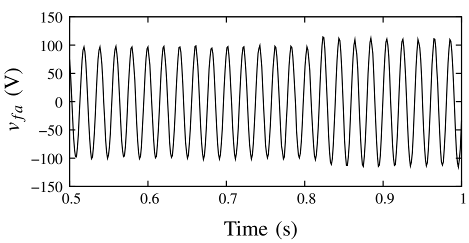


(a) Active power supplied by the UPS

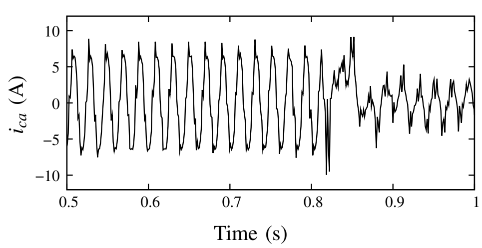


(b) Reactive power supplied by the UPS

Fig. 5. Active and reactive power supplied by the UPS for $m_1 = 0.0125$ rad/(W-s) and $n_1 = 0.015$ V/(VAR)



(a) Phase a output voltage of the UPS



(b) Phase a output current of the UPS

Fig. 6. Output voltage and current of the UPS

current i_{ca} of the UPS. Upon interconnection, the current is seen to decrease as the UPS supplies only a portion of the load power demand while the ac grid supplies the rest. As has been observed from a plot of all three phases v_a, v_b and v_c that these are balanced sinusoids with very little distortion due to the switching frequency harmonics of the inverter.

VII. Conclusions

We presented a detailed examination of the stability of the line interactive UPS connected to the ac grid. The studies in related literature have performed stability analysis for particular topologies of UPS units connected in parallel. However, these studies do not elaborate on how the stability analysis can be extended for larger and more complex systems. In this paper, the method employed to analyze stability begins from the complete mathematical model of the line interactive UPS connected to the ac grid. By simplification, the transfer function of the controlled system has been obtained. The process of simplification can be applied to any system containing UPS units. The plant transfer function has been shown to be a non-minimum phase system. Furthermore, it has been shown that for large p-ω droop coefficients, the controlled system becomes unstable while q-V droop coefficients do not affect the stability of the controlled system. Simulations performed in Scilab verify the conclusions on stability of the controlled system for changes in the droop controller coefficients.

Experimental results show the operation of the line interactive UPS during transients when it is connected to the ac grid after initially being in standalone mode. The results show that the system remains stable following oscillations during the interconnection. Moreover, the power demanded by the load is seen to be shared by the ac grid and the UPS.

References

[1] M. Chandorkar, D. Divan, Y. Hu, and B. Banerjee, "Novel architectures and control for distributed ups systems," in *Applied Power Electronic Conference and Exposition*, vol. 2, February 1994, pp. 683–689.

[2] G. Venkataramanan and M. Illindla, "Microgrids and sensitive loads," in *IEEE Power Engineering Society Winter Meeting*, vol. 1, January 2001, pp. 315–322.

[3] M. Chandorkar, D. Divan, and R. Adapa, "Control of parallel connected inverters in standalone ac supply systems," *IEEE Transactions on Industrial Applications*, vol. 29, no. 1, pp. 136–143, January/February 1993.

[4] E. Coelho, P. Cortizo, and P. Garcia, "Small-signal stability for parallel-connected inverters in stand-alone ac supply systems," *IEEE Transactions on Industrial Applications*, vol. 38, no. 2, pp. 533–542, March/April 2002.

[5] J. Guerrero, L. Vicuna, J. Matas, M. Castilla, and J. Miret, "Output impedance design of parallel-connected ups inverters with wireless load-sharing control," *IEEE Transactions on Industrial Electronics*, vol. 52, no. 4, pp. 1126–1135, August 2005.

[6] C. Sao and P. Lehn, "Autonomous load sharing of voltage source converters," *IEEE Transactions on Power Delivery*, vol. 20, no. 2, pp. 1009–1016, April 2005.

[7] J. Guerrero, L. Vicuna, J. Matas, M. Castilla, and J. Miret, "Wireless-control strategy for parallel operation of distributed-generation inverters," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 5, pp. 1461–1470, October 2006.

[8] J. Guerrero, L. Hang, and J. Uceda, "Control of distributed uninterruptible power supply systems," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 8, pp. 2845–2859, August 2008.

[9] N. Pogaku, N. Prodanovic, and T. Green, "Modeling, analysis and testing of autonoumous operation of an inverter-based microgrid," *IEEE Transactions on Power Electronics*, vol. 22, no. 2, pp. 613–625, March 2007.

[10] J. W. Polderman and J. C. Willems, *Introduction to Mathematical Systems Theory: a Behavioral Approach*. Springer Verlag, 1997.

[11] P. Loh, M. Newman, D. Zmood, and D. Holmes, "A comparative analysis of multiloop voltage regulation strategies for single and three-phase ups systems," *IEEE Transactions on Power Electronics*, vol. 18, no. 5, pp. 1176–1185, September 2003.

[12] S. Iyer, *Second Annual Progress Seminar, IIT Bombay*. http://www.ee.iitb.ac.in/student/∼shivkvi/secondaps.pdf, 2009.