2012 American Control Conference
Fairmont Queen Elizabeth, Montréal, Canada
June 27-June 29, 2012

Flux Estimation from Vanadium and Cobalt Self Powered Neutron Detectors (SPNDs): Nonlinear Exact Inversion and Kalman filter approaches

Krishnan Srinivasarengan, Laxman Mutyam, Madhu N. Belur, Mani Bhushan, A.P. Tiwari, M.G. Kelkar and M. Pramanik

***Abstract*—Self Powered Neutron Detectors (SPNDs), which are widely used in a nuclear reactor for flux measurement, typically have different types of dynamics based on their emitter material: one delayed, second prompt and possibly nonlinear response characteristics. The measurement from the SPNDs hence need compensation to obtain the actual input flux. In this paper, we discuss the modeling of and input estimation design for Vanadium and Cobalt SPNDs. We obtain the structure and parameters of the Vanadium SPND model from its radioactive decay mechanism. We then obtain the other parameters of the model applying system identification tools on the available data corresponding to reactor trip. For Cobalt SPND, we design two model based input/state estimators; the nonlinearity being the key feature: the 'Exact Model Inversion' and the Extended Kalman filter. In the exact model inversion, we demonstrate that input flux can be calculated by solving a third degree polynomial. In the extended Kalman filter estimator, we propose a novel approach to improve the step response of Kalman filter algorithms by 'resetting' the state error covariance matrix. We use Matlab® simulation and reactor data to compare the advantages of the two filters. We show that in both Vanadium and Cobalt SPND cases, Kalman filter based algorithms provide a reasonable balance between speed and noise suppression. While the exact inversion provides an almost prompt, but noisy response to step changes, the modified Kalman filter has a noise-free response with a few minutes of settling time. We also demonstrate the ability of the proposed covariance reset Kalman filter to track step/sudden changes in the input.**

***Index Terms*—Self Powered Neutron Detectors, Kalman Filter, System Identification, Current build-up, Covariance matrix reset, Exact Inversion, Vanadium, Cobalt**

I. Introduction

The power generated by a nuclear reactor is determined by the amount of fission reaction that takes place, which in-turn depends on the number of neutrons available to cause fission. Therefore measurement of neutron flux is essential for estimating reactor power. In case of reactors with large cores, it is also necessary to know power distribution within the reactor core. Hence an accurate estimation of the neutron flux profile inside the reactor core is required for prompt and accurate control of power generation. Self Powered Neutron Detectors (SPNDs) are popularly used as in-core flux detectors for flux mapping, control and protection systems. The current generated by an SPND has typically two components: a prompt part, which is proportional to the neutron flux *now* and a delayed part, which is related to the flux in the near past. Depending on which component is higher, the SPNDs are broadly classified based on the characteristics of their emitter material into prompt and delayed type SPNDs [1]. A thorough modelling of SPNDs is critical to utilize the SPND signal for estimation of the current neutron flux accurately for various reasons, for example, in order to control the reactor for regulation of total generated power.

In this paper we treat problem as an unknown input estimation problem and present results obtained with a nuclear reactor which uses Vanadium (delayed-type) and Cobalt (prompt-type) SPNDs for flux measurement. While the Vanadium SPND model is linear, the Cobalt SPND has a nonlinear model. In Section II, we obtain an analytical model and the model parameters for the Vanadium SPND. In Section III we propose the state error covariance 'reset' approach and describe how this method plays a key role to improve the response of Kalman filter algorithms for sudden changes in neutron flux. In Section IV, we investigate two approaches to design observers for input estimation for Vanadium SPND. Simulation results as well as the results obtained when applied to actual measurement data from the reactor under study are discussed. In Section V, we study two approaches to perform inversion of Cobalt SPND signal. Simulation results and results with actual process data are given. Finally, Section VI summarizes the results.

II. Vanadium SPND: model identification

A. Model structure

A first principle model of Self Powered Neutron Detectors is obtained from the radioactive transition between different isotopes and their β and/or γ emissions. In case of Vanadium SPNDs, such first principle models are not available in the literature. However, by comparing the decay mechanism with a similar type of SPND whose model is readily available in literature, namely, Rhodium SPND in [2] and using [3], the model of Vanadium SPND is obtained as

$$\frac{dN_{51}(t)}{dt} = -\sigma_{51}N_{51}(t)\phi(t) \tag{1}$$

$$\frac{dN_{52}(t)}{dt} = \sigma_{51}N_{51}(t)\phi(t) - \lambda_{52}N_{52}(t) \tag{2}$$

$$i(t) = k_{pv}\sigma_{51}N_{51}(t)\phi(t) + k_{gv}\lambda_{52}N_{52}(t) \tag{3}$$

where

λ_{52} : Decay constant of $^{52}_{23}V$

$i(t)$: Current from the V-SPND

$\phi(t)$: Input flux in the reactor

N_{51} and N_{52} : Atomic densities of $^{51}_{23}V$ and $^{52}_{23}V$ respectively

σ_{51} : Neutron absorption cross-section of $^{51}_{23}V$ (microscopic)

k_{pv}, k_{gv} Probabilities of $^{51}_{23}V$ neutron capture and $^{52}_{23}V$ decay each leading to a current carrying electron respectively

In the above description, the parameters σ_{51} and λ_{52} are obtained from characteristics of the isotopes of Vanadium.

This work was supported by the Board of Research in Nuclear Sciences (BRNS), India. Grant Number: 2009/39/16-BRNS

S. Krishnan and M.N. Belur are with the Department of Electrical Engineering, M. Laxman with Systems & Control Engineering, and M. Bhushan with the Department of Chemical Engineering, Indian Institute of Technology Bombay, India. Corresponding author email: belur@iitb.ac.in

A.P. Tiwari is with Reactor Control Division, Bhabha Atomic Research Center, Mumbai, India.

M.G. Kelkar and M. Pramanik are with Nuclear Power Corporation of India Ltd., Mumbai, India

We used the data available from [3]. Apart from this, the sensitivity of the Vanadium SPNDs obtained using first-principles model [4] and verified against the experimentally measured sensitivity is also available. These are given in the Table I.

TABLE I
MODEL PARAMETERS : V SPND

Parameter	Values & Unit
N_{51}	6.86×10^{22} cm^{-3}
σ_{51}	4.9×10^{-24} cm^2
λ_{52}	0.0036 s^{-1}
S_v	1.415×10^{-20} A·cm^2 · s/n

It has been noted [5] and also observed during simulations that, the burn-up rate of ${}^{51}_{23}V$ is 0.012% per month in a typical thermal neutron flux of 10^{13} n cm^{-2} s^{-1}. Even if the typical thermal neutron flux is two or three times of this value, the burn-up rate can be neglected and hence we assume in this paper that ${}^{51}_{23}V$ is constant: this gives the following *linear* model for the Vanadium SPND

$$\frac{dN_{52}(t)}{dt} = -\lambda_{52}N_{52}(t) + \sigma_{51}N_{51}\phi(t) \quad (4)$$

$$i(t) = k_{pv}\sigma_{51}N_{51}\phi(t) + k_{gv}\lambda_{52}N_{52}(t), \quad (5)$$

which gives the transfer function from input ϕ to output i as

$$G_v(s) = \sigma_{51}N_{51}(k_{pv} + k_{gv})\left[\frac{1 + s\left(\frac{k_{pv}}{\lambda_{52}(k_{pv}+k_{gv})}\right)}{1 + s/\lambda_{52}}\right]. \quad (6)$$

B. System identification approach

Measurement data from Vanadium SPND was available during a reactor shutdown period. Since the reactor shutdown is almost instantaneous, this is considered as a step change. Matlab® system identification tools were used to perform model identification using Prediction Error Methods (PEM) [6] and with the following process model structure:

$$\frac{I(s)}{\Phi(s)} = S_v\frac{T_z s + 1}{T_p s + 1} \quad (7)$$

C. Results

The above system identification steps were applied to data from 100 Vanadium SPNDs. Along with the T_z, T_p values obtained through system identification, the S_v value given in Table I was used and the model parameters obtained as in Table II. These are the computed average values of these model parameters.

TABLE II
COMPUTED V SPND MODEL PARAMETERS

Parameter	Value & unit
T_z	26 s
T_p	313 s
k_{pv}	3.487×10^{-21} A-s
k_{gv}	3.846×10^{-20} A-s

III. COVARIANCE RESET MECHANISM WITH KALMAN FILTER

The Kalman filter and Extended Kalman filter (EKF) algorithms are used for input estimation in Vanadium (Section IV) and Cobalt SPNDs (Section V) respectively. The approach of the implementation is similar to, say [2], [7], in that we consider the input as a constant and this is added in the state equation of the system. This approach has been illustrated to be superior to other approaches in these papers. However, when there are sudden changes in the neutron flux, estimation error is very high due to the constraint of input being constant.

In order to improve the promptness of the response of the algorithm during step changes in neutron flux, we modify the state error covariance matrix when a sudden and significant change in the neutron flux is detected. This detection is done by monitoring the innovation and by *resetting* the state error covariance matrix to a value depending upon the magnitude of change. The resetting is typically done to a 'large' positive definite matrix: this resetting mechanism thus allows for renewed (and temporary) trackability of the input flux despite the sudden change. The subsequent sections describe the result of this modification to Vanadium and Cobalt SPNDs.

IV. VANADIUM SPND: INPUT ESTIMATION

Existing literature on the SPNDs propose different approaches to design compensation for delayed-type SPNDs, but they focus on Rhodium SPNDs. In [2], three approaches: dominant pole method, direct inversion and Kalman filter are studied. In [9], a similar inversion of transfer function is discussed where appropriate inverse transfer functions are realised using Inverse Function Amplifier (IFA). A discrete time Kalman filter is implemented in [7] for Rhodium SPND and the response of the filter is compared with that of 'exact inversion'. In [10], the authors propose an H_∞ filtering approach which reduces 'worst case' estimation error using a a Linear Matrix Inequalities (LMI) formulation. Dynamic compensation techniques using rank order filtering of Vanadium SPNDs have been addressed in [8] and [11].

In this paper we propose two approaches for input estimation for Vanadium SPND in the following subsections: the exact inversion and the Kalman filter methods. These approaches have similarity to the methods proposed in [2] and [7] for a Rhodium SPND, in that we consider the input as one of the states with no dynamics. However, the covariance reset mechanism proposed in this work is not in these references.

A. Exact Model inversion

Since the Vanadium SPND model $G_v(s)$ is linear and proper, inverting the model to obtain the input is possible, with the zeros of $G_v(s)$ determining the stability. The initial delay in convergence of the estimated flux to the actual flux is typically due to difference in initial conditions.

Model

Using equation (7), the state space model of the Vanadium SPND and the corresponding inversion assuming $x := N_{52}$ is:

$$\dot{x}(t) = -\frac{1}{T_z}\, x(t) + \left(\frac{T_z - T_p}{S_v T_z^2}\right) i(t) \quad (8)$$

$$\hat{\phi}(t) = x(t) + \frac{T_p}{S_v T_z}\, i(t) \quad (9)$$

Simulation results

- System Simulation Parameters
  - To make a comparison between the exact model inversion and Kalman filter approaches, the system simulation was performed in the same way considering input also as one of the states of the system.

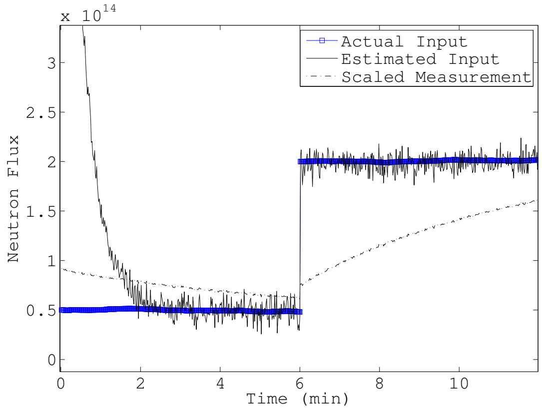

Fig. 1. Input estimation using exact inversion approach

- Process Noise Variance: diag(10^{24},10^{22})
- Measurement Noise Variance: 10^{-16} (we generated noise ranges between 1-5% of the measurement current amplitude)

Figure 1 shows the simulation results for the exact inversion method. Note that, except during the initial period (which is due to inexact initial conditions of the observer), the estimation promptly tracks the unknown input. The drawback is that the filter, due to its lead compensation characteristics, results in noisy estimate. In the simulation results throughout this paper, the term scaled measurement is used to denote output of the sensor scaled by a fixed number. That is, when no compensation is used, the estimated flux is a scaled value of the current measured by the sensor. It shall also be considered as measurement without compensation.

B. Kalman filter

Problem formulation

The Kalman filter implementation for input estimation of Vanadium SPND is based on the idea of assuming the input as one of the states subject to the dynamic constraint of it being constant. The system model (with the first equation the same as (1)) is then given by

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\phi}(t)\end{bmatrix} = \begin{bmatrix}-\lambda_{52} & \sigma_{51}N_{51}\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \phi(t)\end{bmatrix} \tag{10}$$

$$y(t) = [k_{gv}\lambda_{52} \quad k_{pv}\sigma_{51}N_{51}]\begin{bmatrix}x\\ \phi\end{bmatrix} \tag{11}$$

where $y(t)$ is the current generated by the SPND.

A discrete Kalman filter implementation was used for solving this problem which used the discretized system representation given by

$$\begin{bmatrix}x(n+1)\\ \phi(n+1)\end{bmatrix} = A_d\begin{bmatrix}x(n)\\ \phi(n)\end{bmatrix} + \begin{bmatrix}w_1(n)\\ w_2(n)\end{bmatrix} \tag{12}$$

$$y(n+1) = [k_{gv}\lambda_{52} \quad k_{pv}\sigma_{51}N_{51}]\begin{bmatrix}x(n+1)\\ \phi(n+1)\end{bmatrix} + v(n+1) \tag{13}$$

where $A_d := \begin{bmatrix}e^{-\lambda_{52}T_s} & \frac{\sigma_{51}N_{51}}{\lambda_{52}}[1-e^{-\lambda_{52}T_s}]\\ 0 & 1\end{bmatrix}$ and w and v are process and measurement noise respectively.

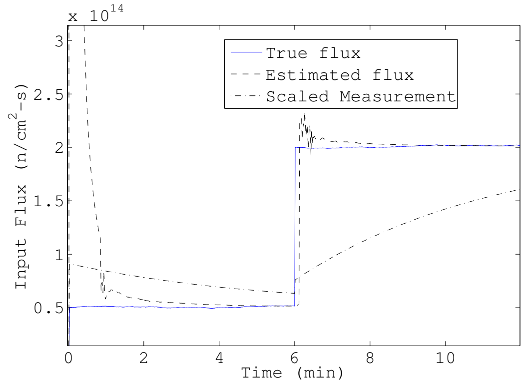

Fig. 2. Input Estimation using Kalman Filtering

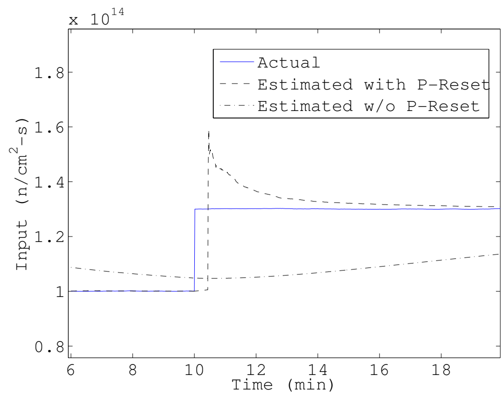

Fig. 3. Illustration of the effect of State Error Covariance Reset in Input Estimation during a step change in flux

Simulation results

The following parameters were used in the Kalman filter and we used the filter over the data generated using the simulation as discussed in Subsection IV-A above.

- Kalman Filter Parameters
  - Process Covariance matrix: diag(1,1)
  - Measurement Covariance matrix: 10^{-16}
  - Initial State Error Covariance matrix: diag(10^{16}, 10^{14}).
- Simulation Results: Figure 2 shows the simulation results of the designed Kalman filter for the above parameters.
  - Settling time ($\pm 2\%$ of final value): ≈ 3.5 minutes.
  - Root Mean Square (RMS) error of flux (after the settling time when the input flux is 2×10^{14} n cm^{-2} s^{-1}) is equal to 10^{12} and decreasing with respect to time.
- As discussed in Section III, a state error covariance matrix reset mechanism was employed in cases where there is a sudden and significant change in the neutron flux. The mean value of the residue of Kalman filter is monitored for a fixed window of 50 seconds. The results of Kalman filter response to a step change with and without covariance reset mechanism is illustrated in Figure 3, where part of a simulation involving step change in flux is shown: the estimate is able to track the input.

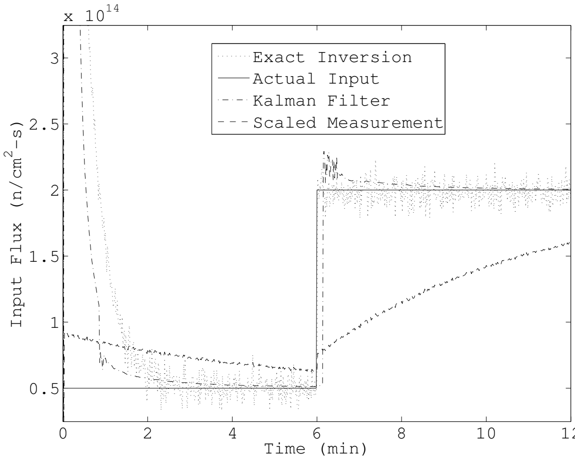

Fig. 4. Comparison of compensation technique responses for simulation data

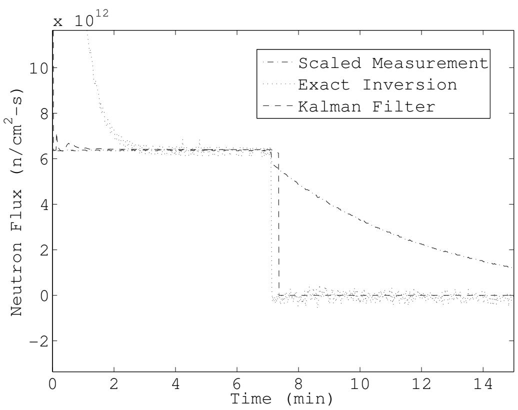

Fig. 5. Comparison of compensation technique responses for reactor trip data

C. *Comparative results for the two approaches*

Figure 4 gives a comparative analysis of the response of Exact Inversion and Kalman filter (along with the scaled measurement) for a simulated data. It shows that the promptness of Kalman filter approach (with the covariance reset) is comparable to that of Exact Inversion approach, but with lesser noise. There is also an overshoot of about 15 % in both the Exact Inversion and the Kalman Filter.

Figure 5 gives qualitative comparison of the two approaches, when applied to actual reactor data dynamics. A lower limit of 10^4 was imposed on the estimated input flux to rule out possible undershoots. The Kalman filter provides a prompt response and is almost noisless compared to Exact Inversion.

V. Cobalt SPND: model based observers

Cobalt SPND model

The model of Cobalt SPND used in this paper is as in [12]. Along with these model parameters, experimentally determined Cobalt sensitivity (S_c), given in the Table III, are used in this paper. The N_{59} burn-up (0.094% per month [5]) is neglected along with the prompt gamma component, to obtain the following reduced model:

TABLE III
Parameters values in Cobalt SPND Model

Parameter	Values & Unit
N_{59}	8.843×10^{22}
σ_{59}	37×10^{-24} cm^2
σ_{60}	2×10^{-24} cm^2
λ_{60}	1.501×10^{-5} hr^{-1}
λ_{61}	0.420 hr^{-1}
k_{60}	1.358×10^{-32} A
k_{61}	3.7996×10^{-27} A
S_c	0.813×10^{-20} A-cm^2-s/n

$$\frac{dN_{60}(t)}{dt} = N_{59}\sigma_{59}\phi(t) - \lambda_{60}N_{60}(t) - N_{60}(t)\sigma_{60}\phi(t) \quad (14)$$

$$\frac{dN_{61}(t)}{dt} = N_{60}(t)\sigma_{60}\phi(t) - \lambda_{61}N_{61}(t) \quad (15)$$

$$i(t) = k_{60}N_{60}(t) + k_{61}N_{61}(t) + S_c\phi(t) \quad (16)$$

In [12], the author shows a detailed analysis of Cobalt SPND model and illustrates the effects of background build-up components and emphasizes the importance of compensating for them to prolong the SPND life. However attempts for this compensation have been limited. In [1], the authors build a Cobalt SPND which has a reduced background buildup. However, effective computational compensation techniques have not been investigated for the problem of reducing the effect due to build-up of current. This paper illustrates the feasibility of designing compensation techniques using model based observers. The challenge in this case is the nonlinearity in Cobalt SPND model. In this section, two approaches to design model based observers for the Cobalt SPND model (14)$-$(16) are described and compared.

A. *Exact Inversion*

This method utilizes the model structure of the Cobalt SPND and performs an inversion with the help of measurement derivatives. To obtain a unique estimate of flux, a dynamic constraint, $\dot{\phi} = 0$ is considered. Unlike the Vanadium SPND which could be modelled fairly accurately using a linear model, the build-up component in the Cobalt SPND, which contributes significantly to the SPND current, causes the Co SPND model to be nonlinear. The Exact Inversion approach cannot yield an inverse 'transfer function' due to this nonlinearity. This subsection proposes a novel approach to deal with the Co SPND nonlinearity: we describe how under the constant flux assumption, the flux satisfies[1] a cubic polynomial that can be found from the measurements and model parameters.

The approach can be summarized as follows. We impose the constraint $\dot{\phi} = 0$. This leads to a 3^{rd} degree polynomial, say $p_0 + p_1\phi + p_2\phi^2 + p_3\phi^3$, one of whose roots is the actual flux. We show that the coefficients of this polynomial are dependent on model parameters and the first and second derivatives of present measurement: thus the coefficients are functions of time. For this cubic polynomial, while one root is guaranteed to be real, extensive simulation shows that exactly one root is close to the actual value of ϕ. The coefficients p_i

[1]For an LTI system excited by a constant and unknown input, with unknown initial conditions, assuming observability of a closely related LTI system, one can use a *linear* combination of the output and its derivatives to give values of the input and the current states immediately.

of the above equation are as follows.
$p_3 := \sigma_{60} g_1 g_2 + \sigma_{59} N_{59} g_1^2$
$p_2 := 2\sigma_{59} N_{59} g_1 g_3 + g_2(\sigma_{60} g_3 + \lambda_{60} g_1) - \sigma_{60} g_1(\dot{y} + \lambda_{61} y)$
$p_1 := \sigma_{59} N_{59} g_3^2 + \lambda_{60} g_2 g_3 - g_1 \ddot{y} - \dot{y}$
$\quad -(\sigma_{60} g_3 + \lambda_{60} g_1 + \lambda_{61} g_1) - \lambda_{61} y(\sigma_{60} g_3 + \lambda_{60} g_1)$
$p_0 := -[g_3 \ddot{y} + (\lambda_{61} + \lambda_{60}) g_3 \dot{y} + g_3 + \lambda_{60} \lambda_{61} y]$.

Simulation parameters

For the purpose of simulation of the continuous time model of Cobalt SPND and for sampling the necessary variables, we used Matlab® ODE solvers: ode45 and ode15s. The derivatives of the measurements were obtained using first and second difference methods. The maximum of the three roots was chosen as the estimated flux.

Simulation results without noise

Figure 6 shows the simulation results for the estimation of input using exact inversion, when no noise is assumed to be present. It expectedly takes three measurements to achieve convergence of estimated input, since the first difference method used to compute first and second derivatives requires three measurements.

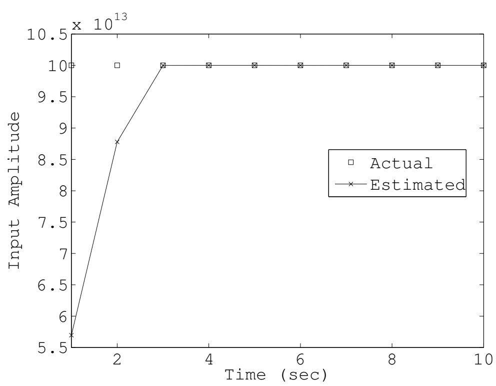

Fig. 6. Estimated Input with Exact Inversion (case for no noise)

Simulation results with noise

It has been observed that the algorithm does not converge when measurement noise of even negligible variance is present. This is expected due to the involvement of differentiation in computing the estimated inputs. Using better differentiation techniques results in a marginal improvement.

One possible option to overcome this problem is, instead of sampling the system every second, the measurement and the computation of estimated input is done at a lower sampling rate. This can reduce the effects that noisy measurements have on the estimated input computation. In Figure 7, a sampling rate of 1 hour was used and the estimated input, though noisy, yielded accurate results. The added measurement noise had variance 10^{-18}. For the same noise characteristics, sampling measurement each second gives unsatisfactory results.

In Figure 8, the estimated output after passing through a first order low pass filter (with a time constant of 10 hours) is shown. This demonstrates that a more refined filtering method achieves better input estimation. We describe the extended Kalman filter in the following subsection.

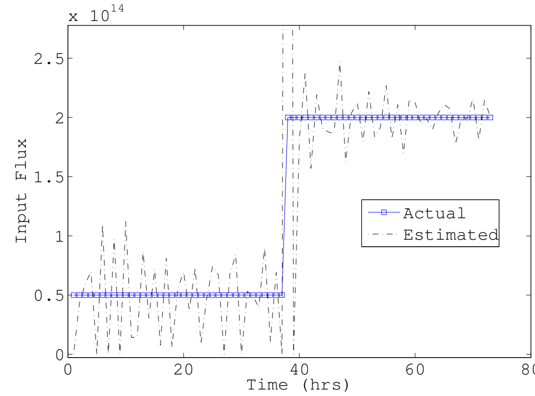

Fig. 7. Estimated Input with a 1 hour sampling rate for simulations with measurement noise

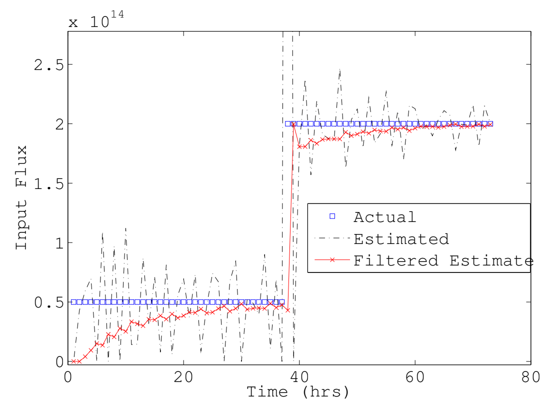

Fig. 8. Estimated and Filtered Input for simulations with measurement noise

B. Extended Kalman filter

Problem formulation

One of the ways to overcome the problem faced by the exact inversion's noisy response is to use a Kalman filter. Since the Cobalt SPND has a nonlinear model, we use an extended Kalman filter (EKF). In formulating the problem for using EKF for input estimation, we consider the augmented system by considering the input as one of the states of the system and imposing the dynamics constraint $\dot{\phi} = 0$.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{\phi}(t) \end{bmatrix} = A_{\mathrm{Co}}(t) \begin{bmatrix} x_1(t) \\ x_2(t) \\ \phi(t) \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \tag{17}$$

$$y(t) = [k_{60} \quad k_{61} \quad S_c] \begin{bmatrix} x_1(t) \\ x_2(t) \\ \phi(t) \end{bmatrix} + v \tag{18}$$

where $A_{\mathrm{Co}}(t) := \begin{bmatrix} -\lambda_{60} - \sigma_{60}\phi_g & 0 & \sigma_{59} N_{59} - \sigma_{60} x_{1g} \\ \sigma_{60}\phi_g & -\lambda_{61} & \sigma_{60} x_{1g} \\ 0 & 0 & 0 \end{bmatrix}$

and w and v are the process and measurement noises. The variables ϕ_g and x_{1g} are the estimated flux and state values from the previous step.

Simulation details and results

- The Kalman filter implementation was in discrete time and hence the linearized system was discretized using the zero-order hold method.

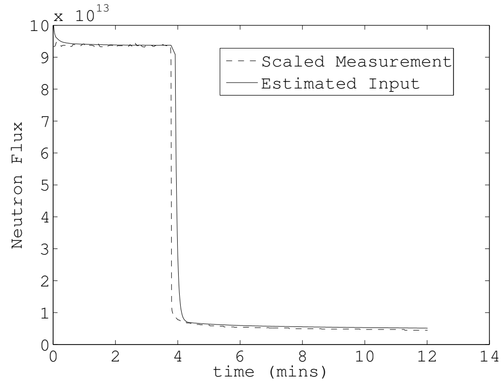

Fig. 10. EKF estimated input and scaled measurement of a reactor trip data

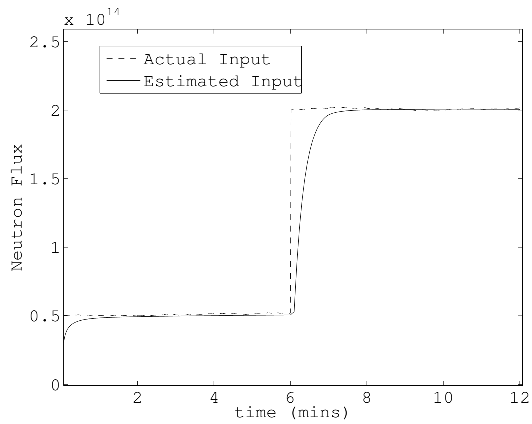

Fig. 9. Input Estimation by EKF Algorithm for a simulated input

- The process covariance matrix used is diag(10^{30},10^{28},10^{22}) .
- The initial conditions used for the system simulations corresponded to the values of N_{60} and N_{61} attained after two years with a constant flux of 10^{14} n cm^{-2} s^{-1}.
- Process and measurement noise variances used were diag(1,1,1) and 10^{-16} respectively.
- The estimated input was restricted to values above 10^4 so as to avoid impractical values. Similarly, the other states were restricted to positive values.
- Figure 9 shows results of EKF algorithm on a simulated data where the initial guess of ϕ to start with was 10^{13}. Settling time (to within 2% of final value) is approximately 2 minutes. The Root Mean Square (RMS) error of flux (after the settling time when the input flux is 2×10^{14}) is approximately 10^{12} $n/\text{cm}^2 - sec$ and decreasing with respect to time.
- In Figure 10, the EKF algorithm is applied to reactor trip measurement data obtained from an actual reactor. Notice that the algorithm is able to track the scaled measurement within the settling time of two minutes.
- Covariance Reset: As elaborated in Section III, to improve the response of EKF output for sudden changes in input flux we propose a covariance reset procedure. As in the Vanadium SPND case, here too the promptness of the response was comparable to that of Exact Inversion.
- Dependence on initial conditions: The input estimation was not affected by the inexact initial estimate of N_{60}, N_{61}. However, such inexact estimates adversely affected the estimation of the states.

VI. Concluding remarks

In this paper, we have discussed the model based observers for Vanadium and Cobalt SPNDs. For Vanadium SPNDs, we derived the analytical model and its parameters and then designed two types of observers based on exact model inversion and Kalman filter. We employed a modification of the Kalman filter algorithm which involves a resetting in the error covariance matrix when the innovation indicates step changes in the input. For Cobalt SPNDs, despite its nonlinearity, we showed that when the flux is constant, the flux satisfies a third order polynomial whose coefficients can be found from the measurement and model parameters: we called this the Exact Inversion method. We also designed an Extended Kalman Filter with covariance reset for the Cobalt SPND. Some of the key results are summarized as follows.

For Vanadium SPND, the response of the exact inversion was prompt, but noisy. The Kalman filter with the covariance reset provided a step response settling time of few minutes with negligible RMS error (3.5 mins and 1% respectively for the case shown in this paper). The results here are shown for a model of Vanadium SPND with average parameter values, and appropriate tuning of this covariance reset for individual SPND models could improve the response time. The modified Kalman filter with covariance reset can be used as the observer for Vanadium SPND.

For Cobalt SPND, the new approach of exact model inversion response was prompt when there was no noise, but was illustrated to be prone to noise due to its dependence on derivatives. The extended Kalman filter algorithm with the covariance reset provided a step response settling time of few minutes with negligible RMS error (2 minutes and 1% respectively for the case shown in this paper). The extended Kalman filter was found to be more suitable as an observer for the Cobalt SPND.

References

[1] N. P. Goldstein and W. H. Todt, "A survey of Self-Powered Detectors - present and future," *IEEE Transactions on Nuclear Science*, vol. 26, pp. 916–923, feb. 1979.

[2] G.-S. Auh, "Digital dynamic compensation methods of Rhodium Self-Powered Neutron Detector," *Journal of the Korean Nuclear Society*, vol. 26, pp. 205–211, 1993.

[3] C. Lederer, J. Hollander, and I. Perlman, *Table of isotopes*. John Wiley and Sons Inc., New york, 1967.

[4] H. Warren, "Calculational model for Self-Powered Neutron Detector," *Nuclear Science and Engineering*, vol. 48, pp. 331–342, 1972.

[5] W. Todt, "Characteristics of Self Powered Neutron Detector used in power reactors," tech. rep., Imaging and sensing technology corporation, Horseheads, Newyork, 1996.

[6] L. Ljung, "Prediction error method: Matlab system identification toolbox," *System Identification Toolbox, Matlab*, v7.3, 2008.

[7] M. Kantrowitz, "An improved dynamic compensation algorithm for Rhodium Self-Powered Neutron Detectors," *IEEE Transactions on Nuclear Science*, vol. NS-34, pp. 562–566, 1987.

[8] R. Kapatral, K. Kumar, A. Das, and S. Kataria, "FPGA implementation of dynamic compensation algorithm for Vanadium self-powered neutron detectors," in *Proceedings of the Symposium on Intelligent Nuclear Instrumentation (INIT)*, pp. 377–382, 2000.

[9] S. Yusuf and D. Wehe, "Analog and digital dynamic compensation techniques for delayed Self-Powered Neutron Detectors," *Nucl. sci. eng.*, vol. 106, pp. 399–408, 1990.

[10] M. Park, Y. Kim, K. Cha, and M. Kim, "H_∞ filtering for dynamic compensation of Self-Powered Neutron Detectors: a Linear Matrix Inequality based method," *Annals of Nuclear Energy*, vol. 26, pp. 1669–1682, 1999.

[11] A. Das, S. Chaganty, and K. Gopalakrishnan, "Dynamic compensation of V SPND using direct inversion and rank order filtering," in *Proceedings of the Symposium on Intelligent Nuclear Instrumentation (INIT)*, pp. 118–123, 2001.

[12] J. Sovka, "Response of Cobalt Neutron Flux Detectors," tech. rep., AECL-3368, 1969.