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Application of model order reduction techniques to Space-time kinetics model of AHWR

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***Abstract*—Advanced Heavy Water Reactor (AHWR), a large nuclear reactor in which heat removal is based on natural circulation of light water in coolant channels, is described by an 80^{th} order model for accurate representation of its core neutronics behavior. The model, besides being nonlinear, is stiff and control design and analysis studies for AHWR are accompanied by serious numerical ill-conditioning problems. Hence, there is a strong motivation for obtaining reduced order model for AHWR. In particular, the application of model order reduction based on Davison's and Marshall's dominant mode retention and Singular Perturbation techniques has been explored. Also, their performance, relative to each other, has been assessed by comparing the characteristics of the three model reduction techniques with the characteristics of the original higher order model. All of these three techniques are found to be very effective in obtaining a lower order simpler models for AHWR.**

I. INTRODUCTION

Description of large-scale systems by mathematical models involves a set of first order differential or difference equations. These models can be used to simulate the system response and predict the behavior. Sometimes, these mathematical models are also used to modify or control the system behavior to conform with certain desired performance. In practical control engineering applications with the increase in need for improved accuracy, mathematical models lead to high order and complexity. Although, the well established modern control concepts are valid for any system order, they may not give fruitful control algorithms in control design. Moreover, working with very high order model involves computational complexity and need for high storage capability. Sometimes, the presence of small time constants, masses, etc. may give rise to an interaction among slow and fast dynamic phenomena with attendant ill-conditioning of stiff numerical problems. When analyzing and controlling these large-scale dynamic systems, it is extremely important to look for and to rely upon efficient simplified reduced order models which capture the main features of the full order complex model.

In literature many different methods have been suggested for model order reduction, such as retaining dominant modes [1] [2], model reduction by aggregation [3] and decomposition of higher order model into slow and fast systems, viz. time-scale methods and singular perturbation analysis [4]. The basic principle in obtaining a reduced order model for a high order system is to neglect those eigenvalues which are farthest from the origin and retain only the dominant eigenvalues. This implies that the reduced order system has only dominant eigenvalues and its overall behavior is also very similar to the higher order model.

Advanced Heavy Water Reactor (AHWR) [5] is a 920 MW (thermal), vertical, pressure tube type, heavy water moderated, boiling light water cooled, natural circulation reactor. The physical dimensions of the AHWR core are large compared to the neutron migration length. Therefore from neutronic viewpoint, the behavior tends to be loosely coupled, due to which a serious situation called 'flux tilt' may arise in AHWR followed by an operational perturbation. Therefore it is necessary to provide on-line monitoring and control schemes during the reactor operation. A reasonably accurate space-time kinetics model for describing the behavior of AHWR is developed for control related studies using nodal methods [6] [7]. An important characteristic of the model based on nodal methods is that the order of mathematical model depends on the number of nodes into which the reactor spatial domain is divided. A rigorous model with more number of nodes will give good accuracy in on-line monitoring and control, but its order is very high. At the same time, it is quite common in nuclear reactor models that they exhibit simultaneous dynamics of different speeds. Such behavior leads the mathematical model exhibiting multiple time scales, which may be susceptible to numerical ill-conditioning. Hence, it is very much essential to develop a suitable mathematical model of lower order which alleviates the dimensionality and numerical ill-conditioning problems in computations.

In the present work, an 80^{th} order linearized model is developed by considering the time dependent core neutronic equations with 17 node scheme and control rod dynamic equations as derived in [7]. Model decomposition based on singular perturbation and time-scale methods and control design have already been attempted in [8]–[13]. However, various other methods are available for model order reduction. Motivated by this, we obtain some simpler models based on retaining dominant modes and compare their performance with Singular Perturbation technique. Also, we present a method to handle the severe numerical ill-conditioning occurring in computations due to the presence of slow control rod dynamics by decoupling higher order model into very slow and fast models. Fast model is used to obtain the simplified model and finally the reduced order model is formulated by augmenting

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the control rod dynamics with the obtained simplified model. The efficacies of the various methods have been evaluated by comparison of the transient performance of the reduced models with that of the original model.

II. Review of model reduction techniques

We consider a large-scale dynamical system described by the linear time invariant model,

$$\dot{x} = Ax + Bu, \quad (1)$$
$$y = Mx \quad (2)$$

where x, u and y denote respectively the n dimensional state, m dimensional input and p dimensional output vectors and A, B and M the system, input and output matrices. For large-scale systems, the order n is quite large, and the intent of model order reduction is to obtain a simplified lower order model which preserves the input and output behavior of the system. The reduced model of order $n_1 < n$, has same response characteristics as that of the original model with far less storage requirements and much lower evaluation time. The resulting model given by,

$$\dot{x_r} = A_r x_r + B_r u, \quad (3)$$
$$y_r = M_r x_r \quad (4)$$

might be used to replace the original description in simulation studies or it might be used to design a reduced order controller or observer. The application of Davison's model order reduction technique, Marshall's model order reduction technique and Singular Perturbation technique has been explored. These techniques are briefly described in the following.

A. Davison's Technique

A structured approach to model order reduction was described in [1], which approximates the original order n of the system to n_1 by neglecting the eigenvalues of the original system that are farthest from the origin and retains only the dominant eigenvalues and hence the dominant time constants of the original system are present in the reduced order model. Initially the system states are rearranged in such a manner that the eigenvectors corresponding to the states to be retained from (1) are placed first.

Let the state vector x is partitioned into dominant and non-dominant parts as x_1 which are considered to be retained and x_2 which are to be ignored. Therefore the partitioned form of (1), (2) is

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix} u, \quad (5)$$
$$y = \begin{bmatrix}M_1 & M_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}, \quad (6)$$

where the order of x_1 is n_1, and the order of x_2 is $n - n_1$. Further consider the representation of the system (5),(6) by the equivalent diagonal form

$$\begin{bmatrix}\dot{z_1}\\ \dot{z_2}\end{bmatrix} = \begin{bmatrix}\tilde{A}_1 & 0\\ 0 & \tilde{A}_2\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix} + \begin{bmatrix}\tilde{B}_1\\ \tilde{B}_2\end{bmatrix} u, \quad (7)$$
$$y = \begin{bmatrix}\tilde{M}_1 & \tilde{M}_2\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}, \quad (8)$$

where order of z_1 is n_1 and that of z_2 is $n - n_1$,

$$\tilde{A}_1 = diag.\begin{bmatrix}\lambda_1 & \lambda_2 & ... & \lambda_{n_1}\end{bmatrix}, \quad (9)$$
$$\tilde{A}_2 = diag.\begin{bmatrix}\lambda_{n_1+1} & \lambda_{n_1+2} & ... & \lambda_n\end{bmatrix} \quad (10)$$

and the eigenvalues λ_i, $i = 1, 2, ..., n_1$ are to be retained in approximate model. Let

$$x = Vz = \begin{bmatrix}V_{11} & V_{12}\\ V_{21} & V_{22}\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix} \quad (11)$$

be the required linear transformation for obtaining the diagonal form representation.

According to Davison's method [1] the modes in z_2 are non-dominant and therefore can be ignored, thus setting $z_2 = 0$ in (11) gives reduced order model (3), (4) where

$$A_r = V_{11}\tilde{A}_1 {V_{11}}^{-1}, \quad (12)$$
$$B_r = V_{11}\tilde{B}_1, \quad (13)$$
$$M_r = \tilde{M}_1 {V_{11}}^{-1}, \quad (14)$$
$$\text{and} \quad x_2 = V_{21}{V_{11}}^{-1}x_1. \quad (15)$$

Thus the original n^{th} order model is approximated by n_1^{th} order model. The first n_1 state variables of the original model are approximated by the state variables of the reduced order model and the $n - n_1$ state variables are expressed in terms of the first n_1 state variables by (15).

B. Marshall's Technique

An alternate method for the computation of reduced order model is proposed in [2], in which it is assumed that $\dot{z_2} = 0$ in (7), which then yields

$$\dot{z_1} = \tilde{A}_1 z_1 + \tilde{B}_1 u \quad (16)$$
$$\text{and} \quad 0 = \tilde{A}_2 z_2 + \tilde{B}_2 u. \quad (17)$$

From (11), we have $z = V^{-1}x = \begin{bmatrix}U_{11} & U_{12}\\ U_{21} & U_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$. Then from (17), we obtain

$$x_2 = -U_{22}^{-1}U_{21}x_1 - U_{22}^{-1}\tilde{A}_2^{-1}\tilde{B}_2 u. \quad (18)$$

Substituting the solution of x_2 from (18) into (5), the reduced order model is obtained as (3) and (4), where

$$A_r = A_{11} - A_{12}U_{22}^{-1}U_{21}, \quad (19)$$
$$B_r = B_1 - A_{12}{U_{22}}^{-1}{\tilde{A}_2}^{-1}\tilde{B}_2, \quad (20)$$
$$\text{and} \quad x_2 = -U_{22}^{-1}(U_{21}x_1 - \tilde{A}_2^{-1}\tilde{B}_1 u). \quad (21)$$

Again the original n^{th} order model is approximated by n_1^{th} order model. The first n_1 state variables of the original model are approximated by the state variables of the reduced order model and the $n - n_1$ state variables are expressed in terms of the first n_1 state variables by (21).

C. Singular Perturbation Technique

In Linear time invariant models of large scale systems, the interaction of slow and fast modes is common feature and it leads the mathematical models to be ill-conditioned in control design. Singular Perturbation analysis [4] provides a simple means to obtain approximate solutions to the original system as well as it alleviates the high dimensionality problem.

In this method both the slow and fast modes are retained, but analysis and design problems are solved in two stages. By a suitable regrouping of the state variables, the original higher order system can be expressed into standard singularly perturbed form in which the derivatives of some of the states are multiplied by a small positive scalar ε, i.e.,

$$\begin{aligned} \dot{x_a} &= F_{11}x_a + F_{12}x_b + G_1u, x_a(0) = x_{10}, && (22)\\ \varepsilon\dot{x_b} &= F_{21}x_a + F_{22}x_b + G_2u, x_b(0) = x_{20} && (23)\\ \text{and} \quad y &= \acute{M}_1x_a + \acute{M}_2x_b. && (24) \end{aligned}$$

where $x_a \in R^{n_1}$ is the slow state vector, $x_b \in R^{n_2}$ is the fast state vector. Let $\sigma(A) = \{\lambda_1, \lambda_2, ...\lambda_n\}$. By setting the parasitic parameter $\varepsilon = 0$ in (23), it yields

$$0 = F_{21}\bar{x}_a + F_{22}\bar{x}_b + G_2u$$

where $\bar{x}_a$, $\bar{x}_b$ are the variables of the system (22), (23) when $\varepsilon = 0$. If F_{22}^{-1} exists, then the solution of $\bar{x_b}$ into (22) results in reduced order model of order n_1 as

$$\begin{aligned} \dot{x_s} &= A_0x_s + B_0u_s, && (25)\\ y_s &= M_0z_s + N_0u_s, && (26) \end{aligned}$$

where

$$\begin{aligned} x_s &= \bar{x_a}, && (27)\\ u_s &= \bar{u}, && (28)\\ A_0 &= F_{11} - F_{12}F_{22}^{-1}F_{21}, && (29)\\ B_0 &= G_1 - F_{12}F_{22}^{-1}G_2, && (30)\\ M_0 &= \acute{M}_1 - \acute{M}_2F_{22}^{-1}F_{21}, && (31)\\ N_0 &= -\acute{M}_2F_{22}^{-1}G_2, && (32) \end{aligned}$$

and a fast system of order $n - n_1$ given by

$$\begin{aligned} \varepsilon\dot{x_f} &= A_{22}x_f + B_2u_f, && (33)\\ y_f &= \acute{M}_2x_f, && (34) \end{aligned}$$

where

$$\begin{aligned} x_f &= x_b - \bar{x}_b, && (35)\\ u_f &= u - \bar{u}. && (36) \end{aligned}$$

Therefore eigenvalues of original system are $\sigma(A) = \sigma(A_0) \cup \sigma(\frac{A_{22}}{\varepsilon})$.

III. Mathematical Model of AHWR

An extensive derivation of AHWR mathematical model is given in [6], [7] and the same has been used for the study carried out in this paper. However, for brevity the model is discussed briefly in the following.

A. Core Neutronics Model

The AHWR core is considered to be divided into 17 nodes, as shown in Fig.1 by the segments labelled from 1 to 17. The top and bottom reflector regions are divided into 17 nodes in identical pattern as the core, labelled form 18 to 51, whereas side reflector is divided into 8 nodes labeled form 52 to 59, giving 59 nodes in total. Ignoring the dynamical effects of xenon and iodine, the following set of nonlinear time dependent equations represents the nodal core model of AHWR.

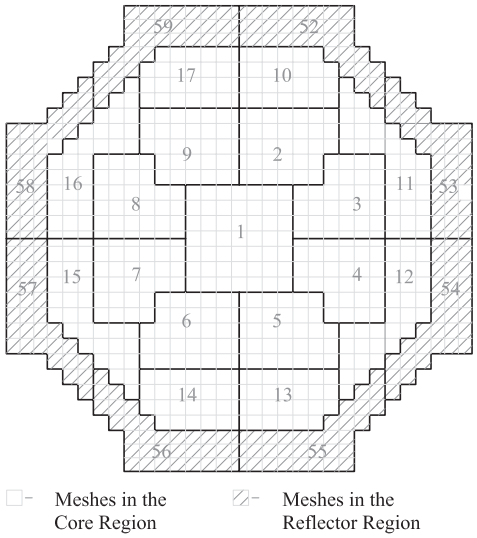


Fig. 1: 17 Node AHWR nodalization scheme with 17 nodes in the core, 17 nodes each in top and bottom reflectors and 8 nodes in surrounding reflector.

$$\begin{aligned} \frac{d\phi_h}{dt} &= -\omega_{hh}\upsilon_h\phi_h + \sum_{k=1}^{N_h}\omega_{hk}\upsilon_h\phi_k + (\rho_h - \beta)\frac{\phi_h}{l} \\ &\quad + \sum_{i=1}^{m}\upsilon_h\lambda_iC_{ih}, h = 1, 2, ...Z_p, && (37)\\ \frac{dC_{ih}}{dt} &= \frac{\beta_i\phi_h}{\upsilon_hl_h} - \lambda_iC_{ih}, i = 1, 2, ...m. && (38)\\ \frac{d\phi_h}{dt} &= -\omega_{hh}\upsilon_h\phi_h + \sum_{k=1}^{N_h}\omega_{hk}\upsilon_h\phi_k, \\ &\quad h = Z_p + 1, ...Z_p + Z_r. && (39)\\ \frac{dH_k}{dt} &= K_{RR}\vartheta_k, k = 2, 4, 6, 8. && (40) \end{aligned}$$

Where ϕ_h is the neutron flux in node h; C_{ih} the delayed neutron precursor concentration for group i; β_i and λ_i denote neutron fraction yield and decay constant for group i; υ_h the mean velocity of neutrons in node h; and l the prompt neutron life time; The coupling coefficients ω_{ij}, depend on the geometry and material composition and characteristic distance between the nodes. The detailed derivations of the above equation are given in [6]. ρ_h, the reactivity contributed by the movement of the RRs around their equilibrium positions, is expressed as,

$$\rho_h = \begin{cases} (-10.234H_k + 676.203) \times 10^{-6} & \text{if } (k = 2, 4, 6, 8) \\ 0 & \text{elsewhere} \end{cases} \quad (41)$$

B. Linearization and State-Space Representation.

The set of equations given by (37)-(40) can be linearized around the steady state operating conditions $(\phi_{h0}, C_{h0}, H_{j0})$. For simplicity, one group approximation of delayed neutron precursors is considered instead of six groups and the linear equations so obtained can be represented in standard state space form. For this, define the state vector as

$$x = \begin{bmatrix} x_{\phi_C}^T & x_C^T & x_{\phi_R}^T & x_H^T \end{bmatrix}^T.$$

where

$$x_{\phi_C} = [\delta\phi_1/\phi_{1_\bullet} \quad ... \quad \delta\phi_{17}/\phi_{17_\bullet}]^T \quad (42)$$

$$x_C = [\delta C_1/C_{1_\bullet} \quad ... \quad \delta C_{17}/\phi_{17_\bullet}]^T \quad (43)$$

$$x_{\phi_R} = [\delta\phi_{18}/\phi_{18_\bullet} \quad ... \quad \delta\phi_{59}/\phi_{59_\bullet}]^T \quad (44)$$

$$x_H = [\delta H_2 \quad \delta H_4 \quad \delta H_6 \quad \delta H_8]^T \quad (45)$$

in which δ denotes the deviation from respective steady state value of the variable. Likewise define the input vector as $u = [\delta v_2 \quad \delta v_4 \quad \delta v_6 \quad \delta v_8]$ and the output vector as $y = [y_1 \quad ... \quad y_{17}]$, where $y_i = \delta\phi_i/\phi_{i_\bullet}$ denotes the corresponding deviation in nodal flux. Then the system of equations (37)-(40) can be expressed in linear standard state space form (1) and (2), with

$$A = \begin{bmatrix} A_{\phi_C\phi_C} & A_{\phi_C C} & A_{\phi_C\phi_R} & A_{\phi_C H} \\ A_{C\phi_C} & A_{CC} & 0 & 0 \\ A_{\phi_R\phi_C} & 0 & A_{\phi_R\phi_R} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (46)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & B_H^T \end{bmatrix}^T, \quad (47)$$

$$\text{and} \quad M = [M_{\phi_C} \quad 0 \quad 0 \quad 0]. \quad (48)$$

where

$$A_{\phi_C\phi_C}(i,j) = \begin{cases} -\omega_{ij}v_i + \frac{\rho_{i0}}{l_i} - \frac{\beta}{l_i} & \text{if } (i=j) \\ \omega_{ij}v_i\frac{\phi_{j0}}{\phi_{i0}} & \text{if } (i\neq j) \end{cases}$$

$$A_{\phi_C C} = -\beta.diag\begin{bmatrix} \frac{1}{l_1} & \frac{1}{l_2} & ... & \frac{1}{l_{Z_P}} \end{bmatrix}$$

$$A_{\phi_C\phi_R}(i,j) = \begin{cases} -\omega_{ij}v_i & \text{if } (i=j) \\ \omega_{ij}v_i\frac{\phi_{j0}}{\phi_{i0}} & \text{if } (i\neq j) \end{cases}$$

$$A_{\phi_C H}(i,j) = \begin{cases} -10.234\times10^{-6}\times\frac{\phi_{i0}}{l} & \text{for } (i=2,4,6,8), \\ & j = i/2 \\ 0 & \text{otherwise.} \end{cases}$$

$$A_{C\phi_C} = diag\,[\lambda_1 \quad \lambda_2 \quad .. \quad \lambda_c]$$

$$A_{CC} = -diag\,[\lambda_1 \quad \lambda_2 \quad .. \quad \lambda_c]$$

$$A_{\phi_R\phi_R}(i,j) = \begin{cases} -\omega_{ij}v_i & \text{if } (i=j) \\ \omega_{ij}v_i\frac{\phi_{j0}}{\phi_{i0}} & \text{if } (i\neq j) \end{cases}$$

$$A_{\phi_R\phi_C} = A_{\phi_C\phi_R}^T$$

$$B_H = diag\,[k \quad k \quad k \quad k]$$

$M_{\phi_C} = I_C, I$ denotes an Identity matrix.

The neutronic parameters, nodal volumes and necessary data under full power operation are given in [6] and [7]. Eigenvalues of system matrix A are shown in Table I. It has 5 eigenvalues at the origin of complex s-plane and the remaining 75 eigenvalues in the left half of s-plane out of which 16 are of the order 10^{-1}, and the rest very large in magnitude.

IV. Application of Model Order Reduction Techniques to AHWR

There are multiple eigenvalues at the origin of the complex s-plane. Hence, diagonalization of the model is not possible. However this difficulty is overcome by rewriting the dynamics of the original system as the following.

$$\dot{x}_M = A_M x_M + A_{\phi_C H}x_H, \quad (49)$$

$$y = M_M x_M, \quad (50)$$

$$\text{and} \quad \dot{x_H} = B_H u \quad (51)$$

TABLE I: Eigenvalues of A

S.No	Eigenvalue	S.No	Eigenvalue
1	0	41	-1.6369×10^{2}
2	0	42	-1.6501×10^{2}
3	0	43	-1.6833×10^{2}
4	0	44	-1.7116×10^{2}
5	0	45	-1.7161×10^{2}
6	-5.1852×10^{-2}	46	-1.7600×10^{2}
7	-5.2002×10^{-2}	47	-1.8084×10^{2}
8	-5.8369×10^{-2}	48	-1.8486×10^{2}
9	-5.8821×10^{-2}	49	-1.9477×10^{2}
10	-5.9777×10^{-2}	50	-2.0282×10^{2}
11	-6.0480×10^{-2}	51	-2.0345×10^{2}
12	-6.0863×10^{-2}	52	-2.0394×10^{2}
13	-6.1191×10^{-2}	53	-2.0586×10^{2}
14	-6.1958×10^{-2}	54	-2.0934×10^{2}
15	-6.2035×10^{-2}	55	-2.1248×10^{2}
16	-6.2324×10^{-2}	56	-2.3111×10^{2}
17	-6.2514×10^{-2}	57	-2.3124×10^{2}
18	-6.2553×10^{-2}	58	-2.3218×10^{2}
19	-6.2712×10^{-2}	59	-2.3271×10^{2}
20	-6.2951×10^{-2}	60	-2.4075×10^{2}
21	-6.2977×10^{-2}	61	-2.4168×10^{2}
22	-8.4578	62	-2.5542×10^{2}
23	-3.8195×10^{1}	63	-2.5569×10^{2}
24	-3.8778×10^{1}	64	-2.6274×10^{2}
25	-6.8742×10^{1}	65	-2.6306×10^{2}
26	-7.6403×10^{1}	66	-2.6394×10^{2}
27	-9.2359×10^{1}	67	-2.6511×10^{2}
28	-9.5878×10^{1}	68	-2.7631×10^{2}
29	-1.0271×10^{2}	69	-2.7746×10^{2}
30	-1.0577×10^{2}	70	-2.7786×10^{2}
31	-1.0861×10^{2}	71	-3.0219×10^{2}
32	-1.1391×10^{2}	72	-3.0289×10^{2}
33	-1.2358×10^{2}	73	-3.2666×10^{2}
34	-1.2424×10^{2}	74	-3.2689×10^{2}
35	-1.3989×10^{2}	75	-3.7870×10^{2}
36	-1.4226×10^{2}	76	-3.9074×10^{2}
37	-1.4783×10^{2}	77	-3.9924×10^{2}
38	-1.4838×10^{2}	78	-4.1423×10^{2}
39	-1.4978×10^{2}	79	-4.7040×10^{2}
40	-1.6005×10^{2}	80	-4.7515×10^{2}

where $x_M = \begin{bmatrix} x_{\phi_C}^T & x_C^T & x_{\phi_R}^T \end{bmatrix}$ is of order 76,

$$A_M = \begin{bmatrix} A_{\phi_C\phi_C} & A_{\phi_C C} & A_{\phi_C\phi_R} \\ A_{C\phi_C} & A_{CC} & 0 \\ A_{\phi_R\phi_C} & 0 & A_{\phi_R\phi_R} \end{bmatrix}, \quad (52)$$

and

$$M_M = [M_{\phi_C} \quad 0 \quad 0]. \quad (53)$$

It can be verified that all the eigenvalues of A_M are distinct. In fact eigenvalues of A_M are similar to the 76 eigenvalues of A, listed at S.No 5 to 80 in Table I. Now the model reduction techniques discussed in section II can be directly applied to the 76^{th} order model given by (49) and (50) for obtaining the simplified models. Finally by augmenting the decoupled state vector x_H, the simplified model for AHWR can be represented as,

$$\dot{x}_R = \begin{bmatrix} \dot{x}_r \\ \dot{x}_H \end{bmatrix} = \begin{bmatrix} A_r & A_{\phi_C H} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_r \\ x_H \end{bmatrix} + \begin{bmatrix} 0 \\ B_H \end{bmatrix} u, \quad (54)$$

$$y = [M_r \quad 0]\begin{bmatrix} x_r \\ x_H \end{bmatrix}. \quad (55)$$

First consider the application of method given in [1] to the model given by (49) and (50) so as to retain the 18 eigenvalues of A, listed at S.No 5 to 22 in Table I. By substituting the matrices $A_r, A_{\phi_C H}$ and M_r in (54) and (55) thus obtained, the following simplified model of order 22 is obtained for AHWR. The eigenvalues retained in simplified model are the first 22 eigenvalues of the original system.

$$\dot{x}_r = A_d x_r + B_d u. \quad (56)$$

$$y = M_d x_r. \quad (57)$$

Similarly, a reduced model of order 21 is obtained by the application of the method described in [2], as

$$\dot{x}_r = A_m x_r + B_m u, \quad (58)$$
$$y = M_m x_r. \quad (59)$$

An observation of the eigenvalues of the system matrix A shown in Table I reveals that the eigenvalues falls into two distinct clusters. First cluster has 21 eigenvalues ranging from -6.2977×10^{-2} to -5.1852×10^{-2} and the second one is of 59 eigenvalues ranging from -4.751×10^{2} to -8.4578. Five eigenvalues are at origin (grouped in the first cluster) whereby the presence of two time scales is indicated. It would therefore, be possible to decompose the model into a slow subsystem of order 21 and a fast subsystem of order 59, by the application of the method presented in Sec.II-C. For carrying out this, the following regrouping of states is suggested:

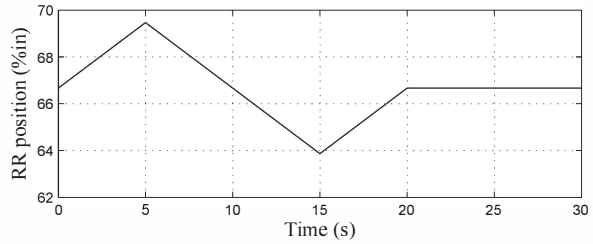
$$x_a = \begin{bmatrix} x_H^T & x_C^T \end{bmatrix} \quad (60)$$
$$x_b = \begin{bmatrix} x_{\phi_R}^T & x_{\phi_C}^T \end{bmatrix} \quad (61)$$

The sub matrices $A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, \acute{M}_1$ and $\acute{M}_2$ in (22),(23) and (24) are obtained by appropriate rearrangement and partitioning of matrices A, B and M which are given respectively by (46), (47) and (48). Using these, we can get a slow subsystem of order 21 with A_0, B_0, M_0 and N_0 represented by (25) and (26).

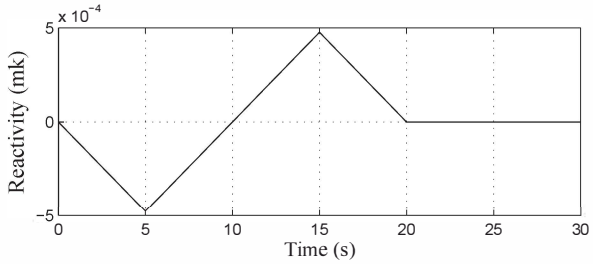
To illustrate the dynamic behavior of the different reduced order linear models, the open loop response is simulated for a short-time control-relevant transient and is compared with the open loop response of the original 80^{th} order model of the AHWR. In simulation, the reactor was assumed to be initially operating at full power and each RR is at 66.1% in position. A control voltage of 1 V was applied to the RR drive in node 2, under which the RR moved linearly into the reactor core. After a short interval of 5s, the control voltage is made -1 V and is maintained at this level for 10s. Then the control voltage is made 1 V for 5s to bring back the RR to its nominal position. Fig.2(a) shows the position of control rod and Fig.2(b) shows the reactivity introduced by it during the transient. From the deviations in the nodal fluxes, the deviation in the core average flux is calculated as

$$\phi_{avg} = \frac{\sum_{i=1}^{17} \delta\phi_i V_i}{\sum_{i=1}^{17} V_i}. \quad (62)$$

The response of different reduced order models shown in Fig.3 reveals that the core average flux obtained from the approximate models is nearly the same as that of original model. Fig.4 shows the variation of neutron flux in node 2, from the respective equilibrium value as obtained by solving the full order model and different reduced order models. Fig.5 shows the variation of neutron flux in node 10 which is neighboring to node 2. Fig.6 shows the variation of neutron flux in node 14, which is far away from node 2. The comparison of responses makes it clear that Davison's technique fails to reproduce the accurate response characteristics as that of original model in node 2, 10 and 14 with the reduced order model obtained by retaining first 22 eigenvalues of A. Marshall and Singular perturbation methods yield better approximation for deviation in core average flux as well as nodal fluxes with an order of 21 compared to Davison's technique. However, the application of Davison's and Marshall's method increases computational burden in obtaining approximate model in reactor applications due to the presence of multiple eigenvalues at origin of the complex s-plane, where diagonalization of AHWR space-time kinetics model is difficult.



(a) Position of RR



(b) Reactivity introduced by RR

Fig. 2: Position of RR and reactivity introduced during the movement of RR.

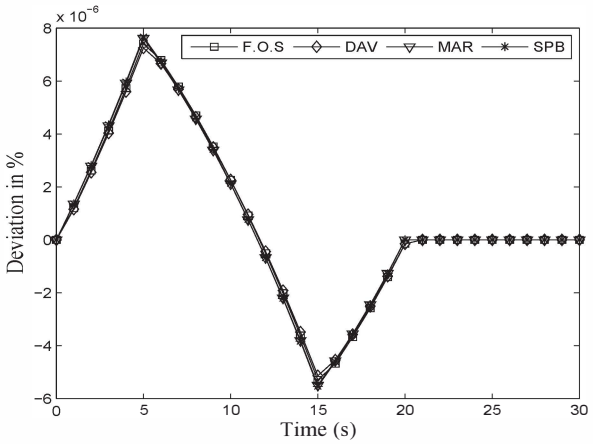


Fig. 3: Deviation in Core average flux during the movement of RR. (F.O.S denotes Full Order System, DAV denotes Davison, MAR denotes Marshall and SPB denotes Singular Perturbation)

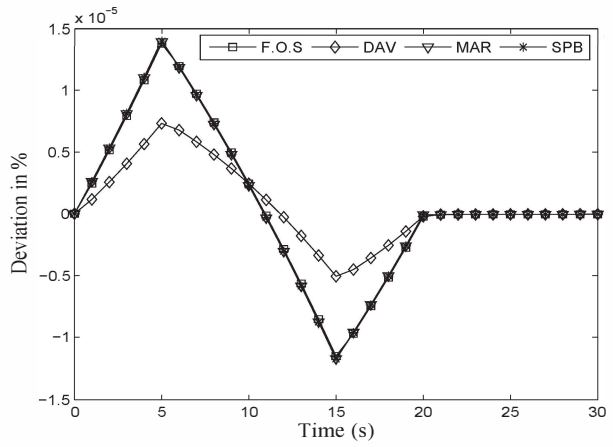


Fig. 4: Deviation in Nodal flux in node 2 during the movement of RR.

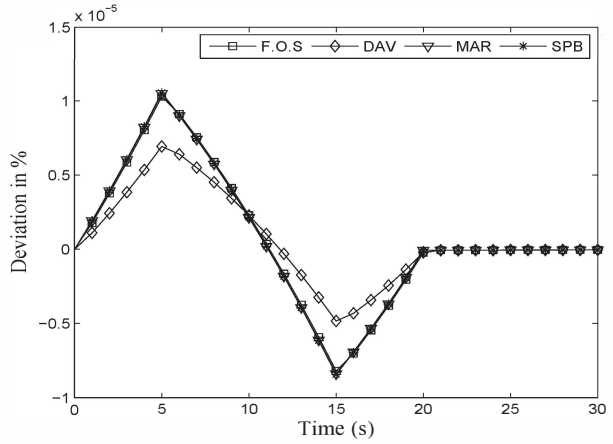


Fig. 5: Deviation in Nodal flux in node 10 during the movement of RR.

V. Conclusion

Application of model order reduction techniques based on Davison's and Marshall's dominant mode retention and model decomposition into slow and fast subsystems based on Singular Perturbation analysis have been successfully explored for AHWR. Davison's and Marshall's dominant mode retention techniques require diagonalization of the original 80^{th} order model while Singular Perturbation techniques requires reordering of state variables and block-diagonalization. The simplified model of AHWR, obtained by application of the Davison's technique is of order 22 while the order of simplified models obtained with Marshall's and Singular Perturbation techniques is 21. Moreover, the transient response of the simplified models based on Marshall's technique and Singular Perturbation technique are in good agreement with the transient response of the original high order model.

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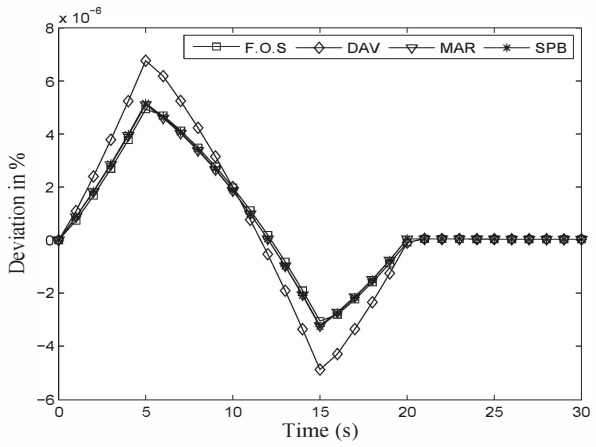

Fig. 6: Deviation in Nodal flux in node 14 during the movement of RR.

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