Adaptive, online models to detect and estimate gross error in SPNDs

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Abstract—In-core flux measurement is critical for monitoring and power regulation of a large core nuclear reactor. Self Powered Neutron Detectors (SPNDs) are used for measuring the neutron flux in a nuclear reactor. In this paper we propose an online method for SPND gross error detection, identification and estimation. The method uses linear models which are extracted from data and which adapt continuously in time. This adaption is made possible by use of recursive PCA. However, unlike existing recursive PCA approaches which make approximations to achieve recursion, our proposed approach does not make any approximation. We term our technique as 'Recursive Principal Component Analysis: Exact Computation' (RPCA-EC). Use of recursion ensures that the computational requirements of RPCA-EC are low thereby facilitating online implementation. Continuous adaption ensures that the model evolves to adequately capture the time varying relationships amongst the SPNDs. The relationships amongst the SPNDs vary with time due to significant variations in the neutron flux profiles in the reactor with varying operating power levels in the reactor.

We apply our proposed method on data taken from an operating nuclear reactor in India and compare results with alternate implementations. Results show that the false alarm rate of our implementation is reasonable thereby indicating that the model adapts to time varying relationships. Performance in presence of gross errors is also satisfactory.

I. INTRODUCTION

Self Powered Neutron Detectors (SPNDs) are sensors which are used to measure neutron flux in a nuclear reactor in real time. Typically, of the order of hundred such SPNDs are placed at different locations in the reactor thereby providing a comprehensive view of the flux profile throughout the reactor. The neutron flux provides a direct measure of the reactor output power. Hence, it is essential to reliably obtain accurate measurements of neutron flux at various in-core locations in the reactor control and monitoring systems.

Over a period of time, an SPND can develop a gross error or a fault due to various reasons such as drop in insulation resistance as a result of moisture ingress, and crack in detector sheath [TSPSAK13]. When such faults occur, the detector readings will be erroneous. Additionally, hardware or software issues involving data-collection and archiving systems can also cause erroneous readings to be recorded. In such cases, if the fault is not correctly detected and diagnosed in a timely manner, then correct inferences about reactor operation may not be made. Depending on the nature of the fault, it may be possible to manually detect some faults just by observing the raw measurement data. Examples are junk value reported by a detector typically due to a communication breakdown, and large or sudden changes in a detector reading not accompanied by change in any other detector. However, faults causing relatively smaller changes in detector readings or evolving slowly over a period of time are more difficult to detect. If a fault is manually detected, then the particular detector reading can simply be ignored and not communicated to the reactor control and monitoring systems. But this will lead to loss of information required to construct the flux map of the reactor and may lead to reactor performance degradation. To rectify this situation, the faulty detector can be replaced by a new one. But this involves unscheduled reactor shut-down which will not only be time consuming but also have cost and personnel safety implications [RBBTKP12]. To avoid such scenarios, it is important to develop real-time automated fault detection and diagnosis techniques that can effectively aide the nuclear reactor operator. Such techniques should: (i) detect a fault as and when one occurs, (ii) identify the faulty detector, and (iii) estimate the true flux value corresponding to the faulty detector.

In the general fault diagnosis literature [VRK03], model based techniques have been used for developing such automated strategies. These models could be either derived from first principles or can be extracted from historical data. For SPNDs in a typical nuclear reactor, first principles models relating measurements of different detectors are not available. However, the measurements for all the detectors are routinely collected at high sampling rates (typically 1 sec or less) and archived. In principle, the historical data can then be used for developing data-driven models which can subsequently be used for performing fault detection and diagnosis in realtime.

In the SPND literature, [RBBTKP12] proposed use of Principal Components Analysis (PCA) technique for developing data-driven models. In their work, they (i) grouped the various detectors in smaller clusters such that detectors in a particular cluster were highly correlated, and (ii) used PCA to develop linear, static models relating measurements of detectors in those clusters. Analysis of the residuals arising from the PCA based linear models enabled them to perform fault detection and diagnosis. Use of linear models ensured that the real-time computational requirement was low thereby ensuring feasibility for online implementation. However, the work of [RBBTKP12] was based on the implicit assumption that the relationships amongst detectors in a given cluster, once obtained, were assumed to remain valid at all time

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instants in the future, i.e. the models were static in nature.

The operation of a nuclear power plant varies significantly over a period of time as a function of power levels, fuel type and quantity, and detector age, etc. [STB12]. As a result, the linear, static relationships obtained for a set of detectors using past data may not be able to satisfactorily capture the behaviour of detectors in the future. Thus, in this work we propose development of data driven, adaptive linear models to capture the time varying relationships amongst various detectors. This adaptation is made possible by recursive use of PCA. At each time instant, our proposed recursive PCA implementation fuses fault-free current measurement vector with the existing model to obtain an updated model that reflects the effect of the current measurement vector. Use of recursion, as opposed to batch processing of data, also ensures that the computational time required for adapting the models is much lower compared to the sampling interval, thereby opening up the possibility of real-time implementation. The utility of our proposed adaptive fault detection and diagnosis method is demonstrated on data collected from an operating nuclear power generating reactor in India.

The rest of the paper is structured as follows: in Section II we present background material for our proposed work. In particular, we summarize the functioning of an SPND, preprocessing involved with data based on which the analysis has been performed, and PCA and gross error analysis. In Section III we present our recursive PCA based model adaptation approach for performing gross error analysis. In Section IV, we build adaptive models and benchmark them based on the performances for a variety of gross error scenarios for data obtained from a nuclear power reactor in India. We conclude the paper in Section V.

II. PRELIMINARIES

A. SPND Description

Self Powered Neutron Detector (SPND) is a sensor for measurement of neutron flux that does not require any external power. It is an in-core sensor, i.e. it is directly exposed to reactor environment of high neutron and gamma field. An SPND generates current whose value is a function of the neutron flux in the reactor. The principle for flux measurement using an SPND is as follows: on being exposed to neutron flux within the reactor, certain material (called emitter) in the detector undergoes neutron capture to produce radioactive nuclei which decay to produce more stable nuclei. Electrons are generated during these processes which form the current output of the detectors [Tod98].

SPNDs can be classified as prompt or delayed depending on whether electron emission is prompt or delayed [AM97]. For example, Inconel and Cobalt SPNDs are prompt but Vanadium and Rhodium SPNDs have a delayed response [B79] with a Vanadium SPND having a time constant of 315 seconds. The current work deals with both delayed (Vanadium) and prompt (Inconel and Cobalt) SPNDs.

B. Data Preprocessing

1) Dynamic Response Matching: The nuclear power reactor for which data has been analyzed in this work, has a combination of both prompt and delayed SPNDs. It should be noted that currents generated by prompt and delayed SPNDs may not appear to be directly correlated even if they are subjected to the same neutron flux because they have different response characteristics as discussed in Section II-A. However, to ensure that we are able to develop models that utilize all the available sensors, similar to literature [RBBTKP14], we filter (or slow down) the response of the prompt sensor by passing its measurement through¹ the delayed response SPND transfer function. This makes prompt sensor data comparable with delayed response SPND data [RBBTKP14] and enables seamless use of measurements of these detectors.

2) Clustering: A typical nuclear reactor consists of hundreds of SPNDs. Measurements made by all these detectors will not be similar as the neutron flux varies spatially throughout the reactor. However, it is expected that smaller groups of SPNDs have similar behaviour. To identify such groups or clusters of highly correlated SPNDs, k-means clustering can be used [RBBTKP14]. In our work, we follow the approach of Rihab et al. [RBBTKP14] for obtaining clusters of SPNDs using k-means.

C. PCA Based Modeling

In this section, we summarize the PCA procedure for obtaining linear, steady state models relating the true values of n variables from the corresponding noisy measurements of those variables. Let $\tilde{x}(t) \in \mathbb{R}^n$ be the vector of true values of variables at any time instant t. Let the measurements $\tilde{y}(t) \in \mathbb{R}^n$ be related to the true values as

$$\tilde{y}(t) = \tilde{x}(t) + v(t) \tag{1}$$

where $v(t) \in \mathbb{R}^n$ is random measurement noise assumed to be Gaussian with zero mean and covariance matrix R.

Consider that a training data-set, consisting of observations at N time instants: $\tilde{y}(1), \tilde{y}(2), \ldots, \tilde{y}(N)$, is available. Here without loss of generality the initial time point of the data-set has been labeled to correspond to the first time instant. Let this data-set be represented by a matrix $\tilde{Y} = [\tilde{y}(1) \ \tilde{y}(2) \ \ldots \tilde{y}(N)]^T \in \mathbb{R}^{N \times n}$. The matrix \tilde{Y} of measurements can then be related to the corresponding true unknown values of the variables as (1)

$$\tilde{Y} = \tilde{X} + V \tag{2}$$

where $\tilde{X} \in \mathbb{R}^{N \times n}$ is the corresponding matrix of the true values and $V \in \mathbb{R}^{N \times n}$ is random measurement error matrix.

The measurement data for different detectors may be available in different units. Thus, the measured data is scaled such that each column vector in the matrix \tilde{Y} is zero mean and unit variance. The resulting data matrix is labeled Y with the scaled measurement vector at time instant t being y(t). The corresponding matrix of scaled true values is labeled X with x(t) being the vector of scaled true values at time instant t. PCA involves computation of the covariance matrix

¹In addition to this, we have explored in [RBBTKP14] two other methods to combine prompt and slow measurements: the procedure of slowing down prompt sensors, as applied in the current work, turned out to perform the best in terms of correlation of the time series.

 $C \in \mathbb{R}^{n \times n}$ of the scaled measurements y(t) from the N D. Gross Error Analysis samples stacked in matrix Y as

$$C = \frac{1}{N} Y^T Y \tag{3}$$

Let $P \in \mathbb{R}^{n \times n}$ be the matrix of eigenvectors of C arranged as columns in increasing order of eigenvalues. The matrix Pcan be decomposed as

$$P = \left[\begin{array}{c|c} Q & T \end{array} \right] \tag{4}$$

where $Q \in \mathbb{R}^{n \times m}$ is the matrix of m eigenvectors corresponding to the *m* smallest eigenvalues, and $T \in \mathbb{R}^{n \times (n-m)}$ is the matrix of the remaining n - m eigenvectors.

The eigenvectors of covariance matrix give the orthonormal directions of data variability and the variance of the data along those directions is given by the corresponding eigenvalues [John07]. Hence, a high eigenvalue implies large variability of the data in the corresponding eigenvector direction. Conversely, a small eigenvalue corresponds to a restriction of variation of the data in that eigenvector direction. This restriction can be used to extract a model relating the given sensors. In particular, the eigenvectors in Q (4) indicate directions of low variability. Hence, we can write

$$AX^T = 0_{m \times N}, \text{ where } A = Q^T$$
 (5)

is the identified model or constraint matrix. Thus, at any given time instant t, the vector x(t) is assumed to satisfy

$$Ax(t) = 0_{m \times 1} \tag{6}$$

In the literature, several methods are available to choose a value of m which decides the number of eigenvectors assigned in matrix Q [RBBTKP12]. In the results to be presented later, we choose m as follows:

$$m = \arg\max_{h} \left(\frac{\sum_{i=1}^{h} \lambda_i}{\sum_{i=1}^{n} \lambda_i} \leqslant w \right)$$
(7)

where $\lambda_i, i = 1, 2, ..., n$ is the i^{th} eigenvalue of matrix C in (3) with $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. $w \in (0,1)$ is a user specified parameter that corresponds to variability in the data due to noise. (7) is based on the idea that starting from the eigenvector corresponding to the lowest variability, keep adding eigenvectors corresponding to increasing variability in the Q matrix till the fraction of variability in those directions exceeds the parameter w. In the results to be presented later, w is taken as 0.01.

It is to be noted that even if the true values x(t) satisfy model (6), the measurements y(t) will not satisfy this model exactly since they are corrupted by noise (1). This in turn is indicated by the fact that the variability in directions of eigenvectors in Q is low but not 0. We can thus compute the residuals $r(t) \in \mathbb{R}^m$ at time instant t as

$$r(t) = Ay(t) \tag{8}$$

Let E be the covariance matrix of the residuals. Since the rows of A (corresponding to eigenvectors in Q) are orthonormal, it follows that the residuals in different equations are uncorrelated with each other. Hence, E is a diagonal matrix with the eigenvalues corresponding to eigenvectors in Q being on the diagonal.

In (1) it was assumed that the measured value is a sum of the true value of SPND and a random error component. The model A is obtained from training data that follows this assumption. Once the A matrix is obtained, the developed model can be used to test for presence of gross errors (or faults) in sensors. In our work, the gross error is modeled as a constant bias in the sensor. In presence of gross error in the j^{th} sensor, (1) can be rewritten as

$$y(t) = x(t) + v(t) + b_j e_j$$
 (9)

where, $b_j \in \mathbb{R}$ is the gross error in the j^{th} sensor, and e_j is the unit vector with 1 at the j^{th} position.

Gross Error analysis

To detect a gross error, the following statistic is computed using the residuals:

$$\gamma(t) = r(t)^T E^{-1} r(t)$$
 (10)

In the absence of gross error, $\gamma(t)$ follows a χ^2 distribution with m degree of freedom (where m is the number of relationships in model constraint matrix) [NM98]. Thus, a gross error is detected at significance level α if $\gamma(t) > \gamma(t)$ $\chi^2_{1-\alpha,m}$ where $\chi^2_{1-\alpha,m}$ is the value from a χ^2 distribution with m degrees of freedom with α being the tail area. Once a gross error is detected, the next step is to identify the faulty sensor and estimate the magnitude of the gross error. This is done using the Generalized likelihood ratio test. Details of this procedure are available in [NM98].

III. PROPOSED WORK

A key assumption made in the PCA based gross error detection approach is that the A and E matrices, once obtained, continue to be valid at all time instants in the future. For most nuclear reactors, this assumption will not hold. This in turn will result in a high false alarm rate, when the A matrix obtained using training data in the past is used to test for faults (gross errors) in the measurement vector obtained in the future. This situation can be rectified by continuously adapting the model matrices with varying data. In principle, this adaptation can be done by simply considering the entire dataset from the initial to the current time instant, and performing PCA on that dataset. But this method is not computationally amenable for online implementation as the data size grows indefinitely. We thus propose to use recursive PCA with exact computations of eigenvalues and eigenvectors (RPCA-EC), to update the model while ensuring that the online computational requirements are not excessive. The updated models are then used to perform gross error analysis. These steps are discussed next.

A. Recursive application of PCA

In literature [XCZ12], [ERP04] have proposed recursive PCA where they use an approximate method for recursively computing the eigenvalues and eigenvectors of the covariance matrix using a first order perturbation analysis thereby avoiding their direct computation. This approach is suitable when the covariance matrix is of large size and hence direct computation of eigenvalues and eigenvectors can be time consuming. However, the use of first order matrix perturbation analysis [XCZ12], [ERP04], which involves ignoring some second order terms to approximately update eigenvalues and eigenvectors may result in erroneous computations. Depending on the application, it is possible that models obtained using such an approximation may not adequately capture the true relationships.

In this work, we propose a modified implementation of recursive PCA where only the covariance matrix and mean vector are recursively updated at each time instant. The eigenvalues and eigenvectors of the covariance matrix are then directly computed without making any approximation. We label this implementation of recursive PCA as recursive PCA with exact computation (RPCA-EC). As detectors are grouped into smaller clusters before developing data-driven models, size of the covariance matrix is not large. Hence, the proposed RPCA-EC is computationally feasible for online implementation. At any time instant, the model obtained using RPCA-EC at the previous time instant is used to check for the presence of the gross error in the current measurement vector. If a gross error is detected, the model is not updated and is instead used to identify the faulty sensor and estimate the magnitude of the bias in that sensor. On the other hand, if a gross error is not detected, the current measurement vector is used to update the existing model. The procedure for updating the model is now summarized. At time instant t+1, the following steps are performed. For better readability, we label the procedure as Model Update.

Model Update: Procedure to update the model using RPCA-EC

1) The mean of the measurement vector is recursively updated as

$$m(t+1) = \left(1 - \frac{1}{t+1}\right)m(t) + \frac{1}{t+1}\tilde{y}(t+1) \quad (11)$$

where $m(t) \in \mathbb{R}^n$ is the mean of the measurement vector computed at time instant t. The variance of the i^{th} measurement is also recursively updated as

$$\sigma_i^2(t+1) = \left(1 - \frac{1}{t+1}\right)\sigma_i^2(t) + \frac{1}{t+1}\left(\tilde{y}_i(t+1) - m_i(t+1)\right)^2 \quad (12)$$

2) The i^{th} measurement at time instant t+1 is normalized:

$$y_i(t+1) = \frac{(\tilde{y}_i(t+1) - m_i(t+1))}{\sigma_i(t+1)}$$
(13)

3) The covariance matrix of the measurements is then recursively updated as

$$C(t+1) = \left(1 - \frac{1}{t+1}\right)C(t) + \frac{1}{t+1}y(t+1)(y(t+1))^{T}$$
(14)

4) Eigenvalues and eigenvectors are computed for the covariance matrix C(t+1). Let $Q(t+1) \in \mathbb{R}^{n \times m}$

be the matrix containing the eigenvectors of C(t+1) corresponding to the m smallest eigenvalues. The model matrix at time instant t+1 is then obtained as $A(t+1) = Q(t+1)^T$, and covariance matrix of residuals E(t+1) is obtained using the m smallest eigenvalues.

B. RPCA-EC Overall Implementation

The proposed RPCA-EC based overall procedure for model update and gross error analysis, suitable for online implementation, is listed below.

RPCA-EC Based Gross Error Analysis

- Pass prompt SPND measurement through a lag compensator (delayed response SPND transfer function) [RBBTKP14].
- 2) Normalize the filtered prompt and raw delayed data using the mean and variance of the respective sensors.
- 3) Obtain the residuals with the normalized data using the existing model constraint matrix as in (8).
- 4) Compute the statistic $\gamma(t)$ as in (10) and compare it with the threshold value obtained from a chi-squared distribution. Depending on the outcome of this comparison, perform the following steps:
 - a) Declare a gross error if $\gamma(t)$ exceeds the chisquared threshold value. Identify the faulty detector and the magnitude of the gross error using generalized likelihood ratio based method [NJ99]. In this case, the model is not updated.
 - b) Otherwise update the model using RPCA-EC based procedure listed in Section III-A.
- 5) Go to step 1 when the measurements at the next time instant become available.

While implementing the above algorithm, the following points need to be noted:

- 1) The above procedure is for a single cluster. The same procedure is followed for all the clusters.
- 2) If at any instant one or more sensor value is missing for a particular cluster, typically due to communication failure, the sensor(s) with missing values are projected out from model matrices to obtain models containing sensors with available measured values [NJ99]. Gross error analysis is then performed using the reduced models. However, even if no gross error is detected, the model is not updated due to lack of complete data.

IV. RESULTS

We now compare the performance of proposed RPCA-EC method with the following alternative implementations:

- Static PCA (SPCA): PCA is used to obtain the model only once from the covariance matrix of training data. The model is subsequently never updated. This implementation is presented in [RBBTKP12].
- Batch PCA (BPCA): PCA is repeatedly performed on the entire data matrix (from initial to current time instant). Each fault-free incoming data vector is appended



Fig. 1. Profiles of SPNDs in the chosen cluster (raw data)

to the existing data matrix and PCA method used for model building as discussed in Section II-C.

The following metrics will be used to compare the performances of various techniques [RBBTKP12]:

- False alarm rate: Percentage of time instants when a bias is detected even when no bias was added.
- 2) Detection rate: Percentage of time instants when a bias is correctly detected.
- 3) Identification rate: Percentage of time instants when the sensor with the bias is correctly identified. This is computed (as a percentage) by dividing the number of instants when correct identification has been achieved by the total number of time instants.
- 4) Mean square estimation error (MSE): It is defined as $\sqrt{(\sum_{t \in C_I} (b - b(t)^*)^2)/(N_{correct})}$, where b is the actual bias introduced, $b(t)^*$ is the estimate of bias at the t^{th} time instant, C_I is the set of time instants at which the biases are correctly identified and $N_{correct}$ is the cardinality of C_I .

A. Data Description

The data used in our work corresponds to a power producing nuclear reactor in India. The reactor consists of a cylindrical core with 42 prompt and 102 delayed SPNDs. Data from the sensors sampled at 1 second interval are available. Of the 144 SPNDs, we removed measurements corresponding to 12 SPNDs as they had zero or missing values for significant number of time instants. We are thus left with 41 prompt and 91 delayed sensors. The corresponding training data of length 1000 instants is used for (i) grouping the detectors in four clusters by using K-means algorithm, and (ii) obtaining the initial models for each of the clusters. One of the clusters contains twenty delayed response detectors and is considered for further analysis. Profiles of these twenty detectors are shown in Figure 1 for illustrative purposes. It can be seen that one of the detectors, namely V86, shows large spikes at a few time instants. However, it was retained in further analysis to enable testing of the various approaches for real datasets. The results listed next are obtained by considering the entire 2000 instants dataset.

B. Comparison of False Alarm Rates (Absence of Gross Errors)

The false alarm rates with the three approaches with significance level $\alpha = 0.05$ are listed in Table I. Since

 $\alpha = 0.05$, gross error should be detected approximately 5% of the time instants even when gross error is not present. It is seen that the false alarm rate is very high for SPCA. This is expected since SPCA does not update the model obtained using the initial training data. Thus, as the reactor operations vary, the new data points are identified to be containing gross error even though no gross error was added. Best performance is obtained with the proposed RPCA-EC with false alarm rate being close to the designed value of 5%. This indicates that the models obtained by RPCA-EC adapt gradually with the time varying measurements of the detectors.

TABLE I False alarm rates for various methods

SPCA	BPCA	RPCA-EC
29.96%	10.60%	3.60%

C. Comparison in Presence of Gross Errors

Bias of various magnitudes (a specified percentage of the mean value of training dataset) is added in detector V-83 at all the 2000 time instants. For all the three approaches, model with m = 18 relationships was used as the initial model. Results are compared with thresholds on γ set in two different ways:

- 1) Case 1: The thresholds for all the three approaches are obtained from the chi-squared distribution with appropriate degrees of freedom with significance level $\alpha = 0.05$. The false alarm rates listed in Table I were obtained with these thresholds.
- 2) Case 2: The thresholds for all of the approaches are manually tuned to get false alarm rates equal to 5%.

The results for the above two cases are discussed next.

<u>Case 1</u>: Thresholds with significance level $\alpha = 0.05$.

From the results for various magnitudes of gross errors listed in Table II it is seen that the detection rate obtained by SPCA is higher than the other two approaches, specially at lower bias magnitudes. This is expected as SPCA method does not involve updating the models. On the other hand, the adaptive approaches (BPCA and RPCA-EC) evolve with varying data and thus detect rate is lower when the gross error magnitude is small. However, their performance significantly improves with larger magnitude gross errors.

<u>Case 2</u>: Thresholds with false alarm rate set at 5%.

Here the thresholds were tuned for each approach to get the observed false alarm rates equal to 5%. The significance levels corresponding to these thresholds are reported in Table III. As expected, the significance level for SPCA is quite low as the threshold has to be significantly increased for SPCA to have 5% false alarm rate. On the other hand, the significance level for RPCA-EC is close to 0.05. Gross error analysis results for different magnitudes of gross errors are listed in Table III. Compared to results of Table II, it is seen that the performance of SPCA has deteriorated while of RPCA-EC has significantly improved. From Table III it can also be seen that performance of RPCA-EC is superior to BPCA for all cases, while it is superior to SPCA for higher bias values.

TABLE II GROSS ERROR ANALYSIS: THRESHOLDS WITH $\alpha = 0.05$

3% bias (magnitude = 0.066)					
	SPCA	BPCA	RPCA-EC		
Detection Rate	59.13%	10.85%	6.2 %		
Identification Rate	41.52%	1.75%	2.00 %		
MSE	0.027	0.026	0.028		
5% bias (magnitude = 0.111)					
	SPCA	BPCA	RPCA-EC		
Detection Rate	92.89%	19.86%	15.20%		
Identification Rate	90.04%	11.65%	10.00 %		
MSE	0.022	0.019	0.019		
6% bias (magnitude = 0.133)					
	SPCA	BPCA	RPCA-EC		
D (C D (07.00%	70.000	(0.000		
Detection Rate	91.99%	/8.08%	60.08%		
Identification Rate	97.49% 97.49%	78.08% 72.38%	60.08% 59.67%		
Identification Rate MSE	97.49% 0.023	72.38% 0.023	60.08% 59.67 % 0.022		
Identification Rate MSE	97.99% 97.49% 0.023	78.08% 72.38% 0.023	60.08% 59.67 % 0.022		
Detection Rate Identification Rate MSE 7% bias	97.49% 97.49% 0.023	78.08% 72.38% 0.023 le = 0.155)	60.08% 59.67 % 0.022		
Detection Rate Identification Rate MSE 7% bias	97.39% 97.49% 0.023 s (magnitud SPCA	78.08% 72.38% 0.023 1e = 0.155) BPCA	60.08% 59.67 % 0.022 RPCA-EC		
Detection Rate Identification Rate MSE 7% bia: Detection Rate	97.99% 97.49% 0.023 s (magnitud SPCA 98.30%	$\frac{78.08\%}{72.38\%}$ $\frac{72.38\%}{0.023}$ $\frac{100}{100} = 0.155$ $\frac{100}{98.30\%}$	60.08% 59.67 % 0.022 RPCA-EC 98.30%		
Detection Rate Identification Rate MSE 7% bia Detection Rate Identification Rate	97.99% 97.49% 0.023 s (magnitud SPCA 98.30% 98.25%	$78.08\% \\ 72.38\% \\ 0.023 \\ \hline BPCA \\ 98.30\% \\ 98.25\% \\ \hline$	60.08% 59.67 % 0.022 RPCA-EC 98.30% 98.25 %		

V. CONCLUSIONS

In this work, a recursive implementation of PCA, labeled RPCA-EC has been proposed for fault detection and diagnosis of SPNDs. The approach involves continuous adaption of the linear models relating SPNDs and is thus suitable for time varying reactor operations. The proposed approach was applied on data obtained from a nuclear reactor and its performance compared with other approaches. It was found that in absence of gross error, the approach gave acceptable false alarm rates thereby indicating that the models could adapt to the time varying behaviour. The gross error performance was also acceptable and improves especially at higher bias magnitudes. The approach was found to be computationally feasible for online implementation. Further improvements can be obtained by implementing gross error analysis strategies that consider data in a window rather than an individual observation. Use of varying forgetting factors in the recursive mean, variance and covariance matrix update expressions can also be investigated for data varying at different rates.

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 TABLE III

 GROSS ERROR ANALYSIS: THRESHOLDS WITH 5% FALSE ALARM RATE

	SPCA	BPCA	RPCA-EC		
α	6.5×10^{-6}	0.0242	0.06676		
3% bias (magnitude = 0.066)					
	SPCA	BPCA	RPCA-EC		
Detection Rate	11.50%	5.60%	8.95%		
Identification Rate	5.80%	0.95%	2.65%		
MSE	0.040	0.025	0.027		
5% bias (magnitude = 0.111)					
570 0	SPCA	BPCA	RPCA-EC		
Detection Rate	35.06%	9.65%	20.01%		
Identification Rate	33.11%	5.55%	13.55%		
MSE	0.029	0.020	0.019		
6% bias (magnitude = 0.133)					
	SPCA	BPCA	RPCA-EC		
Detection Rate	58.17%	27.16%	97.89%		
Identification Rate	57.72%	24.16%	97.24%		
MSE	0.025	0.019	0.023		
7% bias (magnitude = 0.155)					
	SPCA	BPCA	RPCA-EC		
Detection Rate	82.99%	98.29%	98.29%		
Identification Rate	82.94%	98.24%	98.24%		
MSE	0.022	0.023	0.023		

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