An Operations Research based development of a timetabling tool for a metro-rail application with short-turning, inducting and stabling requirements

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Abstract
Timetabling in metro-rail operations is essential for efficient train movement and adapting to variable passenger demand. This study presents an integrated timetabling model that utilizes key operational data from line planning: short-turn ratios and rolling stock availability, to develop an operational schedule. Key challenges include managing headways across peak and off-peak transitions and addressing conflicts due to short-turn rakes and shifts between high-speed and energy-saving coasting modes. Our model deploys the full fleet during peak hours with minimized traversal times to achieve tight headways. For off-peak periods, it adjusts by reducing active rakes and increasing headway through coasting mode, conserving energy and cutting costs. During evening peaks, additional rakes are reintroduced, again focusing on tight traversal to meet demand. To prevent headway clashes, the model applies scheduling adjustments, especially for short-turn operations and speed changes, ensuring service continuity. After establishing the core timetable, start times are assigned to rakes based on backtracking from peak hours, and nighttime stabling is arranged. This approach enhances both service efficiency and resource utilization, offering a relevant solution for urban rail systems.

Keywords
Passenger Demand, Timetable, Rolling Stock, Headway, Short-turning ratio, Congestion management

1 Introduction

For metro-rail operations, a timetable represents the schedule of services with specific arrival and departure times at various stations, including the run times between terminals and the induction or dispatch of trains to and from the depot or mainline stabling or sidings during early morning and late night. Timetables can be either aperiodic (irregular) or follow a periodic pattern (regular), where events recur at fixed intervals. A detailed timetable usually comes after a ”line plan” is made: the line plan decides on the origins and destinations of services on a network, and the planned frequency of an operation. Even on a single line, the line plan needs to be constructed carefully, considering short turn options and different possibilities of service plans. This line plan can be created heuristically or through an optimization model. This paper focuses on converting a feasible line plan to an operationally feasible timetable. This paper uses various numbers that are typical and representative only; these numbers are not actual numbers used in Delhi Metro.

Converting a line plan into a timetable is needed to manage train operations and also to provide detailed information to customers. The timetabling problem in transportation, particularly in railway systems, involves creating a schedule for train arrivals and departures at various stations including induction and dispatch of trains from/to depot based on predetermined lines, frequencies, and stopping patterns established during the line planning stage.

When developing an effective train timetable, several key aspects must be considered to ensure smooth operations and optimal resource management. Automatic route setting/signaling (ARS) and automatic train reversal (ATR) play a crucial role in maintaining the safety and punctuality of train operations. Ensuring that trains reach stations as per the scheduled timetable, without any delays or disruptions, is key to maintaining operational efficiency. Headway management becomes essential, particularly during variable phases such as early morning, morning peak hour, peak-of-peak hour, off-peak hour, evening peak hour, and evening off-peak phases. Proper headway ensures smooth train operations and helps balance the demand for commuter services. Resource optimization, especially during peak hours, involves the efficient utilization of rakes to provide comfortable and convenient service to commuters. Inter-departmental coordination is critical for ensuring rake availability for revenue services, while balancing traffic demand with the supply of car capacity. Regular maintenance, such as the 72-hour fitness check of rakes, ensures that rakes inducted from mainline stabling or siding locations return to their parent depot during night stabling. Additionally, a few rakes inducted from the depot in the morning should be sent to stabling or sidings as needed, within a 72-hour window. Proper return ID management ensures that during the afternoon ramp down, rakes taken out of service return to their parent depot, and during ramp up, rakes are inducted with a different ID to start service efficiently.

Several factors must be carefully considered while designing an effective train timetable to ensure smooth and efficient operations. Special train movements, such as piloting and rusty movement, must be well defined. Piloting trains (single or multiple) are operated manually at low speed in the early morning before service begins, to inspect the entire track section. These trains ensure the track is free of faults or anomalies before regular service starts. Rusty movement occurs at night when trains return to the depot, passing through seldom-used crossings and turnover points to ensure signaling features remain functional. The timetable should also include a buffer to accommodate any unforeseen delays in service, ensuring minimal disruption. It must be passenger-centric, providing adequate capacity to handle peak-hour traffic, referred to as Peak Hour Per Direction Traffic (PHPDT), and ensuring the maximum number of trains are available for passenger service. The carrying capacity of each train, determined by car type (6-car or 8-car) and the number of passengers it can accommodate, is also crucial.

The timetable should be efficient in terms of traction energy consumed in train operations: given that the traffic is not the same, energy-efficiency is priority in off-peak hours. For instance, trains may run at maximum speed during peak hours, while coasting during off-peak times when there is less urgency for passengers. Two distinct speed profiles are typically used: the tight run, a triangular profile involving rapid acceleration and deceleration to maximize rake utilization and cater to high passenger demand during peak hours; and the coasting run, a trapezoidal profile that maintains a constant speed and improves energy efficiency during off-peak hours. Additionally, dwell time, the time a train stops at a station for passenger loading and unloading—must be accounted for, as it varies depending on the station's proximity to residential areas, commercial complexes, or interchange stations. These factors together ensure that the timetable is both efficient and capable of meeting passenger needs while optimizing operational resources.

Planned headway should be based on the actual time taken for train reversals at terminal stations. Headway refers to the time interval between two consecutive trains, and to ensure safe train movements, it must be greater than the safe headway, which depends on factors such as driver reaction time and braking distance. During peak hours, front crossovers are used to reduce turnaround times for rakes when changing direction, helping to maintain the planned headway. Conversely, rear crossovers are utilized during off-peak hours when there is sufficient time to meet headway requirements. Turnaround time, which is the time taken by a train to reverse direction at terminal or intermediate stations, includes several factors such as dwell time at the last station, running time to the siding, operator cab changes, and running time back to the first station in the opposite direction. Additionally, the timetable should not be overly complicated to avoid confusion for passengers, while also accounting for potential bottlenecks like intermediate reversals. The short-turning ratio, defined as the ratio of trains arriving at a station to those that turn back to provide services in the opposite direction, is also a critical factor in optimizing turnaround operations.

The timetable must clearly display train numbers, locations, and times, ensuring trains maintain the same number once they enter the mainline, unless a service disruption occurs. During the ramp-down phase, when off-peak demand is low, the frequency of services is reduced to optimize rake utilization and conserve energy. This is achieved by uniformly pulling out rakes and adjusting the headway to match off-peak demand. Conversely, during the ramp-up phase, before the evening peak, additional rakes are brought into service to meet higher demand, clear passenger rush, and reduce waiting times. Timetables are dynamic, adjusting based on passenger traffic demand. Several factors influence timetabling, such as the number of depots, their capacity and location, traversal time from depots to the mainline, and mainline stabling or siding locations where trains enter or leave service. Additionally, first and last train timings, days of operation (e.g., weekdays, weekends, holidays, or special events like elections or festivals), and events like sports extravaganzas must be considered.

The timetable must also balance passengers' expectations with the operational capacity of staff to ensure safe and efficient service. Passengers generally expect trains to arrive at regular intervals with minimal waiting time, avoid overcrowding, ensure a delay-free and disruption-free journey, and facilitate fast transfers at interchange stations.

The overall purpose of this research is to develop a time-tabling tool that takes passenger demand, rolling stock (rake) availability, rake capacity, turn around time at terminal stations, run time during peak hours and off-peak hours, service start and end time, number and location of depots as inputs and shall create a semi-automatic timetable in a reasonable time. This timetable should meet passenger demand and also see that average coach occupancy is neither too low nor too high, on average. The detailed case study is based on a specific Line of Delhi Metro Rail Corporation (DMRC), one of the earliest lines of DMRC and still one of the most heavily used lines. The approach is a general purpose one and can be usefully applied to any similar operation in metro rail, suburban rail or similar public transport operations.

To create an effective timetable, two critical factors need to be considered: headway and the number of rakes required.

1.1 Headway

Headway is determined by two main parameters: the PHPDT and the train configuration along with its carrying capacity.

PHPDT or Load Data: This represents the passenger demand on a particular section over a given time period. PHPDT is typically calculated using Automatic Fare Collection (AFC) data, which tracks the entry and travel time of passengers between their origin and destination stations.

Train Configuration and Carrying Capacity: This refers to the total number of passengers a train can serve during its service. It is influenced by the train's car type (e.g., 6-car or 8-car) and includes both seating capacity and the number of standing passengers, typically calculated as 6 standees per square meter.

Example 1:

Consider a scenario where the PHPDT is 20,000 passengers, and the trains in operation are configured with 6 cars, each having a carrying capacity of 1,750 passengers (including seating and 6 standees per square meter). The number of 6-car trains required per hour to meet the PHPDT of 20,000 can be calculated as: Number of trains = 20000/1750 = 11.43 $\approx$ 12 trains

Therefore, the headway (the time interval between two consecutive trains) required to meet the demand is: Headway = 60 (minutes) /12 (trains per hour) = 5 minutes

Thus, the carrying capacity provided will be number of train passing per hour $\times$ carrying capacity of one train = 12 $\times$ 1750 = 21000 i.e. greater than the demanded PHPDT of 20000.

1.2 Number of rakes required

Number of trains required to operate on main line for revenue services to cater the demanded PHPDT along with maintenance reserve for a particular line. "Revenue/Traffic Hours" means the period between the time of the start of the running of the first scheduled train in the morning and termination of the last scheduled train at night. Parameters required for the calculations of number of rakes are headway, running time and turn back time at terminals. Running time is the period between the time of the start of the train from one terminal and arrival of the train at other terminal station of the line. Running time includes inter-station run time and dwell time at intermediate stations. Inter-station Run Time is the period between the time of the start of the train from one station and arrival of the train at next station. It is calculated by dividing the length of the line with average speed of the line. Terminal Station means the station at the end of a line. Turn back Time also known as layover time, is the time allowed at destination or terminal stations for passengers to alight and board, and for the Train Operator to change ends. It also depends upon the track layout of terminal station. It is the summation of dwell time at last station, run time from platform to siding travelling time for train operator to change from one end to other end, run time from siding to platform in other direction, dwell time at platform. Cycle Time is the one complete round trip time. It is the summation of running time in up and down line and turn back time at both terminals. Siding means a track of a suitable length provided at the end of the terminal station for the purpose of turn back of a train or for stabling of the defective/spare train.

Example 2 :

Consider above example with headway of 5 min, runtime of 47 min in both up and down line and turn back time of 4.5 min.
Therefore Cycle time = $2 \times 47 + 2 \times 4.5 = 103$ min.
Hence, number of rakes required to operate on main line for revenue services = Cycle Time / Headway = $103 / 5 = 20.6 \approx 21$.
Assuming maintenance reserve of 8 % i.e, $8 \times 21 / 100 = 1.68 \approx 2$. Therefore, total no. of rakes required = $21 + 2 = 23$.

2 Literature Review

Public transport timetabling has been a significant area of study in the Operations Research (OR) literature. Researchers have developed various models and heuristics to address both periodic and aperiodic timetabling problems.

Canca et al. (2016) address demand disruptions in rapid transit systems by proposing an optimized short-turning strategy to manage increased passenger load effectively. Their approach identifies optimal turn-back points and service offsets, aiming to reduce passenger wait times and maintain service quality with minimal cost increases. Computational analyses on a real case demonstrate the feasibility and impact of their tactical model on demand management.

Zhang et al. (2018) focus on optimizing train schedules in urban rail systems with high passenger demand by employing a short-turning strategy. They propose a mixed-integer nonlinear programming (MINLP) model that integrates short-turning and full-length train services with depot considerations to enhance train utilization. By transforming this model into a mixed-integer linear programming (MILP) form, they achieve efficient solvability. Case studies on Beijing Subway Line 4 demonstrate that their approach can reduce train usage by approximately 20%, highlighting its effectiveness in balancing capacity and operational costs.

Zhu and Goverde (2019) developed a railway rescheduling model that integrates flexible short-turning and stopping with retiming, reordering, and cancellations to manage disruptions. Their approach minimizes passenger delays, with case studies on Dutch railways showing that flexible short-turning and stop adjustments effectively reduce delays.

Blanco et al. (2020) propose a MILP optimization model for line planning and timetabling in automated metro networks. Their approach jointly optimizes frequencies, capacities, and short-turning decisions to balance costs and passenger needs, using a matheuristic solution to handle real-world complexities in urban subway systems effectively.

Yuan et al. (2022) presented an integrated optimization approach involving the timetabling, line planning and short tuning strategy. Authors have simultaneously taken into account multiple factors, such as the operation zones, train capacity, turnaround operations, and the availability of trains. To achieve high-quality solutions, they have developed a hybrid algorithm that combines a genetic algorithm with a general purpose solver. The proposed model efficiency and effectiveness was demonstrated on simplified Beijing Line 6.

3 Inputs

The line planning problem provides the frequencies of lines/edges, lines to be used. In this research we require a periodic timetable that repeats itself in given periods (peak, off-peak) and also convert the frequencies obtained from line planning to **short turning ratios** and then various other constraints develop a full-day timetable. For a feasible and effective timetable, various inputs are required about the infrastructure, rakes availability, frequencies, turnaround times etc.

3.1 Short Turning Ratios

Short Turning Ratio is defined as ratio of number of trains arriving at the station to number of trains that turn back to carry services in another direction at that station. They are defined for both directions. For example a ratio of 2:1 in down direction at station B means that for each 2 trains arriving at station B, 1 train turns back to service in up direction. These of terminal stations in direction of approach/arrival should always be 1:1 i.e. for each arrival of train it turns back and 1:0 in direction of departure to ensure that no trains that have turned and are arriving at that terminal station to depart should turn back. Short turning ratios are calculated from frequencies. Short tuning ratios can be integer as well as fractional values.

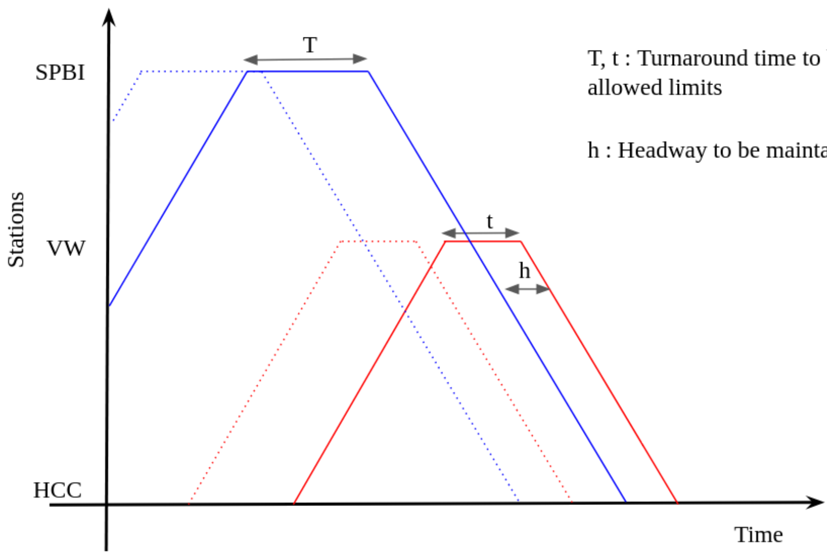

Figure 1: Station-Time chart with Short Turn at VW

3.2 Procedure to change frequencies to short turning ratios

For each direction of approach, if the frequency of rakes is not changing, no short turning would happen. However, if the frequency changes, it indicates that short turning is occurring. When the frequency decreases at a station ahead, it means that short turning is happening at that station and in that direction.

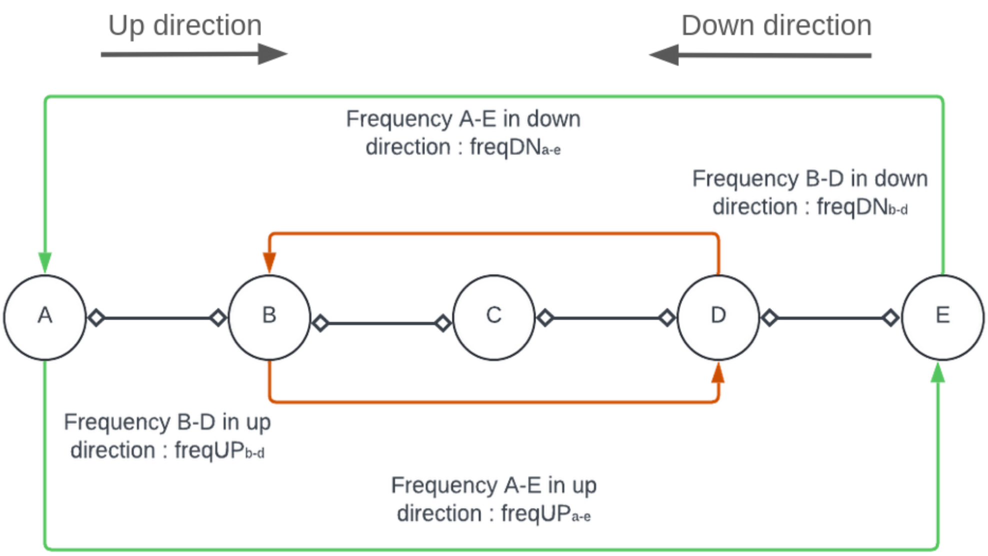

Figure 2: Short Turning Ratio Calculation

For the case shown in Figure 2 , the frequency at station D is given by:

$$\text{freqUP}_{b-d} + \text{freqUP}_{a-e}$$

The frequency at the station ahead of it, i.e., station E, is:

$$\text{freqUP}_{a-e}$$

Since the frequency at station D is greater than the frequency at station E, some rakes are turning at station D.

The short turning ratio is defined as:

$$\text{Short Turning Ratio D in UP direction} = \frac{\text{frequency at Station D}}{\text{frequency at Station D} - \text{frequency at Station E}}$$

Substituting the frequencies, we get:

$$\text{Short Turning Ratio D in UP direction} = \frac{\text{freqUP}_{b-d} + \text{freqUP}_{a-e}}{\text{freqUP}_{b-d}}$$

The short turning ratio for all the intermediate stations that are not turnaround stations will be 1:0. Similarly calculations are carried for the other direction as well.

3.3 Different layouts of overlapping lines and their short turning ratio calculation

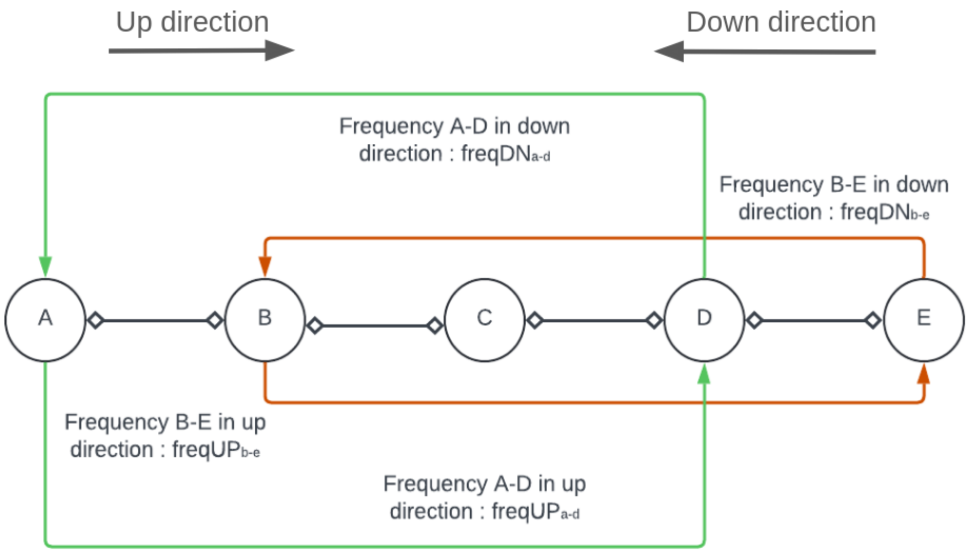

Figure 3: Intersecting Lines with different stations

From Figure 3 it is clear that that stations B and D act as the intermediate stations and A and E are the terminal stations with a short turning ratio of 1:1 in their respective direction of approach. Thus station/section between B-D operate at combined frequency of lines A-D and B-E; thus there will always be a change in values of frequencies when going from D to E or B to A, thus frequency values telling the correct positions of turnaround stations. Using the above same method we can calculate the short turning ratios of all stations.

$$\text{Short Turning Ratio D in UP direction} = \frac{\text{freqUP}_{b-e} + \text{freqUP}_{a-d}}{\text{freqUP}_{a-d}}$$

Figure 4 shows a scenario in which the terminal stations acts as a common station while having one more station B as intermediate turnaround station. Effective frequency in the section of overlaps i.e., B-E will have both lines B-D and A-E combined frequency. Thus short tuning ratio of station B is calculated as :

$$\text{Short Turning Ratio B in DOWN direction} = \frac{\text{freqDN}_{b-e} + \text{freqDN}_{a-e}}{\text{freqDN}_{b-e}}$$

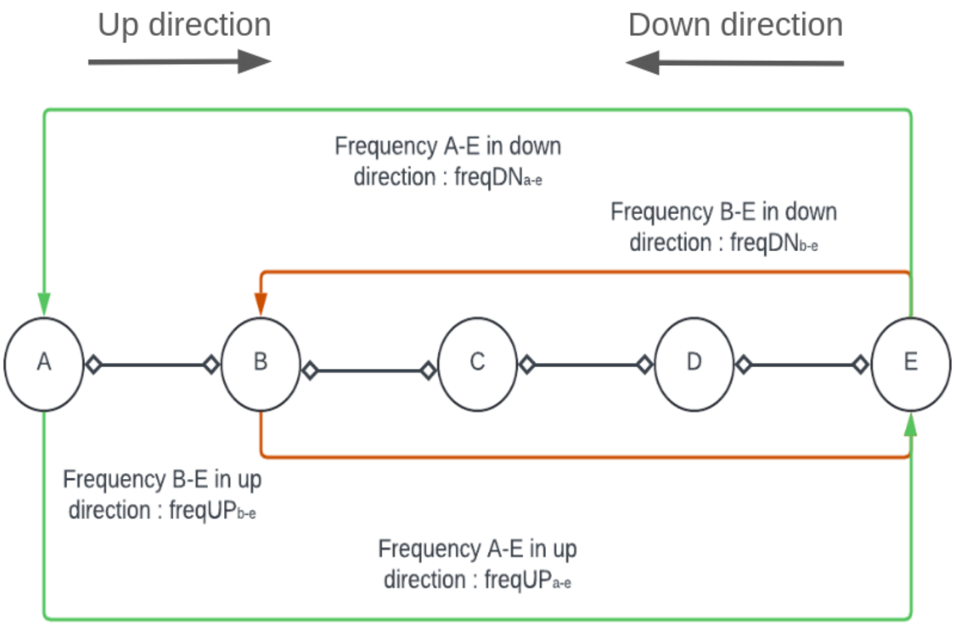

Figure 4: Intersecting Lines with one same station

Short turning ratio in UP direction is 1:0 for all intermediate stations as no turning happens (except at terminal stations) in that direction.

3.4 Infrastructure

Infrastructure is a crucial parameter in building a railway timetable, encompassing various components such as stations, turnaround facilities, capacity, stabling lines, and platforms. These elements significantly influence the speed, performance, and availability of rakes, all of which directly impact the timetable. Given that such detailed infrastructure inputs are not typically available during the initial line planning stages, their incorporation becomes vital for accurate and efficient timetable development. Here's a breakdown of why each component is important:

Stations
Each station's capacity, layout, and facilities determine how many trains can be handled simultaneously, influencing scheduling and potential bottlenecks.

Turnaround Facilities
The availability and efficiency of turnaround facilities at terminal stations and intermediate stations affect the minimum layover time required for each train, impacting the overall frequency and reliability of services.

Capacity
The overall capacity of tracks and stations dictates the maximum number of trains that can run within a given period, influencing both the frequency of services and the potential for delays.

Stabling Lines
Adequate stabling lines are necessary for parking trains when they are not in use; this is critical for maintaining a balanced and reliable service, especially during off-peak hours.

Platforms
The number and length of platforms at each station determine the types and sizes of trains that can be accommodated, affecting scheduling flexibility and the ability to handle peak-hour traffic.

In summary, comprehensive infrastructure information is essential for creating a realistic and efficient railway timetable. Without these inputs, line planning can lead to theoretical schedules that may not be feasible in practice, leading to delays, inefficiencies, and potential service disruptions.

4 Implementation of the IITB-DMRC timetabling tool

The timetable was developed through a simulation-based scheduling method, aimed at satisfying various operational constraints. These constraints include turnaround times, induction times, different headway requirements during peak and off-peak hours, and varying run times throughout the day. To ensure the timetable meets these conditions efficiently, a heuristic approach was employed. This approach integrates four distinct algorithms, each designed to handle specific aspects of the scheduling process, ultimately resulting in a comprehensive full-day timetable that balances demand, resource availability, and operational efficiency. The approach consists of four algorithms integrated with each other -

- A collision removal procedure to rule out train conflicts.
- A procedure to make headway uniform to a resaonable extent.
- A procedure to confine the cascading effect caused by varying run times and headways.
- Post-processing of the outcome to obtain the timetable in user readable format in ascending order of arrival times.

Program workflow: Initially a timetable is generated for a given simulation time assuming all rakes are inducted/initiated from a single terminal station at the intervals of headway. This part of the program also includes the rampdown phase during the afternoon i.e, changing the run type from tight run to coasting run which results in removal of few rakes from the service and an increase in headway, rampup phase during the evening i.e, changing the run type from coasting run to tight run and pulling some rakes into service to maintain the headway required in evening peak period. After a timetable for a specified time period is developed, stating period of the timetable output is trimmed (where all rakes are initialized from a single station) to a particular time instance and backtracking is adopted with the same program where timetable is generated in reverse order such that it meets the desired start time at various start stations and then followed by nightstabling of the rakes at depots, mainline stabling and siding locations.

Algorithm 1 defines the main loop of the heuristic program that includes induction of rakes while trying to maintain uniform headway, checking for collision after each run of rake and resolving those issues, changing the modes and headway calculated from the formula to remove cascading effects of increasing headway and ramping up and down of rakes during desired period of operation. The whole algorithm works for a precomputed run time of rakes.

In the main loop rake objects get initialized and then these objects are called into **Algorithm** 2 that makes the rake run ahead and meanwhile check if any rake has reached station. If stations are reached it gets updated meanwhile it also keeps on updating the short Turning Ratios too. To ensure First Come First Serve scenario a recent time of departures are also calculated and next events off all rakes scheduled accordingly.

After the getNextStn() function the events scheduled needs to be checked for any headway violations and is done by **Algorithm** 3. It returns the recent event of collision among different categories of collision. It returns the rakeEvent which has collision on which a collision removal function would be implemented to resolve the conflicts.

After all events of all the rakes are scheduled and they need to be written into an excel file in increasing order of their departures time at all stations, including some rake that are ramping down and ramping up in between the runs, which creates some empty columns/ rows in the run, due to which a tailored version of topological sorting (**Algorithm** 4) was used. Initially all rake arriving at all stations were sorted and were treated as graph nodes and for all upcoming rakes it was made the pre-requisite for them. Thus for all stations a directed graph was made where edges pointed from the upcoming rake to preceding rake and topological would always provide a unique solution to it if the First Come First Serve (FCFS) is always followed in the timetable.

Algorithm 1 Timetabling Main Loop with all functions

Require: $EventObjects$, $Headway \geq 0$, $offPeakPeriod \geq 0$, $PeakPeriod \geq 0$, $NumberofRakes \geq 0$

$t \leftarrow 0$

while $t \leq endTime$ **do**

  for `all Initialized Rakes` **do**

    if t in **offPeakPeriod then**

      $rampDown$

      $increaseHeadway$

      $changeRunningmode$

    else if t in **Peak-Period then**

      $rampUP$

      $decreaseHeadway$

      $changeRunningmode$

    end if

    getNextStation()

    while $checkForCollisions()$ **do**

      removeCollisions()

    end while

  end for

end while

Algorithm 2 getNextStation () : function for next station with Short Tuning

Require: $eveObj$: $objectofevent$

$eveObj.timeFromBottom + 1 \leftarrow eveObj.timeFromBottom$

if $eveObj.timeFromBottom$ in traversalTime **then**

  $Update\ Ratio$

  $Update\ eveobj.currentStation$

  if Short Turn Ratio Achieved **then**

    $t + turnaroundTime \leftarrow t$

    Turn the train to opposite direction

    $NewRatio \leftarrow Ratio$

    calculate recent Time of Departure

  end if

  $t + DwellTime \leftarrow t$

end if

Algorithm 3 checkCollision () : function to check for collision

Require: $station$, $rakeEventDict$: $eventdicitonary$, $direction$

All events from the particular station are stored from all rakes events

All Events are Sorted in descending order of time

for `All Events` **do**

  check for recent headway violation

  categorize the events of headway violation into categories

  if category $initialized$ **then**

    return colliding event of initialized rake

  else if category $turning$ & $continue$ **then**

    return colliding event of continue rake

  else if category $turning$ & $tunring$ or $contniue$ & $conitue$ **then**

    return colliding event of rake with maximum time

  end if

end for

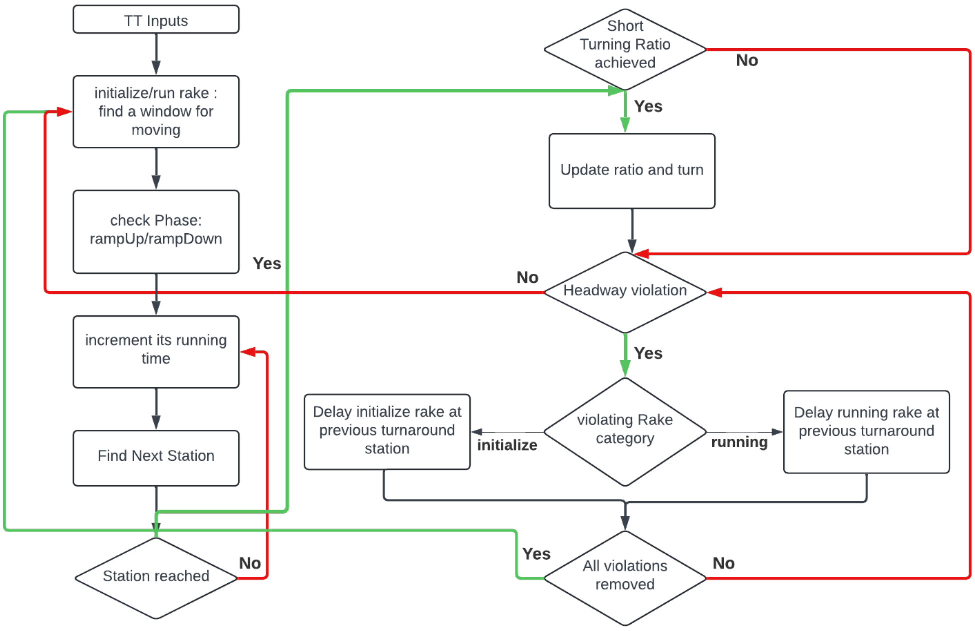

Figure 5: Timetable Heuristic Flowchart

5 Induction/Stabling

A total of 55 rakes are used in the timetable, of which 43 rakes are inducted from the following depots:

- Badli Depot: 11 rakes
- Khyber Pass Depot: 21 rakes
- Sultanpur Depot: 11 rakes.

The rest 12 rakes are inducted from stabling locations that are not depots, but main-lines itself. Major constraints included the first service time and last service time to be followed between the sections. Stabling location rakes to be inducted earlier followed by rakes from depot.

The process of "induction" and "stabling" is explained here. Each night, all the rakes are removed from the operation and halted in an appropriate location: this process of removing a rake from operation is called "stabling": this is either into a maintenance-yard (called the "depot") or on the line-itself (usually the last few rakes). The constraints are about both depot-capacity and the time-duration between consecutive receiving of rakes at depots. The process of "inducting" is, loosely speaking, opposite of "stabling": in inducting, rakes that have been non-operational are pulled into service one by one at appropriate place/time to thus "ramp-up" the services. We now explain the process followed for implementing the inducting and stabling tasks.

5.1 Induction and Stabling of Rakes

After generating the core timetable as detailed in Section 4, where all rakes are initially dispatched from a single depot, the early portion of the timetable is removed up until the start of the morning peak hour. All rakes are then backtracked till the earliest scheduled departure time of the first rake (e.g., if the first service on a line is at 05:00, all rakes are backtracked to this time). The rakes are then mapped to align with the specified first service times and any earlier events before that time are discarded, for rakes to be inducted from mainline stabling and siding locations set by the user. For remaining rakes to be inducted from depot locations, constraint programming is employed. Constraints include that each rake must be assigned to exactly one depot, adhere to depot capacity limits, and maintain the 5-minute headway between consecutive departures from the same depot. This approach is mirrored for night stabling, with the added

Algorithm 4 Full Timetable Generation : Topological Sort

Require: $G(V, E)$: Directed graph with vertices V and edges E

Initialize an empty list L to store the sorted vertices
Compute in-degree for each vertex in V and store in $inDegree$
Initialize a queue Q and enqueue all vertices with $inDegree[v] = 0$
while Q is not empty **do**
    Dequeue a vertex u from Q
    Append u to L
    for each vertex v adjacent to u **do**
        Decrement $inDegree[v]$ by 1
        if indegree == 0 **then**
            Enqueue v into Q
        end if
    end for
end while
if $|L| \neq |V|$ **then**
    return Timetable not following FCFS”
else if then
    return L (Topologically sorted order)
end if

requirement that rakes inducted from mainline stabling or siding locations should return to the depot, while those inducted from the depot should be mapped to stabling or siding locations.

5.2 Challenges and Proposed Solutions

There are various challenges that occur while attempting uniform headway, different headway, changing modes of operations, ramping up rakes, ramping down of rakes etc. Each problem faced with solution implemented are as follows:

- **Uniform headway**
  The problem with a uniform headway is that it is very difficult to maintain given different turnaround times, rake number etc. It is possible that a small headway gap can cause a huge headway difference ahead in time of program. To solve the problem a program was built which allows the rakes to adjust their turnaround time to resolve conflicts and maintain uniform headway. It tries to pack all rakes cycles as close as possible to make rooms for other rakes and strictly follow the headway constraint.

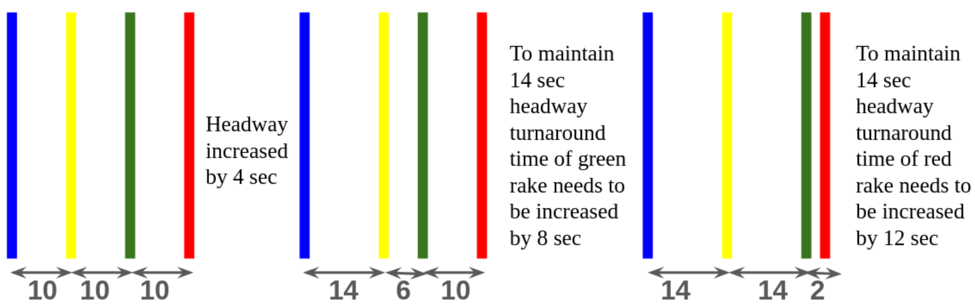

Figure 6: Example of Cascading Effect

- **Changing Headway and modes of operation**
  When peak period to off-peak period and vice-versa , mode changes and transition occurs then rakes require to pulled back to depot or inducted from the depots respectively. It happens to be generate a problem of **cascading**, due to which turnaround times of rakes might rise exponentially, thus a precise algorithm was needed to provide when rakes should be pulled back and inducted when headway is

changing so that cascading effect can be contained. This effect can't be entirely removed but can be contained by below formula. It will cause some disruptions in headway but after some time everything becomes normal.

Let:

- ph := the previous headway.
- nh := the new headway.
- diff := nh − ph (since nh > ph).

When transitioning from peak to off-peak periods, the new headway nh is greater than the previous headway ph. When 1 rake is pulled back, the gap created is $2 \cdot \text{ph}$ as uniform headway is maintained previously.

Let n be the number of rakes after which a rake is to be pulled back for stabling. Then, to confine the cascading effect:

$$\frac{n}{2}\left(2 \cdot 0 + (n-1) \cdot \text{diff}\right) \leqslant 2 \cdot \text{ph}$$

Substituting $\text{diff} = \text{nh} - \text{ph}$ into the inequality, we get:

$$\frac{n}{2}\left((n-1)(\text{nh} - \text{ph})\right) \leqslant 2 \cdot \text{ph}$$

Simplifying inside the parentheses:

$$\frac{n}{2}(n-1)(\text{nh} - \text{ph}) \leqslant 2 \cdot \text{ph}$$

Multiplying both sides by 2 to clear the fraction:

$$n(n-1)(\text{nh} - \text{ph}) \leqslant 4 \cdot \text{ph}$$

Dividing both sides by $(\text{nh} - \text{ph})$:

$$n(n-1) \leqslant \frac{4 \cdot \text{ph}}{\text{nh} - \text{ph}}$$

Thus, the formula for the number of rakes (n) after which rake to be pulled back is:

$$n(n-1) \leqslant \frac{4 \cdot \text{ph}}{\text{nh} - \text{ph}}$$

6 Summary

This study presents a comprehensive approach to metro railway timetabling, emphasizing the algorithms integral to effective schedule generation. The key algorithms include:

- **Main Loop (Algorithm 1)**: A heuristic program that initializes rake initialization and manages headway, checks for collisions, and adjusts modes and headway dynamically to prevent cascading delays during scheduled operation.
- **getNextStation() (Algorithm 2)**: This algorithm advances each rake to the subsequent station, updates station arrivals, calculates departure times, and ensures adherence to the First-Come-First-Serve (FCFS) rule by maintaining recent departure times.
- **checkCollision() (Algorithm 3)**: Designed to detect and identify any headway violations, this algorithm isolates collision events, which enables efficient conflict resolution.
- **Topological Sort (Algorithm 4)**: A customized sorting algorithm that orders events in ascending departure time across all stations, creating a directed graph structure where rakes are nodes and edges represent schedule prerequisites, thereby ensuring a unique FCFS-compliant timetable.

These algorithms collectively streamline the timetabling process, contributing to efficient railway operation and management.

7 Conclusions

In this paper, we illustrated a detailed procedure for generation of a timetable satisfying constraints of run times (different regimes of operation during peak and non-peak hours), turn around times, depot and siding locations and the rakes available for starting services at the beginning of a day. The specific example of Delhi Metro Line has been taken up with all its infrastructural and passenger demand satisfaction constraints. For a particular headway of 169 sec during peak period with a short turn ratio of 2:1 full day timetable was generated using the tool. The constructive procedures developed in the current effort are generalizable to user specified parameters of headway, turn-around time, more than one short turn option and other constraints, but needs to be developed for more general situations involving induction and night stabling of rakes and general network structure of services.

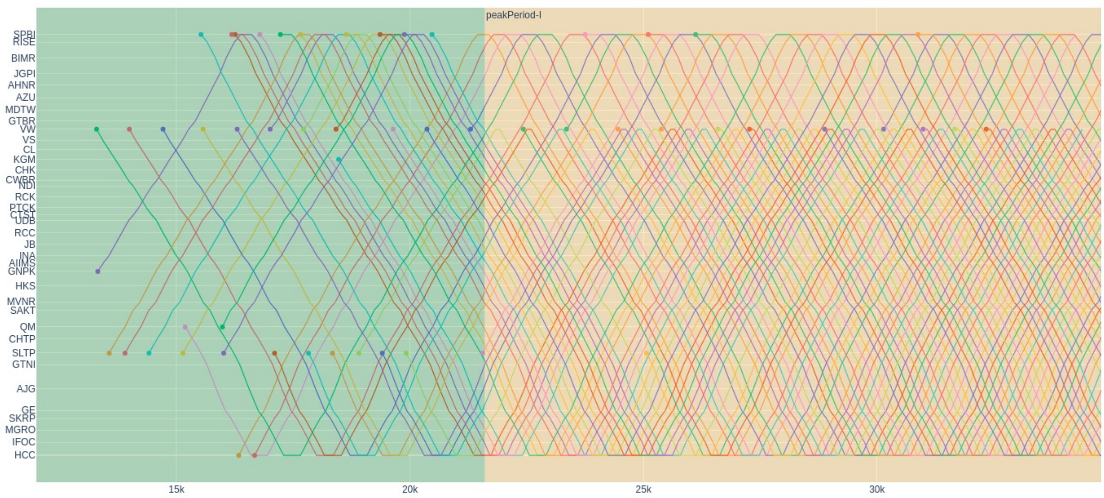

Figure 7: Timetable generated from the program

8 Acknowledgement and Data Usage

The data used in this study is fairly typical and representative, though not exact. The numbers were used for demonstrating the objective and features of the tool and any other inferences.

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