Indian Institute of Technology Bombay
Dept of Electrical Engineering

Handout 7 EE 210 Signals and Systems
Correction August 26, 2015

In the class, I used

$$\int_{|s|>a} \exp(-\frac{\pi s^2}{\delta}) dx \to 0,$$

as δ goes to zero. This note corrects a mistake in the proof given in class. Assume $a > 0$.

$$\int_a^{\infty} \exp(-\pi s^2/\delta) ds = \int_a^{\infty} (\exp(-\pi s^2/\delta)\exp(\pi a^2/\delta)) ds. \exp(-\pi a^2/\delta) \tag{1}$$

$$= \exp(-\pi a^2/\delta) \int_a^{\infty} \exp(-\pi(x^2 - a^2)/\delta) dx \tag{2}$$

$$= \exp(-\pi a^2/\delta) \int_a^{\infty} \exp(-\pi(x-a)(x+a)/\delta) \tag{3}$$

$$\leq \exp(-\pi a^2/\delta) \int_a^{\infty} \exp(-\pi(x-a)(x-a)/\delta) \tag{4}$$

$$= \exp(-\pi a^2/\delta) \frac{\sqrt{\delta}}{2}. \tag{5}$$

In the second last step (the inequality), we replaced $x + a$ by $x - a$, both quantities are positive in the range of interest, and we have $(x + a)(x - a) \geq (x - a)(x - a)$ for $x > a$ whenever a is positive. The last step is from a change of variables $u = x - a$.

The above inequality will ensure the convolution with a limiting Gaussian ($\delta \to 0$) will give us the function value at $t = 0$, hence the proof. Please ignore the alternate arguments that we had in class.