Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 5 EE 210 Signals and Systems

Homework 2 August 21, 2015

Question 1) For an odd function, i.e. $f_o(u) = -f_o(-u)$, for u in the range $[-\pi, \pi]$, show that

$$f_{odd}(u) = \sum_{m \geq 1} A_m \sin(mu)\,,\ -\pi \leq u \leq \pi. \tag{1}$$

Question 2) Let $f(t) = t$ for $-\frac{T}{2} \leq t \leq +\frac{T}{2}$. Find the FS expansion for $f(\cdot)$.

Question 3) Show that every continuous even function defined on $[-\pi, \pi]$ admits an expansion,

$$f_{even}(u) = \sum_{m \geq 0} \hat{A}_m \cos(mu) \tag{2}$$

Question 4) A string is tied straight between two hinges at coordinates $(0, 0)$ and $(L, 0)$ respectively. A point at a horizontal distance of p from origin is given a vertical displacement h initially. Let the initial position be described by the function $f(u)$. We know that the frequencies are multiples of $\frac{2\pi}{L}$, which is called the fundamental frequency or the first harmonic. The higher harmonics are now progressively counted as second, third etc.
a) Find the coefficients A_m if

$$f(u) = \sum_{m \geq 1} A_m \sin(\frac{2\pi}{L} mu).$$

b) Can you expand

$$f(u) = \sum_{m \geq 0} B_m \cos(\frac{2\pi}{L} mu).$$

In this case, find B_m.

c) For what value of p are the even harmonics missing.

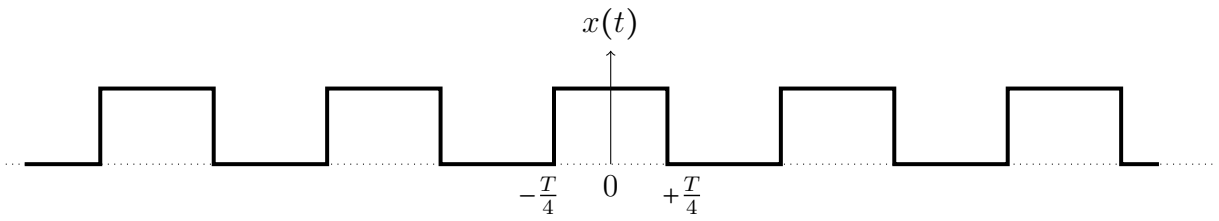
d) Is there a position p such that the odd harmonics are missing.

Question 5) Find the Fourier Series expansion for

$$f(t) = \sin(\theta + 2\pi f t) \text{ where } \theta \in \mathbb{R}. \tag{3}$$

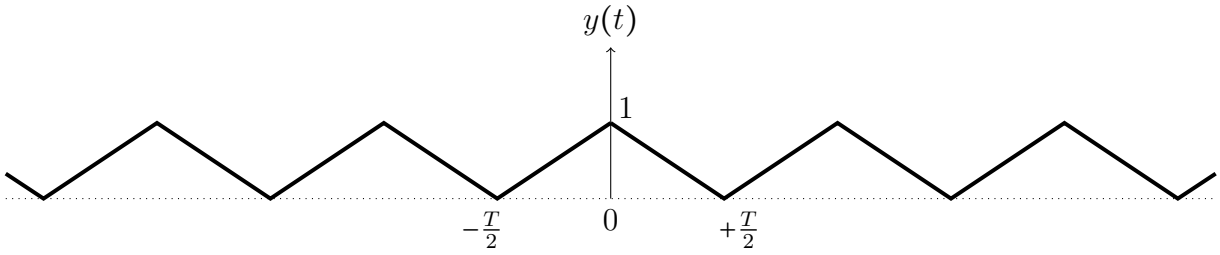
Are the F.S. coefficients continuous in θ?

Question 6) Consider a T-periodic signal $x(t)$ shown in figure. This is known as the rectangular train, where the non-zero amplitude is unity.



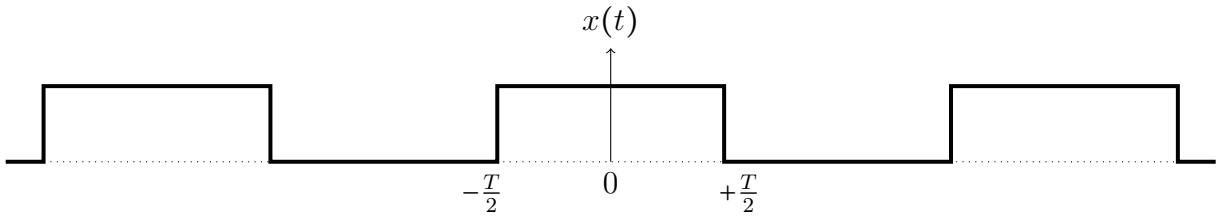
a) Find the Fourier Series coefficients of this signal.

b) Can you find a system $h(t)$ such that $y(t) = x(t) * h(t)$ is the following signal,



c) Can you find the FS coefficients of $y(t)$ by using parts (a) –(b), and without explicitly performing an additional integration.

d) Consider the following $2T$-periodic rectangle train $s(t)$ of height 2 units.



Find the FS coefficients of $y(t)s(t)$, where the multiplication is point-wise for every t.

e) Plot the FS coefficients for parts (c) and (d), assuming $T = 10$.

Question 7) Find the Fourier Transform of the following signal $y(t)$.