Indian Institute of Technology Bombay
Dept of Electrical Engineering

Handout 13 EE 210 Signals and Systems
Home Work 4 Oct 1, 2015

Question 1) Consider a linear interpolator acting on samples $\{x_i\}$, $i \in \mathbb{Z}$ which are apart in time by T units.

a) Write the output $y(t)$ of the interpolator in terms of the input values $x_i, i \geq 0$.

b) Let $T = 2ms$. Suppose instead of samples, we feed the linear interpolator with a periodic cosine waveform of frequency $500Hz$ and amplitude π. What is the output of the interpolator.

c) Let us sample a $500Hz$ cosine waveform of amplitude π at the rate of 1000 samples per second. i.e.,

$$x_i = \pi \cos(\pi i).$$

These samples in turn are fed to the linear interpolator of part (a) with $T = 2ms$. What will be the output waveform.

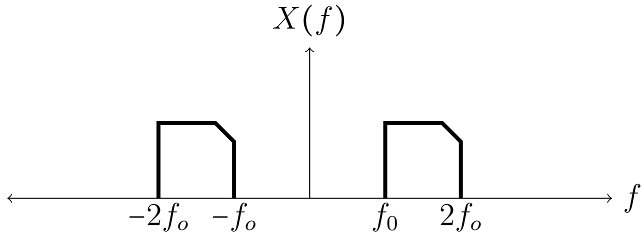
d) Reconcile parts (b) and (c).

Question 2) Find the FT of the following signal

$$f(t) = e^{-at}.\mathbb{1}_{\{t \geq 0\}} \tag{1}$$

where $\mathbb{1}_{\{.\}}$ is the binary (0 or 1) valued indicator function which is unity when the argument is true and zero otherwise. Assume $a > 0$. Plot the magnitude and phase of the FT separately.

Question 3) We have seen Shannon-Nyquist sampling theorem, and the question below will tell you some more details. Consider a signal $x(t)$ whose Fourier transform is as shown in the figure.



(a) What is the maximum frequency present in the system.

(b) Draw the DTFT of the signal $x[n]$ obtained by sampling $x(t)$ at the rate of $4f_o$ samples per second.

(c) Draw the DTFT of the signal $u[n]$ obtained by sampling $x(t)$ at the rate of $2f_o$ samples per second.

(d) Draw the DTFT of the signal $v[n]$ obtained by sampling $x(t)$ at the rate of f_o samples per second.

Question 4) Let

$$x[n] = \frac{\sin(\pi n/8)}{\pi n} \tag{2}$$

and

$$h[n] = \frac{\sin(\pi n/4)}{\pi n}. \tag{3}$$

Compute $y[n] = x[n] * h[n]$.

Question 5) Parseval's Theorem for the DFT: Show that

$$\sum_{k=0}^{N-1} |X[k]|^2 = N \sum_{n=0}^{N-1} |x[n]|^2 \tag{4}$$

where $X[k]$ is the N−point DFT given by,

$$X[k] = \sum_{n} x[n] \exp(-j\frac{2\pi}{N}kn) \tag{5}$$

for $0 \le k \le N-1$.