Indian Institute of Technology Bombay
Dept of Electrical Engineering

Handout 15 EE 210 Signals and Systems
Home Work 5 Oct 21, 2015

Question 1) The Discrete Fourier Transform or DFT is defined as

$$X[k] = \sum_{n=0}^{N-1} x[n] \exp(-j\frac{2\pi}{N}nk). \tag{1}$$

a) What is the DFT of the sequence,

$$x[n] = 1 \ , 0 \leq n \leq N-1$$

b) For the sequence in part (a), find $X[k]$ for $N \leq k \leq 2N-1$. Argue that it is sufficient to consider the DFT for $0 \leq k \leq N-1$.

c) The sequence in part (a) can be viewed as a discrete impulse (Kronecker) train. We can pad the train with zeros in between to generate another train. Let m be a factor of N and for $0 \leq n \leq N-1$,

$$x[n] = \begin{cases} 1, & \text{if } n = lm, l \in \mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Compute the DFT of this sequence.

d) The support of a signal $x[\cdot]$, denoted as $supp(x)$, is the number of non-zero entries in the sequence. Let $x[\cdot]$ be chosen as a zero padded impulse train in part (c). Find the value of m (remember m is a factor of N) for which,

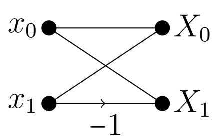
$$\Delta(N) = supp(x) + supp(X) \tag{3}$$

is minimized.

e) Let $N = 100$, give an example of a sequence, for which $\Delta(100)$ is the minimum.

f) What we have seen in (3) is an uncertainty principle. While we have proved it only for a selected class of signals, the results hold with more generality. Justify the name **uncertainty principle** in this question.

Question 2) Recall the basic butterfly network that we studied. The building block of this is the 2−pt butterfly, represented as



For future use, we term an input sequence as **ordered** if the input terminals of the butterfly network are fed with the incoming data sequence without any permutation.Thus x_i is fed to input terminal i for $0 \le i \le N-1$, iff the inputs are in order. The same convention applies to the output sequence. As an example, the FFT structure that we learned in class does not take an ordered input sequence.

a)Draw an 8−pt FFT structure using the basic butterfly blocks (please do not use more than half page for the image). Properly mark the inputs, outputs and the values that you use for multiplication along any branch, if the scaling is different from unity.

b) Consider a 4-pt FFT. Recall that in the network that we studied in EE327 lectures, the input symbols were permuted before feeding it to the butterfly network for FFT computation. Thus the input x_0, x_1, x_2, x_3 first pass through a permutation matrix, and then FFT is taken. The output came out as X_0, X_1, X_2, X_3, i.e. in order. Call the FFT block as Network A, see figure below.

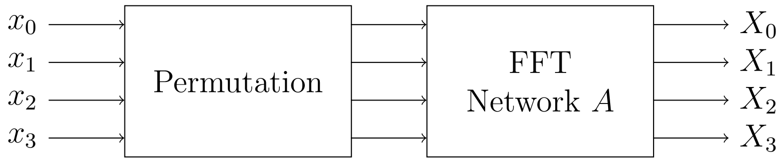


Figure 1: 4-pt FFT block diagram

Consider an alternate $4-pt$ butterfly structure where the input symbols are not permuted, i.e. they are in order, but the output symbols are allowed to be out of order. Name this Network B. Draw the butterfly structure of Network B using the basic blocks, clearly labeling the inputs and outputs. In addition, any scaling other than unity has to be marked on the corresponding branch.

[5 marks]

c) Consider a sequence u_0, u_1, u_2, u_3 which is given as input to Network A. The output of Network A is now fed directly to Network B. Let the ordered output of Network B be v_0, v_1, v_2, v_3. Express v_i in terms of u_i, $i \in \{0, 1, 2, 3\}$.