Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 3 EE 210 Signals and Systems

Tutorial 1 August 7, 2015

It is likely that some notions on signals and systems, when viewed out of the mathematical shell can look confusing, eg: convolution. Let us try to demystify the concept of convolution.

Question 1a) Please write yes/no.

1. I understand convolution.
2. I understand convolution in discrete time.
3. I realize that there are two times in play. One for the signal which varies in time and another for the delay of the system.
4. The notation is causing confusion.

Question 1b) Holiday Package: A travel agency next door, let us name it *AdHoc travels*, made a transparent arrangement for 4 day holiday trips. Here is the deal, a bus takes off everyday 10:00am from Mumbai, some sight-seeing stops in the middle, and reaches Goa by night. Each person (no concession for children) pays 2K for this trip, before boarding. Your stay for the night at a private resort/beach comes at the rate of 4K per person, which you need to pay next morning. You can spend the day at the beaches, and return at night to a country-side traditional living, costs 5K per person (to be paid when you leave), and is also close to many sight-seeing places. The following evening you take another bus back, this time with sleepers, costing 3K per person to be paid at the end of journey, reaching back at 10am. No booking is needed to any of these places prior to your departure. The facilities are large enough to accommodate all who wishes to travel, and facilities are not allowed to talk to each other about the number of tourists and their identity. Further, for simplicity assume that payments are made at 10am of each day.
a)The owners of these facilities (buses+motels) wish to know how much money they are generating from people send through the travel agent every day. Let us say in a sequence of days starting from 1st August 2010,

$$x[\cdot] = (0\ 5\ 15\ 20\ 22\ 7\ 3\ 2\ 14\ 1\ 17\ 11\ 13\ 5\ 5\ 5\ \ 5\ 5\ 5\ 5\ 5)$$

people availed this tour. Compute the amount generated on each day, starting from August 1st.

b) Can you connect this to the operation of convolution.

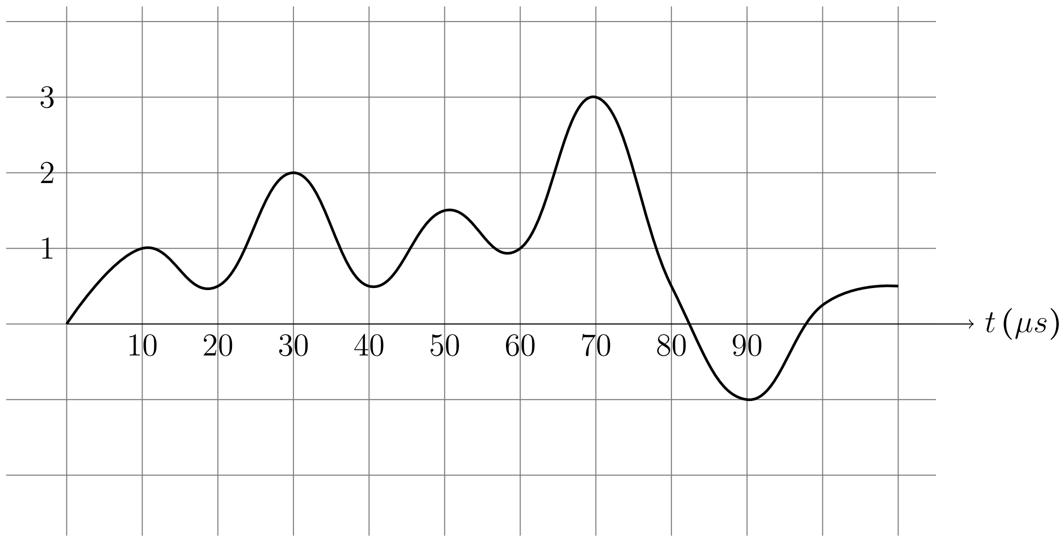
c) If the owners do a 500Rs per person cashback policy, to be paid 24 hours after travel, can you write the system (filter) model which outputs the total amount collected everyday.

d) If you can see the connection in part (b), observe more carefully that there are two kind of time-tables(or clocks) in operation. One is the global calendar, which tells how many customers are arriving each day. And this clock keeps ticking all the time, i.e, everyday there will be zero or more tourists, and the hosts should make them happy, or treat them properly). From the point of view of the hosts (service providers), it is a four day cycle of events that brings them the money from each customer. In other words, the total money they get at any given day comes from four different services (one from Mumbai-Goa service, one from the beach-resort, one from traditional living, and one from the sleeper bus). So this internal four day clock starts ticking for every batch of tourists the moment they enter the system. Suppose that customers are very happy, and business good and steady. Can you suggest something to get more revenue.

e) To understand more about the delays, assume that the facilities simply accept cheques. A cheque given on a day is cashed after k days. What will happen to the revenue(in hard currency).

This is why we didn't worry too much about the time-axis/causality when we drew system representations in the class. You delay the system, it is more like cheques taking a longer time to come out, you will have your money eventually.

Question 2) AM Radio: In AM radio, an audible baseband signal $m(t)$ is multiplied by a carrier signal $\sin(2\pi f_c t)$, where f_c is the carrier frequency (eg: $500kHz$). Shown below is the picture of a baseband signal for some time interval. Assuming $f_c = 500KHz$, sketch the output waveform fed to the transmit antenna.



Can you imagine a system for detecting $m(t)$ from the AM signal received at the receive antenna. It turns out we need a non-linear device. Nevertheless, these are simple mathematical operations and can be done with a computer. After the class, think for a while about incorporating an AM radio in to a computer, purely based on software. Think about what extra device do you need to have this play the local AM stations.

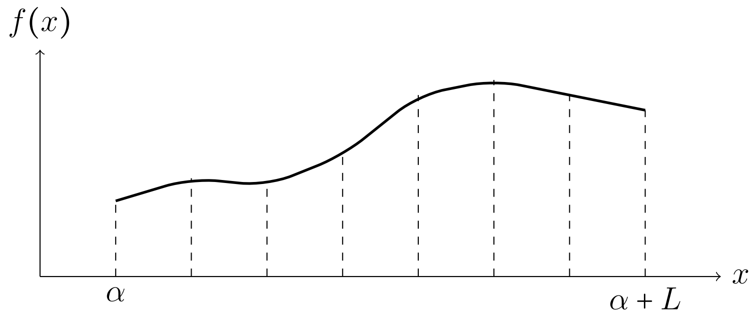
Question 3) Let us refresh our know-how of interpolation.
a) Draw 4 or more cycles of a cosine waveform of $100Hz$ (mark the axis).

b) Assume we pick samples at integer multiples of $5ms$. Find the cubic approximation from this samples and sketch it.

c) Suppose samples are taken at integer multiples of $15ms$. Sketch the cubic interpolated waveform $p(t)$, using dotted lines.

c) What do you think to be the dominant frequency component of $p(t)$. (We haven't started talking about frequency yet. Please make use of whatever idea you have about it.)

Question 4) Piece-wise constants and Riemann's integral. The Riemann Integral is one of the first things you have learnt in Integral calculus (or integration).

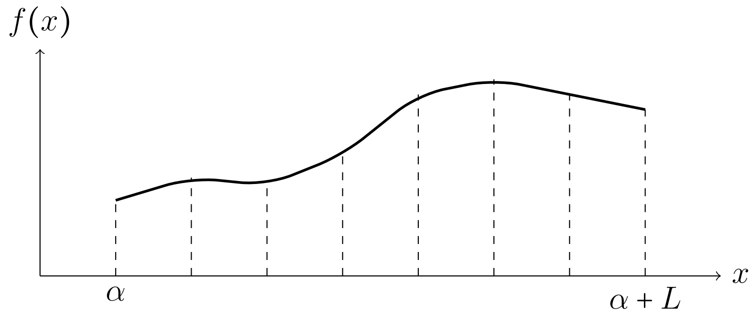


Let us integrate the function $f(x)$ from $x = \alpha$ to $x = \alpha + L$.

$$I = \int_{\alpha}^{\alpha+L} f(x)dx. \tag{1}$$

The idea of Riemann integral is to divide and conquer. As shown in the picture, we divide the $x-$axis to equispaced intervals. If there are m intervals, each will have a width of $\frac{L}{m}$. If we can find the integral in each of these intervals, we can put it together to get the total integral.

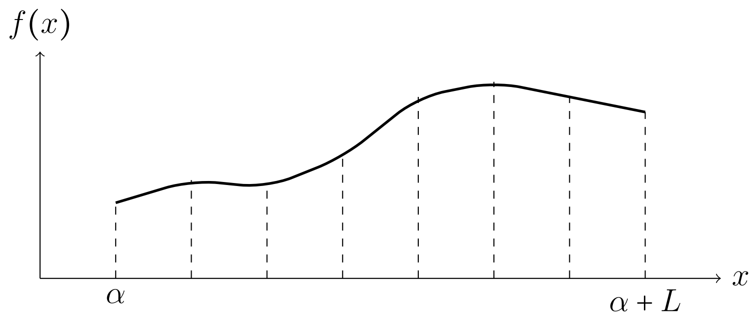
a) Assume for simplicity that the function is continuous. A continuous function will have a maximum and a minimum in any bounded interval (why?). What is the biggest rectangular block that you can inscribe in each interval, draw it in the picture.



b) What is the total area covered by these rectangles, if $f_l(i)$ is the least value in interval i?.

$$S_{lower}(m) = \tag{2}$$

c) Draw the minimal rectangle for each sub-interval which completely contains the function in that interval.



d) What is the total area covered by these minimal rectangles in terms of $f_h(i)$, the highest value in interval i.

$$S_{upper}(m) = \tag{3}$$

e) Notice that $S_{lower}(m) \leq I \leq S_{upper}(m)$. If $S_{lower} = S_{upper}$, then we have the answer to our integral. Look at the picture and argue that they are not equal (though close).

f) The next step is to increase m, say to $m_1 = 2 * m$. Argue why the following is true.

$$S_{lower}(m) \leq S_{lower}(m_1) \leq I \leq S_{upper}(m_1) \leq S_{upper}(m). \tag{4}$$

g) Repeating this procedure with $m_i = 2 * m_{i-1}$, we can make the upper and lower bound arbitrary close (why?).

For $m = 100$ note that,

$$S_{lower}(100) = \sum_{i=1}^{m} f_l(i) \frac{L}{100} \tag{5}$$

For $m = 1000$ note that,

$$S_{lower}(100) = \sum_{i=1}^{m} f_l(i) \frac{L}{1000} \tag{6}$$

Denote $\Delta_m = \frac{L}{m}$, then,

$$S_{lower}(m) = \Delta_m \sum_{i=1}^{m} f_l(m) \tag{7}$$

$$= \sum_{i=1}^{m} f_l(m) \Delta_m \tag{8}$$

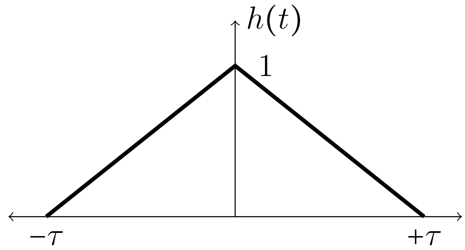
By taking m to infinity, we will have

$$\lim_{m \to \infty} \sum_{i=1}^{m} f_l(m) \Delta_m = \int_{\alpha}^{\alpha+L} f(x) dx \tag{9}$$

Notice that the role of Δ_m in the LHS is taken by dx on the RHS.
So when we use samples to make quick computations/approximations remember that we have to factor in for the Δ_m in the computations. Many a times, we will explicitly scale the sampled values so that their respective heights indeed represent the area of the rectangular block.

Question 5) Analog Convolution:

Consider the filter shown, where $\tau = 1$ ms. Find the output signal when an input $x(t) = 50 \sin(2\pi 1000t)$ is passed through this filter.



Question 6) Let us try to understand superposition and homogeneity in the definition of LTI systems. For brevity let x be the input signal and $h(x)$ be the output signal.
a) First show that superposition implies $\forall n \in \mathbb{Z}^+$ (positive integers),

$$h(nx) = nh(x) \tag{10}$$

b) Use a change of variables to show that superposition implies

$$h(\frac{x}{n}) = \frac{1}{n}h(x) \tag{11}$$

c) Now apply superposition again to show that

$$h(\frac{m}{n}x) = \frac{m}{n}h(x) \tag{12}$$

d) We have shown that superposition implies homogeneity for all rational coefficients ($\mathbb{Q}$). But to extend this to whole $\mathbb{R}$, we need the homogeneity formula to be true on irrational coefficients too. In particular, we need

$$\lim_{\alpha \to a} h(\alpha x) = h(ax)$$

for a rational sequence α tending to a, an irrational number. While this is guaranteed for most functions that we encounter (for example, continuous functions), there are pathological examples where the above equality fails to hold. We omit the details, and assert that while superposition and homogeneity are necessary, one can live with the first for most real-life situations.