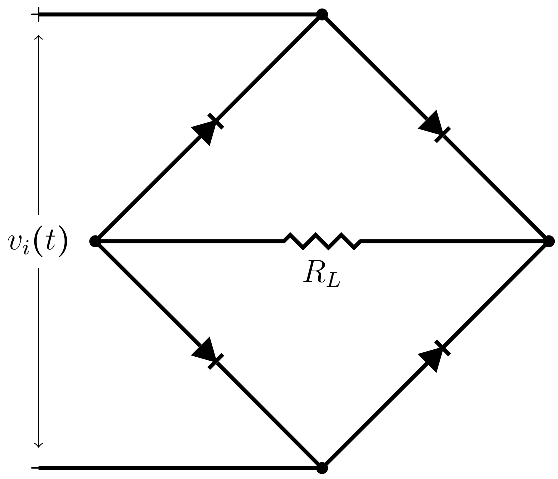
Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 8 EE 210 Signals and Systems

Tutorial 3 August 28, 2015

Question 1) Figure below shows a full wave rectifier, called the bridge rectifier. Assume ideal diodes, each acting like a valve/switch, i.e. the diode is a closed switch when input has the correct polarity, and open otherwise.



(a) Draw the output of the rectifier when the input peak is $20V$ at a frequency is $50Hz$, in the figure below.

Figure 1: Full wave Rectifier

It is clear that the full wave rectifier outputs

$$x_r(t) = |x(t)|. \tag{1}$$

Suppose $x(t) = \alpha \cos(2\pi f_0 t)$ with $f_0 = \frac{1}{T}$. Your laptop battery has such a device, and a filter/regulator which in turn converts the rectified waveform to a *constant* voltage. Let us learn how to derive a 12V dc from a 230V, 50Hz ac supply. Notice that ac-to-dc means suppressing the high frequencies. Ideally we like to have no non-zero frequencies (time variations) at the output. The first stage in this process is a transformer which steps-down the supply voltage to 20V at 50Hz (not shown in Figure). We will now rectify the 20V ac source, by designing a filter which produces dc output from the rectifier. The step by

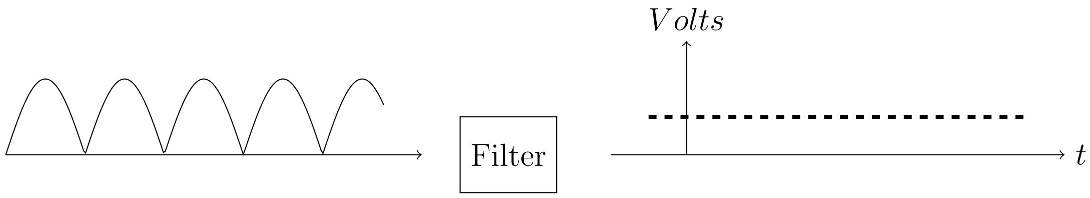


Figure 2: Filtering the Rectifier Output

step procedure that we describe can be used for other filter design problems too. The filter under consideration is an **analog filter** (or continuous time filter).

Step 1: Examine the frequency components of the input.

The knowledge of input frequency components is crucial in designing filters. We can have a lot more insight by into frequencies than time domain (which frequencies are good/bad and what effects they should produce). Thus, *filter design* boils down to the selecting, trimming and manicuring of the spectrum of frequencies. This is like a gardening assignment. Are you a good gardener?.

In the current example, we want to trim off all the non-zero frequencies, that the only remaining signal is zero-frequency (dc value). How do we find the frequencies or frequency spectrum?.

We know that periodic signals have a Fourier series representation, which shows the magnitude of each frequency component. We can also visualize the frequency components in an alternate way, using generalized Fourier Transform, i.e. FS coefficients are scaled samples of the Fourier Transform.

To clarify this concept, let us evaluate the frequency components of the rectifier output using both techniques. First the Fourier Series (FS) approach.

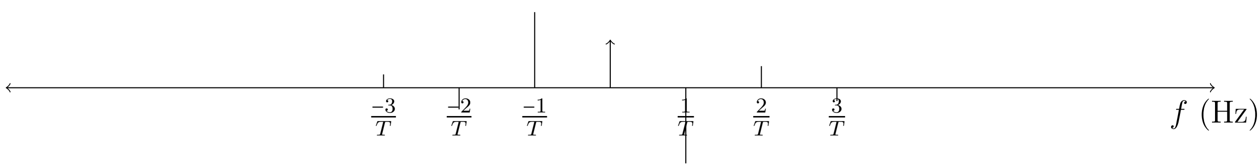
(b) Find the FS series coefficients by the formula.
Solution: For $m \geq 1$, and $T = \frac{1}{100}$,

$$\begin{aligned} a_m &= \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(100\pi t)\cos(\frac{2\pi}{T}mt)dt \\ &= (-1)^m \frac{16}{\pi}\frac{m}{2m^2-1}. \end{aligned}$$

We can also evaluate

$$a_0\frac{2}{\pi}.$$

(c) Plot the FT $X_r(f)$ of the waveform $x_r(t)$. (i.e, FS coefficients at the correct frequencies and marked by impulses of appropriate heights)



(d) Find the FS coefficients using the Fourier Transform of cosine waveforms and Poisson summation formula.

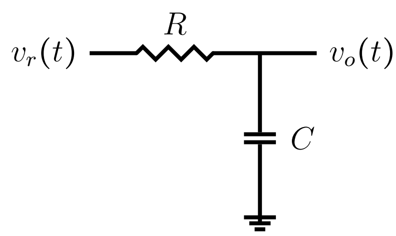
Step 2: Design a filter $H(f)$ to pick/mask the appropriate frequencies. So where will you connect the filter, mark the input to the load as $v_r(t)$ in the first picture.

(e) Consider $V_r(f)$, now suggest(draw) a filter which can avoid all frequencies above 100.01 Hz.

(f) Plot the output of the filter in previous part.

(g) Suggest an ideal filter which will give a dc output. While there are many such filters, argue the advantage of your choice.

(h) Let us assume that the load resistance is very high, and concentrate on a filter which will drive the load. Consider the following filter.



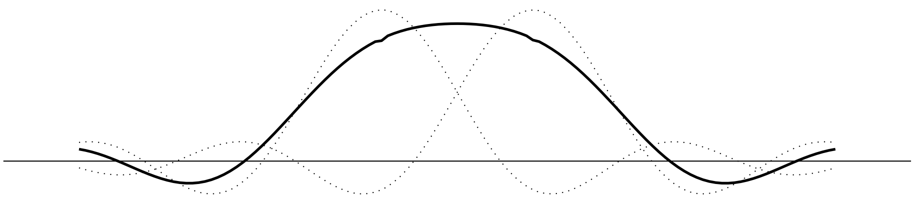
If a unit magnitude sinusoid is input to the filter, what will be the output waveform. From

this, identify the Fourier transform $H(f)$ of this filter.
(i) Tune the filter to obtain a dc waveform, with permitted variations of 10%.

Question 2) Parseval's Theorem: Show that

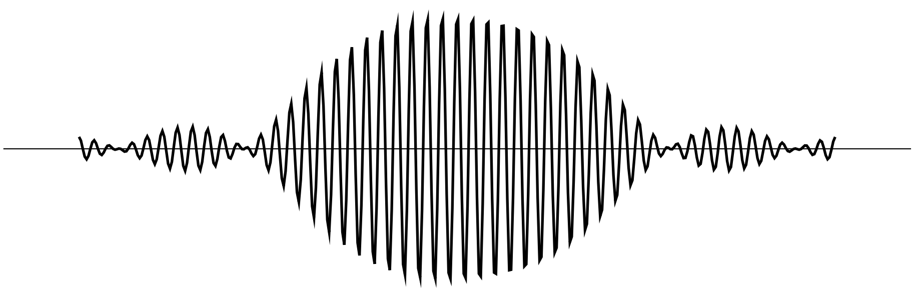
$$\int |x(t)|^2 dt = \int |X(f)|^2 df. \tag{2}$$

Question 3) In figure, the thick line is the sum of the two dotted plots, and each dotted line is a $\alpha \mathrm{sinc}(\frac{2}{\pi}t)$ function, shifted by t_0 units to each side of origin.



a) Compute the FT of this signal.

b) Suppose now the plot is of the form,



What do you expect the FT to be.