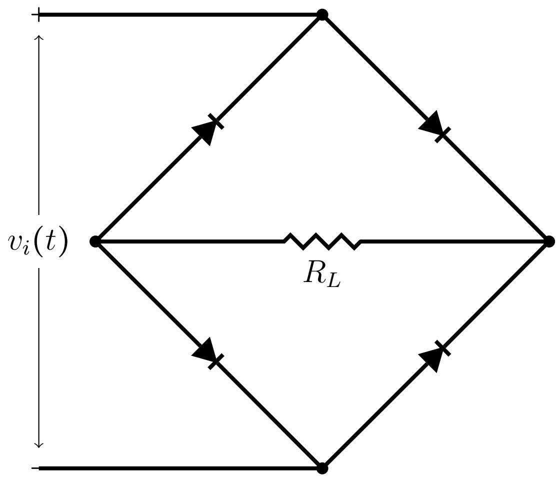
Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 8 EE 210 Signals and Systems

Tutorial 3 August 28, 2015

Question 1) Figure below shows a full wave rectifier, called the bridge rectifier. Assume ideal diodes, each acting like a valve/switch, i.e. the diode is a closed switch when input has the correct polarity, and open otherwise.



(a) Draw the output of the rectifier when the input peak is $20V$ at a frequency is $50Hz$, in the figure below.

Figure 1: Full wave Rectifier

It is clear that the full wave rectifier outputs

$$x_r(t) = |x(t)|. \tag{1}$$

Suppose $x(t) = \alpha \cos(2\pi f_0 t)$ with $f_0 = \frac{1}{T}$. Your laptop battery has such a device, and a filter/regulator which in turn converts the rectified waveform to a *constant* voltage. Let us learn how to derive a 12V dc from a 230V, 50Hz ac supply. Notice that ac-to-dc means suppressing the high frequencies. Ideally we like to have no non-zero frequencies (time variations) at the output. The first stage in this process is a transformer which steps-down the supply voltage to 20V at 50Hz (not shown in Figure). We will now rectify the 20V ac source, by designing a filter which produces dc output from the rectifier. The step by

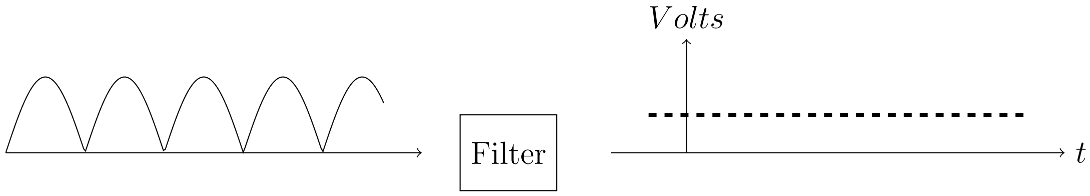


Figure 2: Filtering the Rectifier Output

step procedure that we describe can be used for other filter design problems too. The filter under consideration is an **analog filter** (or continuous time filter).

Step 1: Examine the frequency components of the input.

The knowledge of input frequency components is crucial in designing filters. We can have a lot more insight by into frequencies than time domain (which frequencies are good/bad and what effects they should produce). Thus, *filter design* boils down to the selecting, trimming and manicuring of the spectrum of frequencies. This is like a gardening assignment. Are you a good gardener?.

In the current example, we want to trim off all the non-zero frequencies, that the only remaining signal is zero-frequency (dc value). How do we find the frequencies or frequency spectrum?.

We know that periodic signals have a Fourier series representation, which shows the magnitude of each frequency component. We can also visualize the frequency components in an alternate way, using generalized Fourier Transform, i.e. FS coefficients are scaled samples of the Fourier Transform.

To clarify this concept, let us evaluate the frequency components of the rectifier output using both techniques. First the Fourier Series (FS) approach.

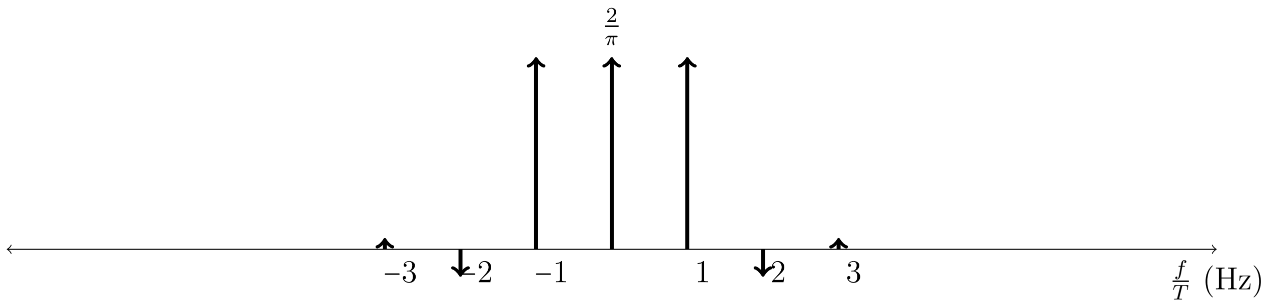
(b) Find the FS series coefficients by the formula.
Solution: Consider a unit amplitude cosine waveform being input to an ideal rectifier circuit. For $m \geq 1$, and $T = \frac{1}{100}$,

$$
\begin{aligned}
a_m &= \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(100\pi t)\cos(\frac{2\pi}{T}mt)dt \\
&= (-1)^m \frac{2}{\pi}\frac{1}{1-2m^2}.
\end{aligned}
$$

We can also evaluate

$$a_0 = \frac{2}{\pi}.$$

(c) Plot the FT $X_r(f)$ of the waveform $x_r(t)$. (i.e, FS coefficients at the correct frequencies and marked by impulses of appropriate heights)



(d) Find the FS coefficients using the Fourier Transform of cosine waveforms and Poisson summation formula.

Solution: The FS expansion of a unit amplitude 50Hz cosine waveform can be written as

$$X(f) = \frac{1}{2}\delta(f - \frac{1}{2T}) + \frac{1}{2}\delta(f + \frac{1}{2T}),$$

where $T = \frac{1}{100}$. Consider one centered half-cycle of the cosine waveform, call it $x_h(t)$. Now

$$x_h(t) = \cos(100\pi t) \times \text{rect}_T(t).$$

Applying the convolution-multiplication formula,

$$\begin{aligned} X_h(f) &= X(f) * T\text{sinc}(fT) &(2)\\ &= \frac{T}{2}\text{sinc}(fT - \frac{1}{2}) + \frac{T}{2}\text{sinc}(fT + \frac{1}{2}). &(3) \end{aligned}$$

Now, the waveform $x_r(t)$ can be obtained by periodically repeating $x_h(t)$, which is nothing but the convolution of $x_h(t)$ with an impulse train having a spacing of T time units. In the frequency domain this implies sampling and scaling of $X_h(f)$, i.e.

$$\begin{aligned} a_m &= \frac{1}{T}X_h(\frac{m}{T}) &(4)\\ &= \frac{T}{2T}\text{sinc}(m - \frac{1}{2}) + \frac{T}{2T}\text{sinc}(m + \frac{1}{2}) &(5)\\ &= \frac{2}{\pi}(-1)^m \frac{1}{1 - 2m^2} &(6) \end{aligned}$$

Notice that this also gives the value for a_0. Compare with the previous parts to see the validity of both derivations.

Step 2: Design a filter $H(f)$ to pick/mask the appropriate frequencies. So where will you connect the filter, mark the input to the load as $v_r(t)$ in the first picture.

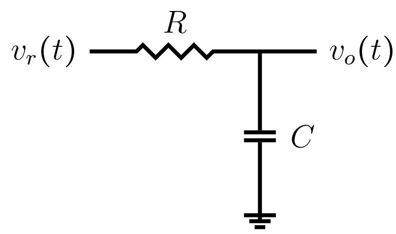
(e) Consider $V_r(f)$, now suggest(draw) a filter which can avoid all frequencies above 100.01 Hz.

Solution: Consider a filter $h(t)$ such that $H(f) = 1$ whenever $|f| \le 100 - \alpha$, and zero otherwise. This is known as a low-pass filter, i.e. the one which passes the lower end of the frequency spectrum and blocks the higher frequencies. For any choice of $0.01 < \alpha < 100$, the filter will block all frequencies above 100.01Hz. An important uncertainty principle dictates that, if $H(f)$ is made wider by frequency scaling, then the corresponding Fourier Inverse $h(t)$ will get narrower in time. Since a short-time filter is preferred, we should pick a higher allowed value of α.

(f) Plot the output of the filter in previous part.

(g) Suggest an ideal filter which will give a dc output. While there are many such filters, argue the advantage of your choice.

Solution: Consider a filter $h(t)$ such that $H(f) = 1$ whenever $|f| \leq \alpha$, and zero otherwise. We can filter to obtain an ideal DC by choosing $0 < \alpha < 100$ as $H(f)$. Notice that the corresponding time domain filter corresponds to a scaled sinc waveform.
(h) Let us assume that the load resistance is very high, and concentrate on a filter which will drive the load. Consider the following filter.



(i) If a unit magnitude sinusoid is input to the filter, what will be the output waveform. From this, identify the Fourier transform $H(f)$ of this filter.
Solution The capacitance offers an impedance of $\frac{1}{j2\pi fC}$ to an input sinusoid of frequency f Hz. The current flowing in the circuit is

$$I = \frac{v_r}{R + \frac{1}{j2\pi fC}}.$$

The voltage drop across the capacitor is $I \times \frac{1}{j2\pi fC}$. Putting it all together, a unit sinsoid will produce an output sinusoid of amplitude

$$A_m = \frac{\frac{1}{2\pi fC}}{R + \frac{1}{j2\pi fC}}.$$

Thus, the filter response is given by

$$H(f) = \frac{1}{1 + j2\pi fRC}.$$

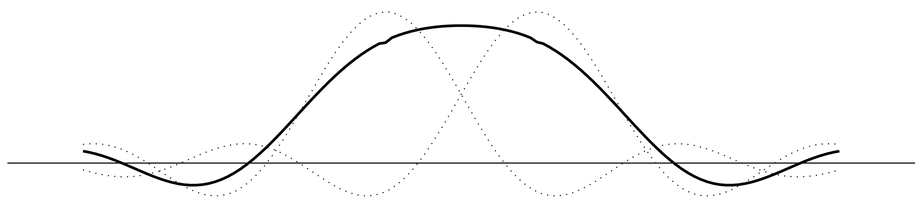
Since Fourier Transform $H(f)$ is nothing but the scaling done by the system on a unit magnitude sinusoid, the $H(f)$ above is indeed the Fourier Transform of the filter/system.

(j) Tune the filter to obtain a dc waveform, with permitted variations of 10%.

Question 2) Parseval's Theorem: Show that

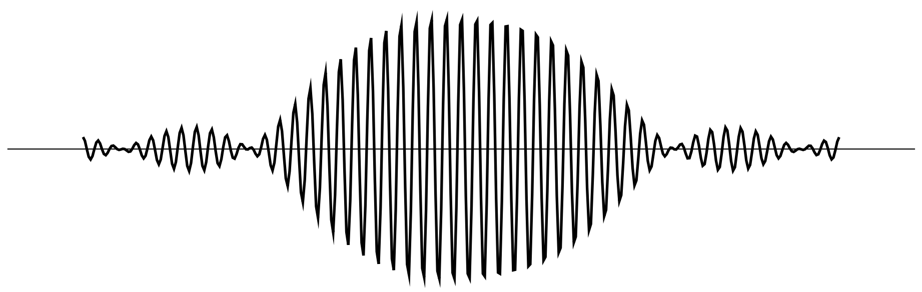
$$\int |x(t)|^2 dt = \int |X(f)|^2 df. \tag{7}$$

Question 3) In figure, the thick line is the sum of the two dotted plots, and each dotted line is a $\alpha \text{sinc}(\frac{2}{\pi}t)$ function, shifted by t_0 units to each side of origin.



a) Compute the FT of this signal.

b) Suppose now the plot is of the form,



What do you expect the FT to be.