Indian Institute of Technology Bombay
Dept of Electrical Engineering

Handout 10 EE 210 Signals and Systems
Tutorial 4 Sep 4, 2015

Question 1) A string is tied from $(0,0)$ to $(\frac{L}{2},0)$. Now, a point at distance p from the origin is given a vertical displacement of h upward. Let $f(t), t \in [0, \frac{L}{2}]$ be the initial position of the string.

(a) Find the coefficients A_m such that

$$f(t) = \sum_{m\geq 1} A_m \sin(\frac{2\pi}{L}mt).$$

(b) Find the coefficients B_m such that

$$f(t) = \sum_{m\geq 0} B_m \cos(\frac{2\pi}{L}mt).$$

(c) Find the coefficients C_m such that

$$f(t) = \sum_{m\in\mathbb{Z}} C_m \exp(j\frac{4\pi}{L}mt).$$

(d) We have learnt that FS coefficients can uniquely identify a continuous function. However, there seems to be three expansions given in parts $(a),(b)$ and (c). How do you reconcile these different expansions.

(e) Which of the expansions above is useful in identifying the harmonics of a vibrating string. Write the first harmonic frequency and suggest a position p for which all the even harmonics are missing.
Hint: 'Fourier Transform+Poisson sum-formula+Convolution-multiplication" can handle Fourier Series

Question 2) Consider a signal $x(t) = \text{sinc}^2(\frac{t}{T})$. It is sampled at the rate of B samples per second to obtain a discrete-time signal $x[n]$. What will be the Fourier representation $X_s(f)$ of $x[n]$. Let us call $\hat{X}(f) := X_s(Bf)$, the latter is also known as the DTFT (Discrete-Time Fourier Transform) of $x[n]$.
(a) Show that $\hat{X}(u)$ is a periodic waveform, and find its period.

(b) We know that locally integrable periodic functions admit Fourier series expansions. What are the FS coefficients of $\hat{X}(u)$.

(c) If $B = \frac{2}{\sqrt{2}T}$, can you get back $x(t)$ from the samples $x[n]$ (Draw $\hat{X}(f)$ and see).

(d) If $B = \frac{2\sqrt{2}}{T}$, give two 'different' reconstruction formulae to obtain $x(t)$ from $x[n]$.

Question 3) Notice that periodic functions have a Fourier-series expansion. Suppose a $T-$periodic function $x(t)$ is sampled to obtain $x_p[n] = x(n\frac{T}{k})$, where k is some integer greater than two. Since $x_p[n]$ is also periodic, we should attempt to find the FS coefficients of $x_p[n]$. Feel free to use the formalisms (Dirac or Impulse formalisms), to find the FS coefficients of $x_p[n]$.