Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 5 EE 603 Digital Signal Processing and Applications

Homework 2 August 12, 2016

Question 1) Let $f(t) = t$ for $-\frac{T}{2} \leq t \leq +\frac{T}{2}$. Find the FS expansion for $f(t)$.

Question 2) Consider a continuous function $f(t)$ in $-\frac{T}{2} \leq t \leq \frac{T}{2}$ with FS coefficients a_m. If $\sum |a_m| < \infty$, show that

$$f(t) = \sum_{m \in \mathbb{Z}} a_m e^{j\frac{2\pi}{T}mt}, \ \forall t \in [-\frac{T}{2}, +\frac{T}{2}]$$

Hint: Apply the uniqueness theorem of FS expansion we did in class. Try to be as precise as possible.

Question 3) A string is tied straight between two hinges at coordinates $(0, 0)$ and $(\frac{T}{2}, 0)$ respectively. A point at a horizontal distance of p from origin is given a vertical displacement h initially. Let the initial position be described by the function $x(u), 0 \leq u \leq \frac{T}{2}$. We know that the frequencies are multiples of $\frac{2\pi}{T}$, which is called the fundamental frequency or the first harmonic. The higher harmonics are now progressively counted as second, third etc.
a) Find the coefficients A_m if

$$x(u) = \sum_{m \geq 1} A_m \sin(\frac{2\pi}{T}mu).$$

b) Can you expand

$$x(u) = \sum_{m \geq 0} B_m \cos(\frac{2\pi}{T}mu).$$

In this case, find B_m.

c) For what value of p are the even harmonics missing.

d) Is there a position p such that the odd harmonics are missing.

Question 4) A guitar string of length $60cm$, when plucked at $10cm$ from one of the ends to a height of $1cm$, produced a set of frequencies, say $f_i, i \in \mathbb{N}$ with respective magnitudes α_i.
a) Suppose the guitarist replaced the string with another one having double the coefficient of tension. He then repeated the above procedure. Choose the option that you expect to happen, and reason your answer in less than 3 lines.

1. The output frequencies remain the same, but amplitudes α_i become higher.
2. Each frequency will get replaced by double the frequency, but the same amplitude.
3. Each frequency gets scaled by $\sqrt{2}$, but no change in amplitude.
4. Each frequency gets doubled, but the amplitudes get multiplied by $\sqrt{2}$.

5. Each frequency gets halved and the amplitude scaled by $\sqrt{2}$.
6. Each frequency gets halved and the amplitude scaled by $\frac{1}{\sqrt{2}}$.

Question 5) Find the Fourier Series expansion for

$$f(t) = \sin(\theta + 2\pi f t) \text{ where } \theta \in \mathbb{R}.$$

Are the F.S. coefficients continuous in θ?

Question 6) A supply voltage of $100\cos(100\pi t)$ is fed to an ideal half wave recitifier to obtian a waveform $x(t)$ at the output of the rectifier. Find the Fourier series expansion of $x(t)$.