Indian Institute of Technology Bombay
Department of Electrical Engineering

Handout 5 EE 703 Digital Message Transmission
Tutorial 2 Aug 29, 2019

Question 1)
(a) For a non-negative valued random variable X, show that

$$P(X \geq a) \leq \mathbb{E}[\frac{X}{a}], \forall a > 0,$$

where $\mathbb{E}$ denotes the expectation.

(b) For a real IID random process $X_n, n \geq 1$ with $\mathbb{E}[|X_1|] < \infty$ and $\mathbb{E}\Big[|X_1|^2\Big] < \infty$, show that

$$P(|\frac{1}{N}\sum_{n=1}^{N} X_n - \mathbb{E}[X_1]| \geq \epsilon) \to 0, \ \forall \epsilon > 0,$$

where the RHS is achieved in the limit as $N \uparrow \infty$.

Question 2) For a sequence of events $A_n, n \geq 1$ belonging to some sigma algebra $\mathcal{F}$, show that

(a)

$$P\left(\bigcup_{n\geq 1} A_n\right) \leq \sum_{n\geq 1} P(A_n).$$

(b)

$$P\left(\bigcup_{1\leq n\leq N} A_n\right) \geq \sum_{n=1}^{N} P(A_n) - \sum_{1\leq n<m\leq N} P\left(A_n \bigcap A_m\right).$$

Question 3) With β a positive constant, consider the family of functions

$$x_k(t) = \operatorname{sinc}\left(\beta(t - \frac{k}{\beta})\right), k \in \mathbb{Z},$$

where $\mathbb{Z}$ stands for the set of all integers (positive as well as negative). For all $k, l \in \mathbb{Z}$, find

$$\int_{\mathbb{R}} x_k(t)x_l(t)dt,$$

and express the answer in terms of appropriate Kronecker delta measure.

Question 4) In a spread-spectrum system using CDMA, an incoming data symbol, say d_n, $n \in \mathbb{Z}$, is first replaced by the vector $d_n\vec{s}$, where $\vec{s} = [s_0, \cdots, s_{N-1}]$. Here N is the spreading factor. Mathematically, the incoming data signal $d(t) = \sum_{n\in\mathbb{Z}} d_n \, \delta(t-nT)$ is spread to obtain a signal $x(t)$, given by

$$x(t) = \sum_{n\in\mathbb{Z}} d_n \sum_{k=0}^{N-1} s_k \, \delta(t - nT - \frac{k}{N}T),$$

where $\delta(\cdot)$ is the Dirac delta measure. Notice that the symbols (impulses) of $x(t)$ are separated by $\frac{T}{N}$ seconds.

The signal $x(t)$ is now passed through an pulse shaping filter $p(t)$ to obtain the output $y(t)$. Assume the pulse shaping filter to be a raised cosine filter with (two-sided) bandwidth

$(1+\alpha)\frac{N}{T}$, where α is the excess bandwidth parameter. Thus the frequency response $P(f)$ is non-zero in the open interval $\left(-\frac{N}{2T}(1+\alpha), +\frac{N}{2T}(1+\alpha)\right)$.

For an arbitrary finite N, let $\hat{D}(f) = \sum_{k=0}^{N-1} s_k \exp(-j2\pi f k)$ denote the DTFT of the sequence $[s_0, \cdots, s_{N-1}]$. Find the power spectral density of the signal $y(t)$, when the incoming data sequence is taken IID from a zero mean distribution of unit variance. (*You can leave the answer in terms of* $P(\cdot)$ *and* $\hat{D}(\cdot)$).

Question 5) Consider a transmitter with a pulse-shaping filter $g(t) = \text{sinc}(\beta t)$. The baseband signal is of the form $X(t) = \sum_k D_k g(t - \frac{k}{\beta})$, where $D_k, k \in \mathbb{Z}$ is an IID zero mean random process having variance P. Assume that the base-band received signal $y_b(t)$ is obtained as the convolution of the baseband input signal with the discrete-time filter $h(t) = \delta(t) - \frac{1}{2}\delta(t - \frac{1}{\beta})$. Find the power-spectral density of the received baseband waveform.

Question 6) Suggest a ISI-free transmit pulse-shaping filter. The filter should be different from the existing ones, and you can give it an appropriate name. Assume an inter symbol duration of T seconds for the data, and an excess bandwidth factor of $\alpha = 0.35$.

Question 7) GNURADIO: Generate and superpose 10 complex sinusoids $x_1(t), x_2(t), \cdots, x_{10}(t)$, where $x_l(t)$ has frequency $10 \times l$ Hz. Sketch (draw with pencil, label, mark axis) and submit the resulting **waterfall** diagram shown by the GNURADIO window (use QT-GUI).