Indian Institute of Technology Bombay

Department of Electrical Engineering

Handout 2 EE 708 Information Theory and Coding

Homework 1 Jan 16, 2018

Question 1) Two simple but useful inequalities

(a) Show that

$$\log_e x \le x - 1, \ \forall x > 0$$

(b) Show that

$$\log_e(1 + x) \ge x - \frac{x^2}{2}, \ \forall x > 0$$

Question 2) Give an example of three random variables being pairwise independent, but not independent.

Question 3) Markov's Inequality: For a non-negative valued random variable X show that

$$P(X \ge a) \le \frac{1}{a}\mathbb{E}[X], \ a > 0,$$

where $\mathbb{E}$ is the expectation operator.

Question 4) State and prove weak law of large numbers (WLLN) for the sequence $X_n, n \ge 1$, generated IID from a distribution with finite mean and variance. (*Hint: the previous question can be used for a proof*)

Question 5) For a random variable X, it is given that

$$\mathbb{E}\left[f(X, A)\right] = Pr(A),$$

for all A from the appropriate Borel field (If this term is new, take it as the collection of all *meaningful* events). Identify the non-negative valued function $f(x, A)$.

Question 6) Given the lengths as $l_1 = l_2 = 5, l_3 = l_4 = 4, l_5 = l_6 = 3, l_7 = l_8 = 2$, does there exist a ternary($D = 3$) prefix-free code with these lengths.

Question 7) Let us do a treasure-hunt in the real line. Consider the unit interval $[0, 1]$. We will divide this into 4 territories, marked as the segments $[0, t_1, t_2, t_3, 1]$. The inner-points t_1, t_2 and t_3 define the boundary points of the adjacent territories. The treasure is buried in the unit interval according to a uniform distribution. We can use a generalized measuring device to ask questions on the location of the treasures. For example: "Is the treasure in the first or third territory?", to which we will get YES/NO answers. Another example question: "Is it in the first territory?".
Let $t_1 = 0.52, t_2 = 0.625, t_3 = 0.74$. Suppose we do this experiment several times and wish to find the territory in the minimum number of questions on the average. Find a strategy.

Question 8) A random variable takes values on an alphabet of K letters, with the probability assignment $p_1, \cdots, p_K$. It is given that $p_i = \rho, 1 \leq i \leq K-1$ and $p_K = \frac{\rho}{2}$, for some constant $\rho \in (0,1)$. These letters are encoded into binary words using the Huffman procedure so as to minimize the average codeword length. Let j and x be chosen such that $K = x2^j$, where j is an integer and $1 \leq x < 2$.

(a) Find the number of codewords having lengths less than j?

(b) In terms of j and x, how many code words have length j?

(c) What is the average codeword length?