Indian Institute of Technology Bombay
Department of Electrical Engineering

Handout 4 EE 708 Information Theory and Coding
Homework 2 Feb 2, 2018

Question 1) For most of our discussion in class, we considered discrete and finite random variables. However, our theory is not limited to this. Consider a geometric random variable X with $P(X=k)=(1-\rho)^{k-1}\rho$, $k\geq 1$.

(a) Compute $H(X)$.

(b) Consider any positive valued random variable U with mean $\mathbb{E}[U]=\frac{1}{\rho}$. For the distribution function $P(\cdot)$ given above, show that $\mathbb{E}\log P(U)=-H(X)$.

(c) Show that $H(U)\leq H(X)$ for any positive valued random variables U and X such that $\mathbb{E}[U]=\mathbb{E}[X]$, and X is geometric distributed.

Hint: No need of big derivations, part (b) and $\log_e(x)\leq x-1$ can quickly take you there.

Question 2) For $x^n\sim\prod_{i=1}^n p(x_i)$, let A_ϵ^n denote the collection of typical sequences, or the typical set. Show that

$$(1-\epsilon)2^{nH(X)(1-\epsilon)}\leq |A_\epsilon^n|\leq 2^{nH(X)(1+\epsilon)},$$

for n sufficiently large enough, for any $\epsilon>0$. Highlight the part of the proof which uses the 'large enough' nature of n.

Question 3) The conditional entropy of X given Y is defined as

$$H(X|Y)=-\mathbb{E}\log p(X|Y)=\sum_{x,y}p(x,y)\log\frac{1}{p(x|y)}.$$

Let X be a binary random variable, according to Bernoulli(α), and $Y=X\oplus Z$, where Z is Bernoulli(β). Here α and β are two parameters from the interval $(0,1)$.

(a) Find $H(X|Y)$.

(b) Is there a value of β for which $H(X|Y)=H(X)$.

Question 4) The weakly typical set is given by

$$A_{w,\epsilon}^n=\{x^n:2^{-n(H(X)+\epsilon)}\leq p(x^n)\leq 2^{-n(H(X)-\epsilon)}\}.$$

Show that

$$(1-\epsilon)2^{n(H(X)-\epsilon)}\leq |A_{w,\epsilon}^n|\leq 2^{n(H(X)+\epsilon)},$$

for n large enough, and for any $\epsilon>0$.

(b) Give an example of a non-trivial probability distribution on $\Omega=(A,B,C)$, for which there is a sequence x^n such that $x^n\in A_{w,\epsilon}^n$, but $x^n\notin A_\epsilon^n$, where the latter set employs the notion of typicality we used in the class.

Question 5) Question 3.11 from the book T. M. Cover and J. A. Thomas, 'Elements of Information theory', second edition, Wiley, 2006.

Question 6) Question for midsem: Formulate and submit a question that you think is suitable for the midsem. You can search through textbook exercises/other web-resources or research publications.