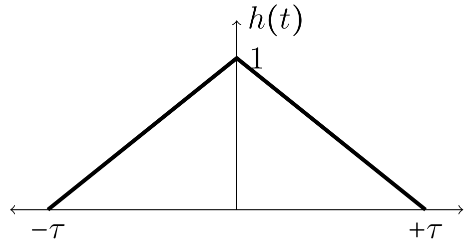
Indian Institute of Technology Bombay
Department of Electrical Engineering

Handout 2 EE 764 Wireless and Mobile Communication
Homework 1 Jan 30, 2015

Question 1) Consider the following filter, with $\tau = 1ms$.



(a) A signal $x(t) = \sum_{i=0}^{9} a_i \delta(t - i \times 10^{-3})$ was passed through the above filter. If $\{a_i, 0 \le i \le 9\}$ is the ordered set $\{1, 2, 3, 4, 2, 5, 1, 3, 2, 1\}$, draw the output of the filter. Can you give this filter a proper name based on the operation that it performs?
(b) Now, $x(t) = 50 \sin(2\pi 1000 t)$ is passed through the above $h(t)$. Find the output of the filter.

Question 2) Are sine and cosine orthogonal?. Let us understand this simple fact. Consider two waveforms

$$x_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f t \ , \ \ x_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f t.$$

(a) Compute $\int_0^T x_1^2(t) dt$.
(b) Compute $\int_0^T x_1(t) x_2(t) dt$.
(c) Orthogonality will imply the dot product in part (b) to be zero. Comment whether $x_1(t)$ and $x_2(t)$ are orthogonal.

Question 3) Throw a random number between 1 and 10 (use any program you like). Let i be this number. You are now required to run the Gnuradio qpsk Example 2 given in the course website. Take the standard deviation of the additive noise as $1/i$, where i is the random number that you have chosen. Choose the line-link marker and sketch the output constellation (approximately is okay, no need to print) with the axis properly marked, and write the value of random number near the sketch.

Question 4) Consider a real random variable $X \sim \mathcal{N}(0, \sigma^2)$, let the probability density function (pdf) of X be denoted as $g(x)$. Consider another real random variable U with zero mean and variance σ^2, having the pdf $f(u)$. For these random variables X and U, answer the four parts below.

(a) Find $E \log g(U)$.

(b) Find $E \log g(X)$.

(c) Using the inequality $\log_e(x) \le x - 1$, $\forall x > 0$, show that

$$-\mathbb{E} \log f(U) \le -\mathbb{E} \log g(X).$$

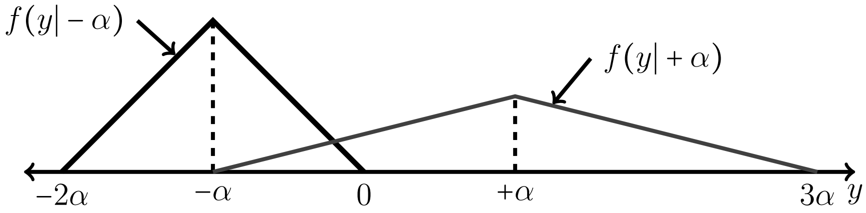
(d) For a random variable Y with pdf $\phi(y)$, the quantity $-\mathbb{E}\log\phi(Y)$ is known as the differential entropy of Y. Of all real-valued centered random variables of variance σ^2, find the pdf which maximizes the differential entropy.

Question 5) Let X_1 and X_2 be two independent and identical Bernoulli distributed binary random variables, with $P(X_1 = 1) = \rho$. Find the conditional expectation

$$\mathbb{E}\left[X_1 | X_1 \oplus X_2\right],$$

where $\oplus$ denotes the XOR operation.

Question 6) Consider a binary hypothesis testing problem to decide between transmitted symbols $X \in \{-\alpha, +\alpha\}$. Noisy observations Y are available to us, which is obtained through the conditional laws $f(y|x)$ as shown below.



If the input $+\alpha$ is chosen with probability $\frac{2}{3}$, specify the MAP decoding rule.

Question 7) Consider a wireless channel where the passband frequency response at time $t = t_0$ is given by

$$H(f) = cos(2\pi\alpha(f - f_c))\mathbb{1}_{\{f_c - 10000 \le f \le f_c + 10000\}}.$$

Suppose our communication band coincides with the above range of frequencies. Assuming slow fading, can you find any value of α such that there are exactly two non-zero taps for the equivalent FIR model, also known as tapped delay-line model (Notice that the model is valid around $t = t_0$ due to our assumption of slow-fading). If you think there is no such α, substantiate with arguments.