Indian Institute of Technology Bombay
Department of Electrical Engineering

Handout 6 EE 764 Wireless and Mobile Communication
Tutorial 1 Feb 17, 2015

Question 1a) Find the characteristic function of a Gaussian random variable. *Note: characteristic function is the Fourier transform of the pdf.*

Question 1b) Given two independent and identical Gaussian random variables X_1 and X_2, show that $X_1 + X_2$ and $X_1 - X_2$ are independent random variables.

Question 2) A real random vector is called Gaussian, if all non-trivial linear combination of its elements are Gaussian. Suppose you are given identical real zero-mean Gaussian random variables X and Y with $E[X^2] = 1$ and $E[XY] = \rho$, $\rho < 1$. Show that (X, Y) is a Gaussian random vector.

Question 3) A Special CDMA: We call the CDMA in this question special because, though we use spreading, the effective bandwidth is still assumed to be within the coherence bandwidth. We will consider wide-band CDMA in a later question (see Question 6).

Consider M frame synchronized users in CDMA, having respective spreading codes $s_1, \cdots, s_M$, each code of length N (these may or may not be orthogonal). Let $x_1, \cdots, x_M$ be the input symbols to be conveyed by the respective terminals.

a)If all the fading gains are unity, and the noise is AWGN, can you write the model representing the N received samples for the given input symbols.

(b) Let us form an $N \times M$ matrix $S = [s_1, \cdots, s_M]$. Assume each user now observes a slow fading link, with respective fading coefficient of $h_i, 1 \le i \le M$. Can you write the received vector for a set of input symbols $x_1, \cdots, x_M$, in terms of S and $h_i, 1 \le i \le L$.

(c) Assume BPSK inputs for each of the symbols. Propose a detector for each input symbol. i.e. the detector should output whether $\hat{x}_i$ is $-a$ or $+a$.

(d) What is the detector structure if s_i and s_j are orthogonal for all $i \ne j$? Find the error probability for each user, if all the links have unit gain.

(e) Consider the case where $M = 2$, and $s_1 = \frac{1}{\sqrt{6}}[1, -1, 1, -1, 1, -1]$ and $s_2 = \frac{1}{\sqrt{6}}[1, 1, 1, 1, 1, 1]$. Assuming BPSK inputs, design a joint decoder which decides both the inputs after N observations of the output. Define the joint error probability in this case, and find a good bound to it.

Question 4) Random Coding: Shannon came up with the idea of random coding. Let us learn some aspects of it for future use. Imagine a system with two equiprobable codewords say u and v. In every n symbols, we sent either u or v and observe the outputs through an AWGN channel of noise variance σ^2, i.e. our sampled model is $y = x + z$.

a) Argue that the error probability depends on the magnitude as well as phase (angle) of the noise vector $\bar{z}$.

b) Typically we characterize the noise by its variance, and we intuitively think that higher noise variance is akin to more probability of error. But, as you can see in the previous part, it is not just the variance which determines the error probability, complicating our analysis.

A way to circumvent this is as follows. Suppose u and v are chosen uniformly at random from all possible $n-$dimensional vectors of power P, i.e. $||x||^2 = nP$ for all codewords. The error event is now dependent on what codewords we choose, as well as the noise vector, let us denote this as $E(\bar{Z}, \bar{X})$. Show that the probability of error

$$P(E(\bar{Z}, \bar{X})) = \mathbb{E}_z P\left[E(\bar{Z}, \bar{X}) | Z = \bar{z}\right]$$

where $\mathbb{E}_z$ denotes the expectation over z. The term inside the expectation is the conditional probability given a fixed noise vector $Z = \bar{z}$. Argue that this conditional probability depends only on $||\bar{z}||$, and hence the average error probability only depends on $||\bar{z}||$.
Note: In the class, we confused the statement 'conditioned on $\bar{Z} = \bar{z}$', with the event that 'it is available at the receiver', these two are separate events.

Question 5) Consider a non-coherent wideband CDMA system, modeled by a $L-$tap delay model with each fading coefficient independent and identical Gaussian of unit variance. Let us find the error probability between two codewords, x_A and x_B which are orthogonal to each other, and also orthogonal to shifted versions of themselves. The length n of the codewords are assumed to be much larger than L.

(a) Consider a detector which simply projects the received vector along the L shifted versions of each codeword, thus generating $r_{a1}, \cdots, r_{aL}$ and $r_{b1}, \cdots, r_{bL}$. Write the equations for r_i.

(b) Find the optimal detector to decode the message from the samples r_i and compute the error probability.

(c) Do exercises 3.31–3.33 in the textbook.