Fading MACs with Distributed CSI and Non-identical Links

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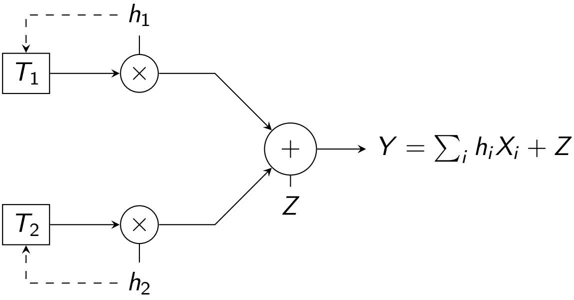
Essential Features



- Block-fading/Opportunistic nature of channel.
- Individual CSI at transmitters + fixed transmit powers.
- Complete CSI at receiver + no outage tolerated.
- Objective: Adaptive Sum-capacity ($\sum_i \mathbb{E}[R_i(h_i)]$) [*ElGamalKim11*].

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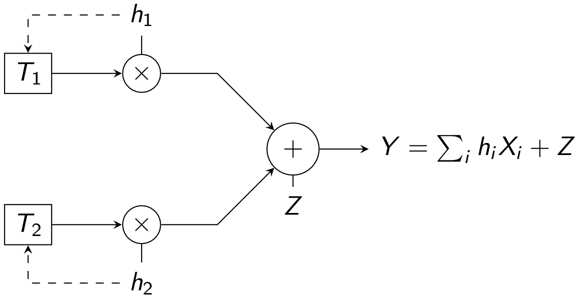

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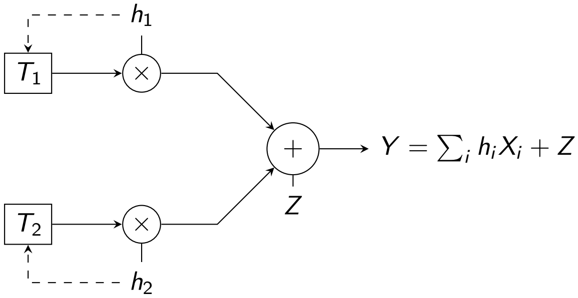

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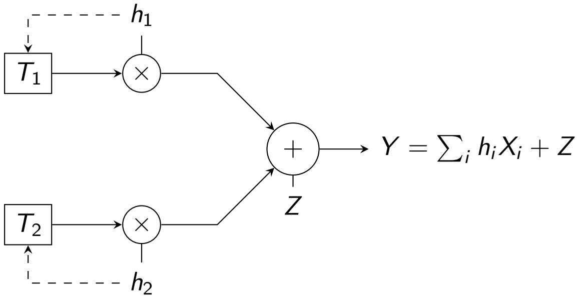

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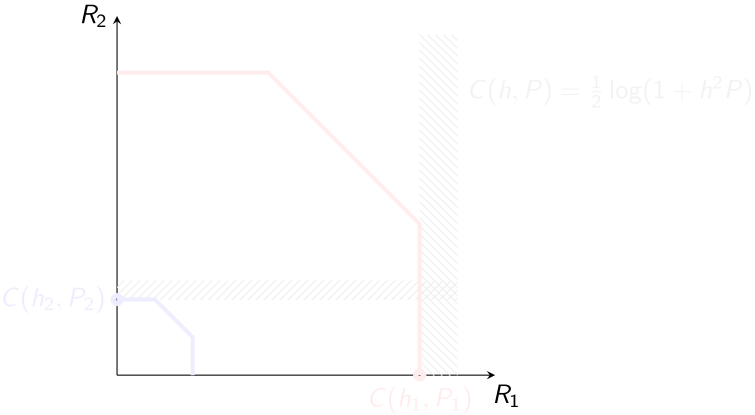

Figure: The users 1 and 2 construct the innermost and outermost symmetric regions respectively, and chooses the rate-pair A. [Deshpande11],[PillaiDey12]

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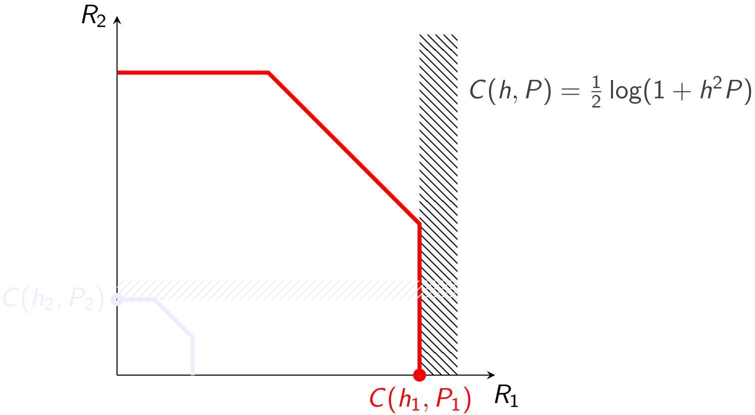

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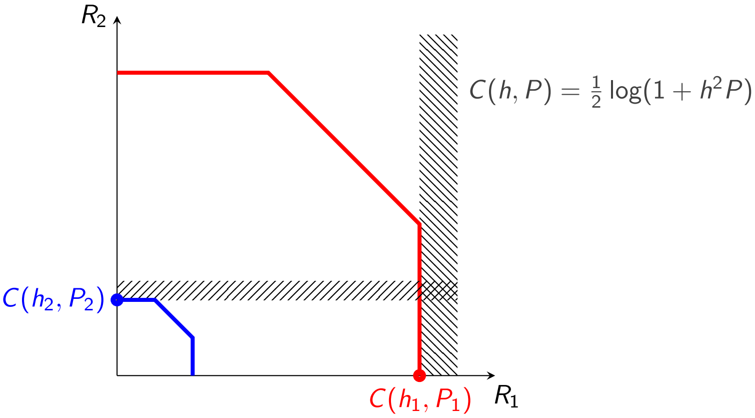

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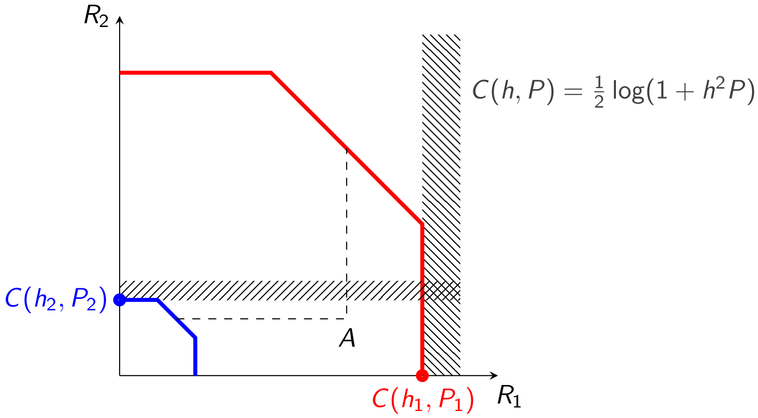

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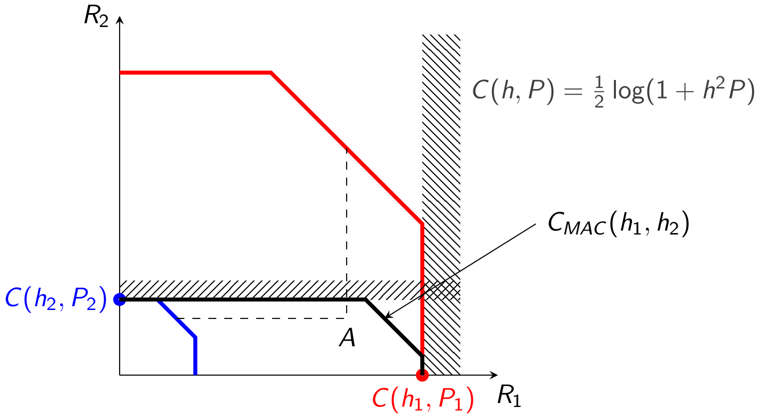

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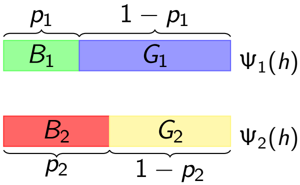
Non-Identical Fading Statistics



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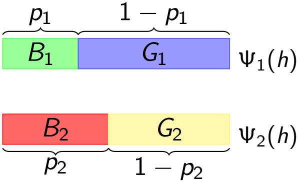

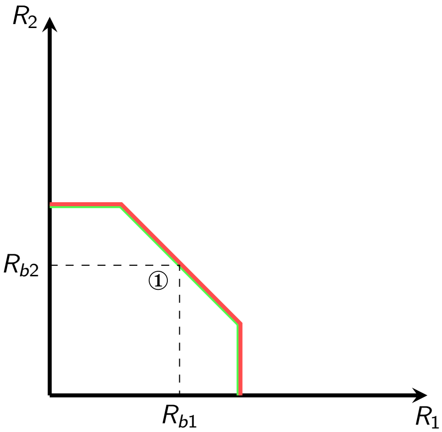

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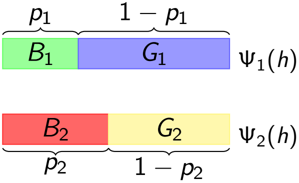

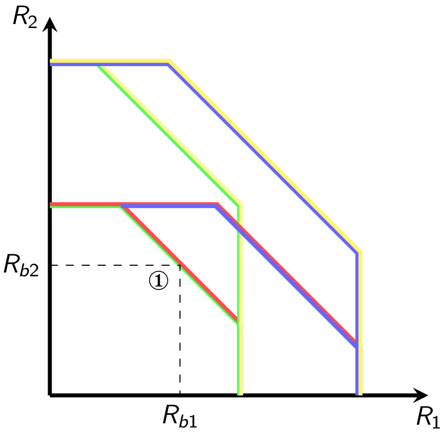

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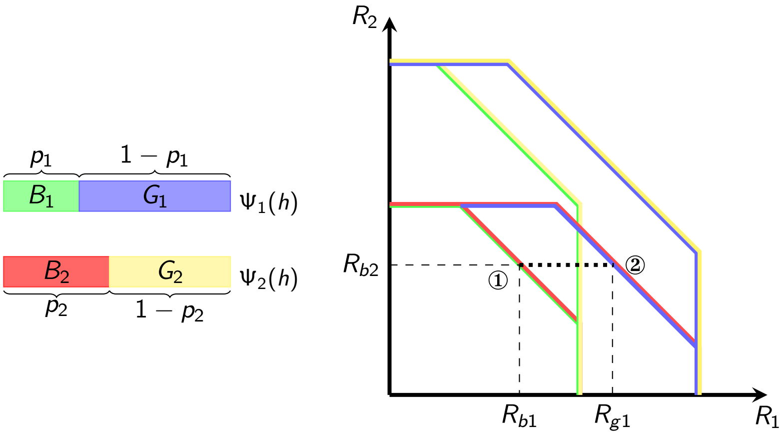

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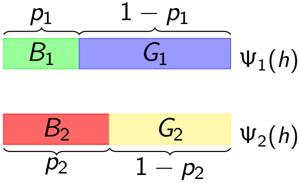

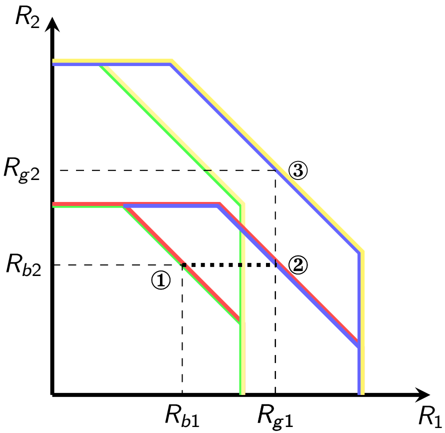

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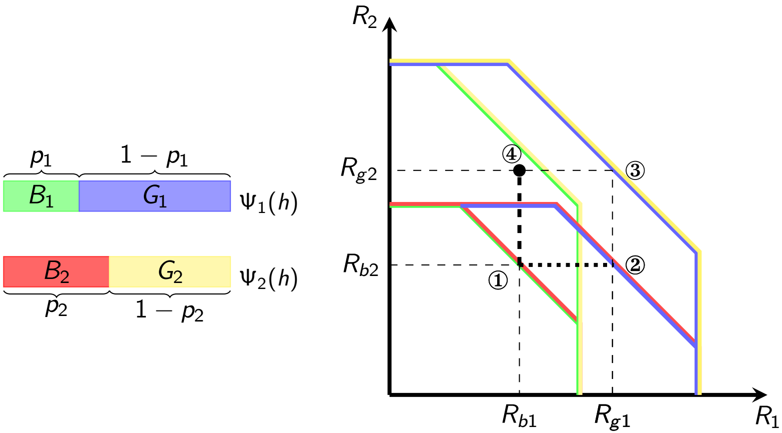
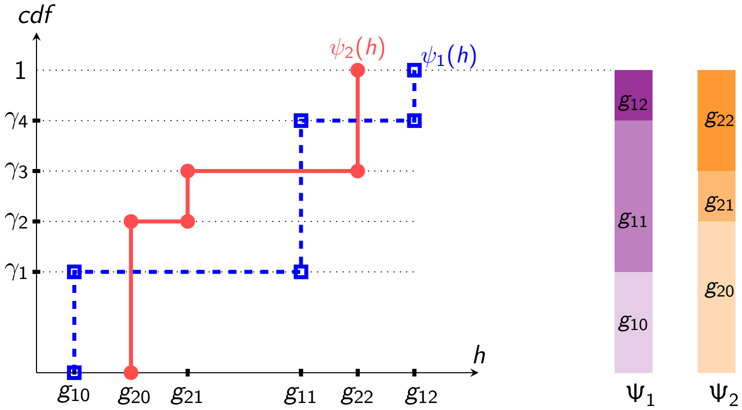


Rate Assignment Demo



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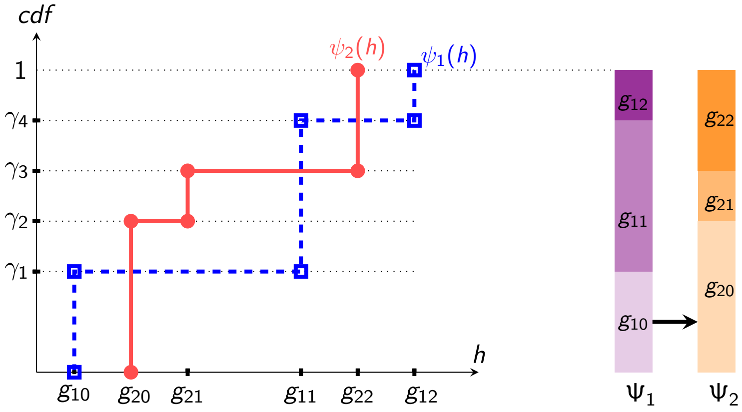

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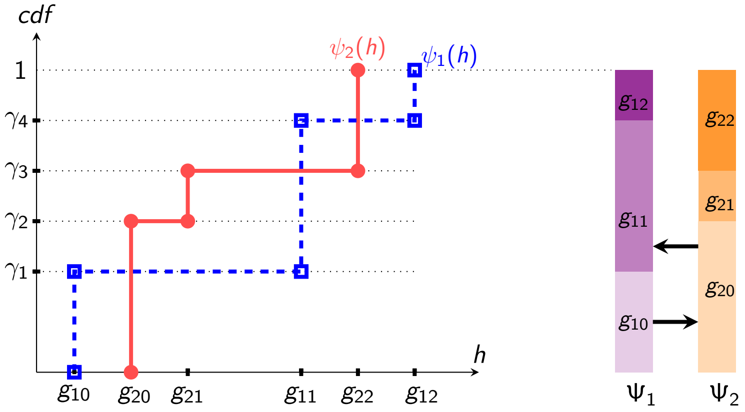

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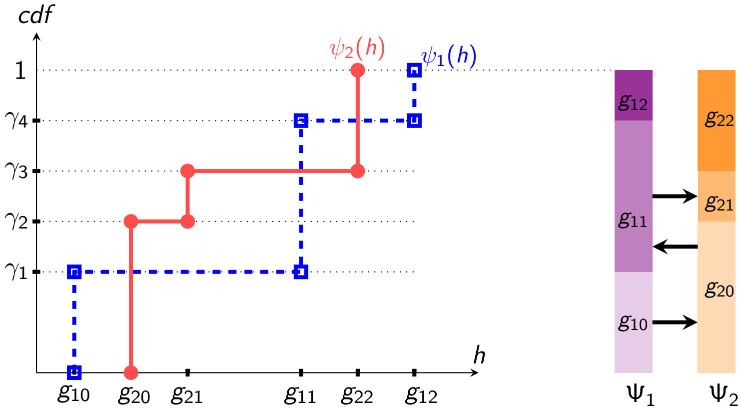

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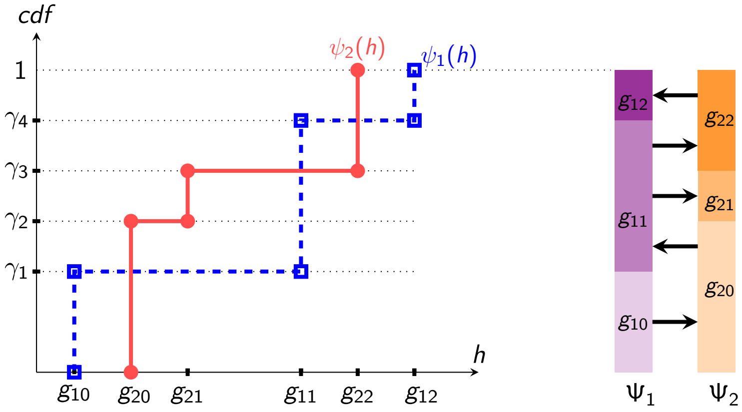

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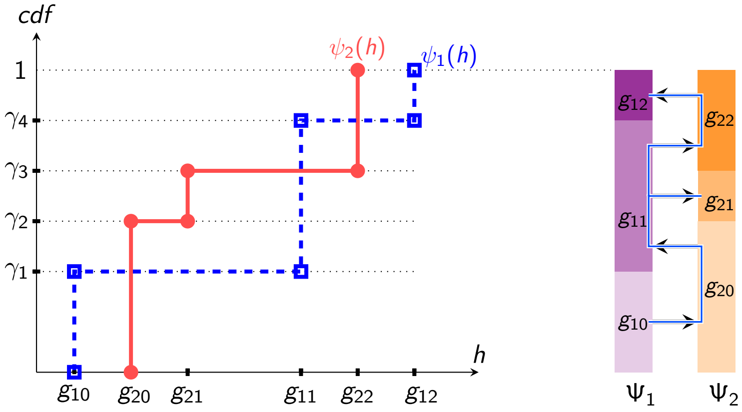

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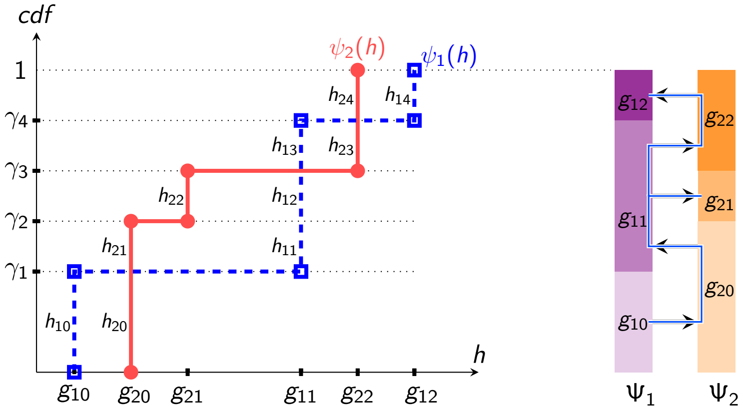


Iterative Assignment

Theorem

The iterative rate assignment

$$R_1(h_{10}) \in C_{MAC}^{sum}(h_{10}, h_{20})$$

$$R_2(h_{2i}) = C_{MAC}^{sum}(h_{1i}, h_{2i}) - R_1(h_{1i})$$

$$R_1(h_{1j}) = C_{MAC}^{sum}(h_{1j}, h_{2(j-1)}) - R_2(h_{2(j-1)})$$

for $0 \leq i < k$, $1 \leq j < k$ *achieves the adaptive sum-capacity.*

- The first expression is about choosing any rate-pair in the dominant face of the minimal pentagon.

Outage-Free

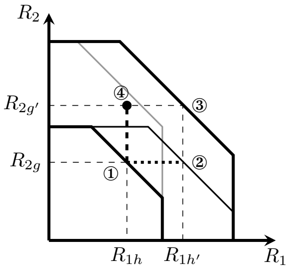
Lemma

Let (h, g) and (h', g') be two state-pairs such that $(h', g') \geq (h, g)$. Assume $(R_{1h}, R_{2g}) \in C_{MAC}(h, g)$ and $(R_{1h'}, R_{2g'}) \in C_{MAC}(h', g')$. If

$$R_{1h'} + R_{2g} = \frac{1}{2} \log(1 + h'^2 P_1 + g^2 P)$$

then

$$R_{1h} + R_{2g'} \leq \frac{1}{2} \log(1 + h^2 P_1 + g'^2 P).$$



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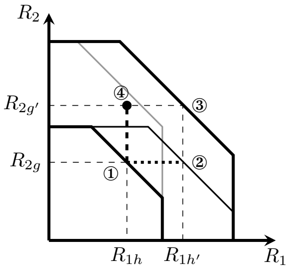
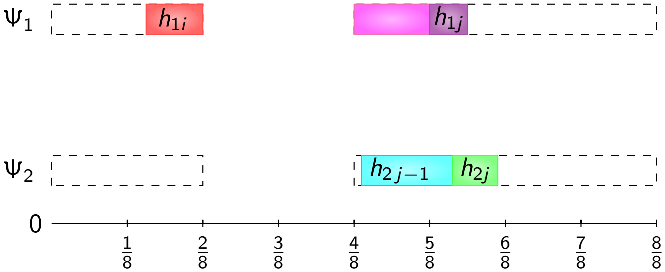


Proof.

$$(1 + h^2 P_1 + g^2 P_2)(1 + {h'}^2 P_1 + {g'}^2 P_2) \leq (1+h^2 P_1+{g'}^2 P_2)(1+{h'}^2 P_1+g^2 P_2).$$

□

Inducting Outage-free



- By induction, rate-choices of all state-pairs are outage-free.
- For every *vertical* state-pair, the rate-choice is on the dominant face.
- Hence the scheme achieves *the adaptive sum-capacity*.

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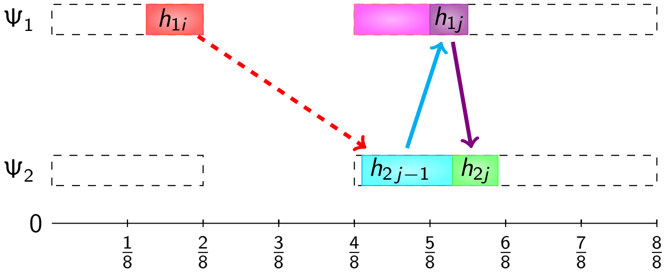

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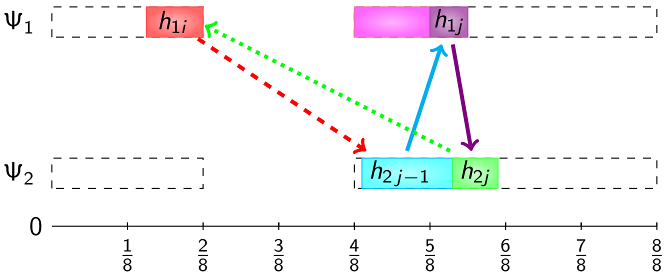

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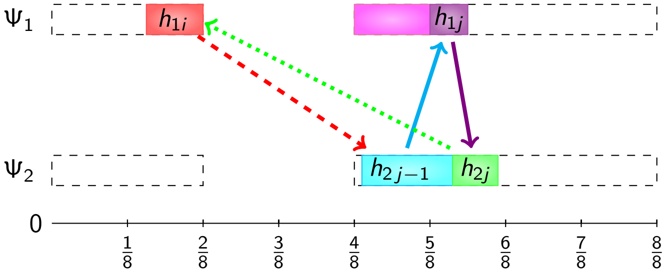

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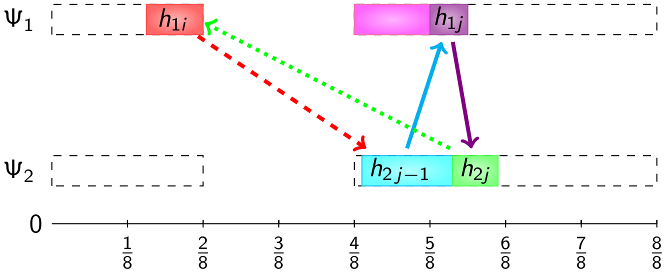

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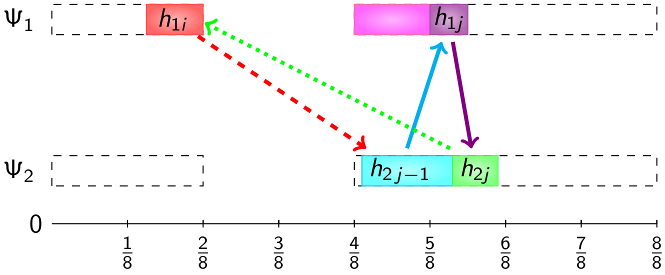

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Continuous-valued Channels

Theorem

For a two user Gaussian MAC with respective fading cdfs $\psi_1(\cdot)$ and $\psi_2(\cdot)$, the adaptive sum-capacity is achieved by the rate-allocation,

$$R_i(h) = \int_0^h \frac{yP_i}{1 + y^2 P_i + (\psi_j{}^{-1}(\psi_i(y)))^2 P_j} \, \mathrm{d}y, j \neq i$$

Numerical Comparison

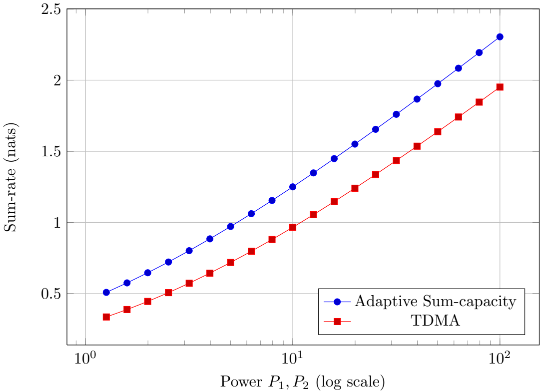

Figure: Two users:Ψ_1-Rayleigh and Ψ_2-Uniform$[0, a]$

Conclusion

- We computed the adaptive sum-capacity of several fading MAC models with individual state information at the transmitters.
- We mention that a simple sequential decoding scheme employing rate splitting can achieve the above rates.
- Our rate-allocations are optimal for any given power-allocation, thus decoupling these two aspects in the optimization. In fact, efficient procedures for power-control can be derived from this.
- Though the two user case was described, extensions to multiple users are straightforward.