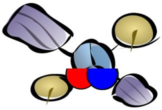
Optimal Scheduling in Distributed MACs under Delay Constraints



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Outline

- System Model with Random Arrivals
- Power Consumption in TDMA
- Optimal Power Allocation Functions
- Generalizations
- Conclusion

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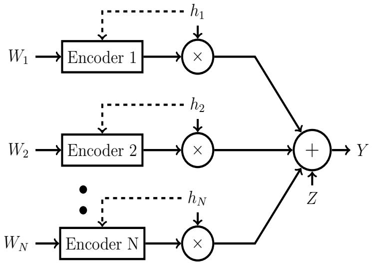
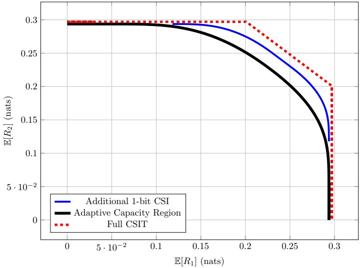
Distributed Multiple Access



- ‘Motivated by practical wireless’ [ElGamal11].
- **Adaptive Capacity** $\mathcal{C}_A$: $\Rightarrow$ No outage rates under blockwise encoding/decoding.
- Unknown for many models[GamalCioffi07]



- Solve $\mathcal{C}_A$ for arbitrary fading distributions.
- An explicit rate-allocation algo is the key.
- Can be generalized to other CSI models.
- SS, BKD, *SRBP*, “Distributed Rate Adaptation and Power Control in Fading MACs”, Submitted to IEEE Trans Infomation Theory, 2014 (also in ITW11, ISIT13, ITW14).

Distributed Access with Arrivals

ALOHA

- G_i: attempt probability of transmitter i.
- Incoming load:

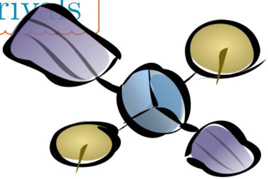
$$\sum_{i=1}^{L} G_i = G$$

- Throughput :

$$\sum_{i=1}^{L} G_i(1-G_i)^{L-1} \rightarrow Ge^{-G}$$

- Maximum= $\frac{1}{e}$.

Distributed Access with Arrivals



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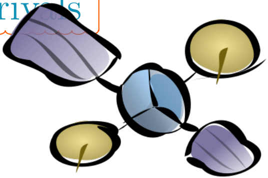

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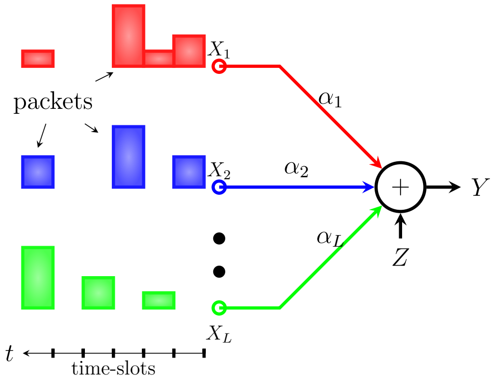


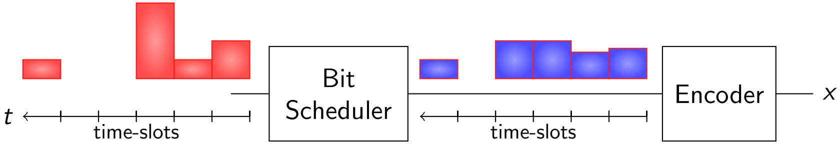
El Gamal et al, " Energy efficient scheduling of packet transmissions over wireless networks", INFOCOM 2002.

1)Is it possible?

2)What is the cost?

Flush packets within the delay + *high success-rate* using distributed info.

Single User Scheduling

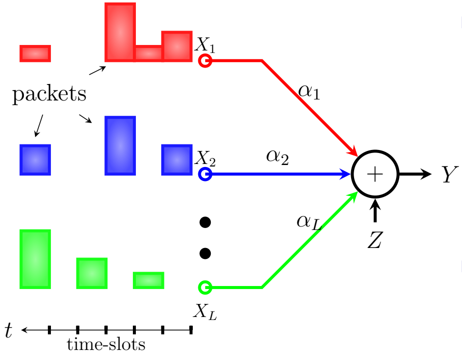


- Bit Scheduler ensures the delay constraint, example max-delay.
- Encoder is also a power scheduler, i.e. for B bits per channel use

$$P(B) = \frac{2^{2B} - 1}{\alpha^2}.$$

- In other words, a $(n, 2^{nB})$ channel code with average power $P(B)$ is employed in that block (*Zero Outage Scheduler*).

Time Division Multiple Access



- Assume $\alpha_i = \alpha, \forall i$.
- User i requires rate $B_i \in \mathcal{B}$,

$$\frac{1}{L} \cdot \frac{1}{2} \log(1 + L\alpha^2 P_i) = B_i$$

- The required power is

$$P_i(B_i) = \frac{2^{2LB_i} - 1}{\alpha^2}$$

Uncoordinated Access

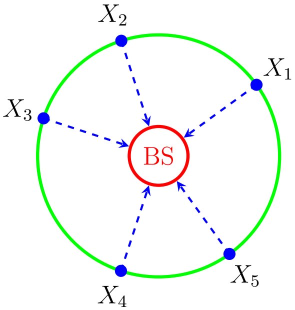


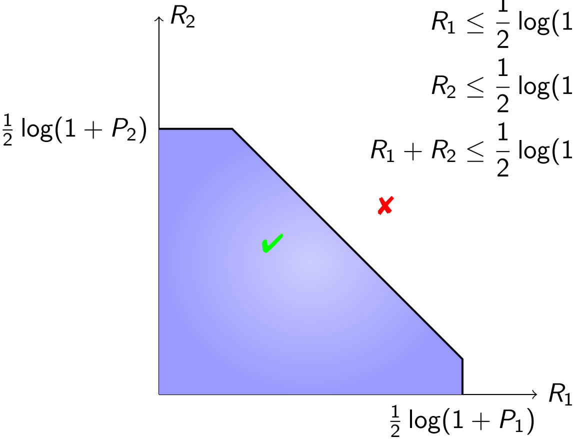
Figure : Symmetric MAC

- User i employs a rate B_i codebook of blocklength n and average power

$$P_i(B_i) = \frac{2^{2LB_i} - 1}{\alpha^2}$$

- Transmissions interfere with each other.
- Can we still decode everyone?

Two User Capacity Region $C_{MAC}(P_1, P_2)$



$$R_1 \leq \frac{1}{2}\log(1+P_1)$$

$$R_2 \leq \frac{1}{2}\log(1+P_2)$$

$$R_1 + R_2 \leq \frac{1}{2}\log(1+P_1+P_2)$$

The region also includes TDMA rates, i.e. our power choice works!

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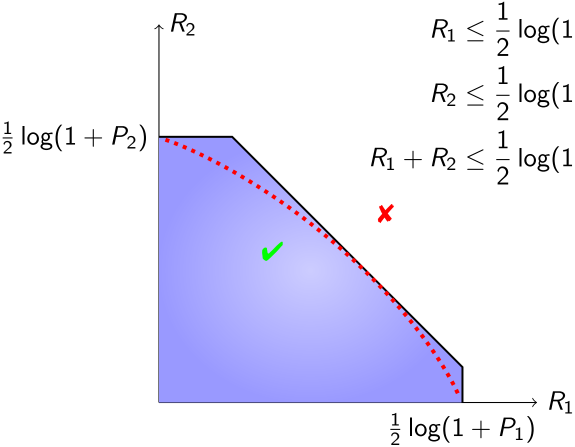

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Optimality

Lemma

With identical link gains and arrival statistics

$$P_{sum}^{min} = \frac{1}{\alpha^2}\mathbb{E}[2^{2LB} - 1].$$

Proof.

$$\begin{aligned}\mathbb{E}\sum_{i=1}^{L} P_i(B_i) &= \mathbb{E}\sum_{i=1}^{L} P_i(B) \\ &= \frac{1}{\alpha^2}\mathbb{E}\left(2^{\log(1+\alpha^2\sum_{i=1}^{L} P_i(B))} - 1\right) \\ &\geq \frac{1}{\alpha^2}\mathbb{E}\left(2^{2LB} - 1\right).\end{aligned}$$

□

Power Optimization

- Minimize average power by choosing $(P_i(B_i), 1 \leq i \leq N)$:

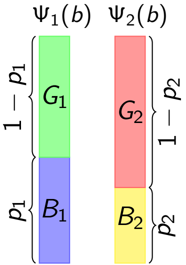
$$\min \mathbb{E} \sum_{i=1}^{N} w_i P_i(B_i)$$

such that $\forall (b_1, \cdots, b_N)$

$$\sum_{i \in S} b_i \leq \frac{1}{2} \log \left(1 + \sum_{i \in S} \alpha_i^2 P_i(b_i) \right), \ \forall S \subseteq \{1, \cdots, N\}.$$

- $\bar{w} = w_1, \cdots, w_N$ is any positive weight vector.

Non-identical Arrivals



- S. Sreekumar, SRBP, BKD, Energy efficient random access under strict delay constraints, ISIT 2014

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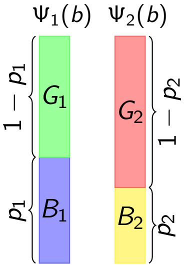

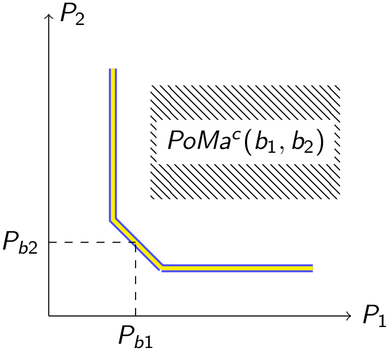

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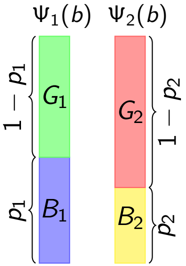

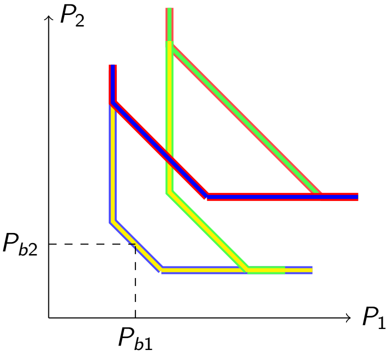

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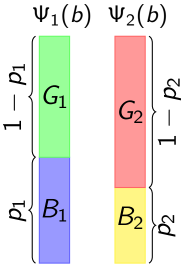

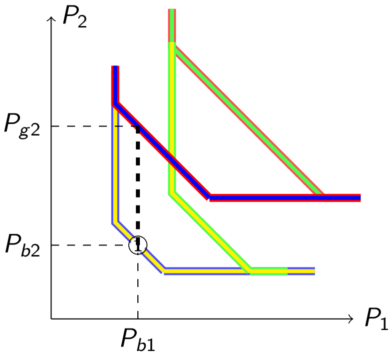

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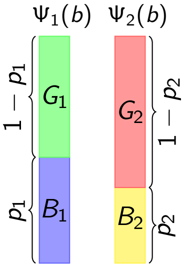

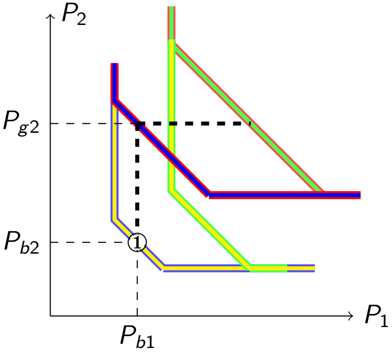

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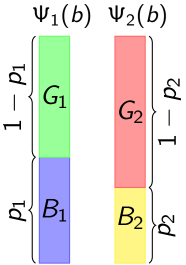

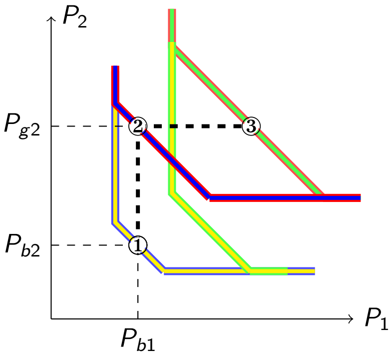

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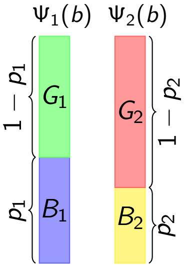

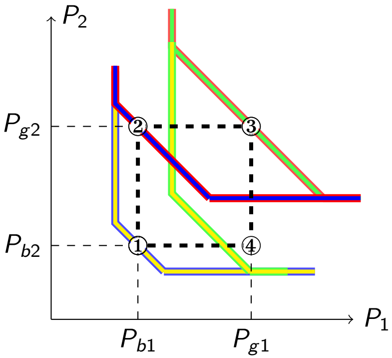

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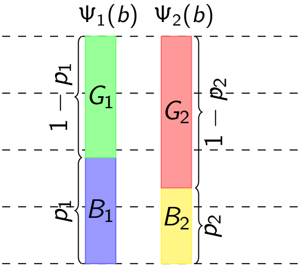

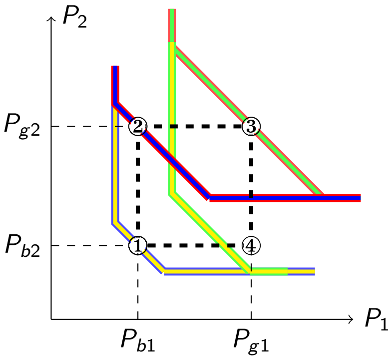

- Allocation not only **successful**, but also **Power Optimal** for

$$\min \sum_{i=1}^{L} \mathbb{E} P_i(B_i) : (P_i(B_i), P_j(B_j)) \in PoMa^c(B_i, B_j) \forall i, j.$$

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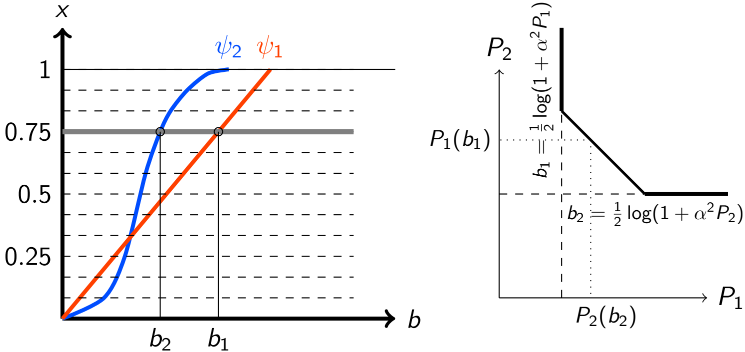

Figure : Optimal Sum-power Allocation

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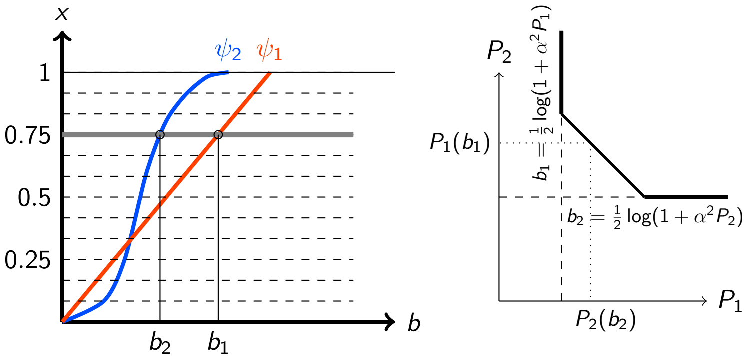
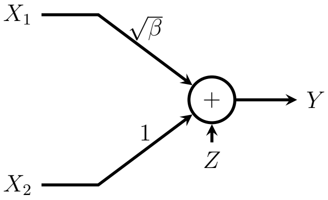


Figure : Optimal Sum-power Allocation

$$P_{sum}^{min} = \int_0^1 \frac{2^{2(b_1(x)+b_2(x))} - 1}{\alpha^2} dx, \text{ where } b_i(x) = sup\{b \mid \Psi_i(b) \leq x\}.$$

Asymmetric Links



- To support a packet-rate pair of (b_1, b_2),

$$\beta P_1(b_1) + P_2(b_2) \geq 2^{2(b_1+b_2)} - 1$$

- **Difficulty:** This does not parse into a suitable constraint on $P_1 + P_2$.

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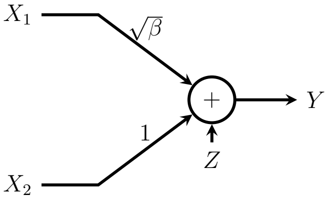

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CDF Transformations

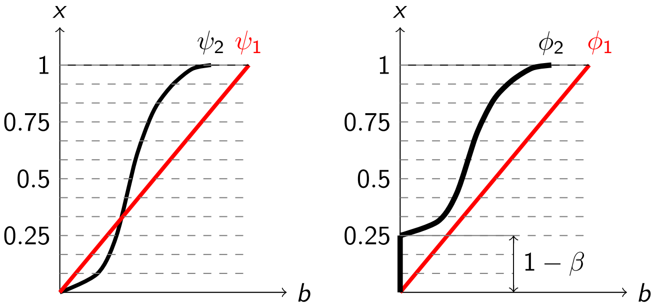

Figure : Optimal Power-allocation for $\beta = 0.75$

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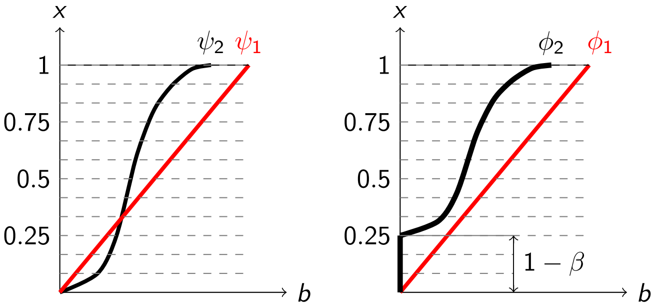

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Theorem

The minimum sum-power for our MAC model is

$$P_{sum}^{min} = \int_0^{1-\beta} \frac{2^{2b_1(x)} - 1}{\beta} dx + \int_0^{\beta} \frac{2^{2(b_1(v+1-\beta)+b_2(\frac{v}{\beta}))} - 1}{\beta} dv$$

Power-Savings

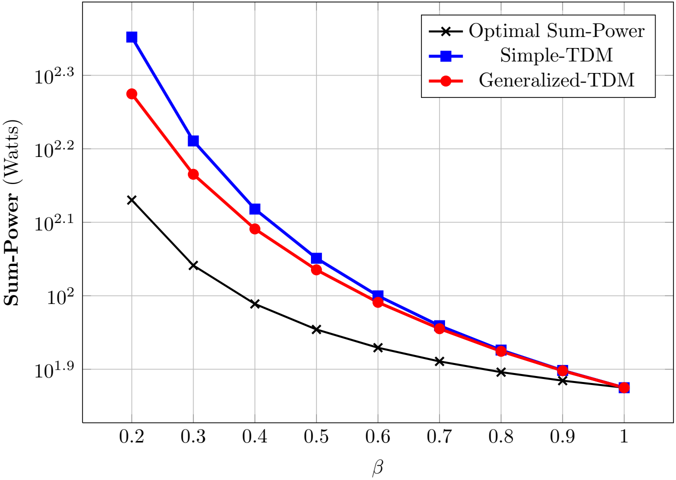
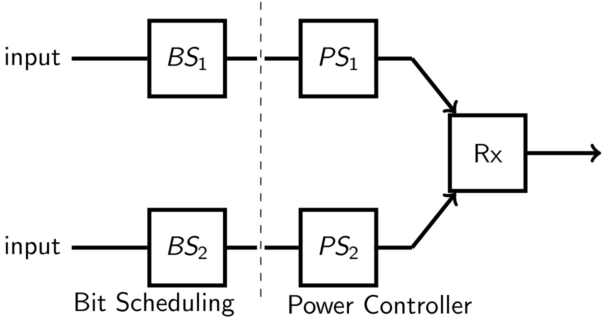


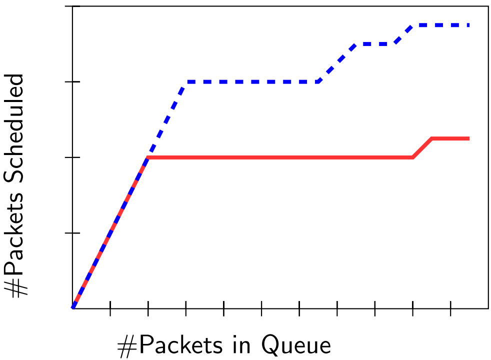
Figure : Power savings for rates in $\{1, 2\}$ with $p(1) = 0.75, \alpha_1 = \beta, \alpha_2 = 1$

Bit Scheduling



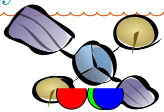
- Source IID + bit scheduler memoryless $\Rightarrow$ o/p stationary.
- Now use the optimal unit-delay power scheduler.

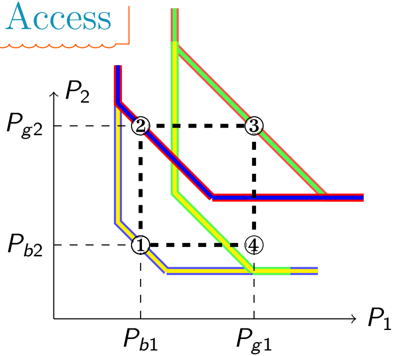
Zero Outage Bit Schedulers

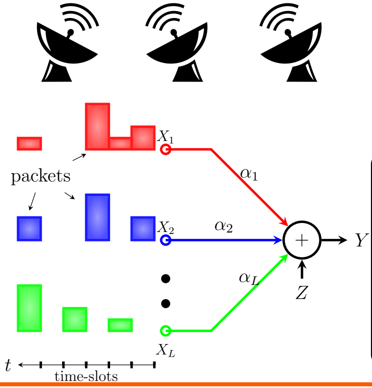


- Deterministic scheduling policies form a basis. [Rajan, Sabharwal, Aazhang, "Delay bounded packet scheduling of bursty traffic over wireless channels", Trans IT 2004].

Energy Efficient Random Access







- Solutions that minimize the average power to transport data [ISIT14].
- "The ultimate goal is to find decentralized schedulers that approach the performance":- [RAA2001]

Reference: SS,**SRBP**,BKD, 'Energy Efficient Random MAC with Strict Delay Constraints', ISIT 2014.

Conclusion

- Optimal power allocations under strict delay constraints were proposed for a MAC with random arrivals.
- The approach can also solve the weighted power minimization problem.
- An iterative algorithm quickly finds the optimal scheduler (observation).
- A dual result can solve the so called *adaptive capacity region* of distributed CSI MACs Chapter 23 of El Gamal and Kim, Network Information Theory, Cambridge 2011.
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