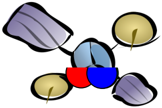
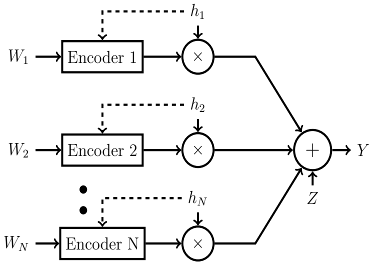
Distributed Multiple Access: Aspects of Throughput, Delay and Energy Efficiency

Sibi Raj B. Pillai

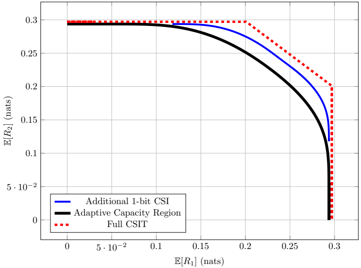


Distributed Multiple Access



- ‘Motivated by practical wireless’ [ElGamal11].
- **Adaptive Capacity** $\mathcal{C}_A$: $\Rightarrow$ No outage rates under blockwise encoding/decoding.
- An open problem for many models[Malkin07]

- Ergodic/long-term capacity region $\mathcal{C}_E$.
- On-Off distributed power ctrl [ShamaiTelatar99]
- Outage based approach also possible.
- *Detailed models in Chapters 23-24 of ElGamal and Kim, ‘Network Information Theory’, Cambridge 2011.*



Mobile Pentagons

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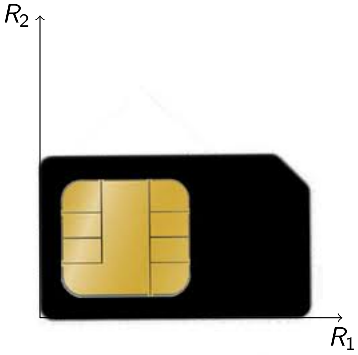

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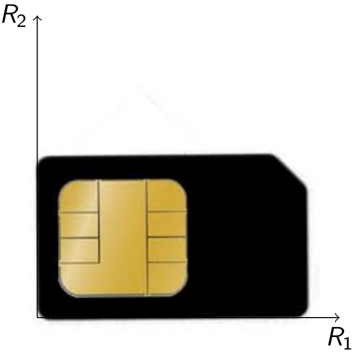
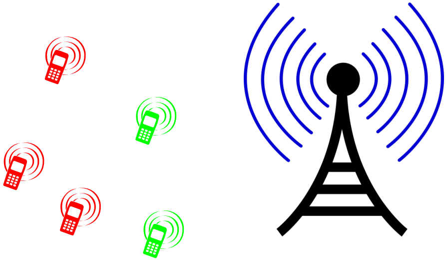


courtesy:techmoran.com, tech255.com

Uplink Multiple Access Channel



- Block-fading channel, Channel State Info (CSI) at receiver.
- Distributed CSI (local knowledge) at the transmitters (CSIT).
- Transmit-power $(P_i(H_i))$/rate $(R_i(H_i))$ adapted in each block.

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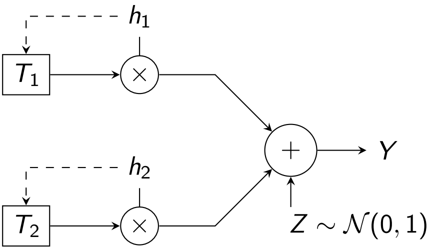

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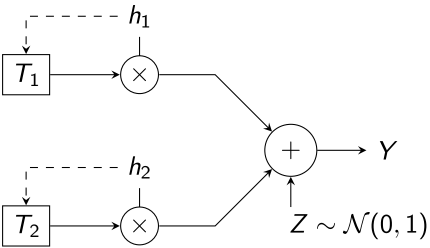

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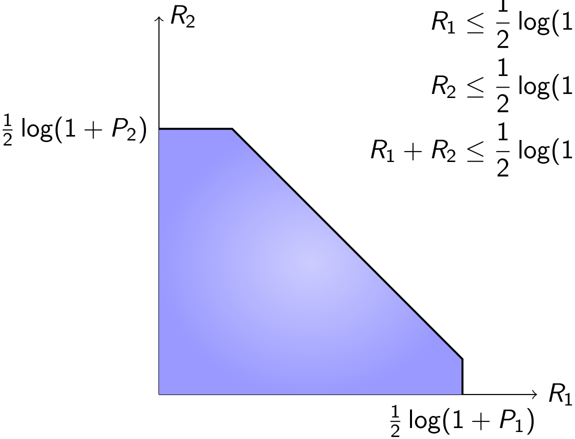
Two User Capacity Region $C_{MAC}(P_1, P_2)$



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$$R_2 \leq \frac{1}{2}\log(1 + P_2)$$

$$R_1 + R_2 \leq \frac{1}{2}\log(1 + P_1 + P_2)$$

The region also includes TDMA, CDMA, FDMA etc

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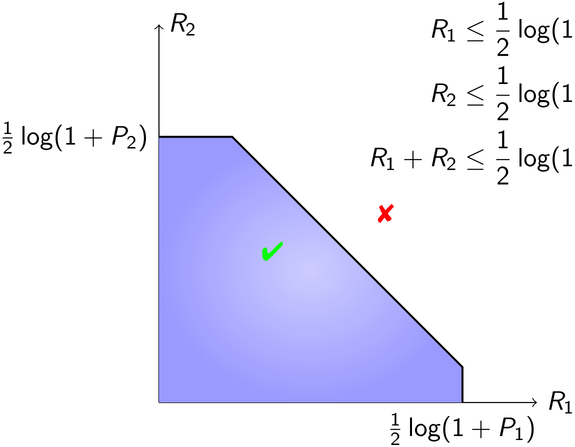

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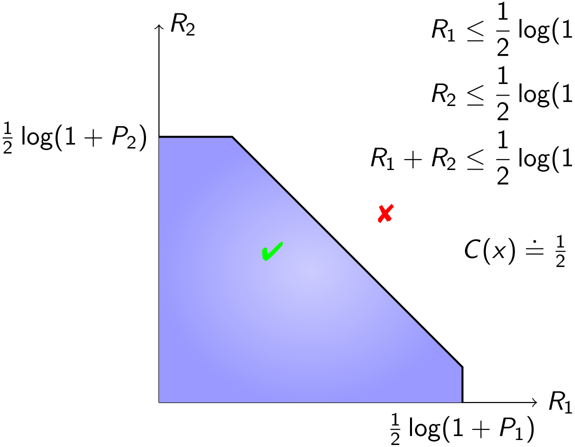

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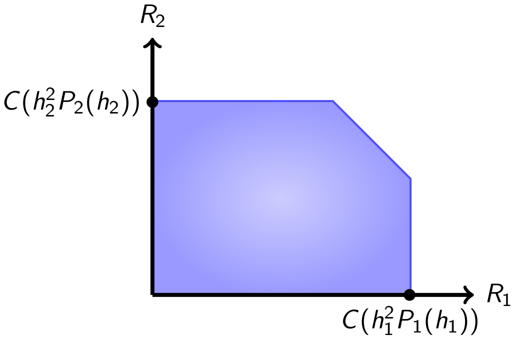
Power-Rate Strategies



- Full CSI: *opportunistic TDMA* (best user) *optimal* [KH95].
- Genrl CSI: adapt rate/power + coding across blocks [DasNarayan02].
- Distributed CSI: Adapt rate/power + block-wise coding and decoding [ElGamalKim11], [DePDe11], [SrDeP15].

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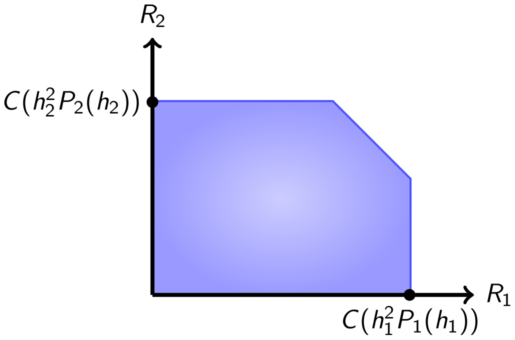

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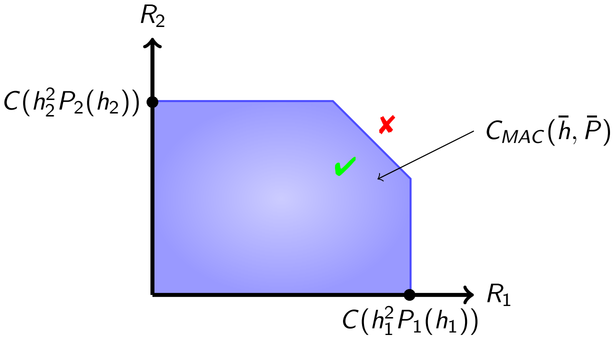

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Adaptive Capacity Region

- Objective to optimize over power-rate tuples $(P_i(H_i), R_i(H_i))$.

$$\max \mathbb{E} \sum_{i=1}^{N} w_i R_i(H_i)$$

such that $\forall (h_1, \cdots, h_N)$

$$\sum_{i \in S} R_i(h_i) \leq \frac{1}{2} \log \left(1 + \sum_{i \in S} h_i^2 P_i(h_i) \right) , \; \forall S \subseteq \{1, \cdots, N\}$$

and

$$\mathbb{E} P_i(H_i) = P_i^{avg} , \; 1 \leq i \leq N.$$

- $\bar{w} = w_1, \cdots, w_N$ is any positive weight vector.

Example: Midpoint Strategy (For Identical Users)

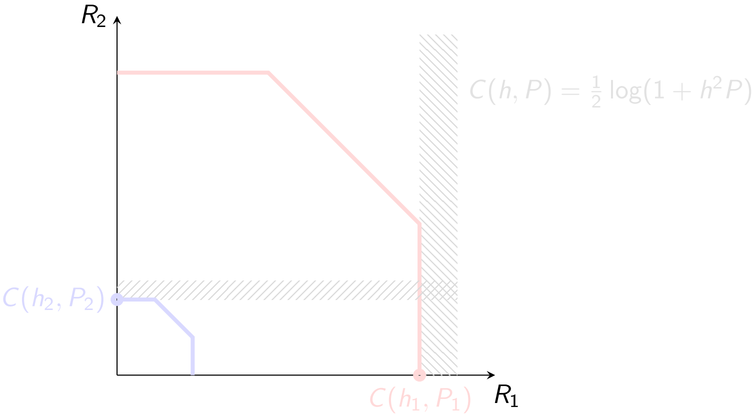

Figure: The users 1 and 2 construct the inner and outer symmetric regions respectively, and chooses A.

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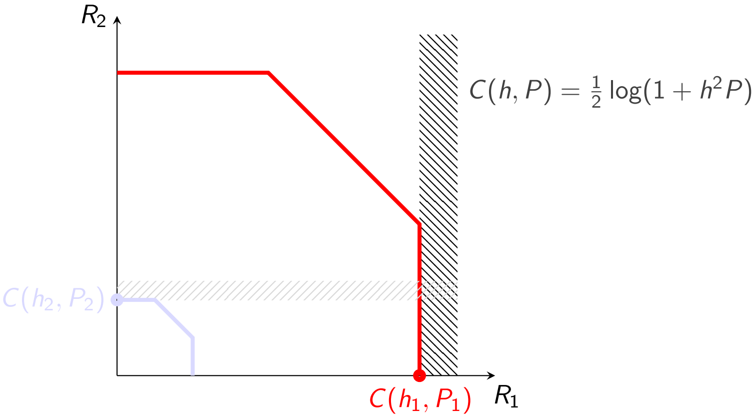

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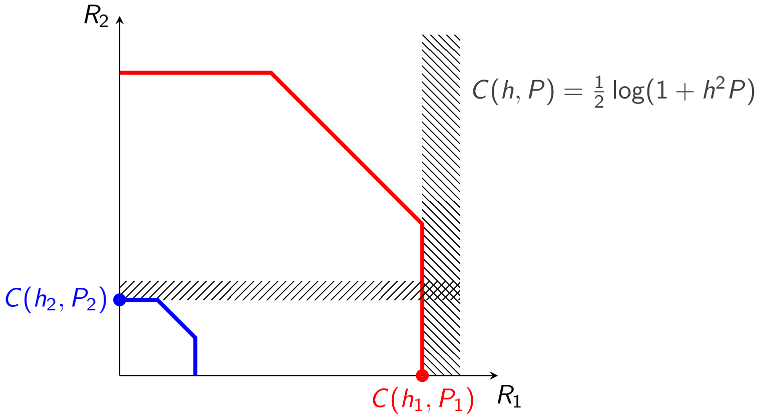

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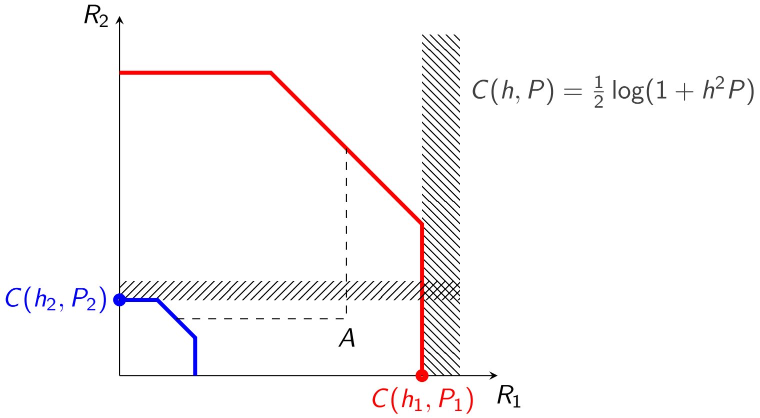

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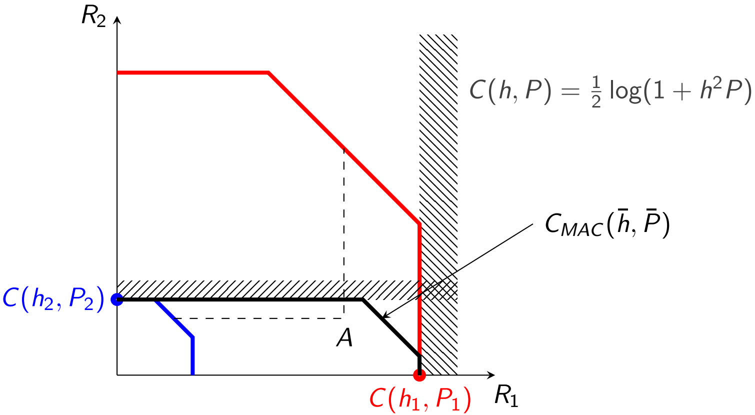

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Theorem (ITW11)

The midpoint strategy achieves adaptive sum-capacity for identical users.

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$$\begin{aligned}\sum_{i=1}^{N} \mathbb{E} R_i(H_i) &= \sum_{i=1}^{N} \int_h R_i(h)\, d\Psi(h) \\ &= \int_h d\Psi \left(\sum_{i=1}^{N} R_i(h) \right) \\ &\leq \frac{1}{2} \int_h d\Psi \log \left(1 + |h|^2 \sum_{i=1}^{N} P_i(h) \right)\end{aligned}$$

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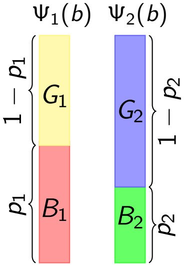
Non-Identical Fading Statistics



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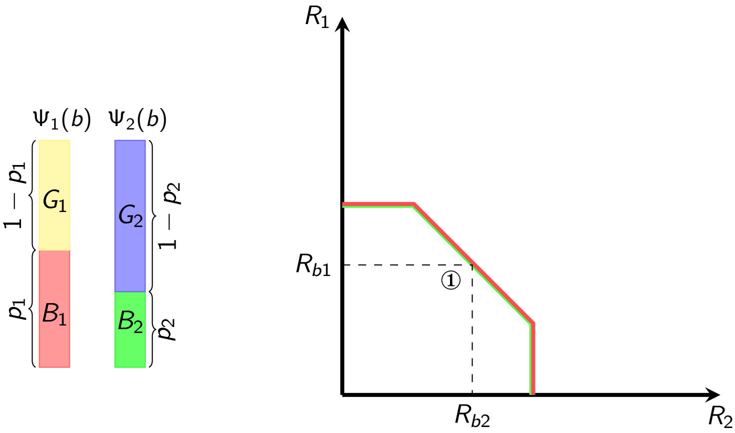

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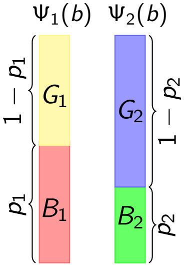

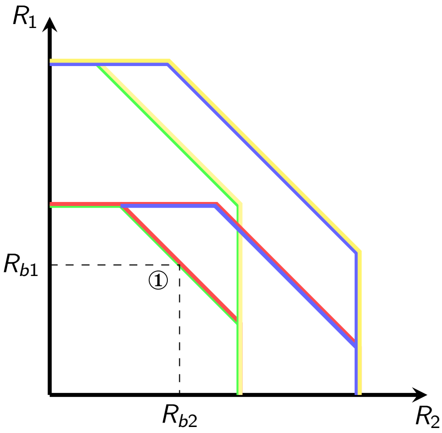

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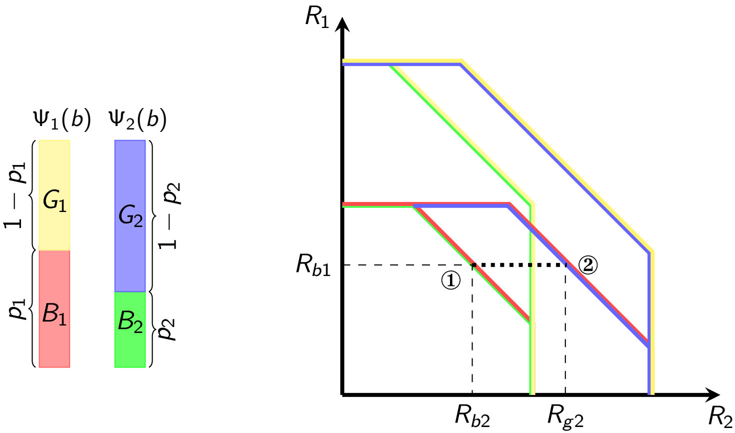

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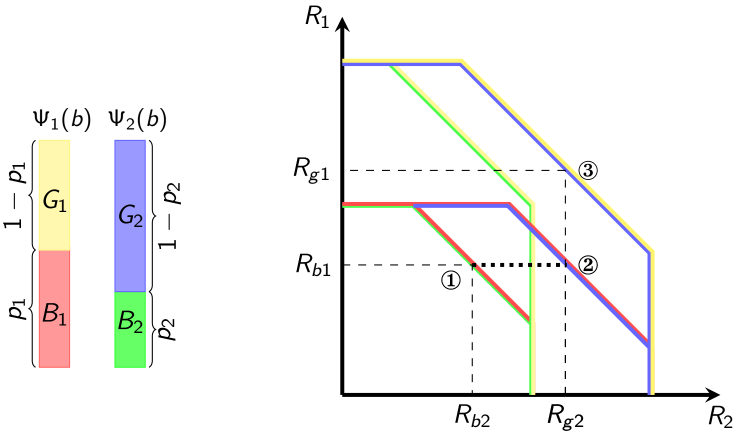

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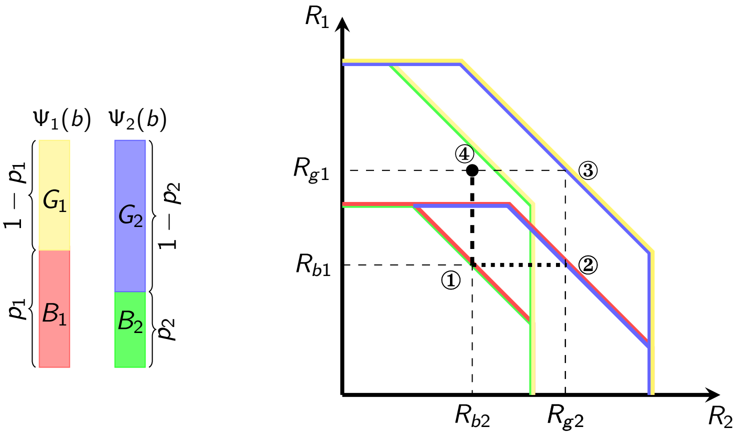

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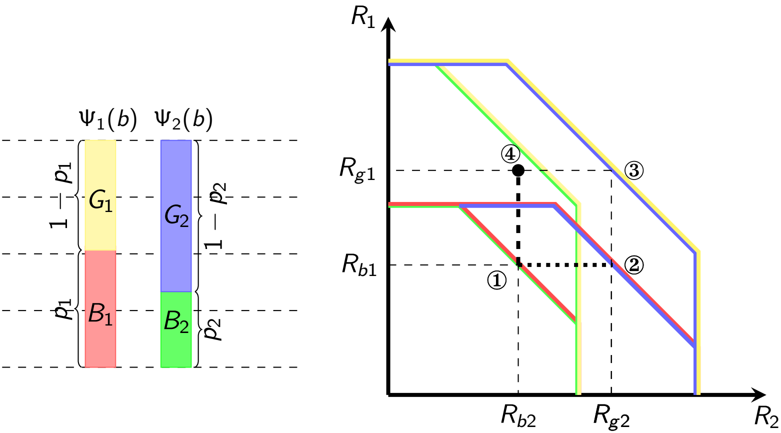

Outage-Free

Lemma

Let (h, g) and (h', g') be two state-pairs such that $(h', g') \geq (h, g)$. Assume $(R_{1h}, R_{2g}) \in C_{MAC}(h, g, \bar{P})$ and $(R_{1h'}, R_{2g'}) \in C_{MAC}(h', g', \bar{P})$. If

$$R_{1h'} + R_{2g} = \frac{1}{2}\log(1 + {h'}^2 P_1 + g^2 P_2)$$

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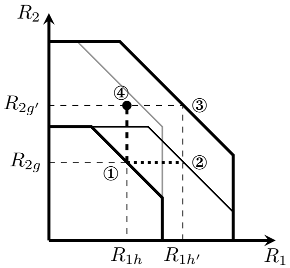

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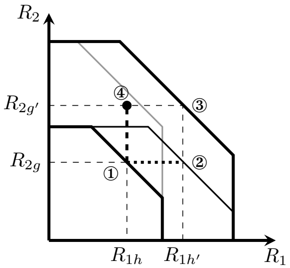
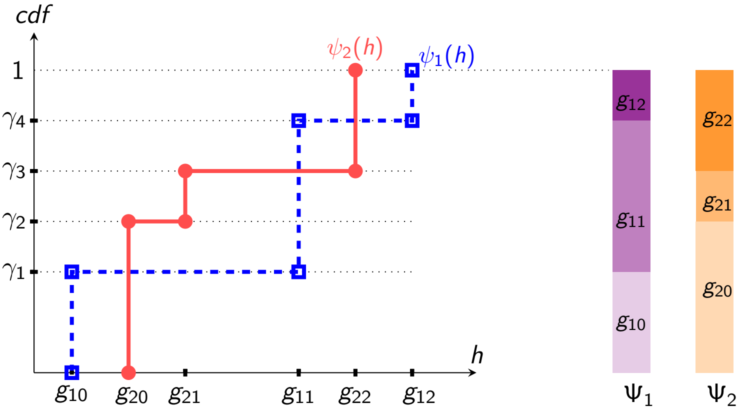


Proof.

$$(1 + h^2 P_1 + g^2 P_2)(1 + {h'}^2 P_1 + {g'}^2 P_2) \leq (1+h^2 P_1+{g'}^2 P_2)(1+{h'}^2 P_1+g^2 P_2).$$

□

Rate Assignment Demo



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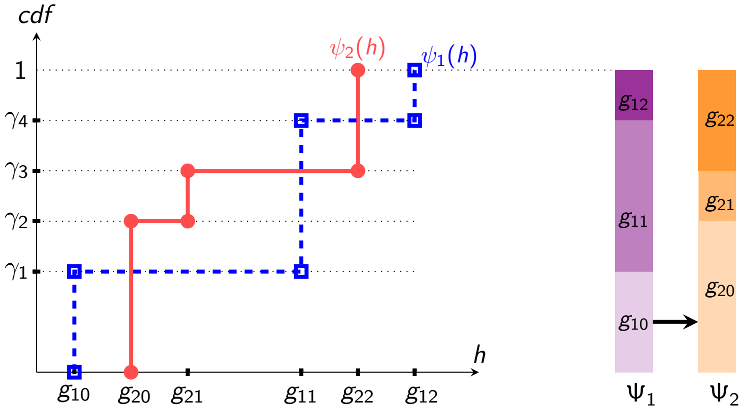

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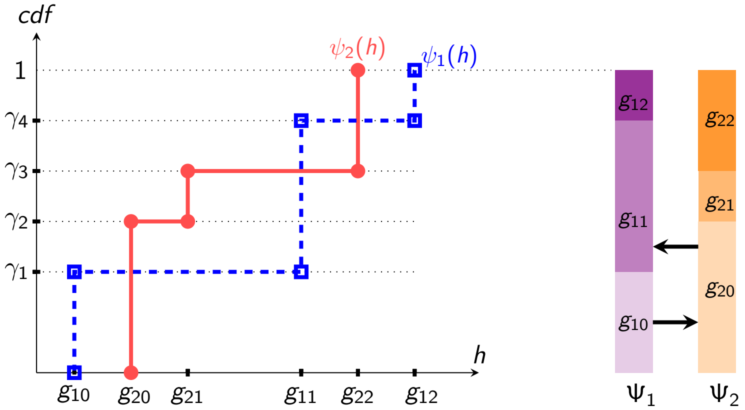

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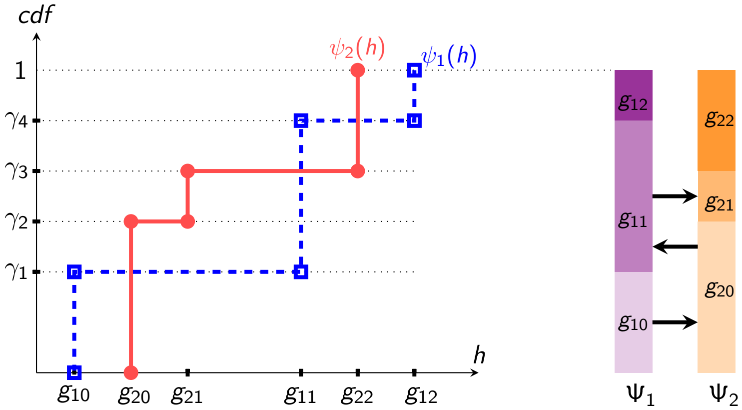

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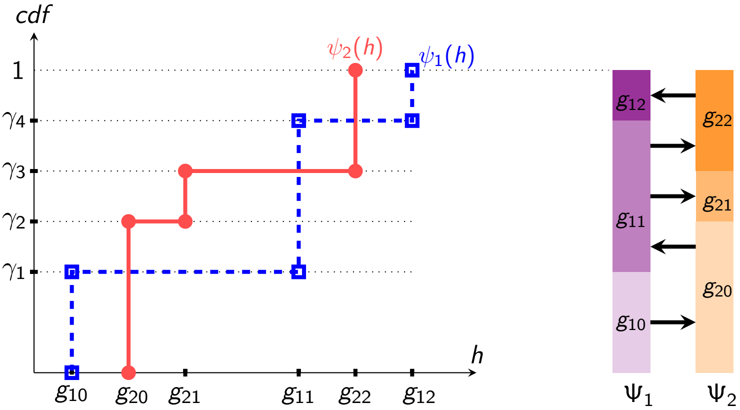

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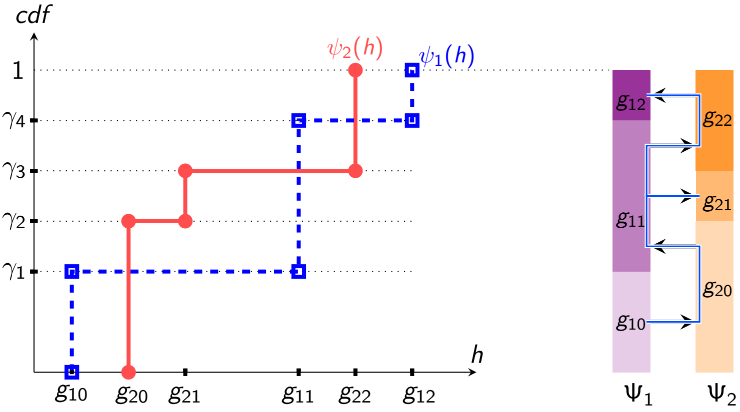

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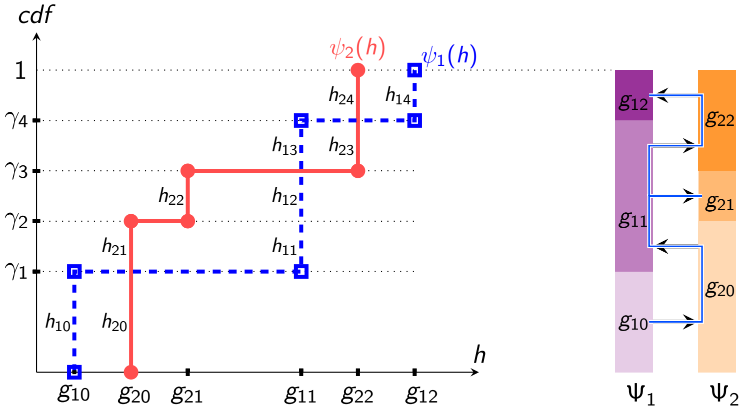


Iterative Assignment

Theorem (ISIT13)

The iterative rate assignment

$$R_1(h_{10}) \in C_{MAC}^{sum}(h_{10}, h_{20}, \bar{P})$$

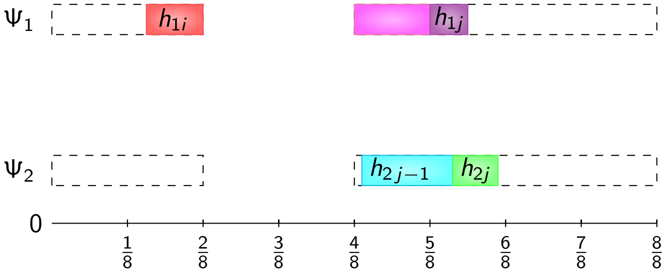
$$R_2(h_{2i}) = C_{MAC}^{sum}(h_{1i}, h_{2i}, \bar{P}) - R_1(h_{1i})$$

$$R_1(h_{1j}) = C_{MAC}^{sum}(h_{1j}, h_{2(j-1)}, \bar{P}) - R_2(h_{2(j-1)})$$

for $0 \leq i < k$, $1 \leq j < k$ *achieves the adaptive sum-capacity.*

- The first expression is about choosing any rate-pair in the dominant face of the minimal pentagon.

Inducting Outage-free



- By induction, rate-choices of all state-pairs are outage-free.
- For every *vertical* state-pair, the rate-choice is on the dominant face.
- Hence the scheme achieves *the adaptive sum-capacity*.

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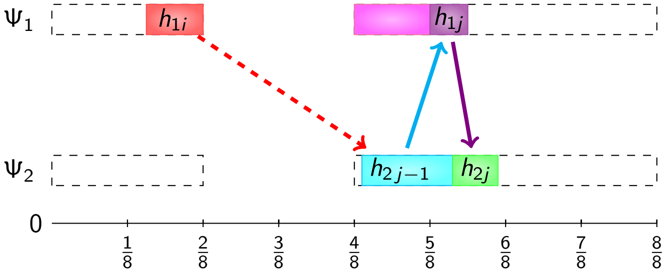

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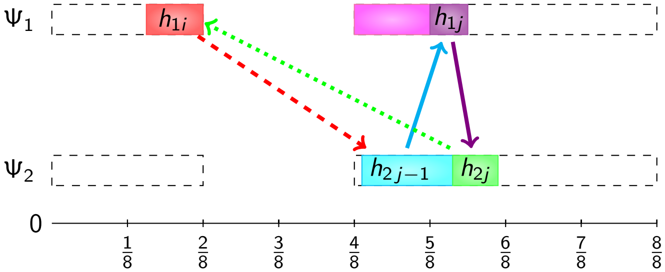

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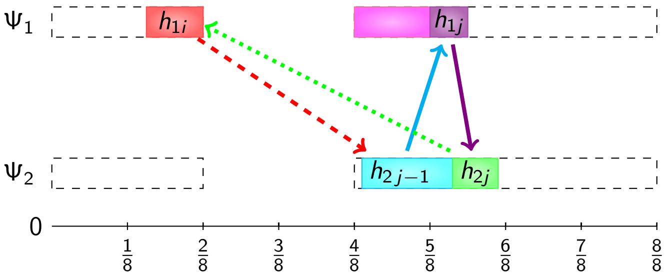

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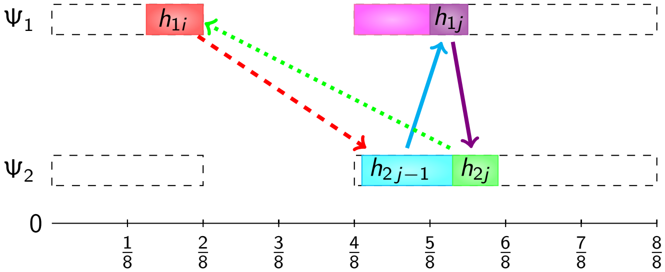

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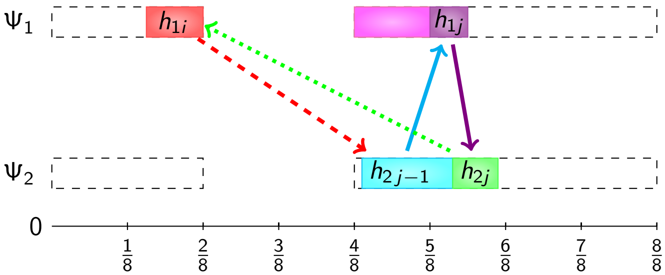

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Numerical Comparison

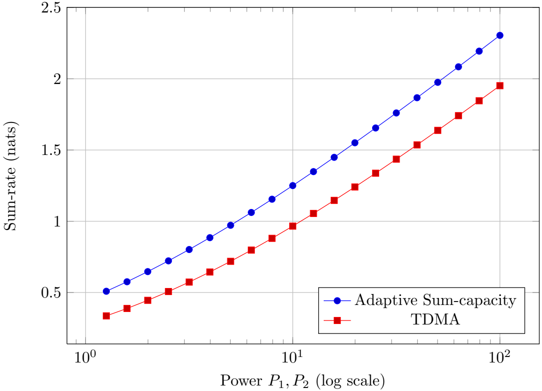


Figure: Two users: Ψ_1-Rayleigh and Ψ_2-Uniform$[0, a]$

Multiuser Generalizations

- For the CDF ψ_i of user $i \in 1, \cdots, N$, define

$$h_j(x) = \psi_j^{-1}(x) := \sup\{h : \psi(h) < x\}$$

Theorem (SsBkd**P**14)

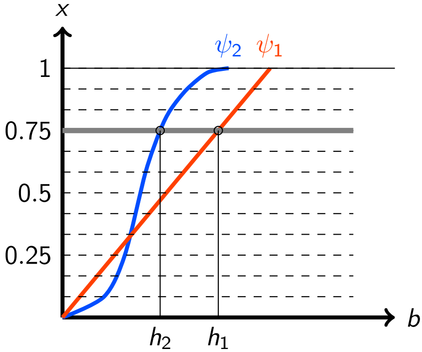
For $h \geq h_i(0)$, the rate allocation

$$R_i(h) = R_i(h_i(0)) + \int_{h_i(0)}^{h} \frac{yP_i}{1 + y^2 P_i + \sum_{j \neq i} (\psi_j^{-1}(\psi_i(y)))^2 P_j} \, \mathrm{d}y, \qquad (1)$$

achieves the adaptive sum-capacity, where

$$\sum_{i \in S} R_i(h_i(0)) \leq \frac{1}{2} \log(1 + \sum_{i \in S} h_i^2(0) P_i), \ \forall S \subseteq \{1, 2, \cdots, N\},$$

Weighted Sum-Capacity



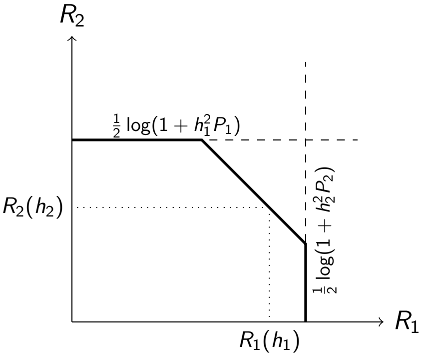


Figure: Optimal Rate-Allocation (weights $w_1 = w_2 = 1$)

CDF Transformations for Weighted Sum-capacity

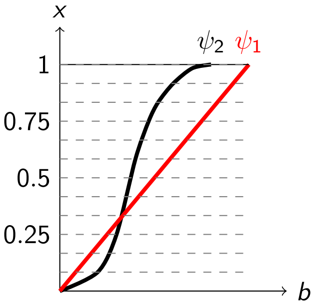


Figure: Optimal Rate-allocation for $w_1 = 1, w_2 = \beta$

Adaptive Capacity Region

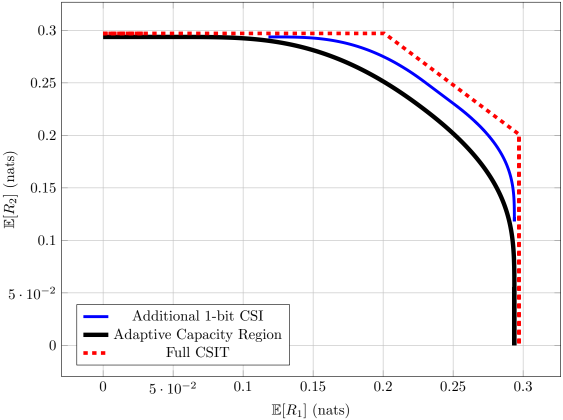
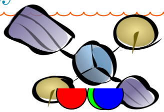
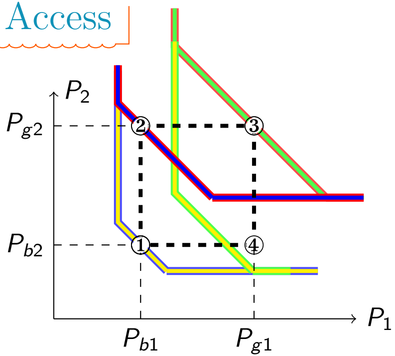
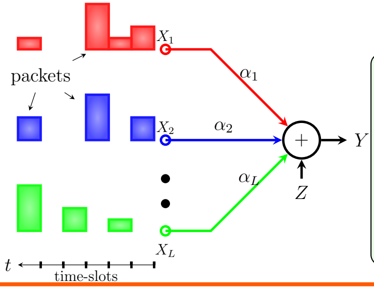


Figure: Adaptive capacity region for IID Rayleigh

Energy Efficient Random Access







- Solutions that minimize the average power to transport data [**SRBP**14].
- "The ultimate goal is to find decentralized schedulers that approach the performance" :- [RAA2001]

Reference: SS,**SRBP**,BKD, 'Energy Efficient Random MAC with Strict Delay Constraints', ISIT 2014.

Power-Savings

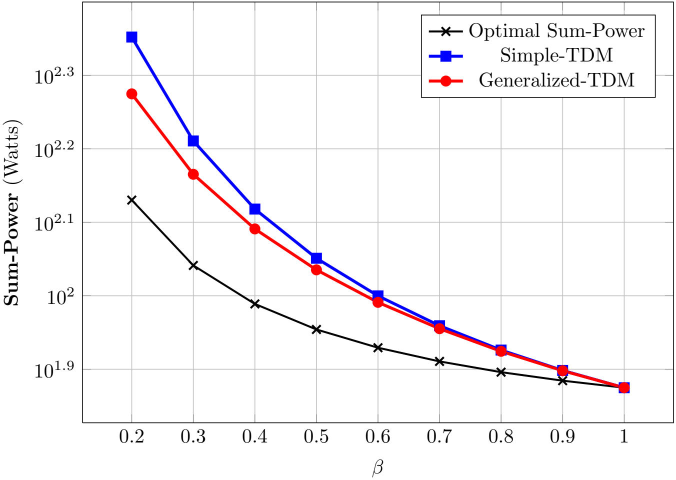
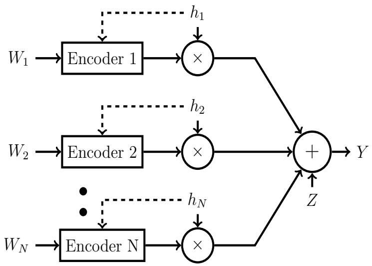


Figure: Power savings for rates in $\{1, 2\}$ with $p(1) = 0.75, \alpha_1 = \beta, \alpha_2 = 1$

Ergodic Sum-Capacity



- 'Long term coding for averages'.
- ShamaiTelatar99, DasNarayan02.
- $$\max \mathbb{E} \log(1 + \sum_{i=1}^{K} v_i P_i(v_i))$$

  $$\text{subj to } \mathbb{E}[P_i(v_i)] \leq P_{avg}$$

KKT conditions

$$v_o \int \frac{dF(\bar{v})}{1 + \sum_{i>1} v_i P(v_i)} = \mu \int \frac{dF(\bar{v})}{1 + \mu P(\mu) + \sum_{i>1} v_i P(v_i)}$$

On-OFF Power Control [ShamaiTelatar99]

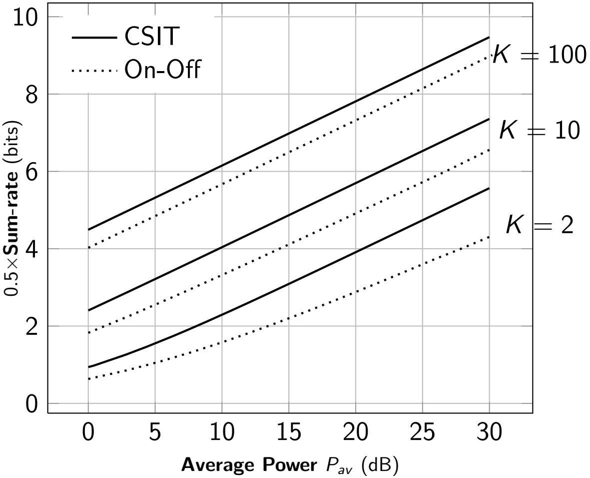
- Mimicks full CSIT

$$P(v) = e^{v_0} P_{avg} \mathbb{I}_{\{v \geq v_0\}}$$

- "Best user transmits" in full CSIT.

- On-OFF suboptimal [KamalP15].

$$P(v) > 0 \Rightarrow P'(\cdot) > 0.$$



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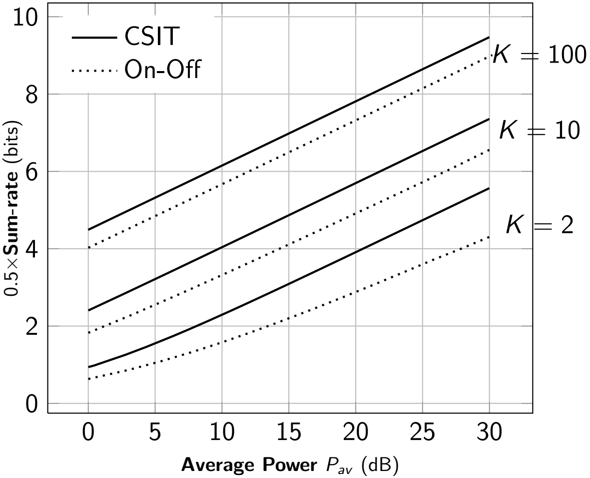

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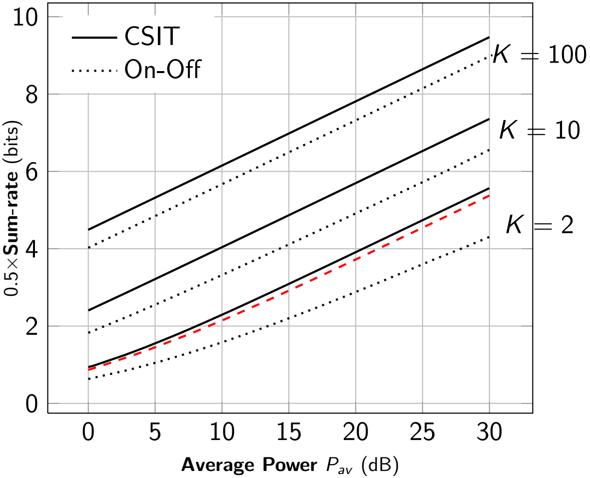

Bounds to C_E

Theorem

For any given power control $Q(v)$, we have

$$0 \leq C_E - \mathbb{E}\log(1 + \sum_{i=1}^{K} v_i Q(v_i)) \leq \max_{P(\cdot)} \mathbb{E}\frac{1 + \sum v_i P(v_i)}{1 + \sum_i v_i Q(v_i)} - 1$$

Proof Outline

$$\begin{aligned} I(x_1 x_2; y) &\leq h(y) - h(Z) \\ &= \int f_y \log \frac{1}{f_y} - h(Z) \\ &\leq \int f_y \log \frac{1}{g_y} - h(Z). \end{aligned}$$

- The RHS is a LP, with $\mathbb{E}P(v) \leq P_{avg}$ and $P(v_i) \leq P(v_{i+1})$.

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A New Power Law

Power Control

$$P(v) = \left(\frac{1}{\lambda} - \frac{1}{v}\right)^{+} G(v - \lambda)$$

where

$$G(x) = 1 + \alpha \exp(-\frac{\beta}{x}).$$

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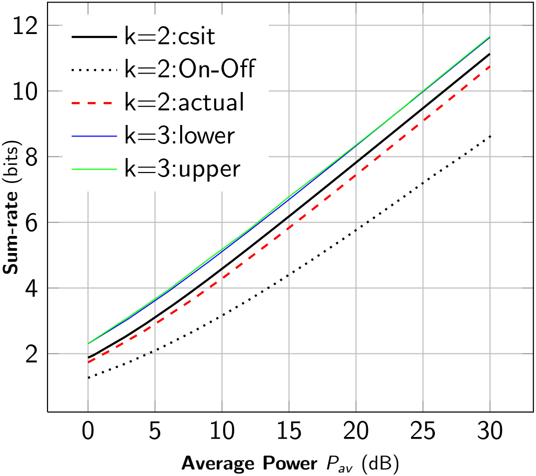

Conclusion

- Adaptive capacity region in the distributed block fading system was characterized.
- Upper and lower bounds to the ergodic capacity was obtained.
- Outage capacity: for a given outage pattern, our techniques will enable the evaluation of outage-capacity in several situations.
- Joint work with Kamal Singh, Sreejith Sreekumar, B K Dey @IITBombay.