On Fading MACs with Asymmetric CSI

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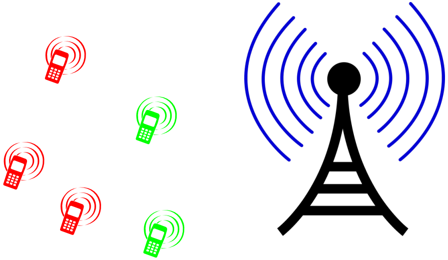
Motivation



- Block-fading channel with *asymmetric CSI*
- Power control with average power constraints
- CSI of some links are *protected/unrevealed* (eg. Cognitive Radios).

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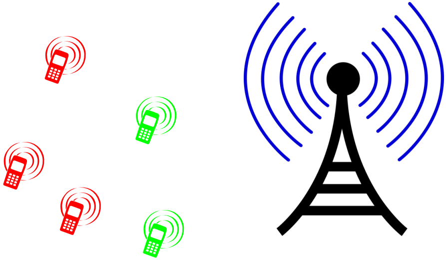

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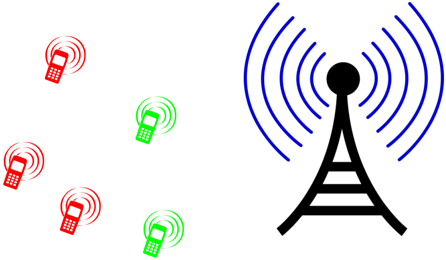

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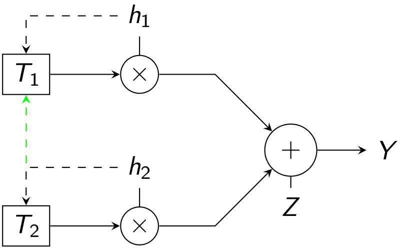

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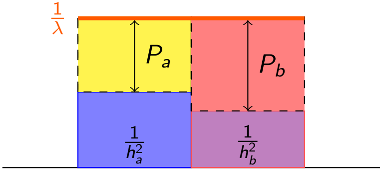
Single User Power-Rate Strategies

- Channel law is $\Psi(h)$.
- Strategy $h \to \left(P(h), C(h^2 P(h))\right)$ where $C(x) \triangleq \frac{1}{2}\log(1+x)$.
- Throughput

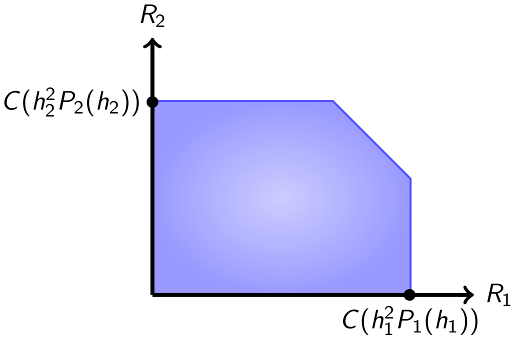
$$C_{sum} = \frac{1}{2}\int \log(1 + h^2 P(h)) d\Psi(h) \text{ subj to } \int P(h) d\Psi(h) = P^{avg}.$$

Optimal Power [Goldsmith97]

$$P^*(h) = \left(\frac{1}{\lambda} - \frac{1}{|h|^2}\right)^+$$



Power-Rate Strategies



- Full CSI: *opportunistic TDMA* (best user) optimal [KH95].
- Genrl CSI: adapt rate/power + coding across blocks [DasNarayan02]
- Individual CSI and slow Fading: Adapt rate/power + block-wise coding and decoding [ElGamalKim11], [PillaiDey12], [Sreekumar13}].

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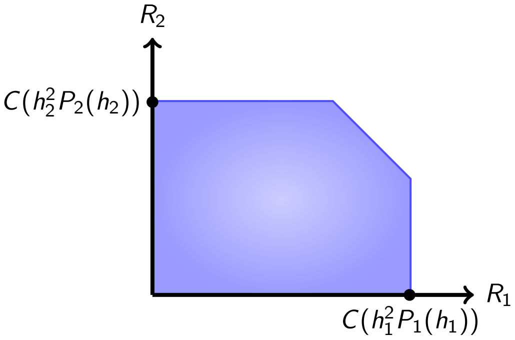

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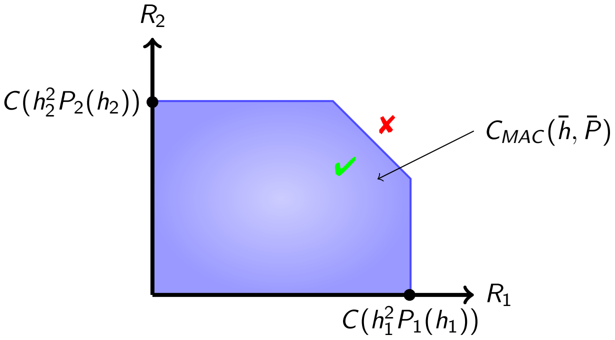

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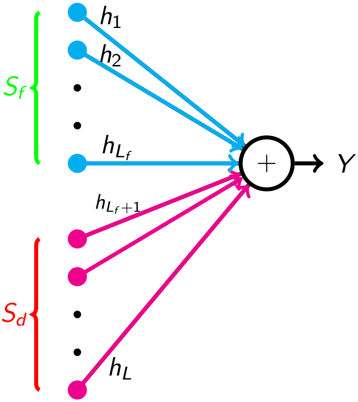
Adaptive Sum Capacity C_Ψ



- $\hat{h}_i = \bar{h}\mathbb{I}_{\{h_i \in S_f\}} + h_i \mathbb{I}_{\{h_i \in S_d\}}$.
- $P_i(\hat{h}_i), R_i(\hat{h}_i)$: power/rate of user i.

$$\max \mathbb{E}[\sum_{i=1}^{L} R_i(\hat{h}_i)]$$

such that

$$\mathbb{E}P_i(h_i) \leq P_i^{avg}, \forall i,$$

and $\forall \bar{h} \in \mathbb{R}^L$, $\bar{P} \triangleq P_1(\hat{h}_1), \cdots, P(\hat{h}_L)$:

$$R_1(\hat{h}_1), \cdots, R_L(\hat{h}_L) \in C_{MAC}(\bar{h}, \bar{P}).$$

Assumptions

- Channels are continuous-valued, admitting respective pdfs.

$$\Psi(\bar{h}) = \prod_{i \in S_f} \Psi_f(h_i) \prod_{i \in S_d} \Psi_d(h_i).$$

- For simplicity: the average powers remain same within each group.
- The paper-version has limited $\Psi_f(\cdot)$ to Rayleigh (*not necessary*).
- Adaptive Sum-capacity

$$C_\Psi = \max \int d\Psi(\bar{h}) \left(\sum_{j \in S_f} R_j(\bar{h}) + \sum_{i \in S_d} R_i(h_i) \right)$$

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- Full CSI arguments [KnoppHumblet95] cannot be invoked as such, as the users in S_d do not know the best user.
- The *best* user in S_f can improve the received signal power, and sum-rate[1].

[1] valid only for identical users

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Claim: From the set S_f, it is sufficient to schedule $\arg\max h_i^2$ for transmission, while remaining users in this group stay silent.

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Two User MAC- Degraded CSI

Theorem

$\exists\ \lambda_1 \geq 0$ and a threshold function $\gamma(h_2)$ such that

$$C_\Psi = \frac{1}{2}\mathbb{E}\left[\log(1 + h_1^2 P_1^*(h_1, h_2) + h_2^2 P_2^*(h_2))\right]$$

where

$$P_2^*(h_2) = \left(\frac{\gamma^2(h_2)}{\lambda_1 h_2^2} - \frac{1}{h_2^2}\right)^+ \mathbb{I}_{\{h_2 \neq 0\}} \text{ and } P_1^*(h_1, h_2) = \left(\frac{1}{\lambda_1} - \frac{\gamma^2(h_2)}{\lambda_1 h_1^2}\right)^+$$

- λ_1 and $\gamma(h_2)$ determined by $\int d\Psi(\bar{h}) P_i^*(\bar{h}) = P_i^{avg}$, $i = 1, 2$.
- Achievability: successive cancellation, user 1 followed by user 2.
- $\gamma(h_2) = \lambda_1$ whenever $P^*(h_2) = 0$, $\Rightarrow$ usual water-filling for user 1.

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Waterfilling Illustration

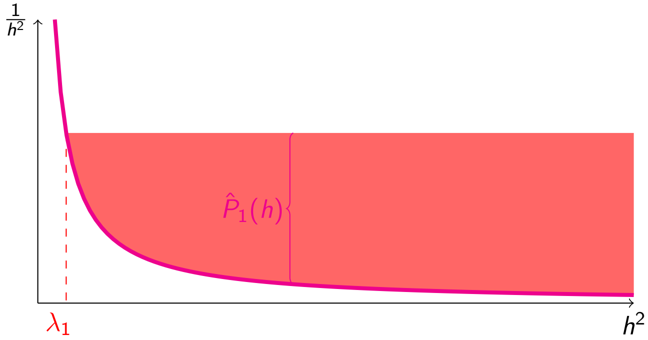

Figure: Power allocation for User 1

The users cooperate to accommodate each other and maximize the sum-rate.

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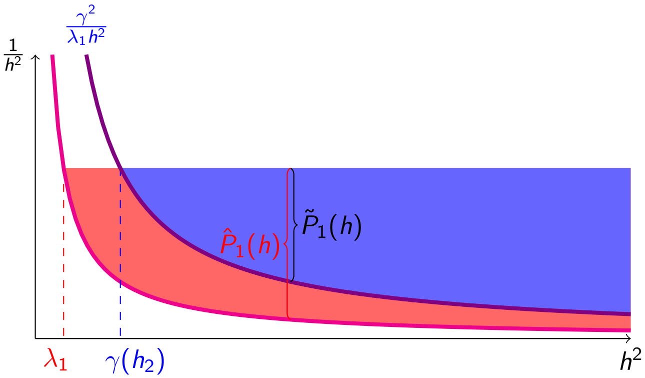

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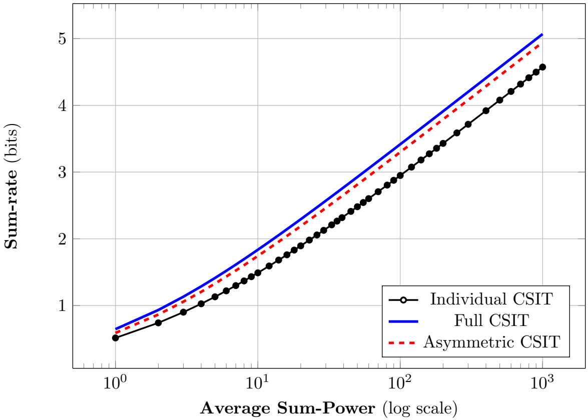
Comparison



Sum-rate (bits)
5
4
3
2
1
10^0
10^1
10^2
10^3
Average Sum-Power (log scale)
Individual CSIT
Full CSIT
Asymmetric CSIT

Many Users

- $C_{\hat{\Psi}}$: the sum-capacity when the power constraints for S_d are

$$\left(\sum_{i \in S_d} P_i^{avg}, 0, 0, \cdots, 0 \right).$$

Lemma

The adaptive sum-capacity obeys $C_\psi \leq C_{\hat{\Psi}}$.

Proof.

$$C_\Psi = \int \sum_{i \in S_f} R_i(\bar{h}) d\Psi(\bar{h}) + \int d\Psi_d(h) \sum_{i \in S_d} R_i(h)$$

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$$= \int \left(\sum_{i\in S_f} R_i(h_1, \cdots, h_{L_f}, h) + \sum_{i\in S_d} R_i(h)\right) d\Psi(h_1, \cdots, h_{L_f}, h)$$

Achievable Scheme

Lemma

$$C_{\Psi} \geq C_{\hat{\Psi}}$$

Proof.

- Under TDM, let user $i \in S_d$ transmit for a fraction $\frac{P_i^{avg}}{\sum_{i \in S_d} P_i^{avg}}$.
- The average rate of $C_{\hat{\Psi}}$ is now achievable in each slot.

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Theorem

C_{ψ} is given by the sum-capacity of a two-user asymmetric CSI MAC with

$$\hat{\Psi}_f(h) = \prod_{i \in S_f} \Psi_f(h) \text{ and } \hat{\Psi}_d(h) = \Psi_d(h)$$

and the respective power constraints of $|S_f| P_f^{avg}$ and $\sum_{i \in S_d} P_i^{avg}$.

Conclusion

- We computed the power-controlled adaptive sum-capacity of some popular Gaussian MACs with asymmetric state information at the transmitters.
- A threshold based power control is optimal for two users, which can be extended to multiple users.
- We believe that the same kind of behavior holds true even when the informed set S_f has non-identical users.