Interference Management in Wireless Networks

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Jointly with: Stephen Hanly *et al*

Outline

- Interference Channels
- Symmetric Model with Many Links
- A Dynamic Power Allocation Problem
- Static Problem with a Peak Constraint
- Conclusion

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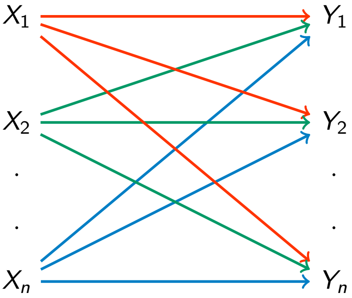
Interference Channels



$$Y_i = h_{ii}X_i + \sum_{i \neq j} h_{ji}X_j + Z_i$$

$$Z_i \sim N(0, \sigma^2)$$

- A long standing open problem
- Wide applications
- Significant progress recently
  ETseW'06, TelTse'07

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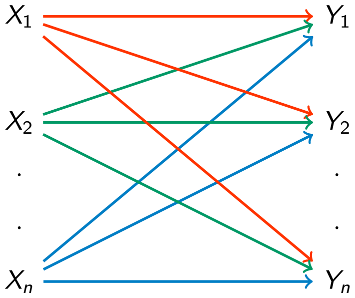

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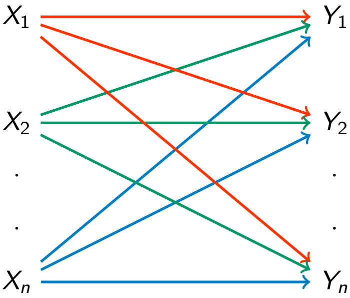

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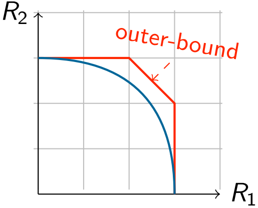

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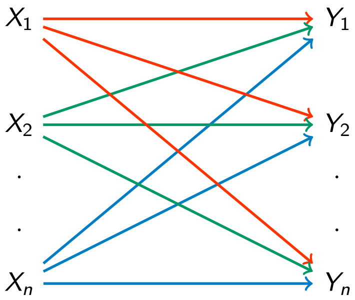

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Symmetric Case

$h_{ii} = 1, h_{ij}^2 = \epsilon$

$E|X_i|^2 = P_i \,;\, \sigma = 1$

Shannon's Theory

Shannon'48

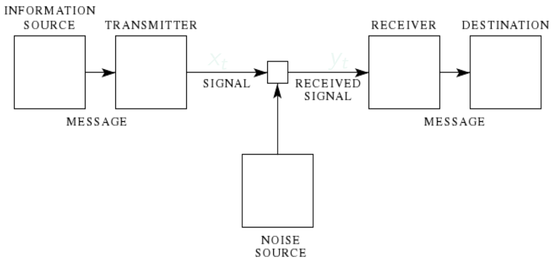

Fig. 1—Schematic diagram of a general communication system.

Discrete AWGN *Capacity* is $\log(1 + SNR)$ Ash'65, Wyner'68

$C(SNR)$ is an engineering quantity

- A particular kind of signals, Sampling, Receiver Structure
- It is a lower bound

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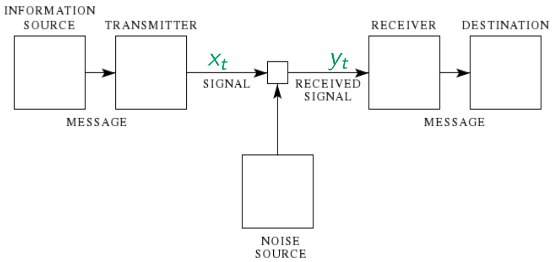

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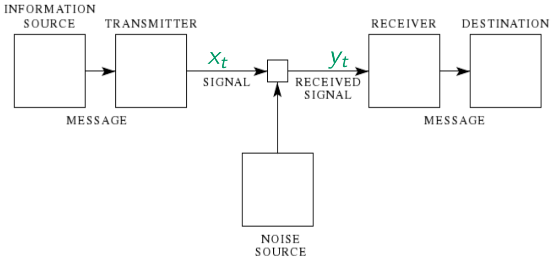

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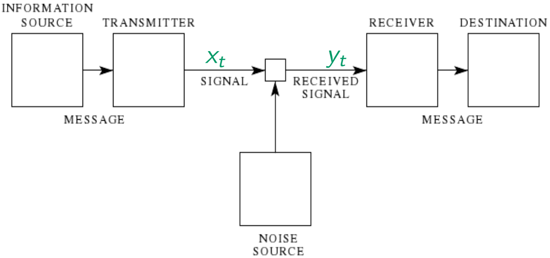

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Dynamic vs Static

Dynamic Schemes

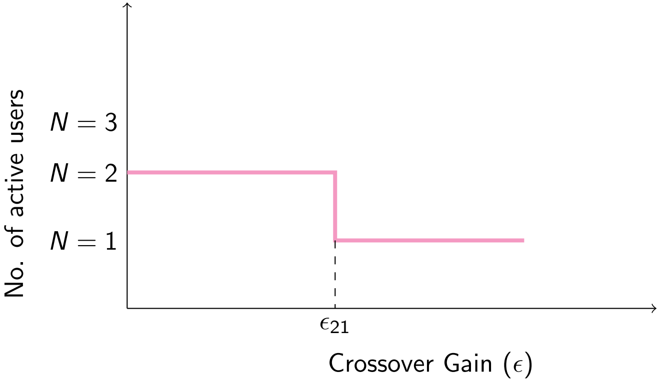
- Dynamically time varying power profile for each user.
- Each user should confine to an average power constraint of P_{avg}.

Static Schemes

- Transmission power is chosen at the start of communication.
- Each user has a peak power constraint of P_{max}.

Motivation

With two users, ON-OFF strategies are optimal *BaEvHa'08*



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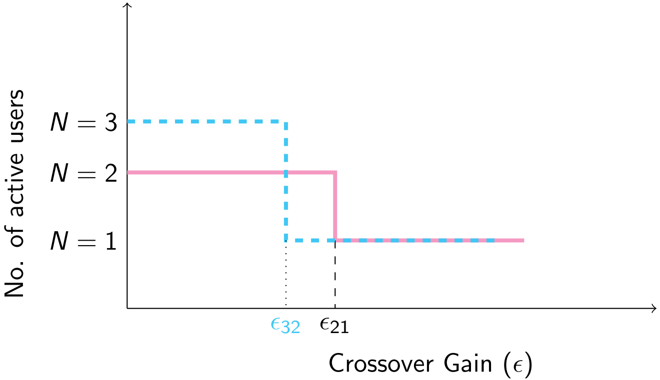

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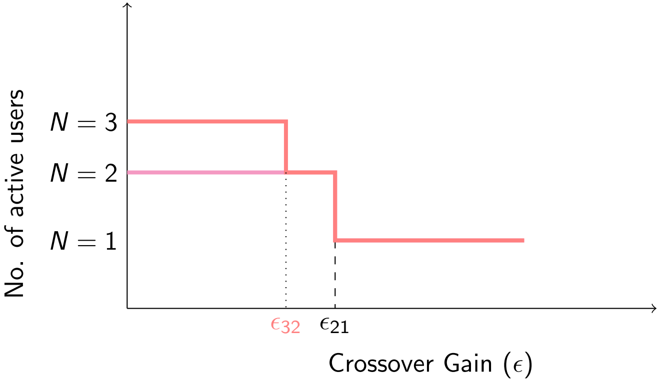

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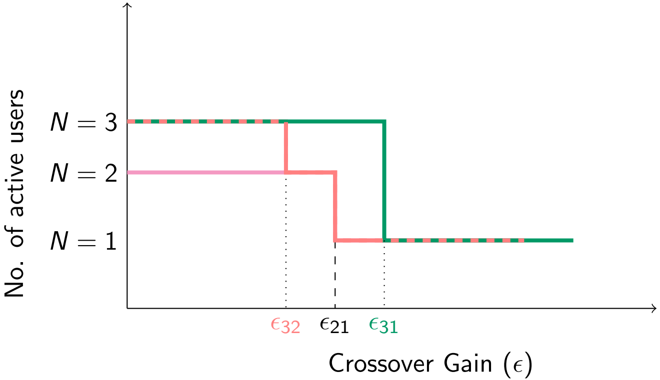
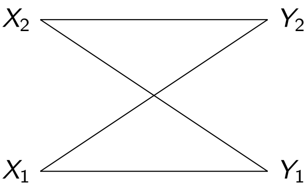


Two User Model



$$\Rightarrow \quad R_2(P_1, P_2) = C\left(\frac{P_2}{1+\epsilon P_1}\right)$$

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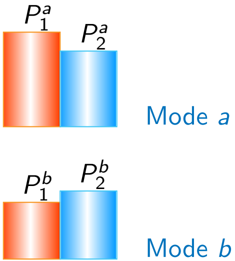
$$\begin{aligned} \text{maximize} \quad & \alpha_a R_a + \alpha_b R_b \\ \text{subject to} \quad & \alpha_a P_1^a + \alpha_b P_1^b \leq P_{avg} \\ & \alpha_a P_2^a + \alpha_b P_2^b \leq P_{avg} \end{aligned}$$



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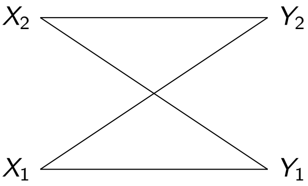

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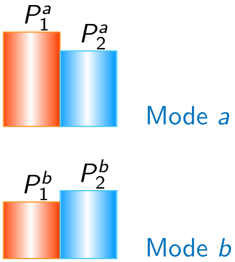
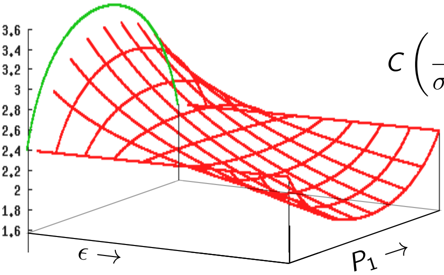


Convex or Concave



$$C\left(\frac{P_1}{\sigma + \epsilon P_2}\right) + C\left(\frac{P_2}{\sigma + \epsilon P_1}\right)$$

$$P_1 + P_2 = \hat{P}$$

Theorem: Convexity of Sumrate $C_{sum}(\hat{P}, \epsilon, \sigma, \cdot)$

- *a concave function for* $\epsilon \leq \epsilon^*$
- *a convex function for* $\epsilon \geq \epsilon^*$
- *a constant function for* $\epsilon = \epsilon^*$

$$\epsilon^* = \sqrt{\sigma}\frac{\sqrt{\sigma+\hat{P}} - \sqrt{\sigma}}{\hat{P}}$$

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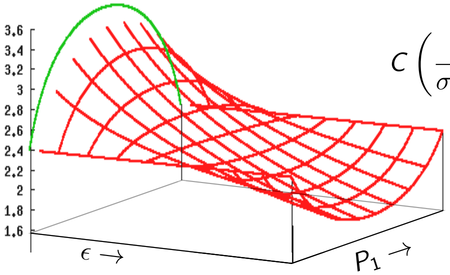

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Dynamic Allocation

$$\alpha_a R_a + \alpha_b R_b \leq \sum_{j \in \{a,b\}} \alpha_j \max\left\{2C\left(\frac{P^j/2}{1+\epsilon P^j/2}\right), C(P^j)\right\}$$

Case I

$$\alpha_a 2C\left(\frac{P^a/2}{1+\epsilon P^a/2}\right) + \alpha_b 2C\left(\frac{P^b/2}{1+\epsilon P^b/2}\right) \leq 2C\left(\frac{P_{avg}}{1+\epsilon P_{avg}}\right)$$

- This rate is achieved by transmitting both users at P_{avg}

Case II

$$\alpha_a C(P^a) + \alpha_b C(P^b) \leq C(2P_{avg})$$

- This rate is achieved by FDM with the active user at $2P_{avg}$

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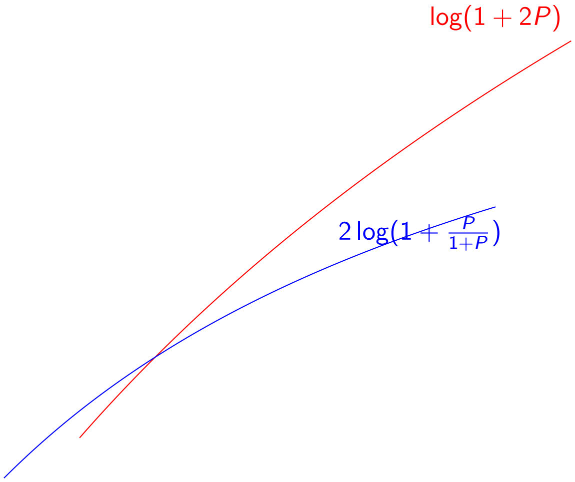

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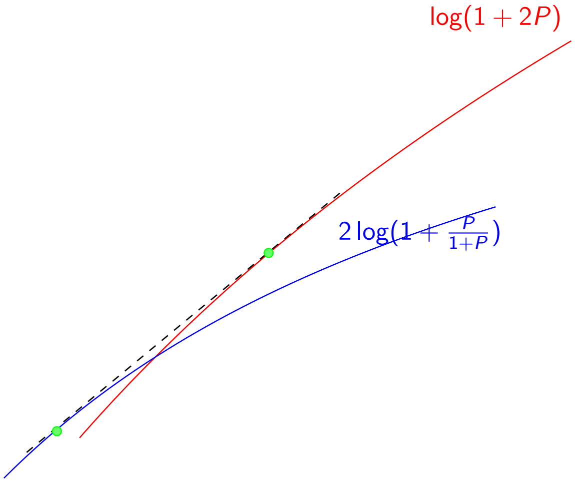

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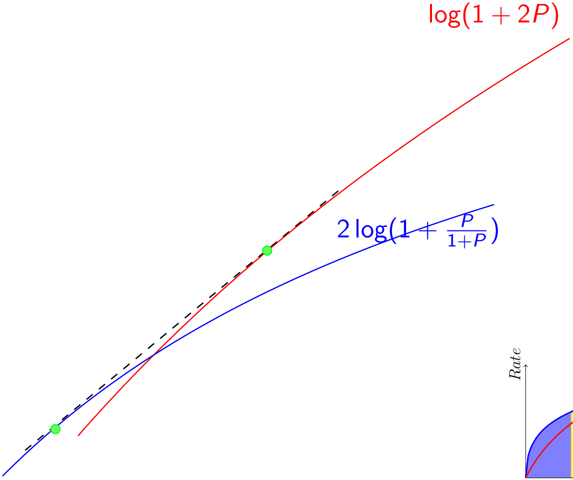

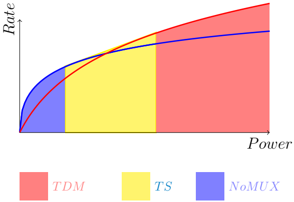

Many Modes, N Users

Theorem: Dynamic Power Allocation

At most $N + 1$ modes are required to achieve the maximal sum-rate.

- *For $\epsilon \leq \epsilon^*$, transmit full blast, with all users at P_{avg}.*
- *For $\epsilon \geq \epsilon^{**}$, use FDM/TDM with the active user at NP_{avg}.*
- *Otherwise, time-share between the above two.*

Static Allocation

- Useful to constrain the peak power in many situations
- Power allocation is chosen at the start of transmission

Majorization ($x \succeq y$) $\sum x_i = \sum y_i$ and $x_1 + \cdots + x_j \geq y_1 + \cdots + y_j$

- *For a Schur-convex function $f(\cdot,\cdot)$, if $x \succeq y$, then $f(x) \geq f(y)$*

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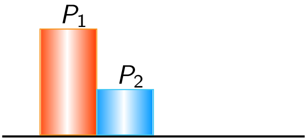

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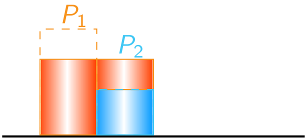

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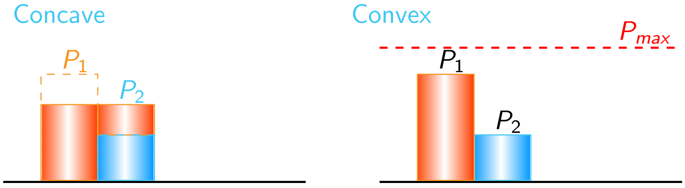

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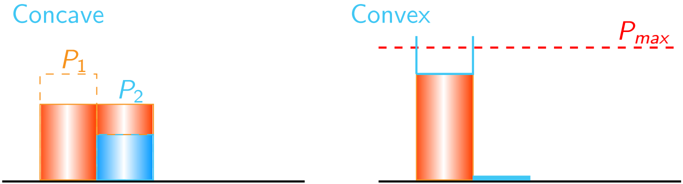

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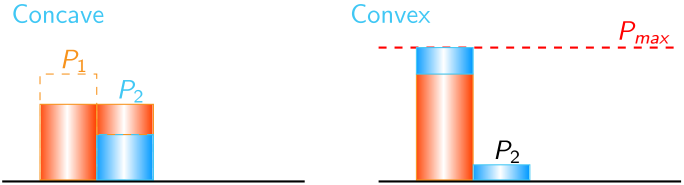

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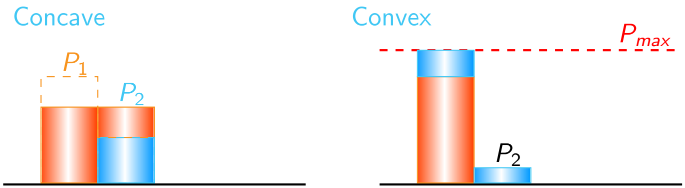

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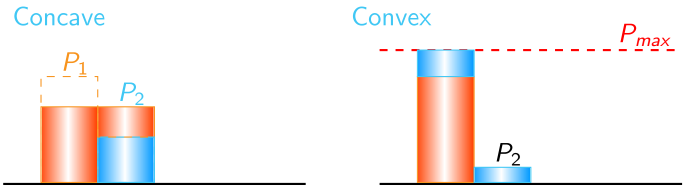

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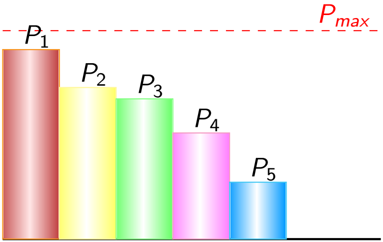

$$\sum_i R_i = \sum_i C\left(\frac{P_i}{1+\epsilon\sum_{j\neq i} P_j}\right)$$

- At most two non-zero power levels to maximize the sum-rate

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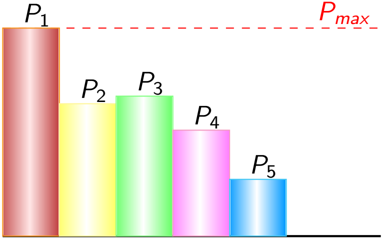

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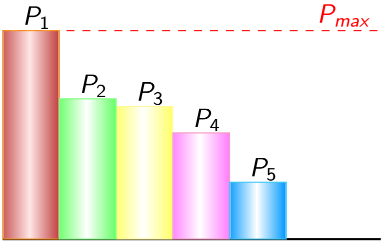

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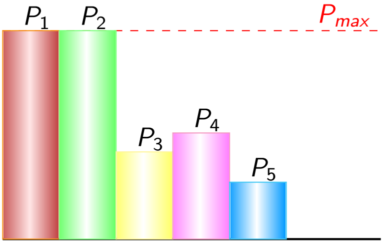

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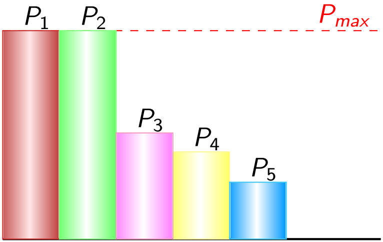

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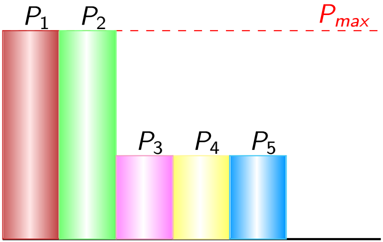

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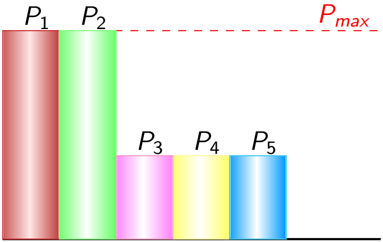

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Two Power Levels

$$J(P,k,l) = kC\left(\frac{P_{max}}{1+\epsilon(k-1)P_{max}+\epsilon lP}\right) + lC\left(\frac{P}{1+\epsilon kP_{max}+\epsilon(l-1)P}\right)$$

$$J'(P,k,l) = \frac{aP^2+bP+c}{\text{Poly}(P)} \quad ; \quad J'(P_{max},k,l) \geq 0$$

- We can find the maximum in $0(N)$ steps.
- Let us assume that there are only two power levels, i.e., $P \in \{0, P_{max}\}$.
- How many **active** users are there in the optimal scheme ?

Two Power Levels

$$J(P,k,l) = kC\left(\frac{P_{max}}{1+\epsilon(k-1)P_{max}+\epsilon lP}\right) + lC\left(\frac{P}{1+\epsilon kP_{max}+\epsilon(l-1)P}\right)$$

$$J'(P,k,l) = \frac{aP^2+bP+c}{\text{Poly}(P)} \quad ; \quad J'(P_{max},k,l) \geq 0$$

- We can find the maximum in $0(N)$ steps.
- Let us assume that there are only two power levels, i.e., $P \in \{0, P_{max}\}$.
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All or One

Theorem: Optimal Active Users (N_{opt})

If $P \in \{0, P_{max}\}$, then

- $N_{opt} = N$ whenever $\epsilon \leq \epsilon_{N1}$.
- $N_{opt} = 1$ whenever $\epsilon > \epsilon_{N1}$.

$$\epsilon_{N1} = \frac{(1 + P_{max}) - (1 + P_{max})^{\frac{1}{n}}}{(n-1)P_{max}[(1 + P_{max})^{\frac{1}{n}} - 1]}.$$

Much more simple to understand the limiting behaviour

$$\lim_{N \to \infty} \epsilon_{N1} = \frac{1}{\log(1 + P_{max})}$$

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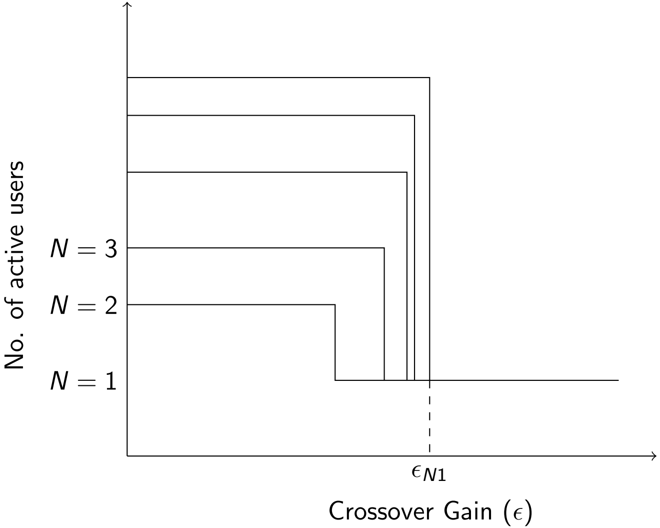
Phase Transition



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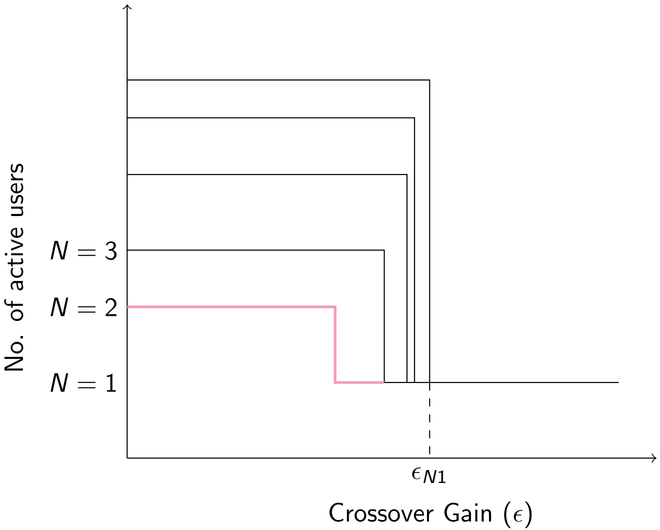

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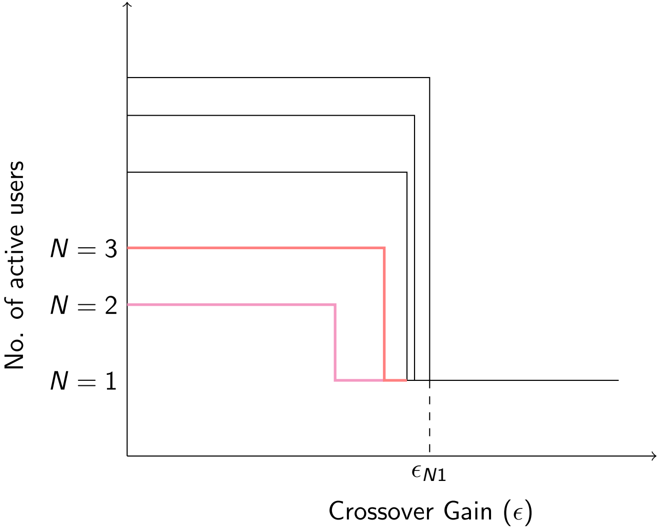

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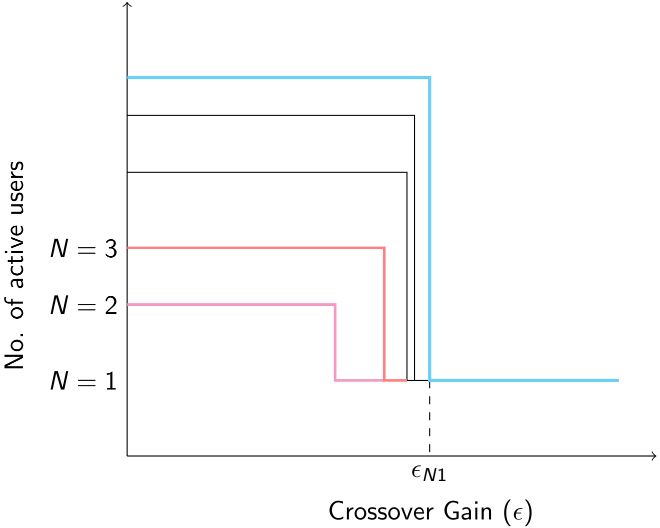

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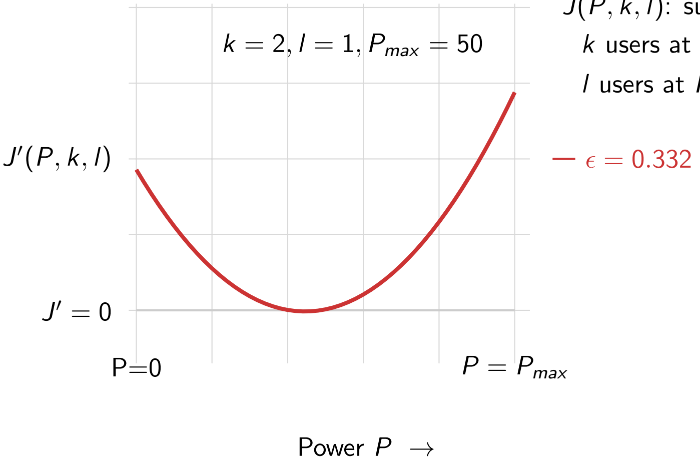
Local Maximas



$J(P,k,l)$: sum-rate

k users at P_{max}

l users at P

- The local maxima prevails when $\epsilon > \epsilon_d$

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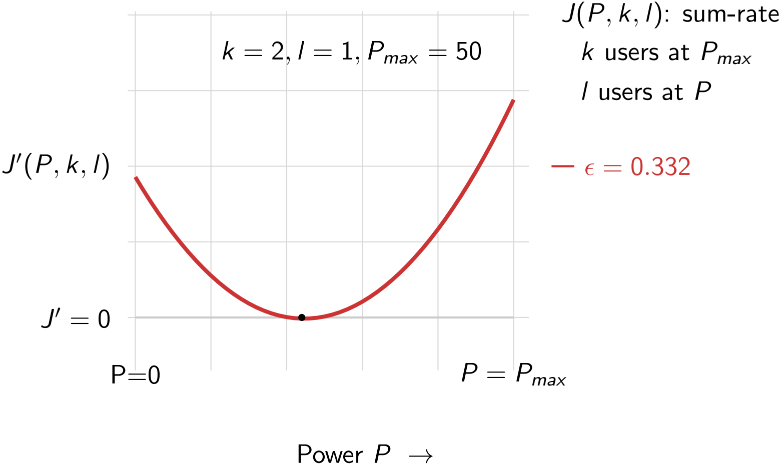

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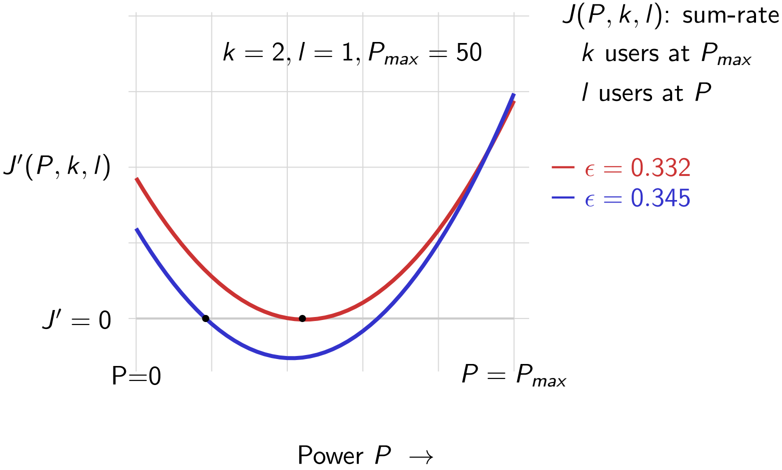

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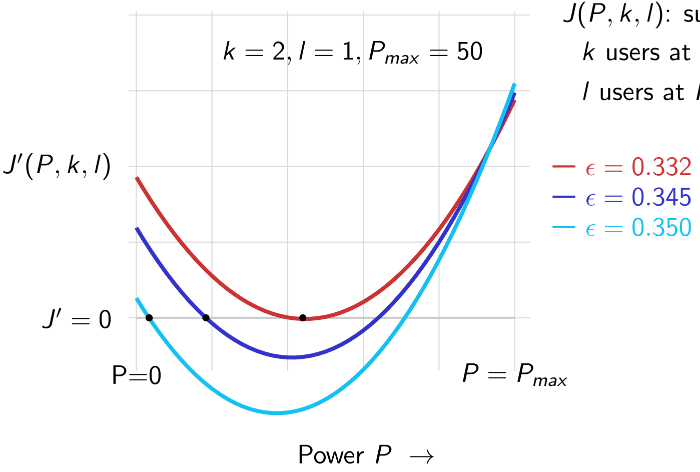

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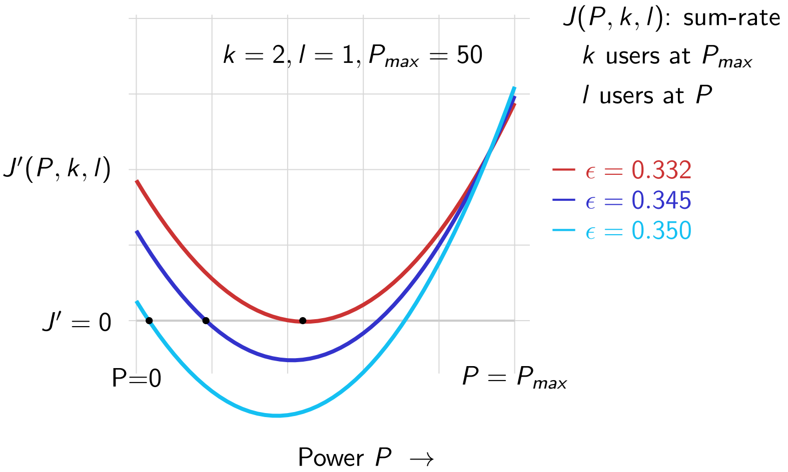

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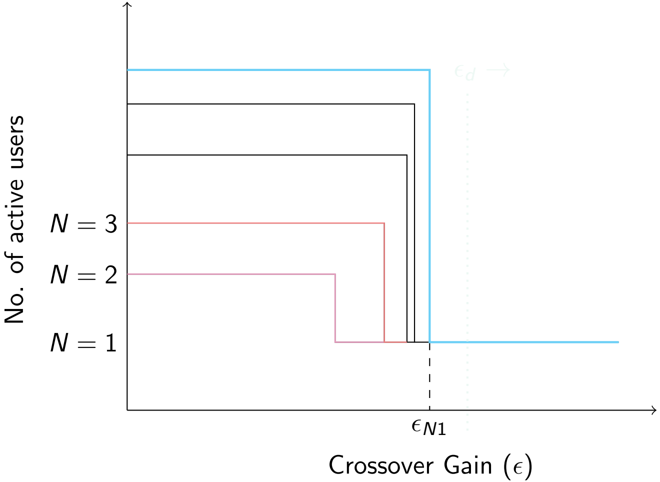
Maximas but ?



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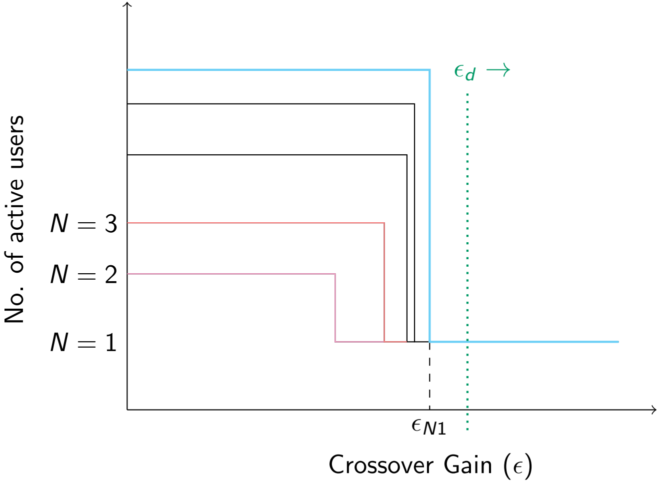

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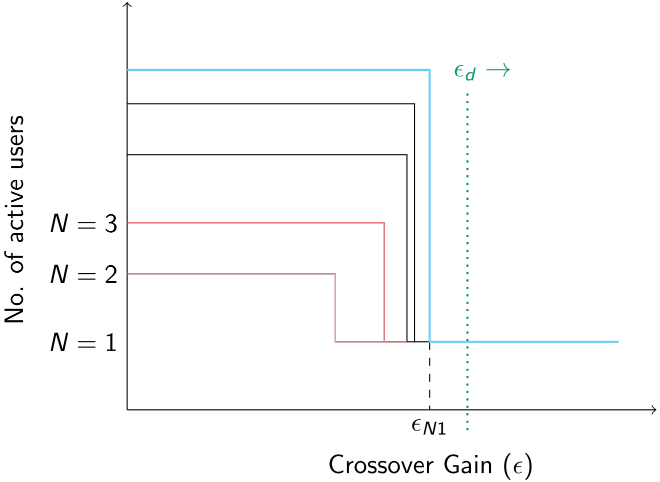

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Conclusion

The interference channel is a model of fundamental significance in multi-user networks.

The capacity region is unknown even in the Gaussian setting.

We proposed an optimal power allocation strategy when each user treats other transmissions as Gaussian noise.

Even if dynamic time-varying transmit powers are allowed, at most $N + 1$ modes are sufficient for maximizing the sum-rate.

Reference

For details

S. R. B. et al,'Maximizing the sum-rate in symmetric n/w of interfering links", IEEE Trans on Information Theory, September 2010.