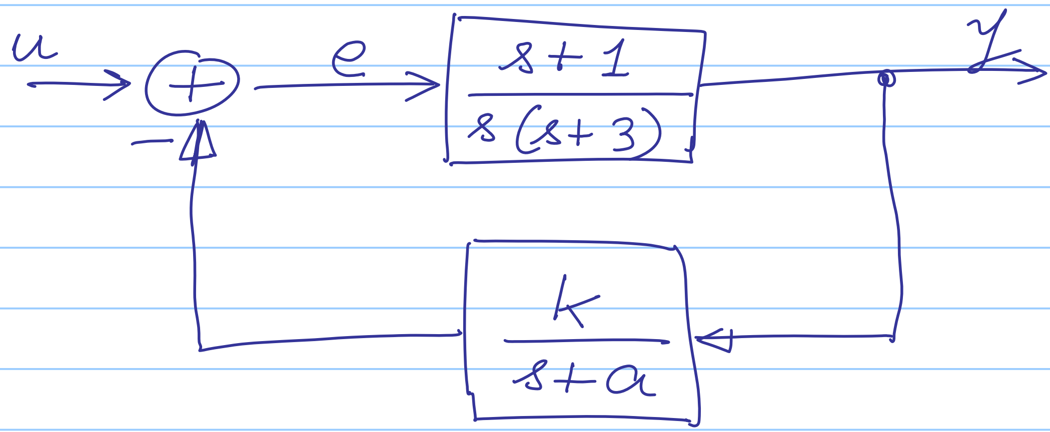
EE 640 - HW3

Note Title 24-07-2008

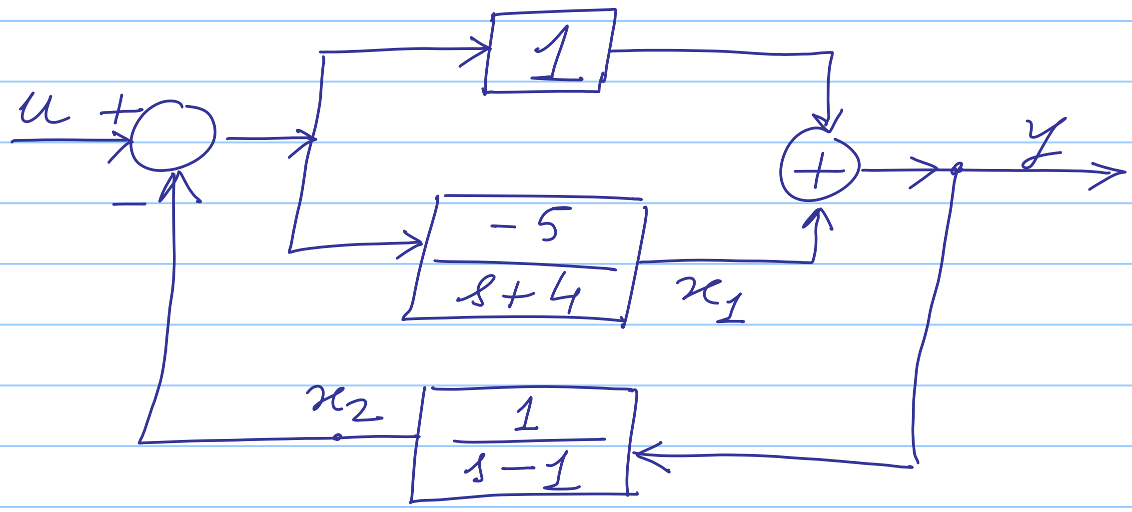
(1) Consider the system:



a) Give a state variable representation of this system

b) Is there any choice of parameters k and/or a for which this system is not controllable and not observable?

(2) Choose state variables as shown:

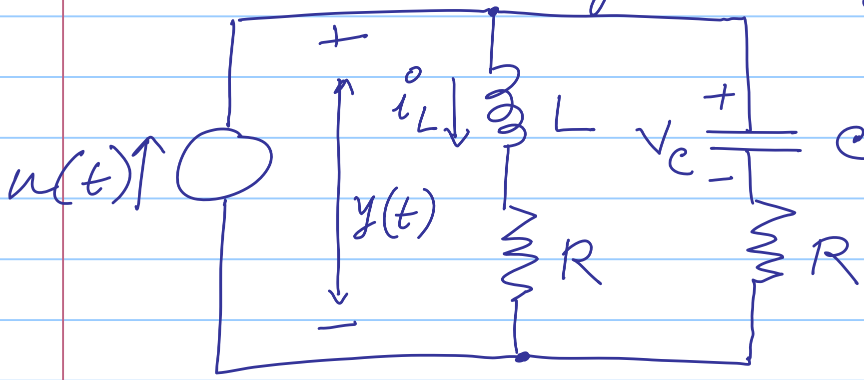


a) Write the state equations

b) Is this system realization controllable? Observable?

c) What is the transfer function from $u(\cdot)$ to $y(\cdot)$?

3) Consider the following circuit



$u(\cdot) \equiv$ current
$y(\cdot) \equiv$ voltage

a) Show that a realization for the circuit can be written as

$$\dot{x}(t) = \begin{bmatrix} -\frac{2R}{L} & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} x(t) + \begin{bmatrix} R/L \\ 1/C \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -R & 1 \end{bmatrix} x(t) + R\,u(t)$$

if we choose $x_1(t) = i_L(t)$ and $x_2(t) = V_c(t)$.

b) Determine what relations between R, L and C are required to make the realization uncontrollable and/or unobservable.

4) a) Show that a pair $\{A, B\}$ where A is diagonal with entries $\{\lambda_i\}$ is controllable if and only if

1) the λ_i's are distinct
2) all components of β are non-zero

b) What are the similar conditions for the observability of a pair $\{\nu, A\}$

c) Try to give simple physical explanations for why, when A is diagonal, repeated eigenvalue cause a loss of controllability and observability.

[Warning: These conditions don't apply when A is not diagonal or cannot be diagonalized]

⑤ Controllability and Observability for Jordan Forms

a) Let

$$J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix} \qquad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \qquad \nu' = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Find necessary and sufficient conditions on the $\{\beta_i\}$ and $\{\nu_i\}$ for non-singularity of $\mathcal{O}(\nu, J)$ and $\mathcal{C}(J, \beta)$

b) Repeat for $J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \mu \end{bmatrix}$

6) Let $\mathcal{C}_k = [b \; Ab \; \cdots \; A^{k-1} b]$

Show that if $\text{rank}(\mathcal{C}_{k+1}) = \text{rank}(\mathcal{C}_k)$ for some k, then

$$\text{rank}(\mathcal{C}_{k+i}) = \text{rank}(\mathcal{C}_k)$$

for all $i \geq 1$.

7) We know that given the input and output (and their necessary derivatives) for an observable system we can determine its state. Prove that for an observable system we can determine the state <u>without knowledge of the input</u> if and only if the first $n-1$ Markov parameters, h_1 to h_{n-1}, are zero. Show that this is equivalent to the fact that the transfer function has no (finite) zeros.

8) $\{A, b, c\}$ is an n^{th} order realization of a given transfer function $\frac{n(s)}{a(s)}$ where $a(s) = s^n + a_1 s^{n-1} + \cdots + a_n$.

Suppose that $\mathcal{C}(A, b) = I$. Show that this information completely determines $\{A, b\}$.