EE 640 — HW 4

Note Title 24-07-2008

1) Consider the diagonal realization

$$\dot{x} = A_d x + b_d u$$
$$y = c_d x$$

where, $A_d = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$, $b_d = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{bmatrix}$

$c_d = [\delta_1 \ \ \delta_2 \ \ - - - \ \ \delta_n]$

Show that the realization is canonical iff

(1) $\gamma_i \neq 0 \quad \forall\, i = 1, \cdots, n$

(2) $\delta_i \neq 0 \quad \forall\, i = 1, \cdots, n$

(3) $\lambda_i \neq \lambda_j \quad \forall\, i \neq j$

Use this fact to prove that the realization is canonical iff

$$\frac{c_d \operatorname{adj}(sI - A_d)\, b}{\det(sI - A_d)}$$ is irreducible.

2) a) Prove that a similarity transformation does not affect controllability of a realization. Derive a relation between the corresponding controllability matrices.

(b) Prove that a similarity transformation does not affect observability of a realization. Derive a relation between the corresponding observability matrices.

3) Given two realizations known to be similar. Find the similarity transform T that connects them (in terms of the $\mathbb{C}$ and $\mathbb{O}$ matrices)
(i) if they are controllable
(ii) if they are observable.

4) If $f(A)$ is a polynomial in A and p an eigenvector of A associated with an eigenvalue λ, then show that,

$$f(A)\,p = f(\lambda)\,p,$$

and thus $f(\lambda)$ is an eigenvalue of $f(A)$ and p the corresponding eigenvector.

(5) Consider the system described by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad (t \geq 0)$$

$$x_1(0) = 1 \qquad x_2(0) = -1$$

We want $x_1(1) = x_2(1) = 0$.
Is there an input $u(t)$ $(0 \leq t \leq 1)$ which accomplishes this? If yes, then calculate such an $u(t)$.

OPTIONAL:

Plot an check in SIMULINK whether the state trajectories are actually reaching 0 at $t=1$.

6) Show that any two controllable realizations (of the same transfer function) with the same characteristic polynomial can be related by a similarity transformation.

Hint: Try with $T = \mathcal{C}_1 \mathcal{C}_2^{-1}$