EE640 — HW5

Note Title 24-07-2008

1) Consider a realization $\{A, b, c\}$ with

$$A = \begin{bmatrix} -0.5 & 1 & 0 \\ -1 & -0.5 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad c^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

a) Is the system observable? If not completely observable, what quantities are unobservable?

b) Is the system controllable? If not completely controllable, what quantities are uncontrollable?

For (a) & (b) above use both the standard decomposition and the PBH rank test to identify the uncontr./unobs. modes.

(2) a) Consider the cascade connections of minimal realizations of $H_1(s)$ and $H_2(s)$ as $H_1(s)H_2(s)$ and $H_2(s)H_1(s)$, where $H_1(s) = 1/s+1$ and

$$H_2(s) = \frac{s+1}{(s+2)(s+3)}.$$

For each interconnections determine the uncontrollable & un-observable modes if any.

b) Repeat for the realizations connected in feedback form, first with $H_1(s)$ in the feedforward path and $H_2(s)$ in the feedback path, and then vice versa.

3) Let $\{A_i, b_i, c_i\ ,\ i = 1, 2\}$ be two minimal realizations with characteristic polynomials $a_i(s) = \det(sI - A_i)$

a) Show that the characteristic poly. of the

i) Series connection is $a_1(s) a_2(s)$

ii) Parallel Connection is $a_1(s) a_2(s)$

iii) Feedback connection, with $\{A_1, b_1, c_1\}$ in forward path & $\{A_2, b_2, c_2\}$ in the feedback path, is $a_1(s) a_2(s) + b_1(s) b_2(s)$

4) a) Prove that $\{A, b\}$ is controllable if and only if $\{A - bK, b\}$ is controllable for all K.

b) Show that $\{A, b\}$ is controllable if and only if the only $n \times n$ matrix X such that $AX = XA$ and $Xb = 0$ is the matrix $X \equiv 0$.

5) Suppose $\{A, b, c\}$ is minimal and $a(s) = \det(sI - A)$ has a repeated root. Prove that A cannot be diagonalized by a similarity transformation.

6 a) If $\{A, b\}$ is given and not controllable, is it always possible to choose c so that $[c, A]$ is observable? A proof or a counter example will suffice.

b) If $\{A, b\}$ is given and is controllable, can we always choose c so that $\{c, A\}$ is observable?