EE640 — Home Work 6

Note Title 24-07-2008

1) Suppose $\{A, b\}$ is controllable and we make a change of state variables such that

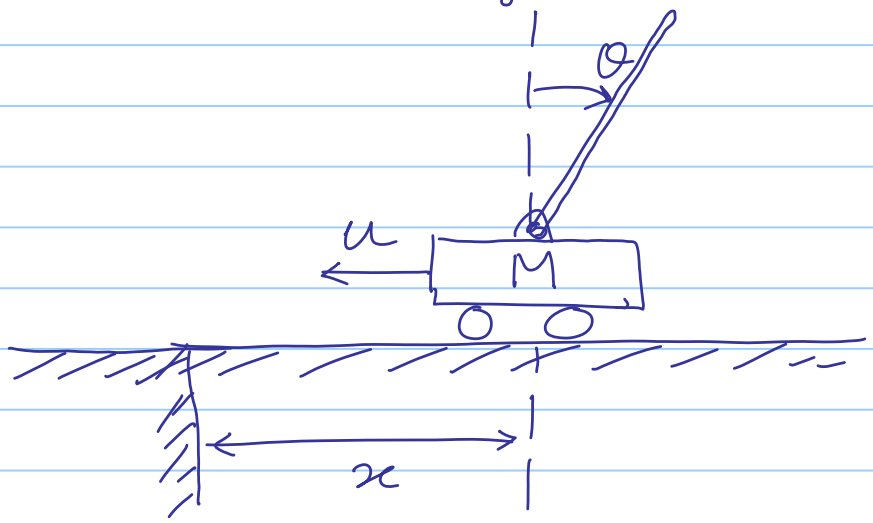
$$T^{-1}b = \bar{b} = [b_1, 0 \cdots 0]^T$$

& $T^{-1}AT = \bar{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ such that A_{11} is a scalar. Show that $\{A_{22}, A_{21}\}$ is controllable.

2) Show that the relative order of a linear system, which is defined as the difference between the degrees of the denominator and numerator polynomials of its transfer function, is not affected by state variable feedback.

3) In appropiate dimensionless units the equations of motion for a cart of mass M with a uniform stick of mass m pivoted on top may be written as (see figure) $\ddot{\theta} = \theta + u$, $\ddot{x} = -\beta\theta - u$ where $\beta = \frac{3}{4}[m/(M+m)]$ = a parameter of the system and u = torque applied to the wheels of the cart by an electric motor. We wish to find a feedback control that will balance the stick (i.e. keep $\theta \approx 0$) and keep the cart near $x = 0$, to do this find gains k_1, k_2, k_3 & k_4 in

the state feedback $u = -k_1\theta - k_2\dot{\theta} - k_3 x - k_4 \dot{x}$ such that the closed loop system has a double pole at $s = -1$ and a pair of complex poles at $s = -1 \pm 1j$.



4) A helicopter near hover can be described by the equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.02 & -1.4 & 9.8 \\ -0.01 & -0.4 & 0 \\ 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 9.8 \\ 6.3 \\ 0 \end{bmatrix} u$$

where x_1 = horizontal velocity
x_2 = pitch rate
x_3 = pitch angle
u = rotor tilt angle

a) Find open loop poles
b) Show that a state-feedback law the move the poles to $s = -2$, $s = -1 \pm j$ is $k = [0.0628 \quad 0.4706 \quad 0.9949]$

5) Using the notation used in class prove that for a controller form realization, $C_c^{-1} = a^T$